Assignment 5

Computational Intelligence, SS2018

Team Members		
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1 Classication/ Clustering

1.1 2 dimensional feature

1.1.1 Perform all of the above-mentioned tasks for the EM algorithm.

In the process of initializing the parameters, we used random function to select m points for calculating means. Therefore, the result is different for each process. We selected the best results from 10 trials.

- Compare the result with the labeled data set (i.e., consider labels as well). Make a scatter plot of the data and plot the Gaussian mixture model over this plot.

```
array([0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
       0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
       0, 0, 0, 0, 0, 0, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
       1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 2, 1, 1, 1,
       1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 2, 2, 2,
       2, 2, 2, 2, 2, 2, 2, 2, 2,| 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2,
       2, 2, 2, 2, 2, 2, 2, 2, 2, 1, 2, 2, 2, 2, 2, 2, 2, 2], dtype=int64)
array([0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
       0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
       0, 0, 0, 0, 0, 0, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
       1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
       1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 2, 2, 2,
       2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2,
       2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2])
```

Figure 1: label data

First array is EM algorithm classification. Second array is answer classification.
Three points are mis-classified.

Figure 2: Three gaussian with scatter data points

EM algorithm suceeded to find three gaussian model and classified the data points well.

- For your tests, select the correct number of components (K = 3), but also check the result when you use more or less components. How do you choose your initialization $\theta 0$? Does this choice have an inuence on the result

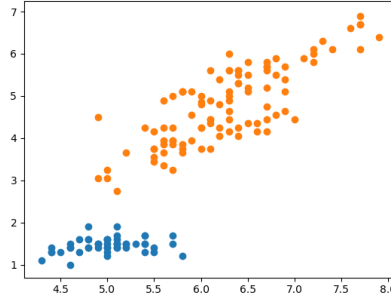When the number of components is 2, the result is the following figure.

Figure 3: K = 2

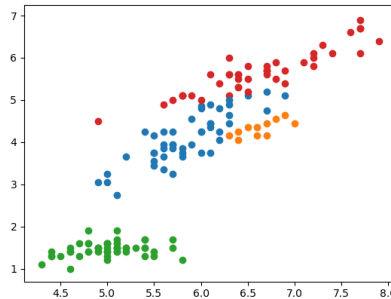When the number of components is 4, the result is the following figure.

Figure 4: K = 4

For initialization $\theta 0$, we refered to the class pdf. The following figure is the reference class pdf.

Figure 5: The initialization process in the class pdf

The initialization process influences the EM algorithm result. When the randomly selected points, for calculating mean value, is well chosen, the result is accurate. That is, the result is accurate when the first random selected points are from first answer label group, the second random selected points are from second answer label group and the third random selected points are from third answer label group.

- plot the log-likelihood function over the iterations! What is the behavior of this function over the iterations?
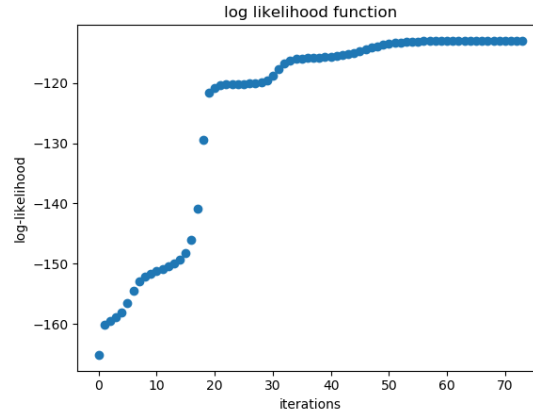
Figure 6: The log-likelihood function over iterations

As shown in Figure 4, the log-likelihood increases over iterations. That is, likelihood increased over iterations. And about 50th iteration, the function looks convering to the value, -112.96270776709655. Therfore, the process stops even though it didn't reach the max iteration number.

