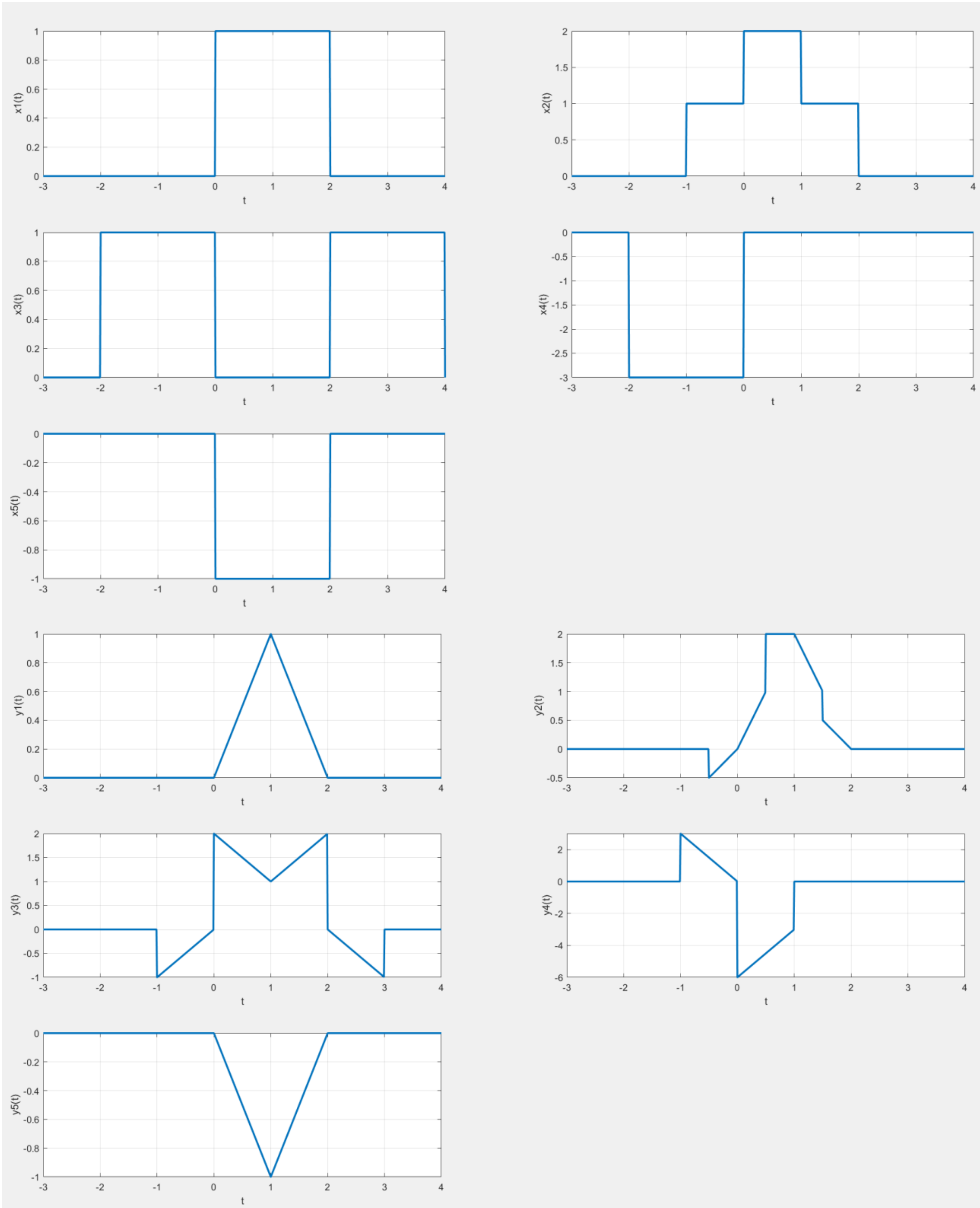
```matlab
clc
clear all
close all

%% Exercitiul 1
% x1,x2,x3,x4,x5
t = -3:0.01:4;
u = @(t)(t>=0);
x1 = @(u,t)(u(t) - u(t-2));
x2 = @(u,t)(x1(u,t) + x1(u,t+1));
x3 = @(u,t)(x1(u,t-2) + x1(u,t+2));
x4 = @(u,t)(-3*x1(u,t+2));
x5 = @(u,t)(-x1(u,t));

figure(1)
subplot(3,2,1);
plot(t,x1(u,t),'LineWidth',2); grid; axis([-3 4 0 1]); xlabel('t'); ylabel('x1(t)');
subplot(3,2,2);
plot(t,x2(u,t),'LineWidth',2); grid; axis([-3 4 0 2]); xlabel('t'); ylabel('x2(t)');
subplot(3,2,3);
plot(t,x3(u,t),'LineWidth',2); grid; axis([-3 4 0 1]); xlabel('t'); ylabel('x3(t)');
subplot(3,2,4);
plot(t,x4(u,t),'LineWidth',2); grid; axis([-3 4 -3 0]); xlabel('t'); ylabel('x4(t)');
subplot(3,2,5);
plot(t,x5(u,t),'LineWidth',2); grid; axis([-3 4 -1 0]); xlabel('t'); ylabel('x5(t)');

