So lets say we want to solve

With IC1 or x(0) being 1
And IC2 of x'(0) being 2

$$x(t) = x(t)''^2 + t$$

Well we can't solve that (at least i don't think we can I didn't try very hard) but what we can solve is an equation that very close to it.

$$x(t) = x(t)''^2 + t + \sigma(t)$$

The concept is that if we solve a differential equation where sigma is very small (has little impact on the rest of the equation) we will have solved differential equation that is very similar thus approximating the differential equation.

First we solve for sigma

$$x(t) - x(t)''^2 - t = \sigma(t)$$

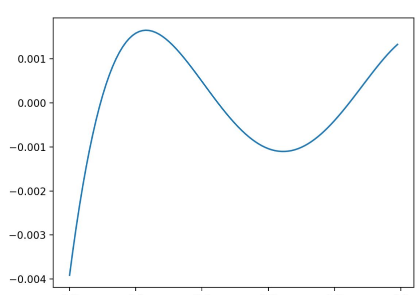
Next we say that x(t) is a fourier series.

$$x(t) = A + B * Cos(t) + C * Sin(t)$$

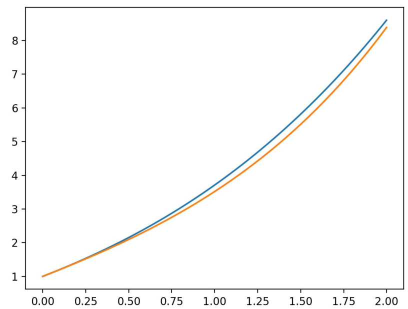
Then we minimize p with respect to A B C or the elements of your fourier series

$$p = \int_0^1 \sigma(t)^2 dt + K * ((x(0) - IC2)^2 + (x'(0) - IC2)^2)$$

Here are the results from my program. With a 10 element fourier series
This is sigma(t)



Orange is step approximation
Blue is forair approximation



There are lots of ways to this the program attached is a rather poor one using spicy optimize even still the result is very good.

If you were to do this for real you would have to turn this into a linear algebra problem. You can do this in most cases where x and its derivatives are a series of polynomials with is why this interested me it looks like it would be useful in the three body problem.
As far as I know this approach is novel I did not find it in book I came up with it.

I have accepted the fact that there may be no solution, no matter how you wright it but with this you can create a fourier series that is very very close.