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TO BE FILLED IN BY THE STUDENT AND THE INVIGILATOR:

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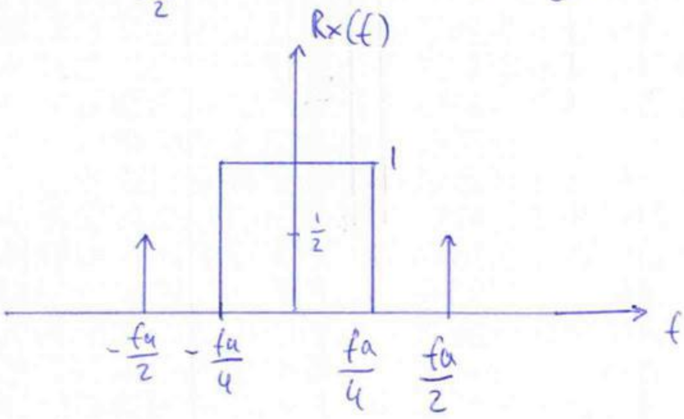
Ex 1

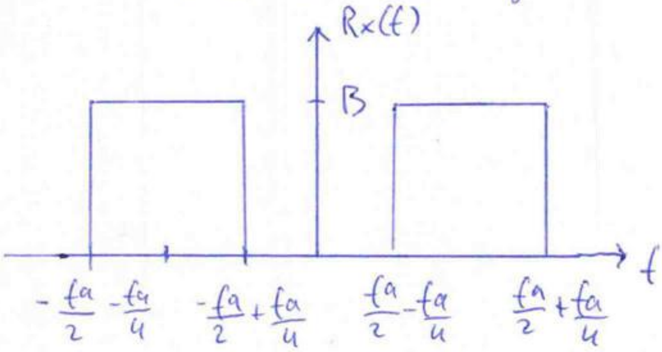
a) $R_X(f) = \mathcal{F}\{r_X(\tau)\} = \mathcal{F}\left\{2B \frac{\sin(\frac{1}{2}\pi f_a \tau)}{\pi \tau} \cos(\pi f_a \tau)\right\}$

$= 2B\left(\mathcal{F}\left\{\frac{\sin(\pi \frac{f_a}{2} \tau)}{\pi \tau}\right\} * \mathcal{F}\left\{\cos(2\pi \frac{f_a}{2} \tau)\right\}\right)$

$= 2B\left(\mathrm{rect}_{\frac{f_a}{2}}(f) * \left(\frac{1}{2}\left(\delta(f - \frac{f_a}{2}) + \delta(f + \frac{f_a}{2})\right)\right)\right)$



By graphical convalution and scaling with a factor = 2B we get:

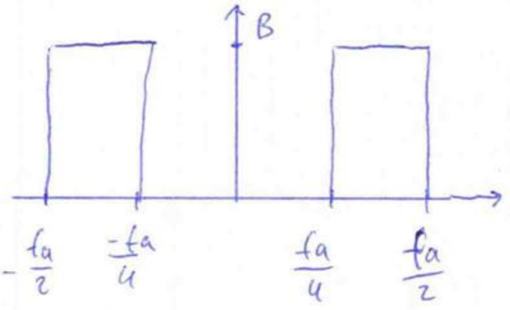


$$R_X(f) = \begin{cases} B & |f_a - f| \leq \frac{f_a}{4} \\ 0 & \text{otherwise} \end{cases}$$

I write on the other side too sorry :(

B) Since by the super formula we have: $R_Y(f) = |A(f)|^2 R_X(f)$ and $A(f)$ is simply a cut off of frequency $\frac{1}{2}f_a$ it is easy to see by looking at the graph in a) that