% y1,y2,y3,y4,y5
y1 = @(u,t)(t.*x1(u,2*t) - (t-2).*x1(u,2*t-2));
y2 = @(u,t)(t.*x2(u,2*t) - (t-2).*x2(u,2*t-2));
y3 = @(u,t)(t.*x3(u,2*t) - (t-2).*x3(u,2*t-2));
y4 = @(u,t)(t.*x4(u,2*t) - (t-2).*x4(u,2*t-2));
y5 = @(u,t)(t.*x5(u,2*t) - (t-2).*x5(u,2*t-2));

figure(2)
subplot(3,2,1);
plot(t,y1(u,t),'LineWidth',2); grid; axis([-3 4 0 1]); xlabel('t'); ylabel('y1(t)');
subplot(3,2,2);
plot(t,y2(u,t),'LineWidth',2); grid; axis([-3 4 -0.5 2]); xlabel('t');
ylabel('y2(t)');
subplot(3,2,3);
plot(t,y3(u,t),'LineWidth',2); grid; axis([-3 4 -1 2]); xlabel('t'); ylabel('y3(t)');
subplot(3,2,4);
plot(t,y4(u,t),'LineWidth',2); grid; axis([-3 4 -6 3]); xlabel('t'); ylabel('y4(t)');
subplot(3,2,5);
plot(t,y5(u,t),'LineWidth',2); grid; axis([-3 4 -1 0]); xlabel('t'); ylabel('y5(t)');
```



