Tema curs 2
Olaru Constantin – Alexandru
Grupa 30121

Algoritm pentru determinarea functiei de transfer in forma minimala:

```
clc
clear all
close all

% crearea si verificarea functiei de transfer
num = [1,1,1,1];
den = [1,3,3,1];

if(num == den) H = 1
else H = tf(num,den)

n = length(pole(H));
m = length(zero(H));

if( n ~= m) fprintf("n este diferit de m\n")
else

% calcularea parametrilor Markov
Markov = filter(num,den,[1, zeros(1,n + m - 1)]);

if(length(Markov) <= 3) temp = Markov((n/2)+1:(n+1));
elseif(length(Markov) <= 4) temp = Markov((n/2)+2:(n+2));
else temp = Markov((n/2)+2:(n+3));
end

% crearea matricei Hankel
Hnn = hankel(Markov(2:(n+1)),temp);

% verificarea rangului
if(rank(Hnn) < n)

% calculele pentru noii alfa si beta
a = -inv(Hnn(1:rank(Hnn),1:rank(Hnn))) * temp(1:rank(Hnn)).';

ah = ones(n);
ah(2:n,2) = a;

B = zeros(n);

C = zeros(n);

for i = 1:n
```

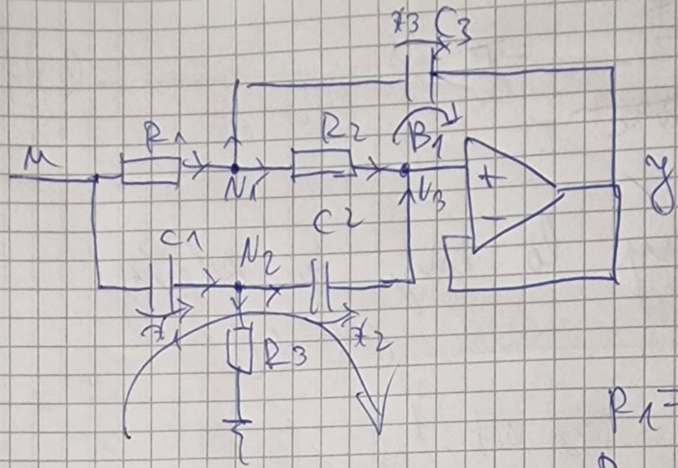
```
    B(i,i) = Markov(1);
    for j = 1:n
    if(i > j) B(i,j) = Markov(i-j+1);
        ah(j,i) = 0;
    end

    end

end

C = ah * B;

C(abs(C) < 1e-10) = 0;

%alfa, beta si functia de transfer in forma minimala
Alfa = ah(n,:);

Beta = C(n,:);

Hmin = tf(Beta,Alfa)

else
    Hmin = H
    fprintf("Hmin = H\n");
end

end

end
```

$$H(s) = \frac{s^3 + s^2 + s + 1}{s^3 + 3s^2 + 3s + 1}$$

```
H =

    s^3 + s^2 + s + 1
  ---------------------
  s^3 + 3 s^2 + 3 s + 1
```

```
Hmin =

     s^2 + 1
  -------------
  s^2 + 2 s + 1
```

Calculul functiei de transfer pentru un FOB in montaj neinversor:



$x_1 = u_{C_1}$

$x_2 = u_{C_2}$

$x_3 = u_{C_3}$

$R_1 = R_2 = R$

$R_3 = R/2$

$C_1 = C_2 = C$

$C_3 = 2C$

$u = x_1 + u_{R_3}$

$u_{R_3} = x_2 + y$

$u = x_1 + x_2 + y$

$y = u - x_1 - x_2$

$$K_I(N_2): \; C_1 \frac{dx_1}{dt} = \frac{u - x_1}{R_3} + C_2 \frac{dx_2}{dt}$$

$$K_I(N_3): \; \frac{x_3}{R_2} + C_2 \frac{dx_2}{dt} = 0$$

$$\Rightarrow \frac{dx_2}{dt} = -\frac{1}{R_2 C_2} x_3$$

$$\Rightarrow \frac{dx_1}{dt} = -\frac{x_1}{C_1 R_3} - \frac{1}{R_2 C_1} x_3 + \frac{1}{R_3 C_1} u$$

$$KCL(N_1): \frac{x_1+x_2-x_3}{R_1} = C_3\frac{dx_3}{dt} + \frac{x_3}{R_2}$$

$$\Rightarrow \frac{dx_3}{dt} = \frac{-1}{C_3}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)x_3 + \frac{u}{C_3R_1}$$