- Make a scatter plot of the data that shows the result of the soft-classication that is done in the E-step
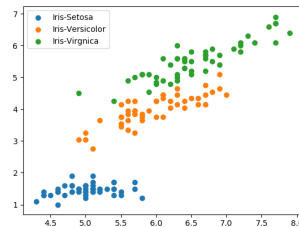
Figure 7: The EM algorithm soft-classification
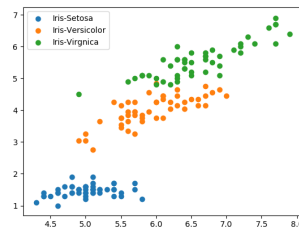
Figure 8: The answer classification

The EM algorithm classifies well the points when it is compared with the answer classification. EM algorithm fails to classify the points near the boundary of iris-Versicolor and iris-Virgnica.

1.1.2 Perform all of the above-mentioned tasks for the K-means algorithm

1.1.3 You may additionally choose any other pair of features; how would this change the classication accuracy

1.2 4 dimensional feature

1.2.1 EM algorithm tasks

- Compare the result with the labeled data set (i.e., consider labels as well). Make a scatter plot of the data and plot the Gaussian mixture model over this plot.

```
array([0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
       0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
       0, 0, 0, 0, 0, 0, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
       1, 1, 2, 1, 2, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1,
       1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 2, 2, 2,
       2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2,
       2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2], dtype=int64)
array([0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
       0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
       0, 0, 0, 0, 0, 0, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
       1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
       1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 2, 2, 2,
       2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2,
       2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2])
```

Figure 9: label data

First array is EM algorithm classification. Second array is answer classification.
Four points are mis-classified.
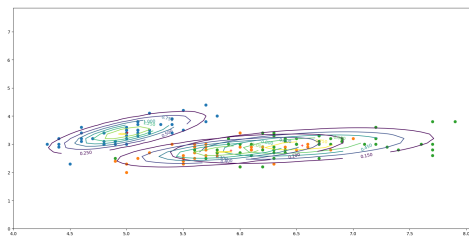
Figure 10: Three gaussian with scatter data points

EM algorithm suceeded to find three gaussian model and classified the data points well.

- For your tests, select the correct number of components (K = 3), but also check the result when you use more or less components. How do you choose your initialization $\theta 0$? Does this choice have an inuence on the result When the number of components is 2, the result is the following figure.
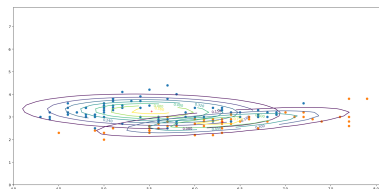
Figure 11: K = 2

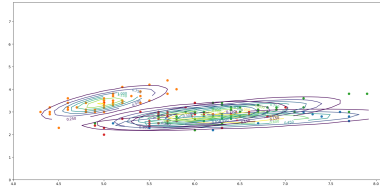When the number of components is 4, the result is the following figure.

Figure 12: K = 4

Initalizaiton $\theta 0$ is the same with the dimension 2.
As written in section 1.1.1, the initialization process affects the result.

- plot the log-likelihood function over the iterations! What is the behavior of this function over the iterations?
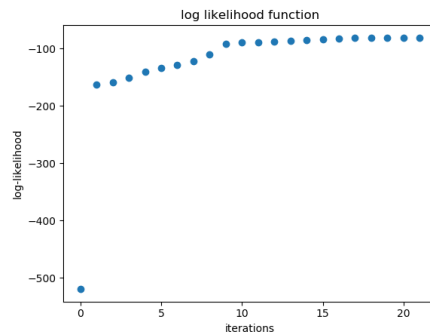

Figure 13: The log-likelihood function over iterations

As shown in Figure 4, the log-likelihood increases over iterations. That is, likelihood increased over iterations. And about 25th iteration, the function looks convering to the value, -87.41158191236445..
Therfore, the process stops even though it didn't reach the max iteration number.

- Make a scatter plot of the data that shows the result of the soft-classication that is done in the E-step
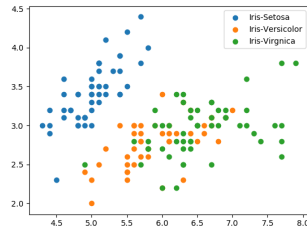
Figure 14: The EM algorithm soft-classification
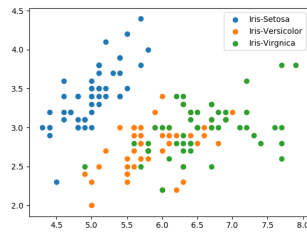
Figure 15: The answer classification

The EM algorithm classifies well the points when it is compared with the answer classification. EM algorithm fails to classify the points near the boundary of iris-Versicolor and iris-Virgnica.

