Building a Policy Case for Pedestrianisation with Braess Theorem

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2024-08-22

In 1968, Dietrich Braess demonstrated a counter-intuitive phenomenon now known as Braess Paradox, where adding a new link to a road network can degrade overall network performance. This paradox is crucial for urban and transport planners to understand, yet it remains poorly understood by the wider public and even amongst professionals. This research project aims to apply a theoretical model of Braess Paradox to Queens Road in Leicester, providing insights for transport and urban policy.

A network model was built incorporating edge attributes such as natural time, usage, capacity, dynamic time, and perceived time. Agents were generated based on origin-destination pairs and various routing algorithms. The model was validated using a classic example and then scaled to more complex scenarios. Our findings indicate that Queens Road exhibits clear Braessian behaviour, leading to an efficiency loss as predicted by Roughgarden, (Roughgarden and Tardos, 2002).

The results suggest that Queens Road should not be considered a through route and should be pedestrianised to better serve the local community. These findings underscore the importance of integrating Braess Paradox considerations into transport planning.

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Introduction

In 1968 Dietrich Braess published a paper which showed that adding a new link to a road network might not improve the overall operation of the network and in some cases may make the network function worse (Braess, 1968). This seemingly counter intuitive result became known as Braess Paradox. A brief generalisable understanding can be formulated as under the condition of minimal flow, adding a new link to a network can improve network performance if that link improves travel time. This part is obvious. However, when increasing flow, a special instance can occur. This is when the new link reduces the effective capacity of the system by increasing the flow through low-capacity links, which then reduces performance (Braess, 1968). This case can still occur even when flow is dynamically managed from the perspective of each user (Roughgarden and Tardos, 2002).

Why is it Important in Transport Networks?

There are many things that urban and transport planners should consider when deciding to add a new road. The logic in Britain has generally been to build for cars (Mitchell and Hinton, 2021). This has seen traffic saw to record levels and is a generalisable result from around the world, which unless you're the majority shareholder in an oil company is generally seen as a bad thing (Mitchell and Hinton, 2021). Obviously, there are many components to this and there are many reasons why traffic has soared. However, anecdotal evidence suggests that Braess Paradox is generally poorly understood by both the wider public and even professionals in the transport and urban planning industry.

What are Potential Practical Applications?

The major practical application is the general principle of reducing access and speed on and to low-capacity routes, to not lead them to become overcapacity. This has a myriad of other benefits to. Further to this, transport planning should not be seen as a one-dimensional quantitative problem that can be easily and exactly solved with a model, especially in the case of cars. It should be understood analytically through the study of a myriad of effects that can be understood quantitatively, and appreciated qualitatively, recognizing that solving transport problems exactly for cars is not feasible. As will be discussed later, predicting braessian edges is an NP hard problem and there is no computationally sensible generalisable solution on large scales (Wardrop and Nagurney, 2022). What then should be taken from this is a rigorous appreciation of the quantitative and how to apply that to the qualitative.

No Exact Generalisable Theoretical Way of Predicting Braessian Edges

The best theoretical work to be done on this topic is often found when Braess paradox is applied to electrical networks. Its beyond the scope of this project to discuss why that is. One reason intuitively is that because the failure of electrical networks is seen as something that can never be allowed to happen; the same is not true for car networks. The obvious analogy is with airline safety and car safety.

However, a good example is the work by Roughgarden 2002, which proved that detecting even the worst manifestations of Braess Paradox is an NP hard problem (Roughgarden and Tardos, 2002). Since then, more substantive work has been done in electrical networks Manik 2022, which showed that an analogous problem is computationally solvable un-exactly at least by considering the rerouting alignment, or in other words how the between centrality changes when adding and edge (Manik, 2022). This is the logic that has been taken forward in finding an approximate solution.