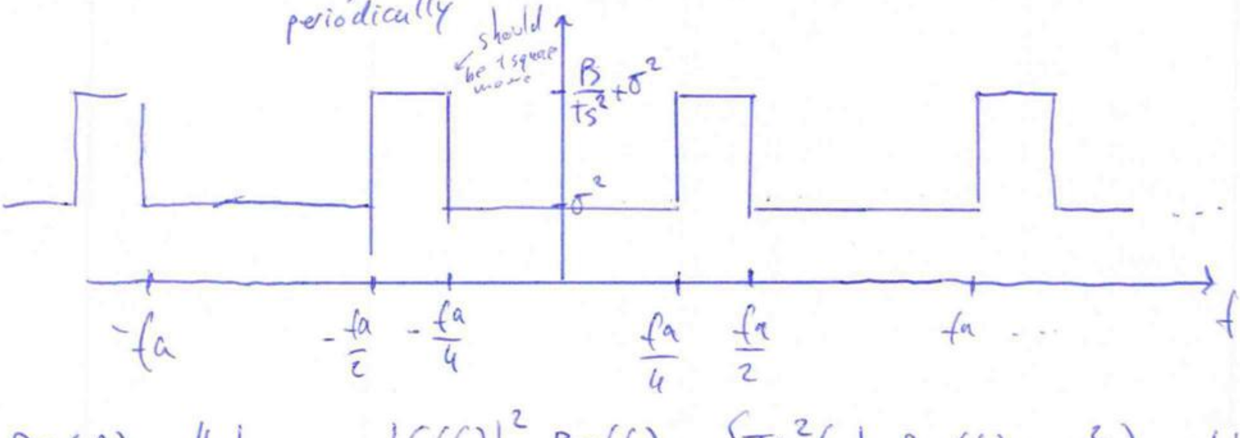
$$R_Y(f) = \begin{cases} 1 & \text{if } -\frac{3f_a}{4} \geq f \geq -\frac{f_a}{2} \text{ or } \frac{f_a}{4} \leq f \leq \frac{3f_a}{4} \\ 0 & \text{otherwise} \end{cases}$$



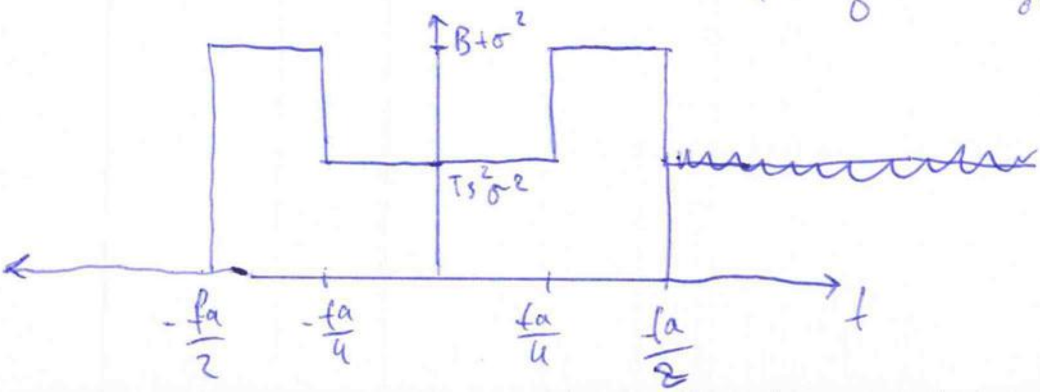
First we take $\lambda \geq 1$ which means that the Niquist theorem is satisfied. Since $W(n)$ is independent of $X(t)$ and $|P(f)| = 1$ and the PAM is an LTI system we have ~~$R_Z = R_Y$~~

$$R_Z(f) = \frac{1}{T_s^2} R_Y(f) + \sigma^2$$

periodically



$R_{\hat{X}}(f)$ will be $= |C(f)|^2 \cdot R_Z(f) = \begin{cases} T_s^2\left(\frac{1}{T_s^2} R_Y(f) + \sigma^2\right) & \text{if } |f| \leq \frac{1}{2}f_a \\ 0 & \text{otherwise} \end{cases}$



