Tema curs 12

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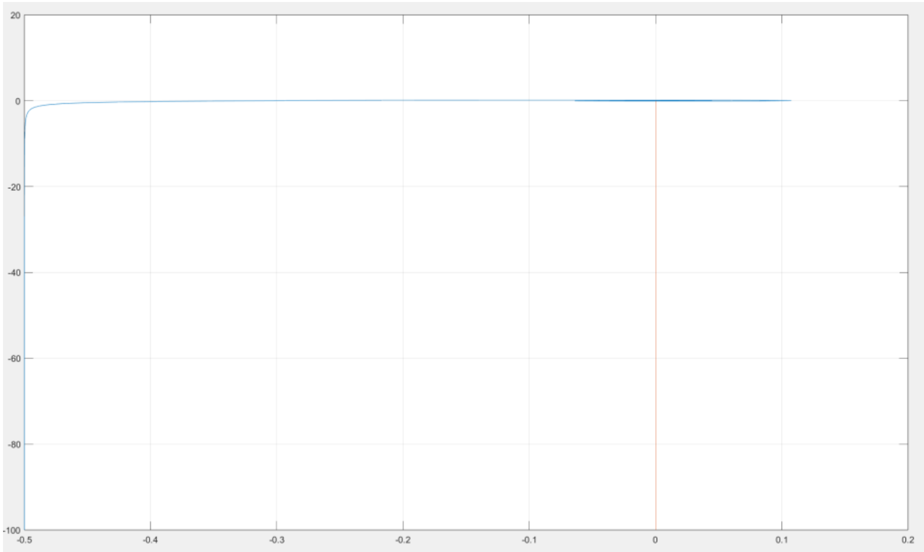
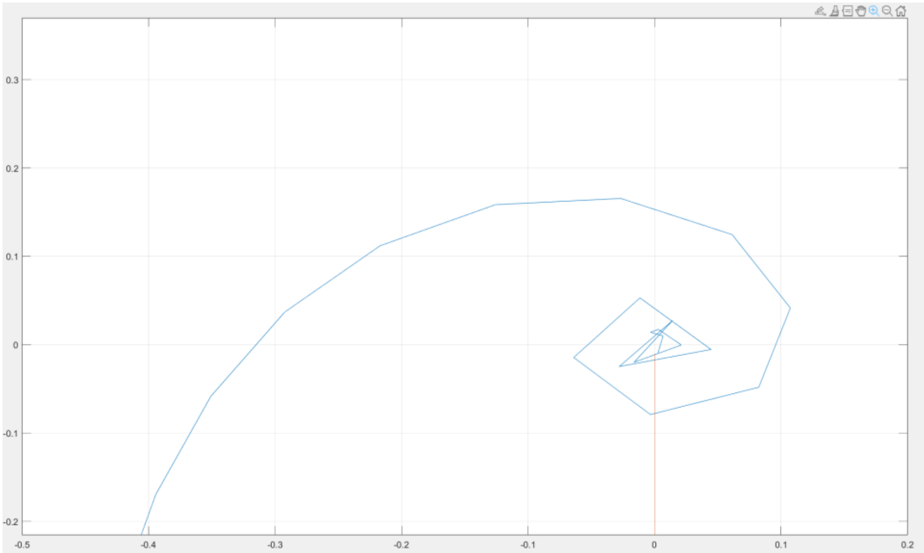
Grupa 30121

Afisarea diagramei Nyquist pentru un numar fix de ω:

```
H = tf(1,[1,0],'iodelay',0.5);
w = logspace(-2,2);
[re_H, im_H] = nyquist(H,w);
re_H_afis = re_H(1,1,:);
im_H_afis = im_H(1,1,:);
plot(re_H_afis(:),im_H_afis(:)), grid, hold on;
H_prim = tf(1,[1,0]);
[re_H, im_H] = nyquist(H_prim,w);
re_H_afis = re_H(1,1,:);
im_H_afis = im_H(1,1,:);
plot(re_H_afis(:),im_H_afis(:));
```