Aims

This research project seeks to apply a theoretical model of Braess paradox to a real-world scenario to add to a body of work relating to the case for a different approach to transport and urban policy. It aims to add to the list of considerations when deciding the appropriate function of a road in a network. The questions this project hopes to answer are, "Should Queens Road be seen as a through route?" and "What practical takeaways can we observe from this analytical understanding?"

Literature Review

In their 2022 work Zhuang and Huang considered dynamic traffic assignment and the effects of junctions on Braess paradox rather than understanding the network in a simplified set of flows model without node interactions (Zhuang and Huang, 2022). They also applied cooperative autonomous vehicles using applied reinforcement learning. More recently, a study by Gao et al. (2021) explored the applications of machine learning in predicting Braess Paradox in transportation networks (Gao et al., 2021). They used neural networks to analyse traffic patterns and identify potential Braessian edges, showing promising results in improving prediction accuracy and reducing computational costs.

Manik (2022), found a new topological understanding of Braess paradox when applied to electrical networks which greatly reduced the computation cost and reduced the intractability, with an overall prediction rate of about 90% (Manik, 2022). This work was built on the work of Shapiro 1987. Colleta and Jaqoud (2016) managed to prove the phenomena on the British power grid to predict the change in network flows as the result of adding an edge (Colleta and Jaqoud, 2016).

A study by Cohen and Horowitz (1991) examined the impact of network changes in the city of Stuttgart, Germany. Their findings confirmed that the addition of new roads led to increased travel times (Cohen and Horowitz, 1991). In another study, Youn, Gastner, and Jeong (2008) analyzed traffic patterns in Boston and demonstrated that removing certain roads could actually improve overall traffic flow (Youn et al., 2008).

Sharma et al. (2023) investigated the paradox within the context of real-time traffic management systems. They used machine learning to analyze the effects of dynamic road closures and traffic rerouting on large-scale traffic networks (Sharma et al., 2023). Their results indicated that under specific configurations, removing key road links significantly improved traffic conditions. Sharma's research pointed to the potential of machine learning in dynamically managing traffic networks to exploit the paradox in favor of improved traffic flow, especially in congested urban areas. In another recent work, Liu et al. (2023) integrated Braess Paradox principles into traffic simulation tools used by city planners in Beijing (Liu et al., 2023). Their model, which incorporated real-time traffic data, revealed that adding high-capacity roads into specific parts of the network

Methodology

1. Network Construction

A directed network was constructed consisting of nodes and edges, where each edge was assigned several key parameters: `natural time`, `usage`, `capacity`, `perceived time`, and `dynamic time`.

- **Natural time (nat_time):** Represents the time taken to traverse an edge under zero traffic conditions.
- **Usage:** Captures the total traffic flow across a given edge.
- **Capacity:** A constant that determines how traffic flow (usage) affects the natural time.
- **Dynamic time:** Computed as a function of natural time, usage, and capacity, representing the time experienced under traffic conditions.
- **Perceived time:** A variable storing the agent's perceived time to traverse an edge, used in routing algorithms. This is a function of both dynamic and natural time.

The model simulated agent movements across the network based on Origin-Destination (OD) pairs. Agents were assigned different routing algorithms, following either natural time, dynamic time, or perceived time, depending on the agent's decision process.

2. Agent Routing and Network Iteration

Agents, generated based on predefined OD pairs, selected routes according to their respective routing algorithms. The model iterated over each agent, allowing them to complete their trip while the edge variables were updated dynamically for subsequent agents. Each agent's route choice and travel time were stored for later analysis.

After all agents completed their routes, the collected data were used to graph route choices and trip times for analysis. These iterations enabled the identification of Braess Paradox effects within the network.

3. Initial Case Study: Simple Network Example

To validate the model, a well-known simple network example was first tested. The network consisted of four nodes and four edges arranged in a parallelogram. A fifth edge was introduced between nodes B and C, creating a new shortest path (A-B-C-D) and reducing the effective network capacity by limiting viable route options. This network arrangement produced Braess Paradox, as the additional edge led to increased congestion and reduced overall system performance.