x1(t)
x2(t)
x3(t)
x4(t)
x5(t)
y1(t)
y2(t)
y3(t)
y4(t)
y5(t)
t

%% Exercitiul 2

2) $y[n] = x[n](g[n] + g[n-1])$

a) $g[n] = 1$

$\Rightarrow y[n] = 2x[n]$

$y_{shifted}[n] = T\{x_{shifted}[n]\} =$

$= T\{x[n-N]\} = 2\,x[n-N]$

$y[n+N] = 2 \cdot x[n+N] \Rightarrow$ S invariant în timp

b) $g[n] = n$

$\Rightarrow y[n] = x[n] \cdot (2n-1)$

$y_{shifted}[n] = T\{x_{shifted}[n]\} =$

$= T[x[n-N]] = x[n-N]\left(\overset{2n-1}{\cancel{2n-2N-1}}\right)$

$y[n-N] = x[n-N] \cdot (2n-2N-1)$

$\Rightarrow$ S nu este invariant în timp

c) $y[n] = -1 + (-1)^n$

$$\Rightarrow y[n] = x[n]\left(-2 + 2\cdot(-1)^n\right)$$

$$y_{shifted}[n] = T[x_{shifted}[n]] =$$

$$= T[x[n-N]]$$

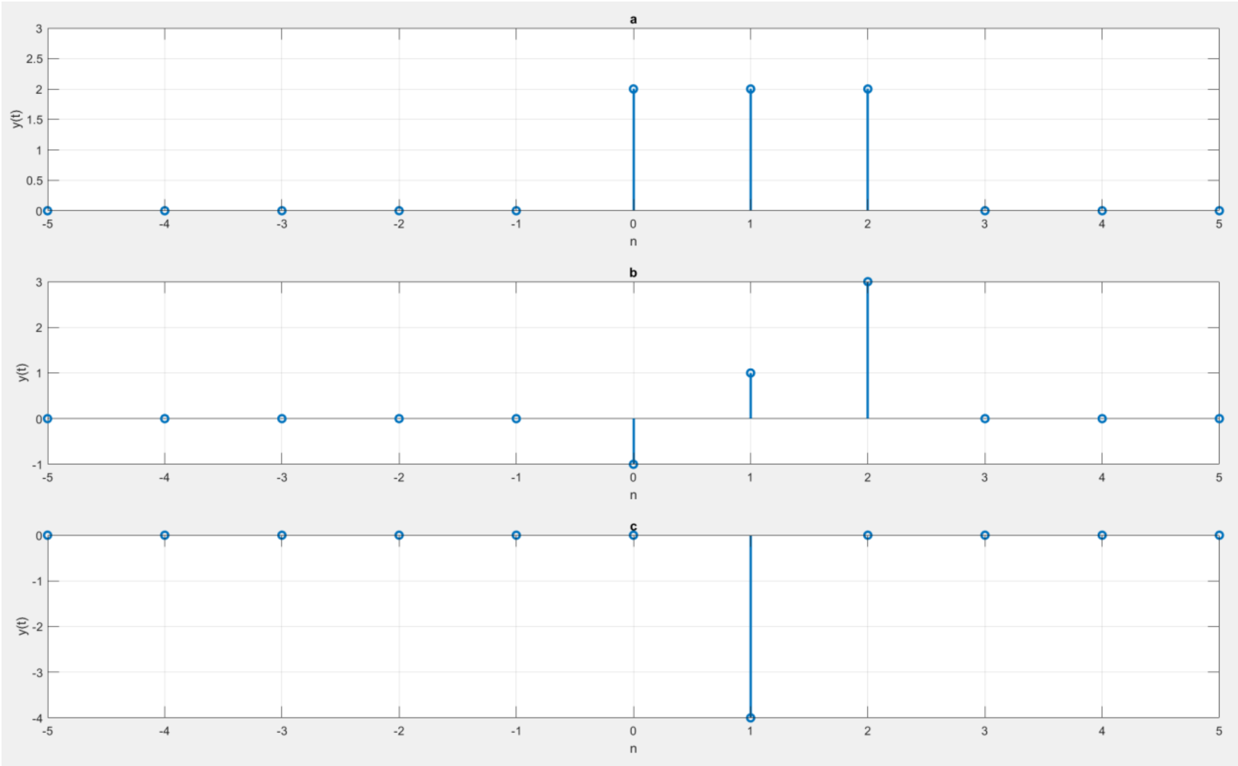
$$= x[n-N]\left(-2 + 2\cdot(-1)^n\right)$$

$$y[n-N] = x[n-N]\left(-2 + 2\cdot(-1)^{n-N}\right)$$

$$= x[n-N]\left(-2 + 2(-1)^n\right)$$

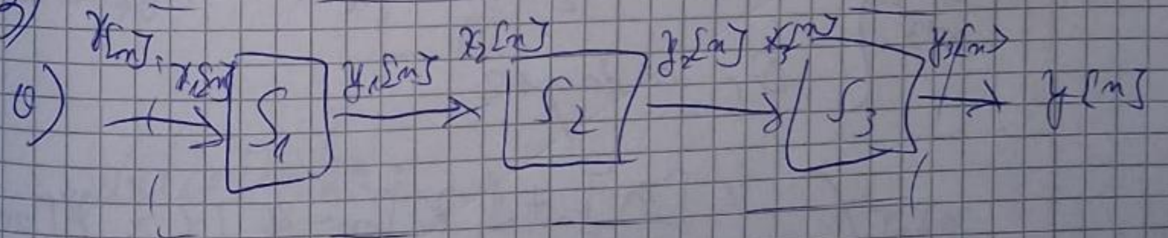
$\Rightarrow$ S este invariant in timp

```
% d

n = -5:1:5;
u = @(t)(t>=0);
x = @(u,n)(u(n) - u(n-3));

y_a = @(u,n)(2*x(u,n));
y_b = @(u,n)(x(u,n).*(2*n-1));
y_c = @(u,n)(x(u,n).*(-2+2*(-1).^n));

figure(3)
subplot(3,1,1);
stem(n,y_a(u,n),'LineWidth',2); grid; axis([-5 5 0 3]); xlabel('n'); ylabel('y(t)');
title('a');
subplot(3,1,2);
stem(n,y_b(u,n),'LineWidth',2); grid; axis([-5 5 -1 3]); xlabel('n'); ylabel('y(t)');
title('b');
subplot(3,1,3);
stem(n,y_c(u,n),'LineWidth',2); grid; axis([-5 5 -4 0]); xlabel('n'); ylabel('y(t)');
title('c');
```