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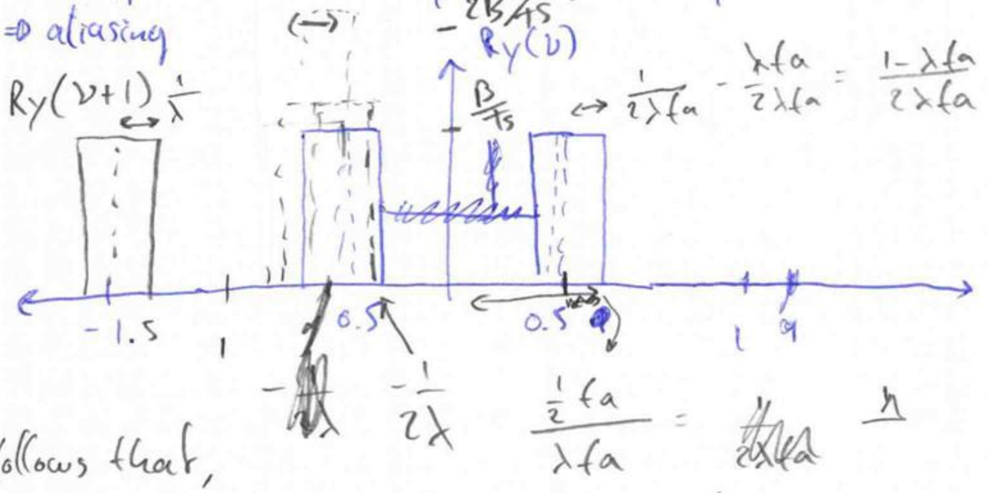
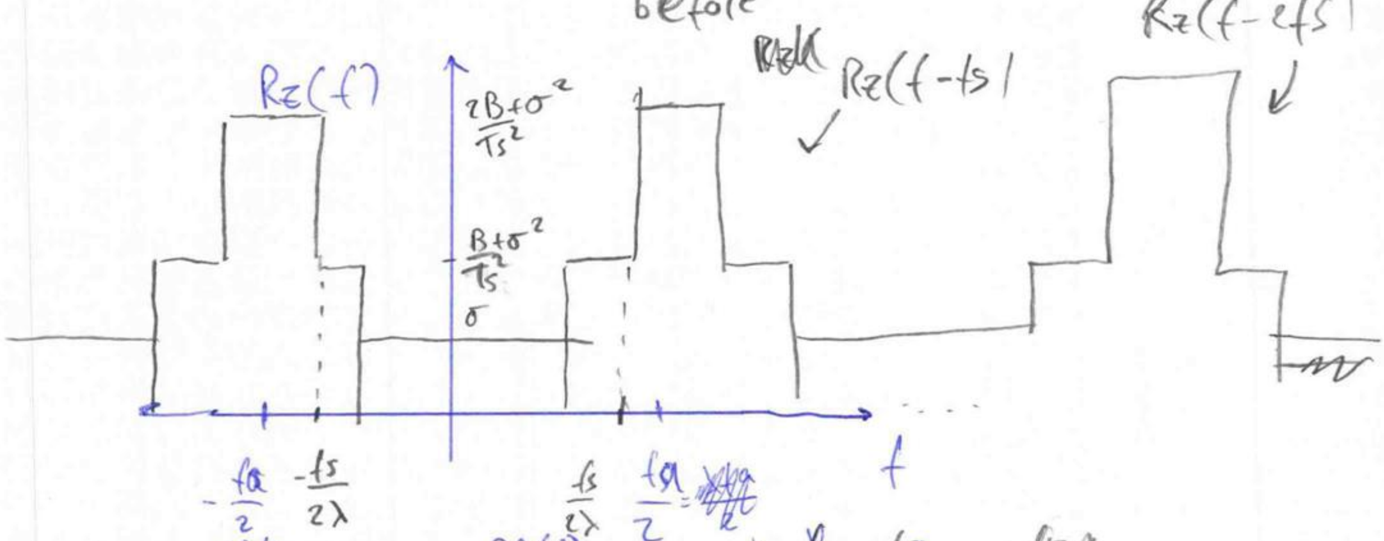
If now we take $\frac{3}{4} \leq \lambda \leq 1$ while sampling, we will have an overlapp since the nyquist ~~frequency~~ theorem is not satisfied
$\Rightarrow$ aliasing

