Tema curs 5

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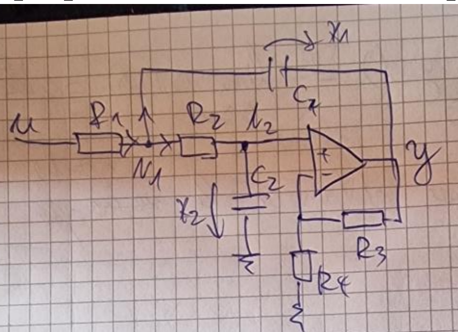
Grupa 30121

Determinarea stabilitatii unui system care are la baza un FTJ cu amplificator operational in montaj neinversor. Am ajuns pana la calculul matricei P, iar functia V(x) este apropiata de o functie care respecta conditiile.



$x_1 = u_{C_1}$

$x_2 = u_{C_2}$

$TK_1(N_1) = \cancel{\frac{u_{R_1} + u_{C_1}}{R_1 \quad C_1}}$

$$i_{R_1} = i_{C_1} + i_{R_2}$$

$$\frac{u - \left(x_1 + \left(1 + \frac{R_4}{R_3}x_2\right)\right)}{R_1} = C_1 \frac{dx_1}{dt} + \frac{\cancel{x_1 - \left(1 + \frac{R_4}{R_3}\right)}}{R_2}$$

$x_1 - \left(x_2 - \left(1 + \frac{R_4}{R_3}\right)\right)$

$$\frac{1}{R_1}u - \frac{1}{R_1}x_1 - \frac{1}{R_1}\left(1 + \frac{R_4}{R_3}\right)x_2 = C_1\frac{dx_1}{dt} + \frac{x_1 + \frac{R_4}{R_3}x_2}{R_2}$$

$$\boxed{\frac{dx_1}{dt} = \frac{1}{C_1}\left(-\frac{1}{R_1} - \frac{1}{R_2}\right)x_1 - \frac{1}{C_1}\left[\frac{1}{R_1}\left(1 + \frac{R_4}{R_3}\right) - \frac{R_4}{R_3 R_2}\right]x_2 + \frac{1}{C_1 R_1}u}$$

$TK_1(N_2) = i_{R_2} = i_{C_2}$

$$\frac{x_1 + \frac{R_4}{R_3}x_2}{R_2} = C_2\frac{dx_2}{dt}$$

$$\boxed{\frac{dx_2}{dt} = \frac{1}{C_2R_2}x_1 + \frac{R_4}{C_2R_3R_2}x_2}$$

$\cancel{P_C(\lambda)}$

$$A = \begin{pmatrix} -\frac{1}{C_1}\left(\frac{1}{R_1}+\frac{1}{R_2}\right) & -\frac{1}{C_1}\left[\frac{1}{R_1}\left(1+\frac{R_4}{R_3}\right)-\frac{R_4}{R_3R_2}\right] \\ \frac{1}{C_2R_2} & \frac{R_4}{C_2R_3R_2} \end{pmatrix}$$

$$P_C(\lambda) = \begin{vmatrix} \lambda+\frac{1}{C_1}\left(\frac{1}{R_1}+\frac{1}{R_2}\right) & \frac{1}{C_1}\left[\frac{1}{R_1}\left(1+\frac{R_4}{R_3}\right)-\frac{R_4}{R_3R_2}\right] \\ -\frac{1}{C_2R_2} & \lambda-\frac{R_4}{C_2R_3R_2} \end{vmatrix}$$

$$= \lambda^2 + \lambda\left(\frac{1}{C_1}\left(\frac{1}{R_1}+\frac{1}{R_2}\right) - \frac{R_4}{C_2R_3R_2}\right) - \frac{R_4}{C_2R_3R_2}\left(\frac{1}{C_1}\left(\frac{1}{R_1}+\frac{1}{R_3}\right)\right)$$