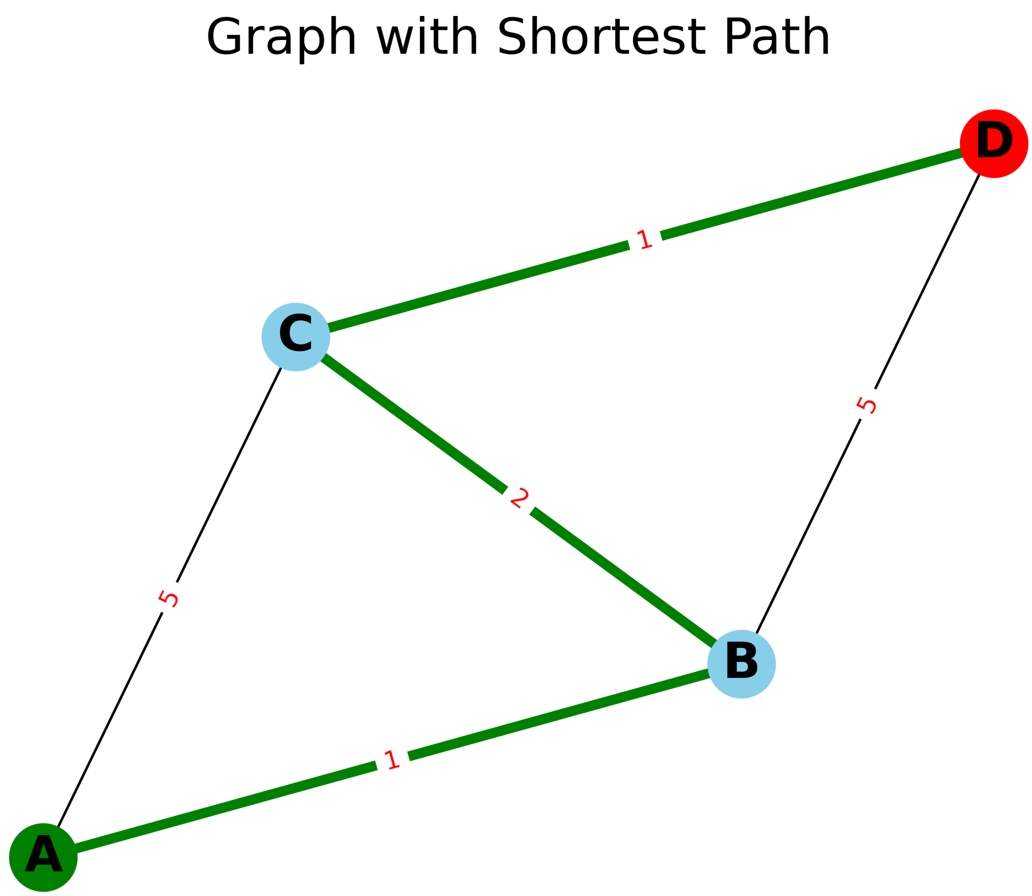


Figure 1: Classic Simple Braess Example.

4. Scaling to a Complex Network

The model was then scaled to a more complex, non-realistic network to further test its predictive capacity. A Braessian prediction algorithm was developed to identify edges that were over capacity. This algorithm examined preceding edges that contributed to congestion, allowing for a dynamic adjustment of routing decisions. However, this scenario yielded mixed success, with limited predictive capabilities.

The underlying cause of these limitations is yet to be determined and could be attributed to the algorithm's design, implementation issues, or physical constraints inherent to the network. For instance, reducing capacity at edge X may alleviate congestion at edge Y but worsen traffic at edges Z and W. This complexity, particularly in highly disordered networks, presents a significant challenge for accurately predicting Braess Paradox behavior.

5. Algorithm Development: Braess Prediction and Data Cleaning

To enhance the model's predictive capabilities, a back-prediction algorithm was developed. The algorithm identified edges that contributed to future over-capacity conditions by examining preceding edges in the network. Though the algorithm showed moderate success, its efficacy in real-world applications remains limited.