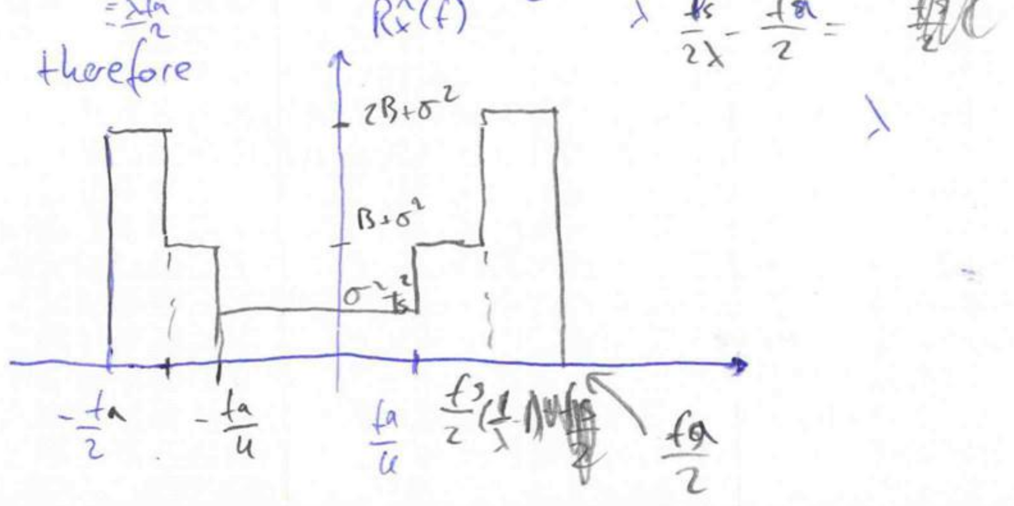


it follows that,

following the same reasonning as $\frac{1}{2\lambda}$ before



and therefore



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c) for $\lambda > 1$ we have $P_e = 2\cdot\left(\frac{f_a}{2}-\frac{f_a}{4}\right)\sigma^2 + \left(\frac{f_a}{4}-\left(-\frac{f_a}{4}\right)\right)T_s^2\sigma^2$

$$= \frac{f_a}{2}\sigma^2 + \frac{f_a}{2}\sigma^2 T_s^2 = \frac{f_a}{2}\sigma^2(1+T_s^2)$$

$$P_s = 2\cdot\left(\frac{f_a}{2}-\frac{f_a}{4}\right)B = \frac{f_a}{2}B$$

$$SnR = \frac{\cancel{\frac{f_a}{2}}B}{\cancel{\frac{f_a}{2}}\sigma^2(1+T_s^2)} = \frac{B}{\sigma^2(1+T_s^2)}$$

for $\lambda < 1$ we have $P_e =$ = same as before

$$= \frac{f_a}{2}\sigma^2(1+T_s^2)$$

$\frac{f_s}{2}\left(\frac{1}{\lambda}-1\right) = \frac{\lambda f_a}{2\lambda} - \frac{\lambda f_a}{2} = \frac{f_a}{2} - \frac{\lambda f_a}{2}$ ← which is notcorrect since when we pick $\lambda = 1$ we have 0

$$P_s = 2\left(\frac{f_a}{2}-\frac{f_s}{2}\left(\frac{1}{\lambda}-1\right)\right)2B + 2\cdot\left(\frac{f_s}{2}\left(\frac{1}{\lambda}-1\right)\right)B$$

$= \ldots = \lambda f_a 2B + (1-\lambda)f B \Rightarrow SNR = \frac{P_s(\lambda)}{P_e}$

something like that

d) If we see the SNR as SNR(λ) then we simply have to find when

$$\frac{\partial\, SNR}{\partial\lambda}(\lambda) = 0$$

and verify that the sollution λ_s verify

$$\frac{\partial^2}{\partial\lambda^2}SNR(\lambda) < 0$$

But I dont have any time left

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$T \sim Exp(\lambda)$

$$f_T(t) = \lambda e^{-\lambda t} \quad t \geq 0$$

a) $\mathbb{E}[T] = \int_{-\infty}^{\infty} t f_T(t)\,dt = \int_0^{\infty} \lambda t e^{-\lambda t}\,dt = -\frac{t}{e^{-\lambda t}}\Big|_0^{+\infty} + \left[-\frac{1}{\lambda} e^{-\lambda t}\right]_0^{\infty}$

$\lambda \quad -\frac{1}{\lambda} e^{-\lambda t} \qquad \to 0$

$= 0 + \left[0 - \left(-\frac{1}{\lambda}\right)\right] = \frac{1}{\lambda}$

b) We have to derive $P_r(T \leq \tau \mid T \geq \tau_s) = \frac{P_r(T \leq \tau \cap T \geq \tau_s)}{P_r(T \geq \tau_s)}$