Determinarii stabilitatii unui sistem cu RN folosind RF:

$$H_d(s) = \frac{k}{s+0,5} \; ; \; H_r(s) = e^{-\tau_m s}$$

a) $$H_{des}(s) = \frac{k}{s+0,5} \cdot e^{-\tau_m s}$$

$$k = 2 \; ; \; \tau_m = 0,5$$

$$H_{des}(s) = \underbrace{\frac{2}{s+0,5}}_{H'_{des}} \cdot e^{-0,5 s} \qquad \begin{array}{l} \gamma = 0 \\ \hat{\omega}_{k1} = 0,5 \end{array}$$

$$H'_{des}(s) = \frac{2}{s+0,5} = \frac{4}{2s+1}$$

$$CFJ: \omega >> \min_i \{\omega_{ki}\} = 0,5$$

$$H'_{des}(j\omega)\Big|_{\omega<<0,5} = H'_{des}(s)\Big|_{\substack{s=j\omega \\ \omega<<0,5}} =$$

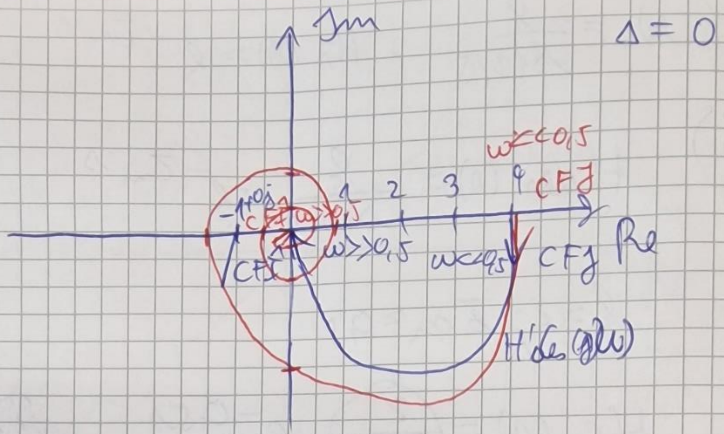
$$= \frac{4}{2s+1}\Big|_{\substack{s=j\omega \\ \omega<<0,5}} = \frac{4}{2j\omega+1}\Big|_{\omega<<0,5} =$$

$$= \frac{-8j\omega+4}{1+4\omega^2}\Big|_{\omega<<0,5} = \frac{-8j\omega}{1+4\omega^2} + \frac{4}{1+4\omega^2}\Big|_{\omega<<0,5}$$

$$Re\{H(j\omega)\}\Big|_{\omega<<0,5} = \frac{4}{1+4\omega^2}\Big|_{\omega<<0,5} = 4$$

$$Im\{H(j\omega)\}\Big|_{\omega<<0,5} = \frac{-8\omega}{1+4\omega^2}\Big|_{\omega<<0,5} = 0_+$$

$\Delta = 0$



CF Î: $\omega >> \max\{\omega_{f_i}\} = 0,5$

$$H(j\omega)\Big|_{\omega>>0,5} = H(s)\Big|_{\substack{s=j\omega\\ \omega>>0,5}} = \frac{-8j\omega}{1+4\omega^2}\Big| + \frac{4}{1+4\omega^2}\Big|_{\omega>>0,5}$$

$$Re\{H(j\omega)\}\Big|_{\omega>>0,5} = \frac{4}{1+4\omega^2}\Big|_{\omega>>0,5} = 0_+$$

$$Im\{H(j\omega)\}\Big|_{\omega>>0,5} = \frac{-8\omega}{1+4\omega^2}\Big|_{\omega>>0,5} = 0_-$$

$$H_{des}(s) = \frac{2}{s+0,5}\cdot e^{-0,5s} = \frac{4}{2s+1}\cdot e^{-0,5s}$$

CF J: $\omega \cancel{<<} 0,5$

$$H_{des}(j\omega)\Big|_{\omega<<0,5} = H_{des}(s)\Big|_{\substack{s=j\omega\\ \omega<<0,5}} = \frac{4}{2j\omega+1}\cdot e^{-0,5j\omega}$$

$$= \frac{4-8j\omega}{1+4\omega^2}\cdot\left(\cos(-0,5\omega) + j\sin(-0,5\omega)\right)\Big|_{\omega<<0,5}$$

$$= \frac{4-8j\omega}{1+4\omega^2}\cdot\left(\cos(0,5\omega) - j\sin(0,5\omega)\right)\Big|_{\omega<<0,5}$$