Additionally, data cleaning was performed on the network data downloaded from Open Street Map:

- **Speed Variable:** Street speeds provided in miles per hour (mph) were cleaned and converted into meters per second (ms^-1). If no speed data were available, highway type was used to estimate speed, with a default value of 20 mph applied in the absence of both variables.
- **Natural Time:** This variable was computed as the length of an edge divided by its speed, resulting in travel time in seconds.
- **Capacity:** Capacity was assigned based on highway type using a dictionary structure. A default capacity was set for tertiary roads, and lists were handled by selecting the first element.
- **Usage:** Initially, usage was set to 0 to reduce complexity and improve model clarity.

6. Application to a Realistic Network

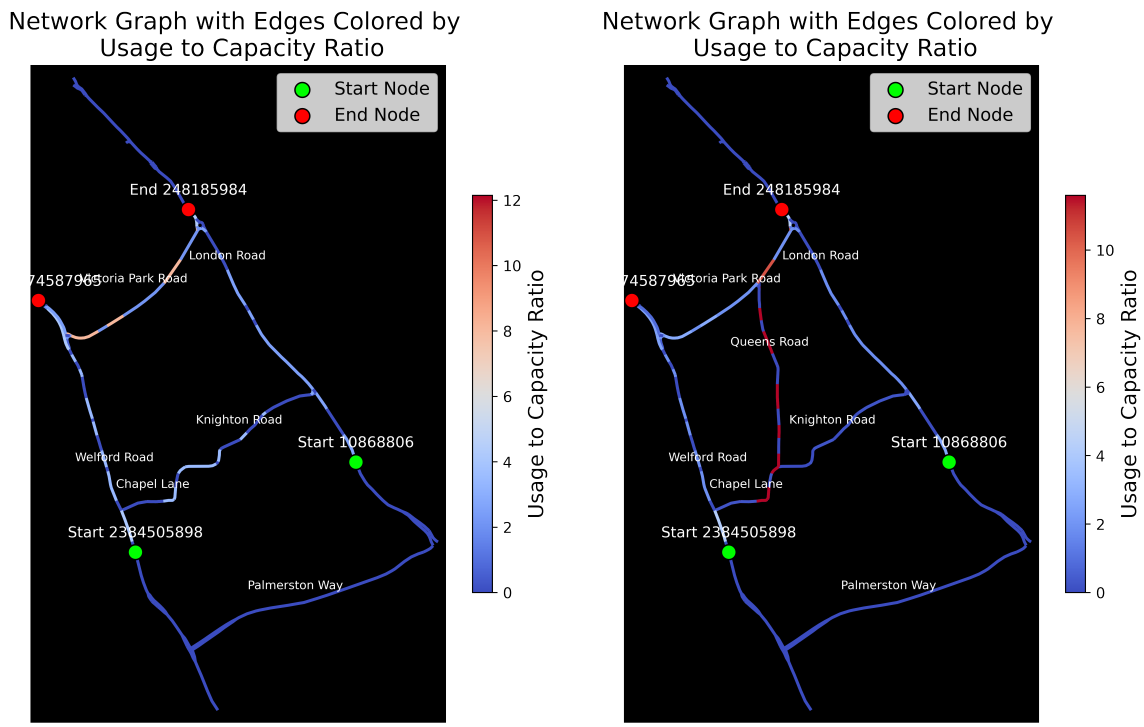
After testing the model in theoretical networks, it was applied to a realistic scenario using data from the Open Street Map network for Leicester. Queens Road, a potential Braessian edge, was selected due to its characteristics as a shortcut between major roads with OD pairs leading into central Leicester. The road's lower capacity, relative to adjacent A-roads, combined with its shorter travel time, suggested it could exhibit Braessian behavior.

The network was processed by trimming edges with low centrality to simplify the model and retain only the critical route network. A set of OD pairs was tested, and the average path time was recorded as the main metric. The results, along with the data cleaning process, are discussed in later sections.