%% Exercitiul 3

3)



$y[n] = y_3[n]$

$x_3[n] = y_2[n]$

$x_2[n] = y_1[n]$

$x_1[n] = x[n]$

S_1: $y_1[n] = \begin{cases} x_1[\frac{n}{2}], & n \text{ par} \\ 0, & n \text{ impar} \end{cases}$

S_2: $y_2[n] = x_2[n] + \frac{1}{2} x_2[n-1] + \frac{1}{4} x_2[n-2]$

S_3: $y_3[n] = x_3[2n]$

$\Rightarrow y_3[n] = y_2[2n]$

$y_3 = x_2[2n] + \frac{1}{2} x_2[2n-1] + \frac{1}{4} x_2[2n-2]$

$y_3 = y_1[2n] + \frac{1}{2} y_1[2n-1] + \frac{1}{4} y_1[2n-2]$

$y_3 = \begin{cases} x_1[n] + \frac{1}{2} x_1[n-\frac{1}{2}] + \frac{1}{4} x_1[n-1], & \text{par} \\ 0, & n \text{ impar} \end{cases}$

$y[n] = \begin{cases} x[n] + \frac{1}{2} x[n-\frac{1}{2}] + \frac{1}{4} x[n-1], & n \text{ par} \\ 0, & n \text{ impar} \end{cases}$

$y_{shifted}[n] = T[x_{shifted}[n]] =$

$= T[x[n-N]]$

$\neq$

$$= \begin{cases} x[n-N] + \frac{1}{2}x[n-N-1] + \frac{1}{4}x[n-N-2], & n \text{ par} \\ 0, & n \text{ impar} \end{cases}$$

$$y[n-N] = \begin{cases} x[n-N] + \frac{1}{2}x[n-N-1] + \frac{1}{4}x[n-N-2], & n \text{ par} \\ 0, & n \text{ impar} \end{cases}$$

$\Rightarrow y[n]$ invariant in timp

omogenitatea:

$$T[k\,x[n]] = \begin{cases} k\,x[n] + \frac{k}{2}x[n-1] + \frac{k}{4}x[n-2] \\ 0 \end{cases}$$

$$k\,T[x[n]] = \begin{cases} k \cdot x[n] + \frac{k}{2}x[n-1] + \frac{k}{4}x[n-2] \\ 0 \end{cases}$$

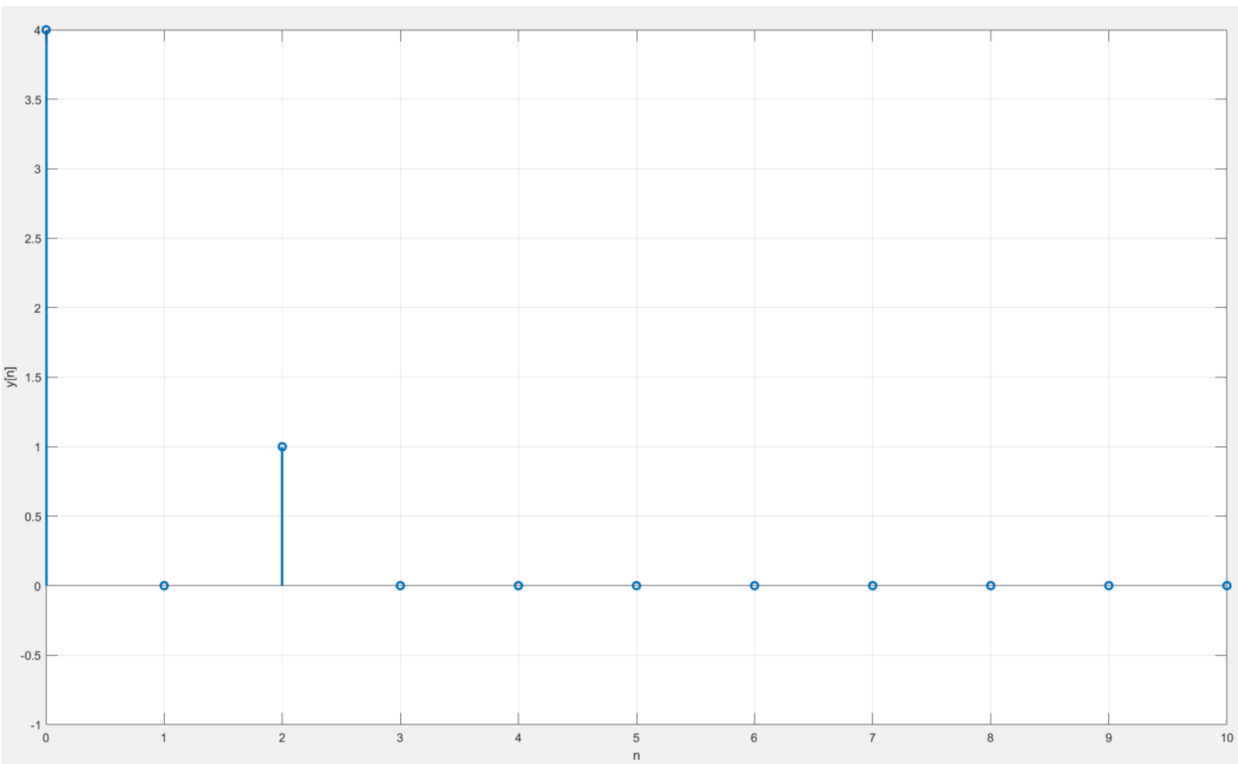
aditivitatea:

$$T[x_1[n] + x_2[n]] = \begin{cases} x_1[n] + x_2[n] + \frac{1}{2}(x_1[n] + x_2[n]) + \\ \quad + \frac{1}{4}(x_1[n] + x_2[n]) \\ 0 \end{cases}$$

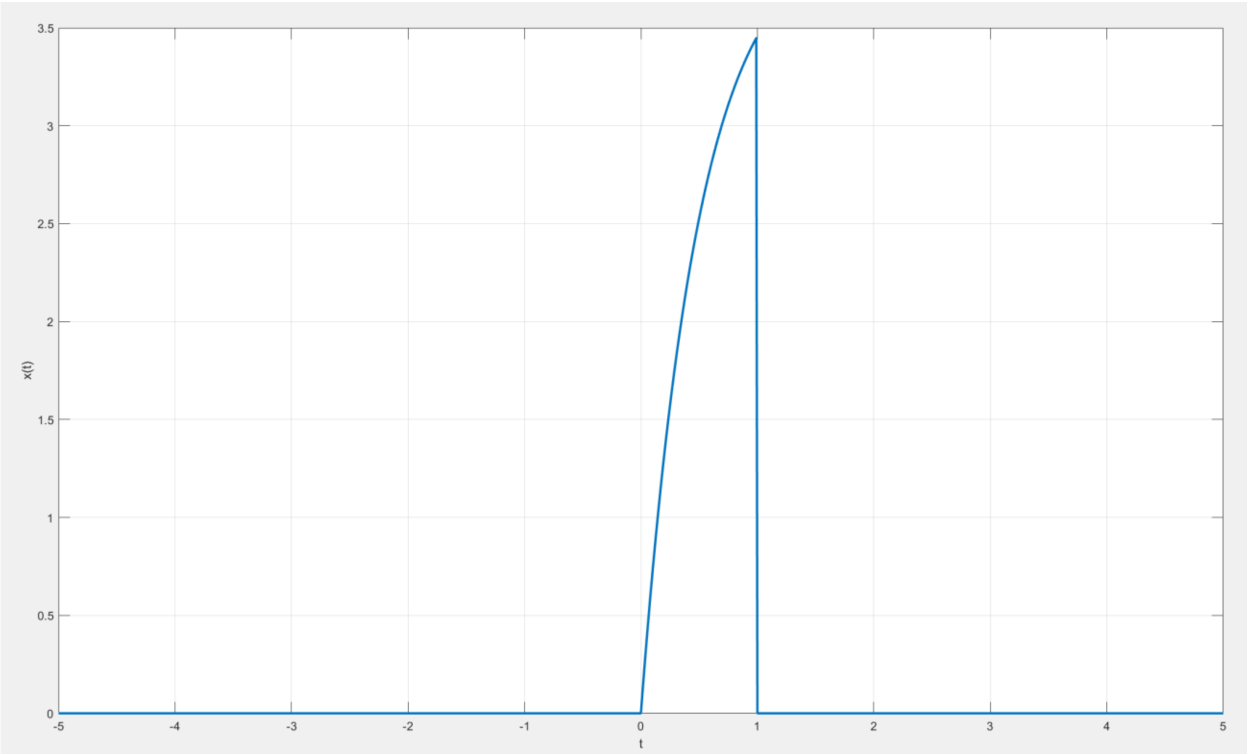
$$T[x_1[n]] + T[x_2[n]] = \begin{cases} x_1[n] + \frac{1}{2}x_1[n] + \frac{1}{4}x_1[n] + \\ \quad + x_2[n] + \frac{1}{2}x_2[n] + \frac{1}{4}x_2[n] \\ 0 \end{cases}$$