$\Rightarrow$

$$\mathrm{Re}\{H_{0des}(j\omega)\} = \frac{4}{1+4\omega^2}\cdot\cos(0{,}5\omega) + \frac{8\omega}{1+4\omega^2}\cdot\sin(0{,}5\omega)\Big|_{\omega \ll 0{,}5}$$

$$= 4$$

$$\mathrm{Im}\{H_{0des}(j\omega)\} = \frac{-4}{1+4\omega^2}\sin(0{,}5\omega) - \frac{8\omega}{1+4\omega^2}\cos(0{,}5\omega)\Big|_{\omega \ll 0{,}5}$$

$$= 0_-$$

CFI: $\omega \gg 0{,}5$

$$\mathrm{Re}\{H_{0des}(j\omega)\}\Big|_{\omega \gg 0{,}5} = 0_-$$

$$\mathrm{Im}\{H_{0des}(j\omega)\}\Big|_{\omega \gg 0{,}5} = 0_-$$

$$\mathrm{Re}\{H_{0des}(j\omega)\} = 0 \Rightarrow \frac{4}{1+4\omega^2}\cdot\cos(0{,}5\omega) - \frac{8\omega}{1+4\omega^2}\sin(0{,}5\omega) = 0$$

$$4\cos(0{,}5\omega) - 8\omega\sin(0{,}5\omega) = 0$$

$$\cos(0{,}5\omega) - 2\omega\sin(0{,}5\omega) = 0$$

$$tg(0{,}5\omega) = \frac{1}{2\omega}$$

$$\omega = 2\cdot arctg\frac{1}{2\omega}$$

$Im\{H_{des}(j\omega)\} = 0$

$-4\sin(0{,}5\omega) - 8\omega\cos(0{,}5\omega) = 0$

$\sin(0{,}5\omega) + 2\omega\cos(0{,}5\omega) = 0$

$\omega = -2\,\operatorname{arctg}\frac{1}{2\omega}$

$\Delta = (N_0 + 2N_+)\cdot\frac{\pi}{2} = 0 \Rightarrow$ sistem stabil

~~b~~) $k = 2$, $\tau_m = ?$ a.î. sistemul H_0 să rămână stabil

$$H_{des}(s) = \frac{2}{s+0{,}5}\cdot e^{-\tau_m s}$$

$$\left|H_{des}(s)\right|\Big|_{s=j\omega} = 1$$

$$\left|\left|\frac{2}{s+0{,}5}\right|\cdot e^{-\tau_m s}\right|\Bigg|_{s=j\omega} = 1$$

$$\left|\frac{2}{j\omega+0{,}5}\cdot e^{\tau_m s}\right|\Bigg|_{\cancel{s=j}} = 1$$

$$\cancel{\frac{2}{j\omega}}\left|\frac{4-8j\omega}{1+4\omega^2}\right|\cdot\left|e^{\tau_m s}\right| = 1$$

$$\sqrt{\frac{16}{(1+4\omega^2)^2} + \frac{64\omega^2}{(1+4\omega)^2}} = 1$$

$$\frac{4}{\sqrt{1+4\omega^2}} = 1$$

$$\Leftrightarrow \sqrt{1+4\omega^2} = 4$$

$$1+4\omega^2 = 16$$

$$4\omega^2 = 15$$

$$\omega^2 = 3,75$$

$$\omega = \sqrt{3,75} \simeq 1,93$$

$$\angle H_{0d}(j\omega)\Big|_{\omega=1,93} = \angle \frac{2}{j\omega+0,5} \cdot e^{-\tau_m j\omega}\Big|_{\omega=1,93}$$

$$= \angle 2 + \angle e^{-\tau_m j\omega} - \angle j\omega + 0,5\Big|_{\omega=1,93}$$

$$\cancel{= 0 - 3 = 0 - 1,93}\ \omega$$

$$= 0 - 1,93\,\tau_m - \angle j \cdot 1,93 + 0,5$$

$$\angle H_{0d}(j\omega) = -\pi + 2\pi k \Rightarrow \qquad \approx 45^\circ = \frac{\pi}{4} + \frac{\pi}{6}$$

$$\tau_m = \frac{\pi + 2k\pi - \operatorname{arctg}\frac{1,93}{0,5}}{1,93}$$

$$\tau_m = \frac{\pi - \frac{\pi}{4} + \frac{\pi}{6}}{1,93} = \frac{22\pi}{46,32} \simeq 0,94$$