1.2.2 How do the convergence properties and the accuracy of you classication change in comparison to scenario 2.1?

- EM-algorithm
The convergence value of log likelihood function increased. In the scenario 2.1, the value was -112.96. However, in the scenario 2.2, the value increased to -87.41. Also, the total iteration decreased in the scenario 2.2. The value decreased from 75 to 26.
Also, we compared the scenario 2.1 and 2.2 with each 10 trials. Overall, the EM classifies the data points better in 2.2 scenario. The accuarcy of 2.2 scenario is higher than that of 2.1 scenario. Because the EM algorithm in 2.2 scenario has more additional information about the dataset.

1.2.3 Within your EM-function conne the structure of the covariance matrices to diagonal matrices! What is the inuence on the result.

1.3 Processing the data with PCA

1.3.1 How much of the variance in the data is explained this way?

- original variance(sum of eigenvalues) - 4.499157046979866

- associated eigenvalues - 4.15886089, 0.23573307

- the amount of explained variance - 0.9767594057574307

1.3.2 How does the performance of your algorithms compare to scenario 2.1 and scenario 2.2?

Scenario 2.1 has no difference at performance for both algorithms. Actually it takes more time to do PCA. The amount of explained variance is 1. The data from PCA is rotated original data that eigenvectors are the axes.
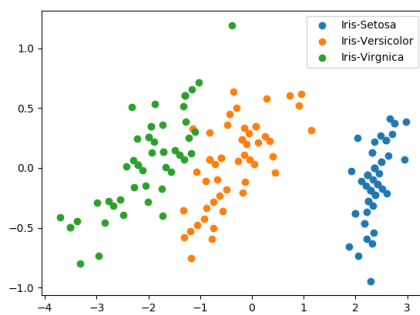
Figure 16: answer for pca scenario 1
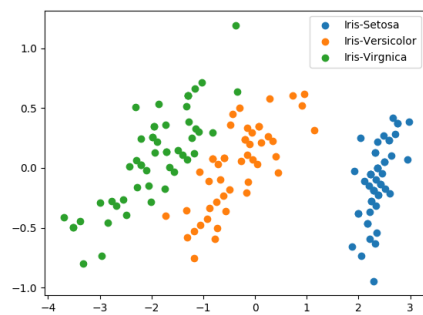
Figure 17: scatter plot for scenario 1. EM
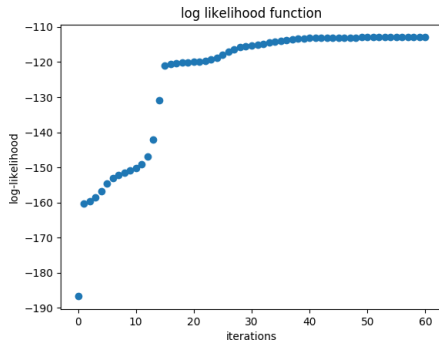
Figure 18: log-likelihood plot for scenario 1. EM
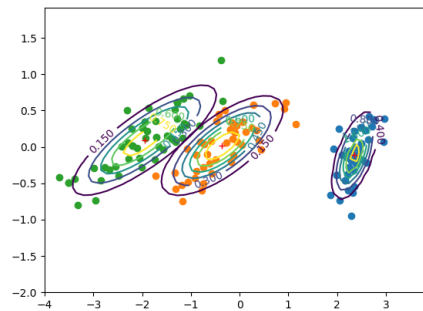
Figure 19: scatter plot with Gaussian for scenario 1. EM
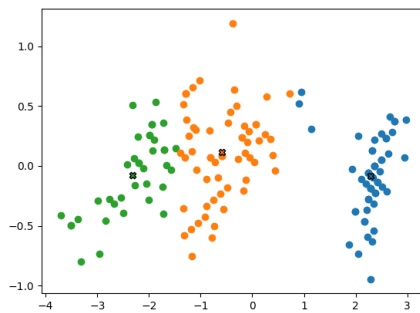
Figure 20: scatter plot for scenario 1. K-means
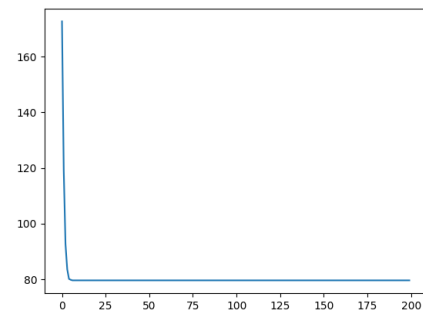
Figure 21: cumulative distance for k-means plot for scenario 1. K-means