$\Rightarrow S$ este liniar

```
% c

n = 0:1:10;

u = @(t)(t==0);
x = @(u,n)(4*u(n));
y = zeros(size(n));

for i = 1:length(n)

    if mod(n(i),2) == 0
        y(i) = x(u,n(i))+1/2 * x(u,n(i)-1) + 1/4 * x(u,n(i)-2);
    else y(i) = 0;
    end

end

figure(4)
stem(n,y,'LineWidth',2); grid; axis([0 10 -1 4]); xlabel('n'); ylabel('y[n]');
```



```
%% Exercitiul 4

t = -5:0.01:5;
u = @(t)(t>=0);
x = @(u,t)(4*u(t) - 4*u(t-1));
y = @(u,t)((1-exp(-2*t)).*x(u,t));

figure(5)
plot(t,y(u,t),'LineWidth',2); grid; axis([-5 5 0 3.5]); xlabel('t'); ylabel('x(t)');
```



%% Exercitiul 5

5)

a) $y(t) = t x(t)$

$y_{shifted}(t) = T[x_{shifted}(t)] = T[x(t-t_0)] =$

$= \cancel{(t-t_0)x} \; t \cdot x(t-t_0)$

$y(t-t_0) = (t-t_0) \cdot x(t-t_0)$

$\Rightarrow$ S nu este invariant în timp

$T[k \cdot x(t)] = t \cdot k \cdot x(t)$

$k \cdot T[x(t)] = k t \cdot x(t)$

$T[x_1(t) + x_2(t)] = t(x_1(t) + x_2(t))$

$T[x_1(t)] + T[x_2(t)] = t x_1(t) + t x_2(t) =$

$= t(x_1(t) + x_2(t))$

$\Rightarrow$ S liniar

b) $y(t) = x(t-2) + x(2-t)$

$y_{shifted}(t) = T[x_{shifted}(t)] = T[x(t-t_0)]$

$= x(t-t_0-2) + x(2-t+t_0)$

$y(t-t_0) = x(t-t_0-2) + x(2-t+t_0)$

$\Rightarrow S$ invariant in timp

$T[\ell \cdot x(t)] = \ell \cdot x(t-2) + \ell x(2-t)$

$\ell T[x(t)] = \ell x(t-2) + \ell x(2-t)$

$T[x_1(t)+x_2(t)] = x_1(t-2) + x_1(2-t) + x_2(t-2) + x_2(2-t)$

$T[x_1(t)] + T[x_2(t)] = x_1(t-2) + x_1(2-t) + x_2(t-2) + x_2(2-t)$

$\Rightarrow S$ liniar

c) $y(t) = \sin(x(t))$

$y_{shifted}(t) = T[x(t-t_0)] = \sin(x(t-t_0))$

$y(t-t_0) = \sin(x(t-t_0))$

$\Rightarrow S$ invariant in timp

$T[\ell x(t)] = \sin(\ell \cdot x(t))$

$\ell \cdot T[x(t)] = \ell \sin(x(t))$

$\Rightarrow S$ neliniar

d) $y(t) = |x(t) - x(t-1)|$

$y_{shifted}(t) = T[x(t-t_0)] = |x(t-t_0) - x(t-t_0-1)|$

$y(t-t_0) = |x(t-t_0) - x(t-t_0-1)|$

$\Rightarrow S$ invariant in timp

$T[k_1 x(t)] = |k(x(t)) - k(x(t-1))|$

$k\,T[x(t)] = k\,|x(t) - x(t-1)| = |k(x(t)) - k x(t-1)|$

$T[x_1(t) + x_2(t)] = |x_1(t) - x_1(t-1) + x_2(t) - x_2(t-1)|$

$T[x_1(t)] + T[x_2(t)] = |x_1(t) - x_1(t-1)| + |x_2(t) - x_2(t-1)|$

$\Rightarrow$ S ~~nu~~ neliniar

e) $y[n] = x[-n]$

$y_{shifted}[n] = T[x[n-N]] = x[-n+N]$

$y[n-N] = x[-n+N]$

$\Rightarrow$ S invariant in timp

$T[k\,x[n]] = k\,x[-n]$