$\Rightarrow$ sistem stabil pentru $\tau_m < 0,94$

1) $\tau_m = 0{,}5$, $k = ?$ a.î. sistemul să rămână stabil

$$H_{des}(s) = \frac{k}{s+0{,}5} \cdot e^{-0{,}5s}$$

$$\left| H_{des}(s) \right|\Big|_{s=j\omega} = 1$$

$$\left| \frac{k}{j\omega+0{,}5} \right| \cdot \underbrace{\left| e^{-0{,}5s} \right|}_{1} = 1$$

$$\cancel{\left| \frac{-k j\omega + k\,0{,}5}{} \right.} \quad \left| \frac{2k}{2j\omega+1} \right| = 1$$

$$\left| \frac{-4j\omega k + 2k}{1+4\omega^2} \right| = 1$$

$$\sqrt{\frac{4k^2 + 16\omega^2 k^2}{(1+4\omega^2)^2}} = 1$$

$$\sqrt{\frac{4k^2}{1+4\omega^2}} = 1$$

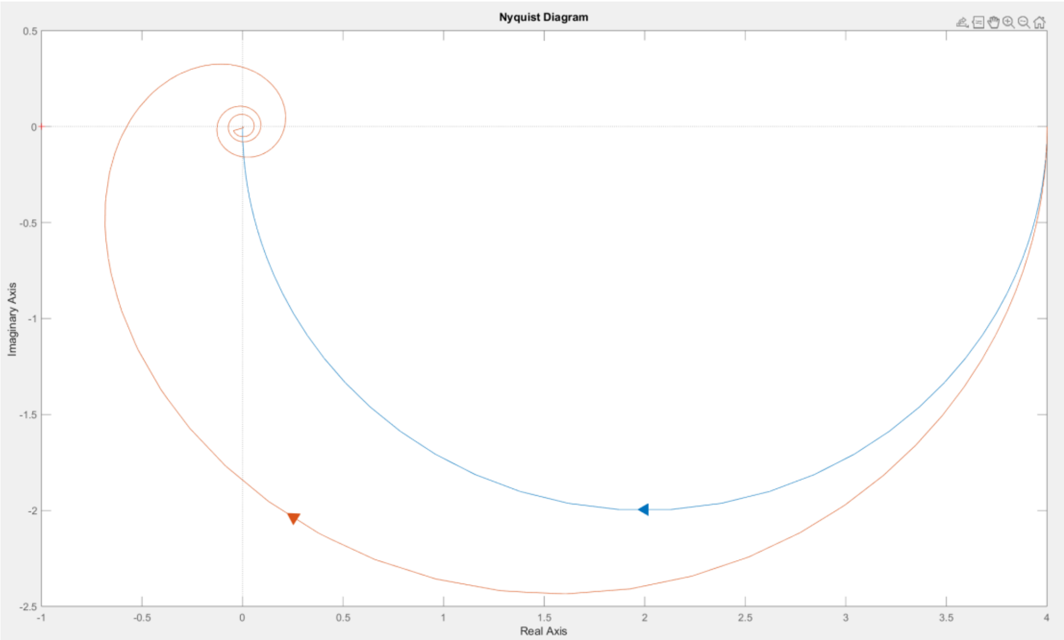
$$\sqrt{4k^2} = \sqrt{1+4\omega^2}$$

$$\cancel{2}k^4 = \sqrt{1+4\omega^2}$$

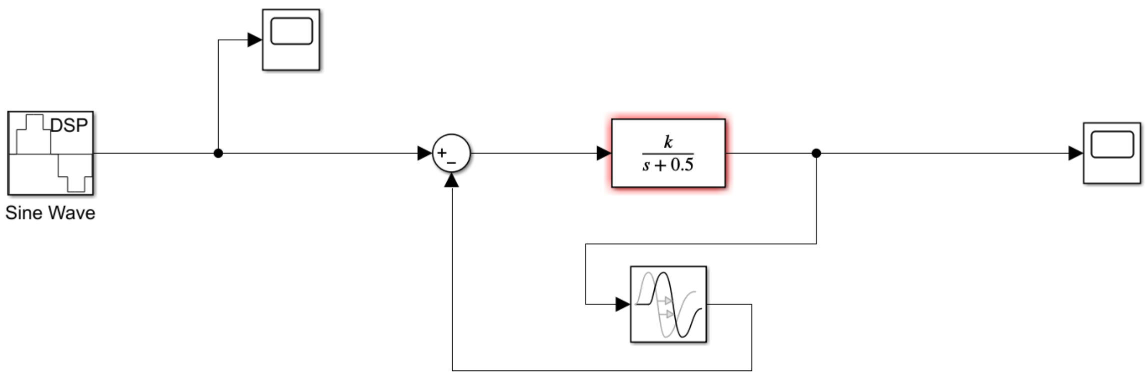
$$k = \cancel{\tfrac{1}{2}+\omega^2} \; \frac{\sqrt{1+4\omega^2}}{2}$$