EM showed better performance in scenario 2.2. PCA reduced the dimension of data to 2 so it took less time and showed more accuracy.
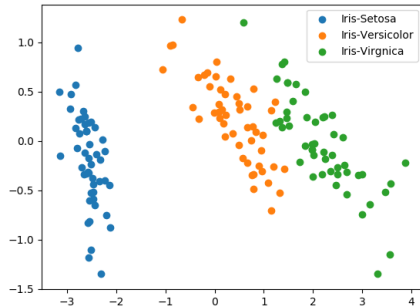

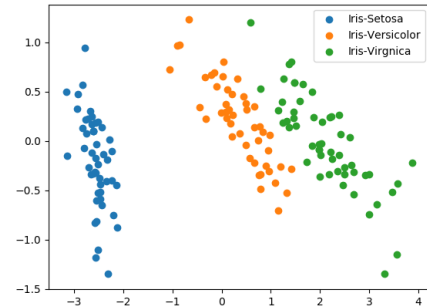Figure 22: answer for pca scenario 2

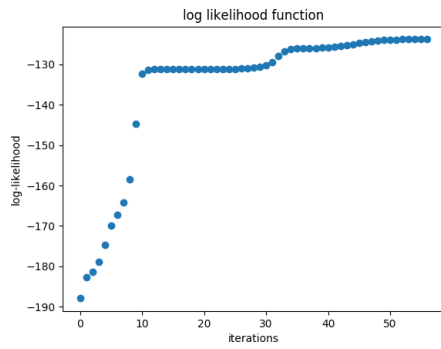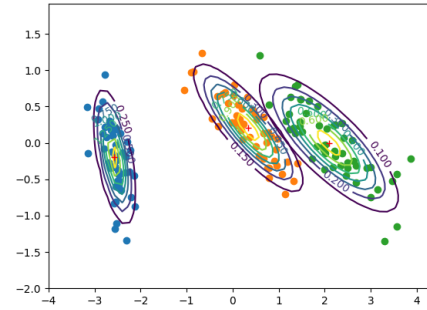Figure 23: scatter plot for scenario 2. EM

Figure 24: log-likelihood plot for scenario 2. EM

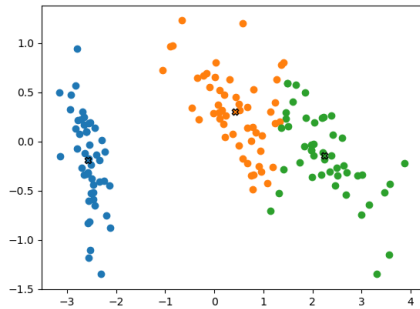Figure 25: scatter plot with Gaussian for scenario 2. EM

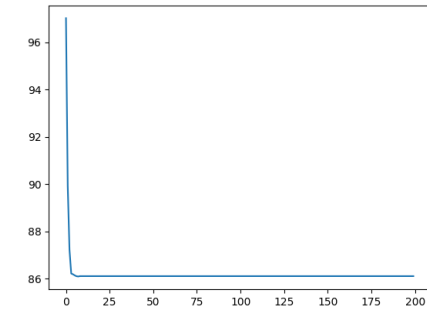Figure 26: scatter plot for scenario 2. K-means

Figure 27: cumulative distance for k-means plot for scenario 2. K-means

1.3.3 Apply PCA with whitening, so that the transformed data has zero mean and a unit covariance matrix. How does this influence the choice of your initialization?

2   Samples from a Gaussian Mixture Model

2.1   **Write a function Y = sample-GMM(alpha, mu, cov, N)**

2.2   **Using a GMM of your choice($K > 3$), demonstrate the correctness of your function**