$k\,T[x[n]] = k \cdot x[-n]$

$T[x_1[n] + x_2[n]] = x_1[-n] + x_2[-n]$

$T[x_1[n]] + T[x_2[n]] = x_1[-n] + x_2[-n]$

$\Rightarrow$ S liniar

f) $y[n] = 2x[n] + 3$

$y_{shifted}[n] = T[x[n-N]] = 2 \cdot x[n-N] + 3$

$y[n-N] = 2 \cdot x[n-N] + 3$

$\Rightarrow$ S invariant in timp

$T[k\,x[n]] = 2k\,x[n] + 3$

$\ell\, T[x[n]] = 2\ell\, x[n] + 3\ell$

$\Rightarrow S$ neliniar

g) $y[n] = x[2n]$

$y_{shifted}[n] = T[x[n-N]] = x[2n-N]$

$y[n-N] = x[2n-2N]$

$\Rightarrow S$ variant în timp

$T[\ell \cdot x[n]] = \ell \cdot x[2n]$

$\ell\, T[x[n]] = \ell \cdot x[2n]$

$T[x_1[n] + x_2[n]] = x_1[2n] + x_2[2n]$

$T[x_1[n]] + T[x_2[n]] = x_1[2n] + x_2[2n]$

$\Rightarrow S$ liniar

h) $y[n] = n\, x[2n]$

$y_{shifted}[n] = T[x[n-N]] = n \cdot x[2n-N]$

$y[n-N] = (n-N) \cdot x[2n-N]$

$\Rightarrow S$ variant în timp

$T[\ell\, x[n]] = \ell n\, x[2n]$

$\ell\, T[x[n]] = \ell n\, x[2n]$

$T[x_1[n] + x_2[n]] = n\, x_1[2n] + n\, x_2[2n]$

$T[x_1[n]] + T[x_2[n]] = n\, x_1[2n] + n\, x_2[2n]$

$\Rightarrow S$ liniar

i) $y[n] = e^{-2n} x[n]$

$y_{shifted}[n] = T[x[n-N]] = e^{-2n} x[n-N]$

$y[n-N] = e^{-2(n-N)} x[n-N] \Rightarrow S$ variant in time

$T[k x[n]] = \cancel{k} e^{-2n} k x[n]$

$k T[x[n]] = k \cdot e^{-2n} x[n]$

$T[x_1[n] + x_2[n]] = e^{-2n}(x_1[n] + x_2[n])$

$T[x_1[n]] + T[x_2[n]] = e^{-2n} x_1[n] + e^{-2n} x_2[n]$
$= e^{-2n}(x_1[n] + x_2[n])$

$\Rightarrow S$ linear

j) $y[n] = \sum_{k=-\infty}^{n} x[k]$

$y_{shifted}[n] = T[x[n-N]] = \sum_{k=-\infty}^{n} x[k-N]$

$y[n-N] = \sum_{k=-\infty}^{n-N} x[k]$

$\Rightarrow S$ variant in time

$T[k x[n]] = \cancel{k} \sum_{k=-\infty} \cancel{k} x[k]$

$\cancel{k} T[x[n]] = \cancel{k} \sum_{k=-\infty}^{n} \cancel{k} \cdot x[k]$

~~$\Rightarrow S$ nelinior~~

$$T\{x_1[n] + x_2[n]\} = \sum_{k=-\infty}^{n} \left(x_1[k] + \sum_{k=-\infty} x_2[k]\right)$$

$$T\{x_1[n]\} + T\{x_2[n]\} = \sum_{k=-\infty}^{n} x_1[k] + \sum_{k=-\infty}^{n} x_2[k]$$

$$= \sum_{k=-\infty}^{n} \left(x_1[k] + x_2[k]\right)$$

$\Rightarrow$ S liniar

b) $y[n] = x[n] + 3u[n+1]$

$y_{shifted}[n] = T\{x[n-N]\} = x[n-N] + 3u[n+1]$

$y[n-N] = x[n-N] + 3u[n-N+1]$

$\Rightarrow$ S variant in timp

$T\{k x[n]\} = k x[n] + 3u[n+1]$

$k\, T\{x[n]\} = k x[n] + k 3 u[n+1]$

$\Rightarrow$ S neliniar