$$\dot{X} = \begin{pmatrix} -\frac{1}{C_1R_3} & 0 & -\frac{1}{R_2C_1} \\ 0 & 0 & -\frac{1}{R_2C_2} \\ \frac{1}{R_1C_3} & \frac{1}{R_1C_3} & -\frac{1}{C_3}\left(\frac{1}{R_1}+\frac{1}{R_2}\right) \end{pmatrix} X + \begin{pmatrix} \frac{1}{R_3C_1} \\ 0 \\ 0 \end{pmatrix} u$$

$$y = \begin{pmatrix} -1 & -1 & 0 \end{pmatrix} X + (1)\cdot u$$

$$H(s) = C\cdot(sI_3-A)^{-1}\cdot B + D$$

$$= \begin{pmatrix} -1 & -1 & 0 \end{pmatrix}\cdot\frac{\left(adj(sI_3-A)\right)}{det(sI_3-A)}\cdot\begin{pmatrix} \frac{2}{R_3C_1} \\ 0 \\ 0 \end{pmatrix} + 1$$

$$\cancel{adj(sI_3-A)} = \begin{pmatrix} \frac{R}{12C^2R^2} & \times & 0 \\ \frac{-1}{12C^2R^2} & \times & 3CR \\ \cancel{0} & \times & 0 \end{pmatrix}$$

$$sI_3 - A = \begin{pmatrix} s+\frac{2}{RC} & 0 & \frac{1}{RC} \\ 0 & s & \frac{1}{RC} \\ -\frac{1}{2RC} & -\frac{1}{2RC} & s+\frac{1}{RC} \end{pmatrix}$$

$$adj(sI_3 - A) = \begin{pmatrix} \Gamma_{11} & X & X \\ \Gamma_{12} & X & X \\ \Gamma_{13} & X & X \end{pmatrix}$$

$$\Gamma_{11} = s\left(s+\frac{1}{RC}\right) + \frac{1}{2R^2C^2} = \frac{2R^2C^2s^2 + 2RCs + 1}{2R^2C^2}$$

$$\Gamma_{12} = \frac{1}{2R^2C^2}$$

$$\Gamma_{13} = -\frac{1}{2RC}$$

$$H(s) = \frac{(-1 \ -1 \ 0) \begin{pmatrix} \frac{2R^2C^2s^2+2RCs+1}{2R^2C^2} & X & X \\ \frac{1}{2R^2C^2} & X & X \\ -\frac{s}{2RC} & X & X \end{pmatrix} \begin{pmatrix} \frac{2}{RC} \\ 0 \\ 0 \end{pmatrix}}{det(sI_3 - A)} + 1$$

$$= \frac{(-1 \ -1 \ 0) \cdot \begin{pmatrix} \frac{2R^2C^2s^2+2RCs+1}{R^3C^3} \\ \frac{1}{R^3C^3} \\ -\frac{s}{R^2C^2} \end{pmatrix}}{det(sI_3 - A)} + 1$$

$$= \frac{-\frac{2R^2C^2s^2+2RCs+1}{R^3C^3} - \frac{1}{R^3C^3}}{\det(sI_3 - A)} + 1$$

$$\det(sI_3 - A) = \left(s+\frac{2}{RC}\right)s\cdot\left(s+\frac{1}{RC}\right) + 0 + 0 + \frac{s}{2R^2C^2} - 0 + \frac{1}{2R^2C^2}\left(s+\frac{2}{RC}\right)$$

$$= \frac{(RCs^2+2s)(RCs+1)}{R^2C^2}\cancel{2RC} + \frac{s}{2R^2C^2}\cancel{RC} \cdot \cancel{\frac{RCs+2}{RC}}$$

$$\cancel{= \frac{2R^3C^3s^3 + 2R^2C^2s^2 + 4R^2C^2s^2 + 4RCs + RCs^2 + 2s}{2R^3C^3}}\,\cancel{RC}$$

$$+ \frac{s}{2R^2C^2} + \frac{RCs+2}{2R^3C^3}$$

$$= \frac{2R^3C^3s^3 + 2R^2C^2s^2 + 4R^2C^2s^2 + 4RCs + RCs + RCs + 2}{2R^3C^3}$$

$$= \frac{R^3C^3s^3 + 3R^2C^2s^2 + 3RCs + 1}{R^3C^3}$$

$$\Rightarrow H(s) = \frac{-2RCs}{R^3C^3s^3+3R^2C^2s^2+3RCs+1}$$

$$H(s) = \frac{-2RCs\,\cancel{(RCs+1)}}{(RCs+1)^{2}} + 1$$

$$= \frac{-2RCs + (RCs+1)^2}{(RCs+1)^2}$$

$$= \frac{R^2C^2s^2 + 1}{R^2C^2s^2 + 2RCs + 1}$$

$$H(s) = \frac{s^2 + \frac{1}{R^2C^2}}{s^2 + \frac{2}{RC}s + \frac{1}{R^2C^2}}$$

$$s^2 + \frac{1}{R^2C^2} \quad \Big| \quad s^2 + \frac{2}{RC}s + \frac{1}{R^2C^2}$$

