Proof a Day — 2026

Alexander Crabtree

Contents

1 Weeks 1–13 **2**
1.1 Week 1 (Jan 1–Jan 7) 2
2026-01-01 — Division 2

1 Weeks 1–13

1.1 Week 1 (Jan 1–Jan 7)

2026-01-01 Division

Theorem 1.1. *If $a|c$ and $b|c$ and $\gcd(a,b) = d$, then $ab|cd$. $a, b, c \in \mathbb{Z}$*

Proof. As $d = \gcd(a,b)$, we have that $d = au + bv$ for some $u, v \in \mathbb{Z}$. We now have that $cd = c(au + bv) = cau + cbv$. Since $a|c$ and $b|c$, $c = ax = by$ for some $x, y \in \mathbb{Z}$. Substituing c results in $cd = cau + cbv = (by)au + (ax)bv = (ab)(yu + xv)$. As $u, v, x, y \in \mathbb{Z}$, $yu + xv \in \mathbb{Z}$. By definition of divisibility, $ab|cd$. □