$$+ \frac{1}{C_2R_2C_1}\left[\frac{1}{R_1}\left(1+\frac{R_4}{R_3}\right)-\frac{R_4}{R_3R_2}\right] = 0$$

$$\begin{array}{c|cc}
s^2 & 1 & \frac{1}{C_2R_2C_1}\left[\frac{1}{R_1}\left(1+\frac{R_4}{R_3}\right)-\frac{R_4}{R_3R_2}\right] \\
s^1 & \frac{1}{C_1}\left[\left(\frac{1}{R_1}+\frac{1}{R_2}\right)-\frac{R_4}{R_2R_3C_2}\right] & 0 \\
\hline
s^0 & \frac{1}{C_2R_2C_1}\left[\frac{1}{R_1}\left(1+\frac{R_4}{R_3}\right)-\frac{R_4}{R_3R_2}\right] & 0
\end{array}$$

dacă $R_4 = R_3 \Rightarrow$ ~~sistem~~ nici o schimbare de semn

dacă $R_4 < R_3 \Rightarrow$ nici o schimbare de semn

dacă $R_4 > R_3 \Rightarrow$ o schimbare de semn $\Rightarrow$ sistem instabil

$$E(x) = \frac{C_1}{2}x_1^2 + \frac{C_2}{2}x_2^2 \qquad (u(t)=0)$$

$$\frac{dE}{dt} = C_1 x_1 \dot{x}_1 + C_2 x_2 \dot{x}_2$$

$$= \cancel{C_1} x_1 \left[\left(\frac{1}{\cancel{C_1}}\left(-\frac{1}{R_1}-\frac{1}{R_2}\right)\right)x_1 - \frac{1}{\cancel{C_1}}\left[\frac{1}{R_1}\left(1+\frac{R_4}{R_3}\right)-\frac{R_4}{R_3R_2}\right] x_2\right] + \cancel{C_2} x_2 \left(\frac{1}{\cancel{C_2}R_2}x_1 + \frac{R_4}{\cancel{C_2}R_3R_2}x_2\right)$$

$$= x_1^2\left(-\frac{1}{R_1}-\frac{1}{R_2}\right) - x_1x_2\left[\frac{1}{R_1}\left(1+\frac{R_4}{R_3}\right)-\frac{R_4}{R_3R_2}\right]$$

$$+ x_1x_2\frac{1}{R_2} + x_2^2\frac{R_4}{R_3R_2}$$

$$V(x) = \frac{R_1C_1}{2}x_1^2 + \frac{R_2R_3}{R_4}\cdot\frac{C_2}{2}x_2^2$$

$$\dot{V}(x) = x_1^2\left(-1-\frac{R_1}{R_2}\right) + x_1x_2\left(R_2\frac{R_3}{R_4}+R_2-1\right)$$

$$+ \frac{R_3}{R_4}x_1x_2 + x_2^2 < 0$$

$$V(x) = X^T\cdot P\cdot X = (x_1\ \ x_2)\begin{pmatrix}\frac{R_1C_1}{2} & 0\\ 0 & -\frac{R_2R_3}{R_4}\frac{C_2}{2}\end{pmatrix}\cdot\begin{pmatrix}x_1\\ x_2\end{pmatrix}$$

$= \ldots C_2R_2 P_{12}$

$$\left(\left[-\frac{1}{R_1C_1}\left(1+\frac{R_4}{R_3}\right)+\frac{R_4}{R_3R_2C_1}\right]P_{11}+\frac{R_4}{R_2R_3C_2}P_{12}\right) \times \left[-\frac{1}{R_1C_1}\left(1+\frac{R_4}{R_3}\right)+\frac{R_4}{R_3R_2C_1}\right]$$

$$\times\; P_{12}+\frac{R_4}{R_2R_3C_2}P_{22}\Big) + \left(-\frac{1}{C_1}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)P_{11}+\frac{1}{C_2R_2}P_{12} \times -\frac{1}{C_1}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)P_{12}+\frac{1}{C_2R_2}P_{22}\right)$$

