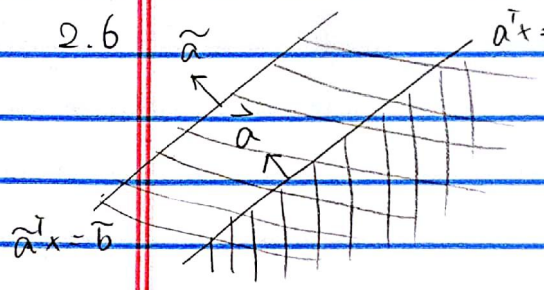
EE 364a HW 1

2.6



$\exists\ \lambda > 0$ s.t. $\tilde{a} = \lambda a$

$\Rightarrow \tilde{a}^T x \le \tilde{b}$

$\Rightarrow \lambda a^T x \le \tilde{b}$

$\Rightarrow a^T x \le \frac{\tilde{b}}{\lambda}$

For $H \subseteq \tilde{H}$ $\quad \frac{\tilde{b}}{\lambda} \ge b \Rightarrow \tilde{b} \ge b\lambda$

For $H = \tilde{H}$ $\quad \tilde{a} = \lambda a \Rightarrow \tilde{b} = b\lambda$

2.7. $\{x \mid \|x-a\|_2 \le \|x-b\|_2\}$

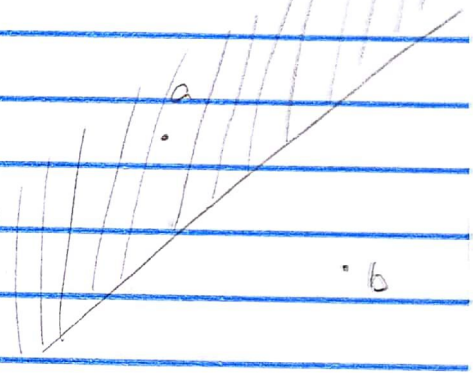
square on both sides

$(x-a)^T(x-a) \le (x-b)^T(x-b)$

$\Rightarrow x^Tx - 2x^Ta + a^Ta \le x^Tx - 2x^Tb + b^Tb$

$\Rightarrow 2(b-a)^Tx \le b^Tb - a^Ta$

in the form of $c^Tx \le d$



2.12

a) slab ✓

b) rectangle ✓

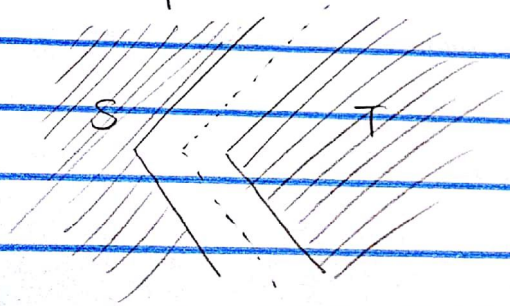
c) edge ✓

d) $\{x \mid \|x-x_0\|_2 \le \|x-y\|_2$ for $\forall y \in S\}$ ✓

$\Leftrightarrow \bigcap_{y_0 \in S} \|x-x_0\|_2 \le \|x-y_0\|_2$ → half space

$\Leftrightarrow$ intersection of convex is convex

e) X



Counter Example

A1.7. a) $K = \{0\}$

For any $x \in \mathbb{R}^2$

$x \cdot 0 = 0 \geq 0$

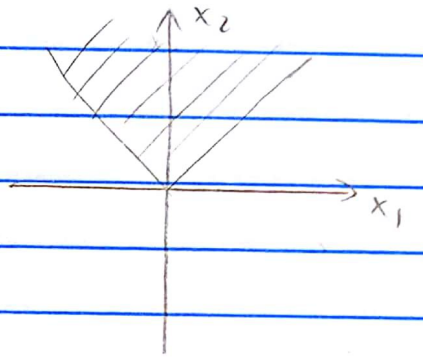
$\Rightarrow$ Dual cone $K^* = \mathbb{R}^2$

b) $K = \mathbb{R}^2$

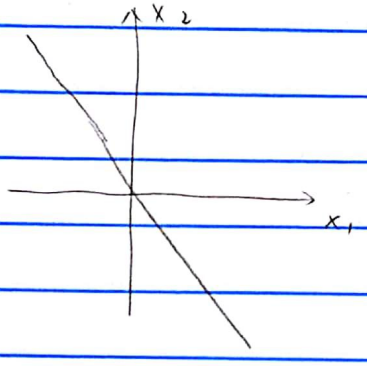
$\Rightarrow K^* = \{0\}$

c) $K = \{(x_1, x_2) \mid |x_1| \leq x_2\}$

$K = K^*$



d) $K = \{(x_1, x_2) \mid x_1 + x_2 = 0\}$



$K^* = \{(x_1, x_2) \mid x_1 - x_2 = 0\}$