$= \frac{\int_{\tau_s}^{\tau} \lambda e^{-\lambda t}\,dt}{\int_{\tau_s}^{\infty} \lambda e^{-\lambda t}\,dt} = \frac{\left[-e^{-\lambda t}\right]_{\tau_s}^{\tau}}{\left[-e^{-\lambda t}\right]_{\tau_s}^{\infty}} = \frac{e^{-\lambda \tau_s} - e^{-\lambda \tau}}{e^{-\lambda \tau_s}} = 1 - e^{-\lambda \underbrace{(\tau_s - \tau)}_{\tau_0}}$

$\Rightarrow$ Probability of waiting τ_0 more minutes follows $Exp(\lambda)$ since $\frac{\partial}{\partial \tau_0}\left(1 - e^{-\lambda \tau_0}\right) = \lambda e^{-\lambda \tau_0}$ so indep. of τ_s

c) $Z = X + Y \qquad f_Z(z) = \int_x f_{X,Y}(z - x, x)$

$Y(t) \geq 0 \Rightarrow X(t) \leq 0$

is negative

neg

$f_{X(t)Y(t)}(x,y) = f_{X|Y}(x|y) \cdot f(y) = f_{X|Y}(x|y) f_Y(y)$

y is positive

$+ f_{X|Y}(x,y) f(y)$

neg

$f_{Y|X}(y|x) \cdot f(x) +$

$\mathbb{E}[W(n) V(n)] =$

$R(v)$ $r(6) = \sigma^2 +$

$\mathbb{E}[V(t) \cdot \mathbb{E}$

$\mathbb{E}[$

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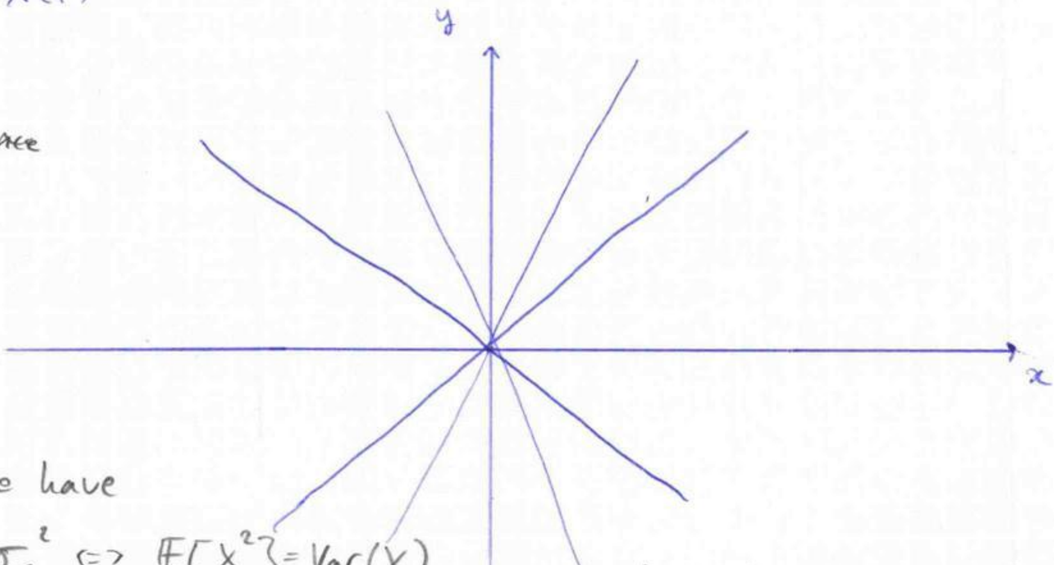
$X(t)$

Ex 3

Since

a)



Since we have

$R_X(t) = \sigma_x^2 \iff \mathbb{E}[X^2] = Var(X)$

$m_X = 0$ same for m_Y

and $f_{X,Y}(x,y) = \frac{f(x|y)}{f(y)} = \frac{f(y|x)}{f(x)}$

It is

if we know that y ~~is negatif~~ then the probability will lies on the positive side for $f(x)$

b)

$r_{XY}(\tau) = \mathbb{E}[X(t+\tau)Y(t)] = \mathbb{E}[X(t+\tau)]\mathbb{E}[Y(t)]$

$= 0 \quad \forall \tau$ since $\mathbb{E}[X(t)] = 0$ and $\mathbb{E}[Y(t)] = 0$

and the fact that they are uncorrelated

$r_{XY}(0) = \mathbb{E}[X(t)Y(t)]$

c)

Yes ~~as~~ Since the addition of two gaussian variable is still gaussian

and if we scale a Gaussian variable it will remain Gaussian

d)