$$\times \left(\left[-\frac{1}{R_1C_1}\left(1+\frac{R_4}{R_3}\right)+\frac{R_4}{R_3R_2C_1}\right]P_{12}+\frac{R_4}{R_2R_3C_2}P_{22}\right)$$

$$\Rightarrow \begin{cases} -\dfrac{2}{C_1}\left(\dfrac{1}{R_1}+\dfrac{1}{R_2}\right)P_{11}+\dfrac{2}{C_2R_2}P_{12}=-1 \\[2ex] \left[-\dfrac{1}{R_1C_1}\left(1+\dfrac{R_4}{R_3}\right)+\dfrac{R_4}{R_3R_2C_1}\right]P_{11}+\dfrac{R_4}{R_2R_3C_2}\cancel{R_{11}}P_{12} - \\[2ex] -\dfrac{1}{C_1}\left(\dfrac{1}{R_1}+\dfrac{1}{R_2}\right)P_{12}+\dfrac{1}{C_2R_2}P_{22}=0 \\[2ex] \left[-\dfrac{2}{R_1C_1}\left(1+\dfrac{R_4}{R_3}\right)+\dfrac{2R_4}{R_3R_2C_1}\right]P_{12}+\dfrac{2R_4}{R_2R_3C_2}P_{22}=-1 \end{cases}$$

$$P_{22}=P_{12}\cdot\left[\frac{R_2\cancel{R_3}C_2}{R_1C_1}\left(\frac{\cancel{R_3}+R_4}{\cancel{R_3}}\right)+1\right]+\frac{R_2R_3C_2}{2R_4}$$

$$P_{11}=-\frac{C_2\cancel{R_2}}{C_1}\left(\frac{R_1+R_2}{R_1\cancel{R_2}}\right)P_{11}+\frac{C_2R_2}{2}$$

$$\left[-\frac{1}{R_1C_1}\left(1+\frac{R_4}{R_3}\right)+\frac{R_4}{R_3R_2C_1}\right]\left[-\frac{C_2}{C_1}\left(\frac{R_1+R_2}{R_1}\right)P_{12}+\frac{C_2}{2}\right.$$

$$+\frac{R_4}{R_2R_3C_2}P_{12}-\frac{1}{C_1}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)P_{12}+$$

$$+2\cdot P_{12}\cdot\left(\frac{R_3+R_4}{R_1C_1}+1\right)+\frac{R_3}{R_4}=0$$

$$\frac{C_2}{R_1C_1^2}\left(1+\frac{R_4}{R_3}\right)\left(1+\frac{R_2}{R_1}\right)P_{12}+\frac{R_4C_2}{R_3R_2C_1^2}\left(1+\frac{R_2}{R_1}\right)P_{12}$$

$$+\frac{C_2R_2}{R_1C_1}\left(1+\frac{R_4}{R_3}\right)+\frac{C_2R_4}{2R_3C_1}+$$

$$+\frac{R_4}{R_2R_3C_2}P_{12}-\frac{R_1+R_2}{C_1R_1R_2}P_{12}+2P_{12}\left(\right.$$

$$\left.\frac{R_3+R_4+R_1C_1}{R_1C_1}\right)+\frac{R_3}{R_4}=0$$

$$P_{12}\left[\frac{C_2}{R_1C_1^2}\left(1+\frac{R_4}{R_3}\right)\left(1+\frac{R_2}{R_1}\right)-\frac{R_4C_2}{R_3R_2C_1^2}\left(1+\frac{R_2}{R_1}\right)\right.$$

$$+\frac{R_4}{R_2R_3C_2}-\frac{1}{C_1}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)+2\cdot\left(\frac{R_3+R_4+R_1C_1}{R_1C_1}\right)$$

$$\left.\right]=-\frac{C_2R_4}{2R_3C_1}-\frac{R_3}{R_4}$$