$$\underbrace{1}_{\gamma_0} \underbrace{-\frac{2}{RC}}_{\gamma_1} s^{-1} \underbrace{+\frac{4}{R^2C^2}}_{\gamma_2} s^{-2} \underbrace{-\frac{6}{R^3C^3}}_{\gamma_3} s^{-3}$$

$$-s^2 - \frac{2}{RC}s - \frac{1}{R^2C^2}$$

$$/ \quad -\frac{2}{RC}s \quad /$$

$$\frac{2}{RC}s + \frac{4}{R^2C^2} + \frac{2}{R^3C^3}s^{-1}$$

$$/ \quad + \frac{4}{R^2C^2} + \frac{2}{R^3C^3}s^{-1}$$

$$-\frac{4}{R^2C^2} - \frac{8}{R^3C^3}s^{-1} - \frac{4}{R^4C^4}s^{-2}$$

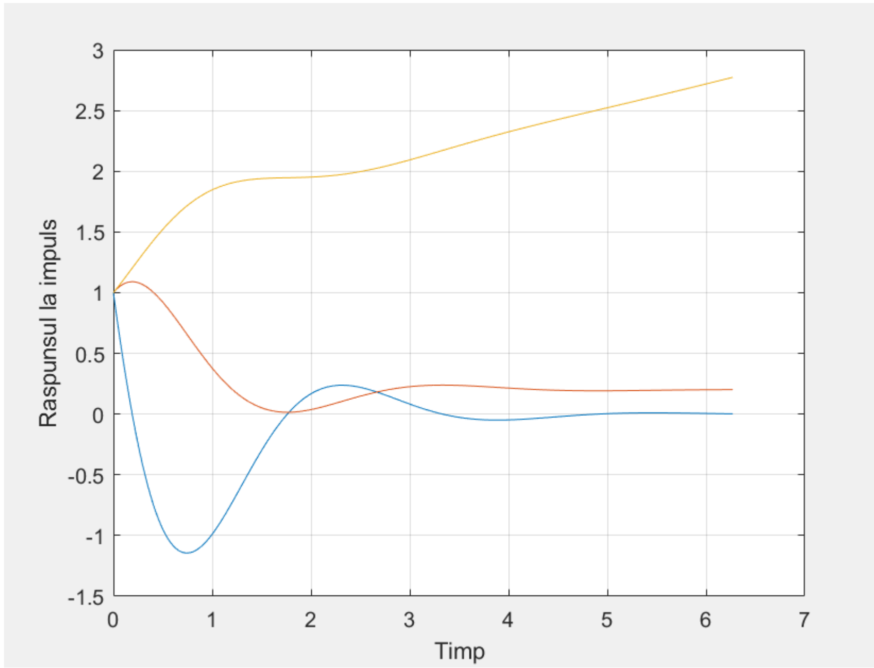
$$\mathcal{H}_{22} = \begin{pmatrix} \gamma_1 & \gamma_2 \\ \gamma_2 & \gamma_3 \end{pmatrix} = \begin{pmatrix} -\frac{2}{RC} & \frac{4}{R^2C^2} \\ \frac{4}{R^2C^2} & -\frac{6}{R^3C^3} \end{pmatrix}$$

$$\text{rang } \mathcal{H}_{22} = 2$$

Raspunsul unui circuit la impuls folosing odexy:

```
imp.m

% functia pentru ode

function h = imp(t,x)
num = [2];
den = [1, 2, 5, 0];
[A, B, C, D] = tf2ss(num, den);

u = 1;

h = A * x + B .* u;

end

clc
clear all
close all

% crearea functiei de transfer
num = [2];
den = [1, 2, 5, 0];
H = tf(num, den);

% crearea spatiului starilor folosind tf2ss
[A,B,C,D] = tf2ss(num,den);

%timpul si conditiile initiale nenule (1 1 1)
t = 0:1/10/pi:2*pi;
CI = [1 1 1];

[t,x] = ode45('imp',t,CI);

plot(t,x); grid on;
xlabel('Timp'); ylabel('Raspunsul la impuls');
```

$$H(s) = \frac{2}{s(s^2 + 2s + 5)}$$



Raspunsul la impuls
Timp
3
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
0
1
2
3
4
5
6
7