$\Rightarrow k > 1$ sistem instabil

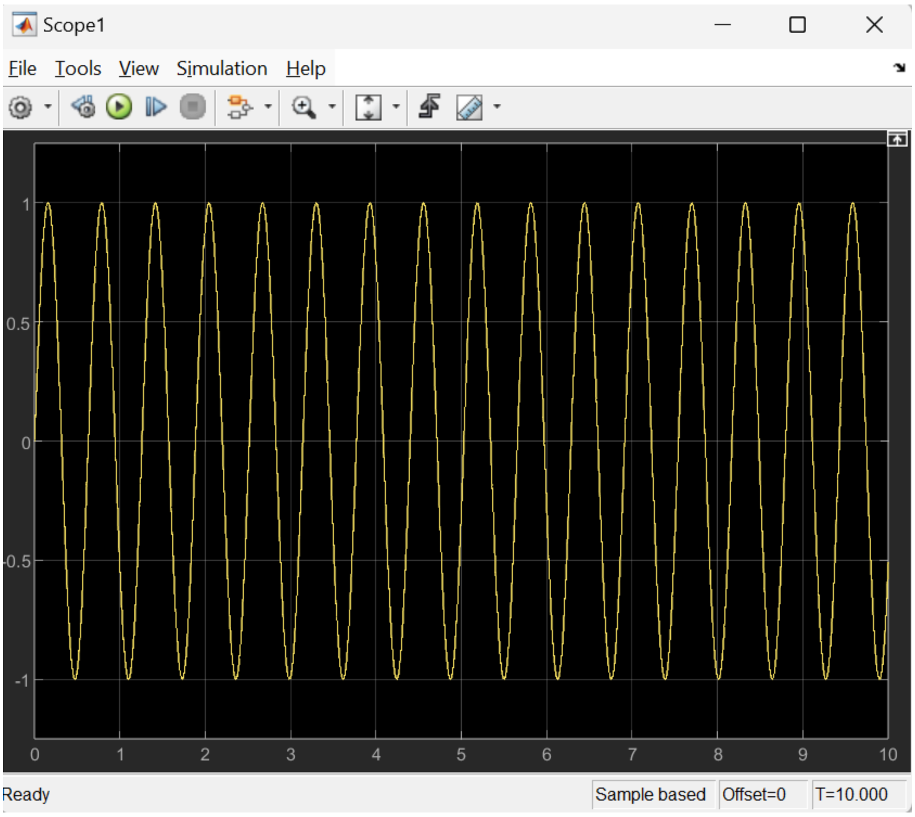
Diagrama Nyquist pentru sistemul cu RN (rosu) si pentru sistemul cu RNU (albastru):



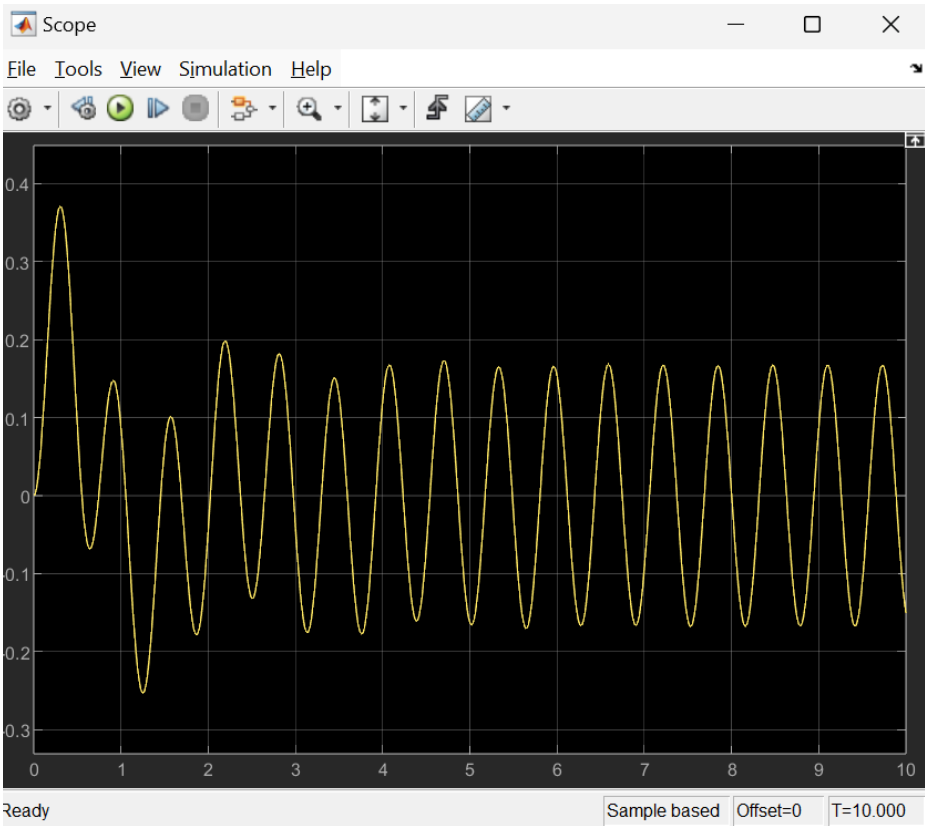
Schema de verificare din Simulink. Am folosit pentru intrare sinusoidala cu ω = 10/2π rad/s blocul „Sine Wave Function", am creat functia de transfer un bucla inchisa folosind un sumator, pe calea directa blocul „Transfer Fcn" unde am introdus functia de transfer de pe calea directa, pe calea de reactie am pus blocul „Transport Delay".



