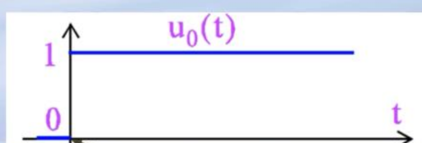
Semnale și sisteme

Lucrare de laborator nr. 4

Semnale elementare

Funcția treapta unitara $u_0(t)$

$$u_0(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

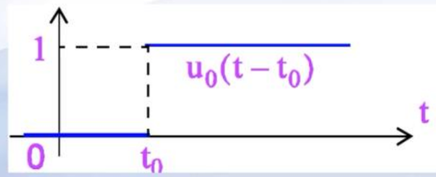


- schimbare brusca a valorii din 0 in 1 la momentul $t = 0$

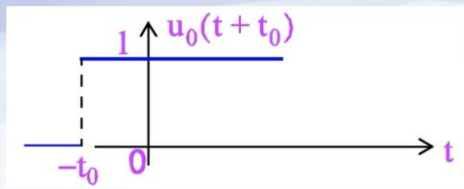
Functia treapta unitara $u_0(t)$

Functia treapta unitara deplasata:

$$u_0(t - t_0) = \begin{cases} 1, & t \geq t_0 \\ 0, & t < t_0 \end{cases}$$



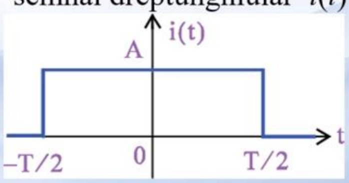
$$u_0(t + t_0) = \begin{cases} 1, & t \geq -t_0 \\ 0, & t < -t_0 \end{cases}$$

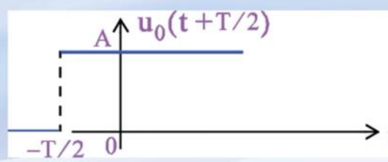


Functia treapta unitara $u_0(t)$

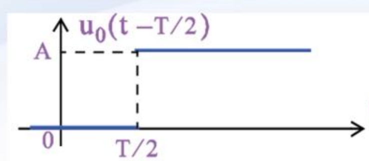
<u>Exprimarea altor semnale in termeni de functii treapta unitara</u>

- semnal dreptunghiular $i(t) = f(u_0(t))$






```
u0 = @(t)(t>=0)
A = 2; T = 1; t = -2:0.001:2;
i = A*u0(t+T/2) - A*u0(t-T/2);
plot(t, i, 'LineWidth', 2)
axis([min(t), max(t), min(i)-0.01, max(i)+0.01])
xlabel('t'); ylabel('i(t)'); grid
```

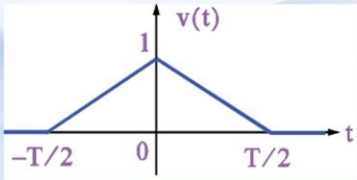


$$i(t) = A\, u_0(t + T/2) - A\, u_0(t - T/2)$$

Functia treapta unitara $u_0(t)$

Exprimarea altor semnale in termeni de functii treapta unitara

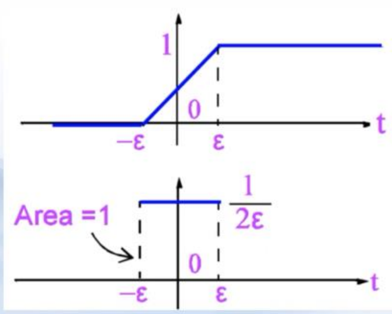
- semnal triunghiular $v(t) = \mathrm{f}\,(u_0(t))$



$$v(t) = \begin{cases} \frac{2}{T}t + 1, & -\frac{T}{2} \le t < 0 \\ -\frac{2}{T}t + 1, & 0 \le t \le \frac{T}{2} \\ 0, & \text{in rest} \end{cases}$$

$$v(t) = \left(\frac{2}{T}t + 1\right)\left(u_0\left(t + \frac{T}{2}\right) - u_0(t)\right) + \left(-\frac{2}{T}t + 1\right)\left(u_0(t) - u_0\left(t - \frac{T}{2}\right)\right)$$

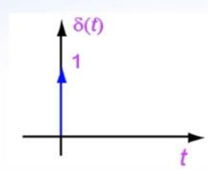
Semnalul impuls unitar $\delta(t)$



$\varepsilon \to 0$ ➔ functia treapta unitara

$\varepsilon \to 0,\ 1/(2\varepsilon) \to \infty$ ➔ functia impuls

Reprezentarea grafica a impulsului unitar:
sageata la $t = 0$ cu lungimea egala cu aria impulsului



Semnalul impuls unitar $\delta(t)$. Proprietati

Proprietatea de esantionare: inmultind o functie $f(t)$ cu functia delta $\delta(t\text{-}a)$ se obtine functia $f(t)$ esantionata la momentele de timp unde functia delta nu este zero:

$f(t)\delta(t-a) = f(a)\delta(t-a)$

Proprietatea de cernere: inmultind o functie $f(t)$ cu $\delta(t-a)$, si integrand de la - ∞ la ∞, se obtine valoarea lui $f(t)$ evaluata la $t = a$:

$\int_{-\infty}^{\infty} f(t)\delta(t-a)dt = f(a)$

<u>Exemplu</u>: Sa se evalueze urmatoarele expresii: a) $3t^4\delta(t-1)$, b) $\int_{-\infty}^{\infty} t\delta(t-2)dt$

a) Proprietatea de esantionare $f(t)\delta(t-a) = f(a)\delta(t-a)$. In acest exemplu, $f(t) = 3t^4$, $a = 1$,

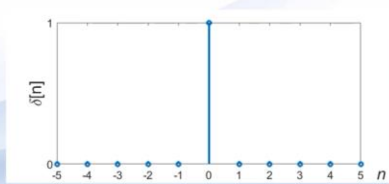
$3t^4\delta(t-1) = \{3t^4|_{t=1}\}\delta(t-1) = 3\delta(t-1)$

b) Proprietatea de cernere $\int_{-\infty}^{\infty} f(t)\delta(t-a)dt = f(a)$. In acest exemplu, $f(t) = t$, $a = 2$,

$\int_{-\infty}^{\infty} t\delta(t-2)dt = f(2) = 2$

Impulsul unitar discret $\delta[n]$
Semnalul treapta discret $u[n]$

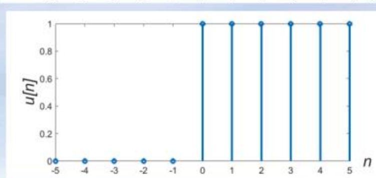
$$\delta[n] = \begin{cases} 1, n = 0 \\ 0, n \neq 0 \end{cases}$$



Proprietatea de esantionare a lui δ[n]:

$x[n]\delta[n-n_0] = x[n_0]\delta[n-n_0]$

$$\mathrm{u}[n] = \begin{cases} 1, n \geq 0 \\ 0, n < 0 \end{cases}$$



Relatii intre $\delta[n]$ si $u[n]$:

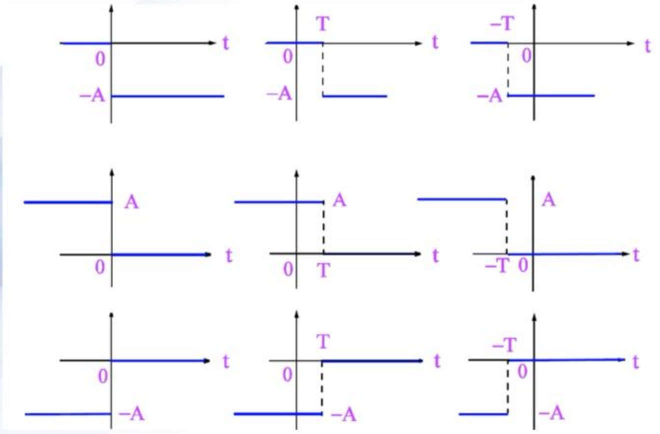
$$u[n] = \sum_{k=0}^{\infty} \delta[n-k]$$

$$\delta[n] = u[n] - u[n-1]$$

Exercitii

Exercitiul 1:

Exprimati fiecare dintre functiile din figura in termeni de functii treapta unitara. Reprezentati functiile in Matlab.



Exercitii

Exercitiul 2: Evaluati urmatoarele expresii:

a) $\int_{-4}^{7} \sin(\pi t)\, \delta(t-1) dt$

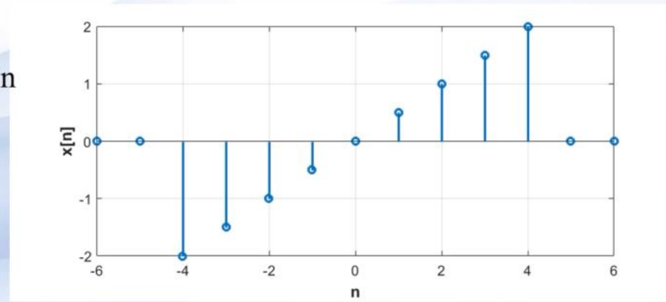
b) $\int_{0}^{2} \sin(\pi t)\, \delta(t-3) dt$

c) $e^{-t} \cos(10t) \delta(t)$

d) $\sin(2\pi t) \sum_{k=0}^{\infty} \delta(t-k), \quad k \in Z$

Exercitii

<u>Exercitiul 3</u>: Determinati o expresie pentru semnalul x[n] din figura alaturata utilizand functii treapta unitara.



Reprezentati urmatoarele semnale in Matlab:
a) $x[2-n]$ b) $x[n+2]$ c) $x[-n]u[n]+x[n]$
d) $x[n+2]+x[-1-n]$ e) $x[3n]\delta[n-1]$
f) $x[n+1](u[n+3]-u[-n])$ g) $(\mathrm{u}[\mathrm{n}-4]-\mathrm{u}[\mathrm{n}-3])x[n]$