Results

The model could validate the classical simple model of Braess theorem. The model struggled with predicting complicated scenarios, however to this date there is still no generalisable analytical solution to this. The model did well in forming the policy case around Queens roads use as a through road.

Results Queens Road



(a) No Queens Road

(b) With Queens Road

Figure 2: Two graphs showing road usage with high traffic. Queens Road on the left, Queens Road removed on the right.

This is a graph of Queens Road with over capacity edges shown in red, Queens road shows clear braessian behaviour from a qualitative perspective. There are two high-capacity routes being bypassed for the shorter more direct route, whilst at the same time causing traffic on the two lateral routes needed to go from the bottom of Welford Road to the top of London Road or vice versa. This is borne out in the data below, which shows the model being run with a different number of agents and how the path length changes. Note there are 4 total routes that can be taken in this scenario, the OD pair which shows the braessian behaviour is the bottom of Welford Road to the top of London road.

There are two graphs above, one with the purely braessian causing OD pair and one with a set of all possible pairs. The graphs show clear braessian behaviour showing the shorter route does improve average journey times at low capacities but once the system gets strained it vastly negates performance. It also nicely and neatly shows the theoretical efficiency loss from this behaviour as 4/3 which was proven by Roughgarden 2002 for linear latency functions.

Policy Recommendation

Queens road should not be thought of as a quick through route between the bottom of Welford Road and the top of London Road or vice versa. It there for should only be a road for local people and should be pedestrianised accordingly. It is a thriving hub for the local community given it is very walkable from dense urban housing around it and its opposite major religious institutions as well as big colleges, the university, and the park. For these reasons traffic calming, and elimination measures should follow.

Conclusion

This project managed to develop a simple transport model which was able to demonstrate Braess paradox on a real road network in Leicester, these findings if developed on have clear implications for transport planning. There is still no generalisable solution that has been found in detecting braessian edges in transport networks. This paper seeks to emphasise the difficult nature of making transport models and perhaps seeks to push for a better understanding of the quantitative. This can show the limits of trying to solve anything exactly and push for a more rounded and thoughtful understanding of transport as well as urban life.

Discussion and Limitations

There is a myriad of limitations to this project given its complex theoretical nature applied to a practical problem in relatively short time. The biggest limitations come with the transport model itself, it is relatively simple, which was done for good reason. It goes without saying the duration of the project was part of the reasoning. However, probably the overriding reason was if there were too many competing high-level interactions - given that braessian edge detection is already thought to be a somewhat intractable NP hard problem, that having too many second order interactions, would have made this project potentially a lot harder. There are also limitations about the generalisability of the project, again there is no generalisable detection algorithm to present. There are only the embers of intuition coming from a centrality algorithm which detects which edges form the shortest paths between sets of OD pairs, and the "ranked previous edges" algorithm which ranks the edges which feature in the paths leading to over capacity edges. The model failed to demonstrate braessian behaviour once the graph became too complicated, this is probably due to the nature of the problem. Some Braess-like behaviour was found, such that you can delete links which have non minimal flow an retain roughly equal performance. The data was also a limitation, Open Street Map is not entirely complete or reliable, and even so, many basic assumptions were made about capacity, without much academic rigour. This was beyond the scope of this limited project. This makes the findings less applicable to the real-world example and certainly an area of improvement if the model was to be used as part of a policy consideration. These decisions were made as the topological relationism was considered to dwarf the specific conditions of the roads.

Further work

The project should hope to build on the work of Zhuang, by adding in nodal interactions. By definition adding and edge adds at least two extra nodes which reduces the overall capacity of the system, so for applied purposes the semi braessian situation of nodes should be investigated. Another piece of Zhuang's work which should be commended on this topic is the dynamic interactions of vehicles instead of treating them as flow in a pipe. This could be added to the model. The models data could be improved as discussed above. The most important piece of work would be to try and find generalisable set of principles to aid transport planners.

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