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$$\hat{r}_x(k) = \frac{1}{2N}\sum_{n=-N}^{N-k} x(n+k)x(n)$$

Ex 4

$$\hat{R}_x(\omega) = \sum_{k=1}^{1} \hat{r}_x(k)e^{-j\omega k}$$

a) $$\hat{R}_x(0) = \sum_{k=-1}^{1} \hat{r}_x(k) = \hat{r}_x(-1) + \hat{r}_x(0) + \hat{r}_x(1)$$
$$= \hat{r}_x(0) + 2\hat{r}_x(1)$$

$\hat{R}_x(0) < 0$ when $\hat{r}_x(0) + 2\hat{r}_x(1) < 0$

when $\frac{1}{2}\hat{r}_x(0) < -\hat{r}_x(1)$

No, it doesn't make sense:

We can define the PSD as $R_x(\nu) = \lim_{N\to\infty} \mathbb{E}\left[\frac{1}{N}\left|\sum_{n=0}^{N} x(n)e^{-j2\pi\nu n}\right|^2\right]$ which is greater than 0 for all values of ν. therefore $\hat{R}_x(\nu)$ should be greater than 0.

b) It holds that for an AR(1) process, $r_x(k) = \alpha^{|k|}\frac{\sigma_x^2}{1-\alpha^2}$

Since $\hat{R}_x(0) = \frac{\sigma_x^2}{1-\alpha^2} + 2\alpha\frac{\sigma_x^2}{1-\alpha^2} = (1-2\alpha)\frac{\sigma_x^2}{1-\alpha^2}$ if we admit that $r_x(k)$ is known

if We have ~~then~~ that

$1-2\alpha > 0$

and $1-\alpha^2 > 0$ $\Rightarrow$

~~the~~ ~~R~~ r_x

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b) Yes, since the ~~P~~SD estimator is only intersted of the value ~~for which~~ $\hat{r}_x(1)$ $\hat{r}_x(0)$ and with an AR(1) process we fix a conditition on those 2 samples that differe from on time unit we can fix a strong contraint involving α and σ^2

c)

d) We have with that

$$\underbrace{\begin{pmatrix} \hat{r}_x(0) & \hat{r}_x(1) & \dots & \hat{r}_x(P-1) \\ \hat{r}_x(1) & \hat{r}_x(0) & & \\ \vdots & & \ddots & \\ \hat{r}_x(P-1) & & & \hat{r}_x(0) \end{pmatrix}}_{R\hat{x}} \underbrace{\begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_P \end{pmatrix}}_{a} = - \underbrace{\begin{pmatrix} \hat{r}_x(1) \\ \vdots \\ \hat{r}_x(P) \end{pmatrix}}_{-\hat{r_x}}$$

We have to solve:

~~a=~~ $a = -R_Y^{-1} r_Y$

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e) It will happen that we will have a worse approximation ~~a~~ of the process, since the coefficents ~~will~~ will try to math the greater values

f)

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$Z_{ni} = Y_{1i} + \dots + Y_{ni}$ $\quad Y_{ni}$ has zero mean

Ex5

Let Z_{nm} a random seq $m = 1, 2 \dots$

a) ① $\mathbb{E}[Z_{ni}] = \mathbb{E}[Y_{1i} + \dots + Y_{ni}] = \mathbb{E}[Y_{1i}] + \dots + \mathbb{E}[Y_{ni}]$

$= 0 \quad \forall i$

② $MMSE_{[Z_{nm}]} = \hat{Z}_{n,m} = Z_{n,m-1} = \mathbb{E}[Z_{nm} \mid Z_{nm-1}, \dots, Z_{n1}]$

Proof:

$\mathbb{E}[Z_{nm} \mid Z_{nm-1}, \dots Z_{m1}] = \mathbb{E}[Y_{1m} + \dots Y_{nm} \mid Y_{1m-1} + \dots + Y_{nm-1}, Y_{11} + \dots + Y_{n1}]$

$\mathbb{E}\left[\sum_{n=1}^{m-1} Z_{ni}\right]$ => all are indemendent

and mean 0 so it is equivalent for

$\hat{Z}_{n,m}$ to pick on of the possible Z_{ni} $\forall i < m$

So we will pick $Z_{n,m-1}$ hence it is martingal

b) We follow the same steps a