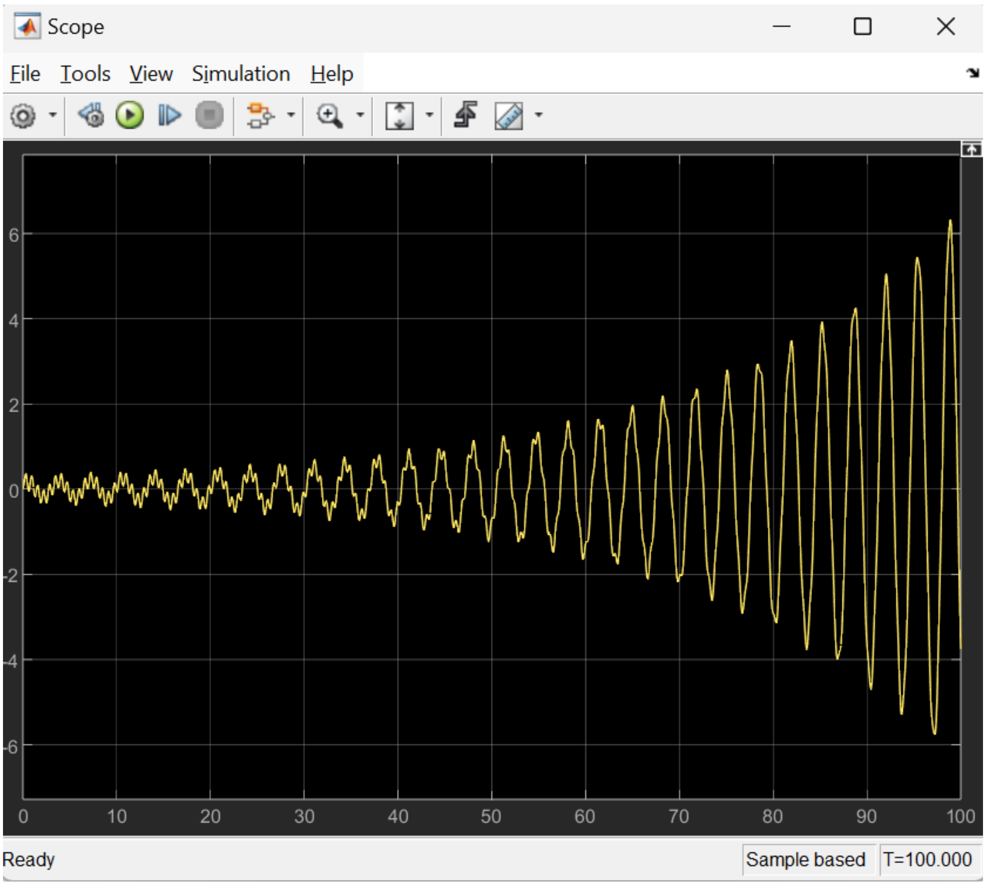
Intrarea:



Iesirea pentru τ_m= 0.5 si k = 2:



Iesirea pentru un τ_m (= 1) in afara domeniului de stabilitate:



Iesirea pentru un k (= 50) in afara domeniului de stabilitate: