Assignment 1

Wednesday, October 8, 2025 2:05 PM

A three-phase two-level VSC is connected to a constant DC voltage source on its DC-side, and its AC-side is connected to an infinite bus through an LR filter. The VSC system has the following specifications:

- Grid voltage: 580 V (line-to-line, RMS) $= V_{LL,RMS}$
- Grid frequency: 60 Hz
- DC-link voltage: 1000 V
- Rated power: 100 kVA $= S_{base}$
- Modulation strategy: Sinusoidal PWM
- Control mode: GFL PQ control
- Switching frequency: 3.06 kHz

For all subsequent questions, you may use any design procedure to obtain your controller parameters. Bode plots, root locus, MATLAB SISOTOOL, Nyquist plot, state-space designs are all accepted, but trial-and-error methods are not acceptable.

<u>You do not need to design the outer PQ controller loop</u>: instead, divide your power set-points by grid voltage to obtain inner current controller references.

You can simulate the VSC system in either MATLAB/Simulink, PSCAD, or PLECS. Converter latency and higher-order harmonics can be neglected.

1. [**10 points**] Design the LR filter values for the VSC, using the procedure developed in class. Then use these values in all subsequent questions.

$S_{base} = 100\text{kVA}$ $\quad \omega_0 = 2\cdot\pi\cdot 60 = 376.99$

$V_{LL,RMS} = 580\ V$

$$Z_{base} = \frac{V_{LL,RMS}^2}{S_{base}} = R_{base} = X_{base} = \frac{580^2}{100\times10^3} = 3.364\ \Omega$$

$$L_{base} = \frac{X_{base}}{\omega_0} = \frac{3.364}{\omega_0} = 8.9\ mH$$

- $L \sim 10\% - 20\%$ of L_{base}

$$\Rightarrow L = 0.15\,L_{base} = 0.15(8.9\,mH) = 1.3\,mH \qquad 3e^{-3}$$

- Quality factor: $Q = \frac{\omega_0 L}{R} = \frac{X_L}{R}$ $\Rightarrow$ ideally $Q > 75\%$
- calculate $R = \frac{\omega_0 L}{Q}$

↳ set $Q = \omega_0 = 377$

$\Rightarrow R = L = 1.3\ m\Omega$

2. [**40 points**] Design PI current controller and PLL controller for the VSC, using the net control block diagram models developed in class.
   a. Show your detailed design procedure for current controller (e.g., provide bode plot/root locus plot of your system, any calculations you have done, and/or any code, and <u>explain why you have chosen the final controller parameters</u>) and provide the final PI controller design in the format of $C(s) = k_p + \frac{k_i}{s}$

loop gain: $\ell(s) = \left(\frac{K_p}{L_s}\right)\frac{s+\frac{K_i}{K_p}}{s+R/L} \Rightarrow \ell(s) = \frac{K_p}{L_s}$

Closed-loop transfer function: $\frac{i_d(s)}{i_d^{ref}(s)} = G_i(s) = \frac{1}{\tau_i s+1}$, choose $K_p = \frac{L}{\tau_i}$ (proportional gain) & $K_i = \frac{R}{\tau_i}$ (integral gain)

- Choose τ_i so its small for fast current controller response, but large enough s.t. $\frac{1}{\tau_i}$ (B.W) is about 10× smaller than switching frequency (f_{sw} = 3.06 kHz)

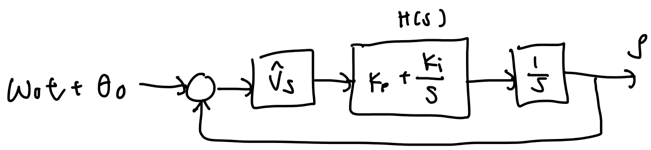
$\Rightarrow \tau_i = 3\ ms$ & $\frac{1}{\tau_i} \sim 0.3\ kHz$

$$K_p = \frac{L}{\tau_i} = \frac{1.3\ mH}{3\ ms} = 0.43\ [V/A]$$

$$K_i = \frac{R}{\tau_i} = \frac{1.3\ \Omega}{3\ ms} = 0.43\ [V/A]$$

$$C(s) = 0.43 + \frac{0.43}{s}$$

b. Show your detailed design procedure for PLL controller (e.g., provide bode plot/root locus plot of your system, any calculations you have done, and/or any code, and <u>explain why you have chosen the final controller parameters</u>) and provide the final PI controller design in the format of $H(s) = k_{pPLL} + \frac{k_{iPLL}}{s}$



- Goal: regulate V_{sq} at zero
  - $\omega(t) = H(s) V_{sq}(t)$ [rad/s)/V]
  - $\frac{d\rho}{dt} = H(s)\hat{V}_s \sin(\omega_0 t + \theta_0 - \rho)$
  - $\Delta\omega = \frac{d\rho}{dt} = H(s)\hat{V}_s(\omega_0 t + \theta_0 - \rho)$

$$\hat{V}_s = \left(\frac{580V}{\sqrt{3}}\right)(\sqrt{2}) = 473.6\ V$$

- Since linearizing around steady state ω_0, $\omega(0) = \omega_0$ & $\omega_{min} \le \omega \le \omega_{max}$
- $[\omega_{min}, \omega_{max}] \sim [55, 65\ Hz]$ → close to ω_0 but not too close
- closed-loop transfer function: $\frac{\rho}{\rho_{ref}} = \frac{G(s)}{1+G(s)}$ where $G(s) = \hat{V}_s\left(K_p + \frac{K_i}{s}\right)\left(\frac{1}{s}\right)$

$$\frac{\rho}{\rho_{ref}} = \frac{\hat{V}_s\left(K_p + \frac{K_i}{s}\right)\left(\frac{1}{s}\right)}{1 + \hat{V}_s\left(K_p + \frac{K_i}{s}\right)\left(\frac{1}{s}\right)} = \frac{\hat{V}_s K_p s + \hat{V}_s K_i}{s^2 + \hat{V}_s K_p s + \hat{V}_s K_i}$$

Denominator: $s^2 + \hat{V}_s K_p s + \hat{V}_s K_i$

$$\omega_n^2 = \hat{V}_s K_i \Rightarrow \omega_n = \sqrt{\hat{V}_s K_i} \Rightarrow K_i = \frac{\omega_n^2}{\hat{V}_s}$$

damping ratio: $\zeta = \frac{V_s K_p}{2\sqrt{\hat{V}_s K_i}}$ $\quad K_p = \frac{\zeta 2\sqrt{\hat{V}_s K_i}}{V_s}$

choosing:

{ set $BW = 20\ Hz = 2\cdot\pi\cdot 20\ Hz = 125.66$ rad/s

$$BW = \omega_n\left[1 + 2\zeta^2 + \sqrt{(1+2\zeta^2)^2 + 1}\right)^{1/2}$$

$$\Rightarrow \omega_n = BW \cdot \left(1 + 2\zeta^2 + \sqrt{(1+2\zeta^2)^2 + 1}\right)^{-1/2}$$

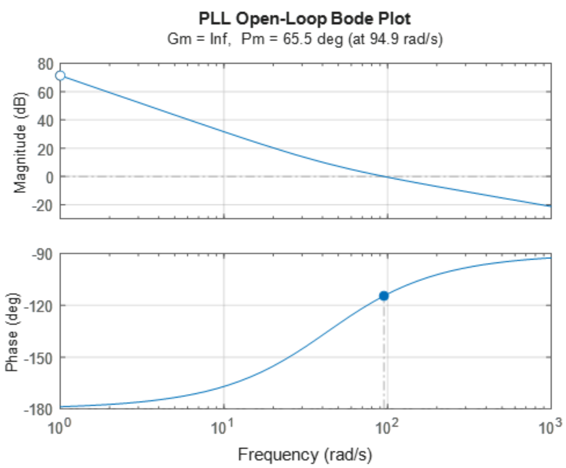
$\omega_n = 61.1$ rad/s

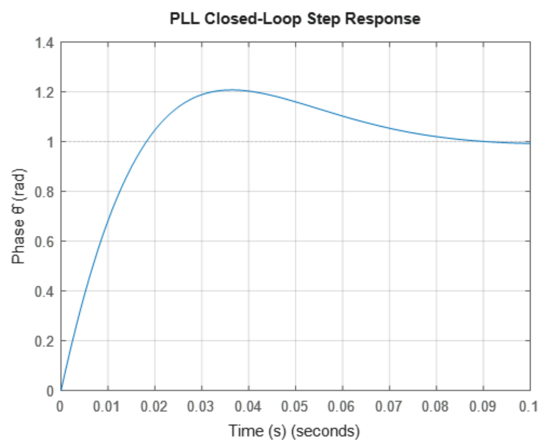
choosing: $\zeta = 0.707$

$$\Rightarrow K_i = \frac{\omega_n^2}{\hat{V}_s} = \frac{(61.1)^2}{473.6} = 7.88$$

$$\Rightarrow K_p = \frac{(0.707)(2)\left(\sqrt{(473.6)(7.88)}\right)}{473.6} = 0.182$$

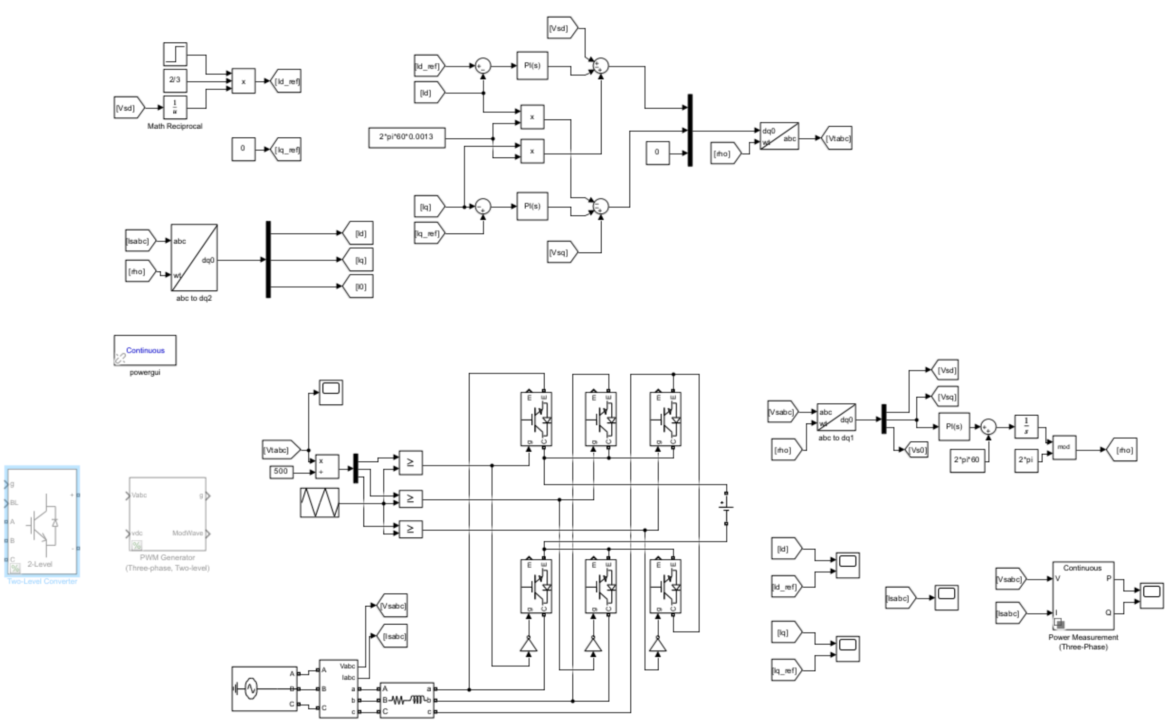
Final PI control design: $H(s) = K_{P,PLL} + \frac{K_{i,PLL}}{s} = 0.182 + \frac{7.88}{s}$





c. Simulate the GFL VSC with your controllers: In the beginning, set both P^{ref} and Q^{ref} to 0, and subsequently, step up P^{ref} to 100 kW. Plot the following and briefly comment on the results (make sure you plot the waveforms for a total of 6 cycles, i.e., 2 cycles before step change and 4 cycles after step change)

i. i_d and i_d^{ref} on the same plot
ii. i_q and i_q^{ref} on the same plot
iii. 3-phase AC currents on the same plot
iv. Measured real and reactive powers at the PCC on the same plot



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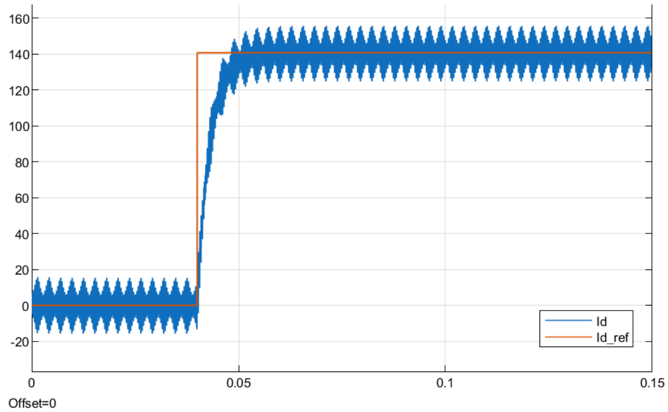
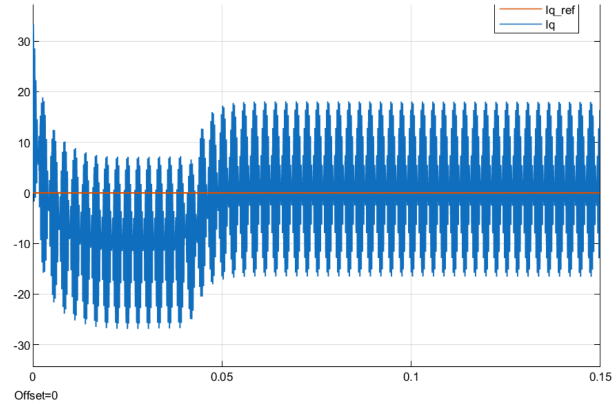
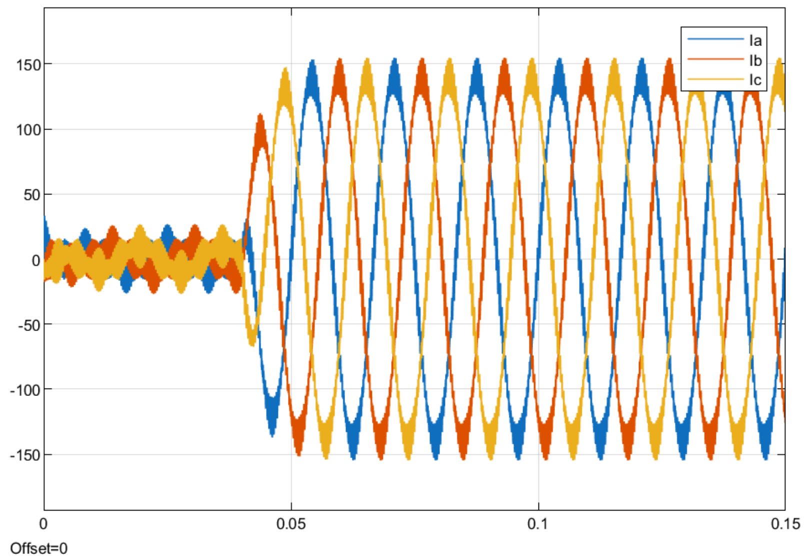
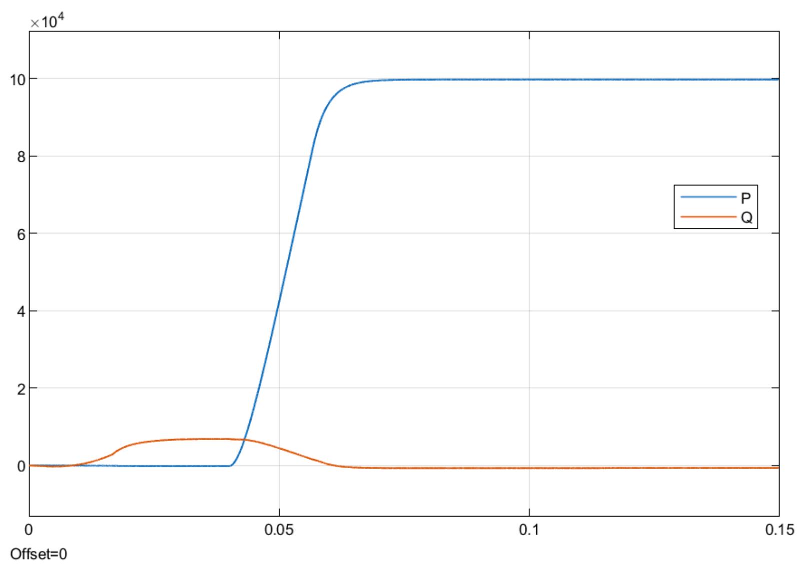








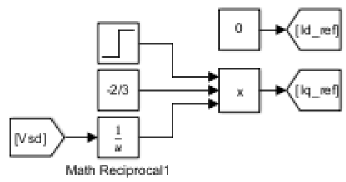
i. Id shows that it quickly catches up to Idref after the disturbance after about 0.01s. There is noise in Id, but the measurements are stable around the new Idref.

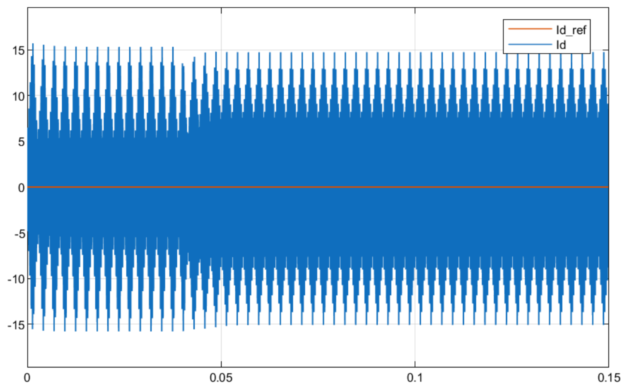
ii. Before the disturbance, Iq averages a value slightly below Iqref (which is zero). After the disturbance, Iq seems to follow the reference better at zero. The fact that Iq seems to be affected by the real power step up shows that Iq and Id are not completely decoupled- however, it does not seem to affect the accuracy of Iq.

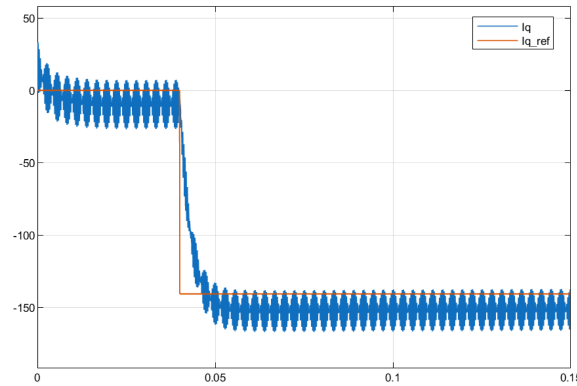
iii. The three phase abc currents increase in amplitude after the step up of the real power.

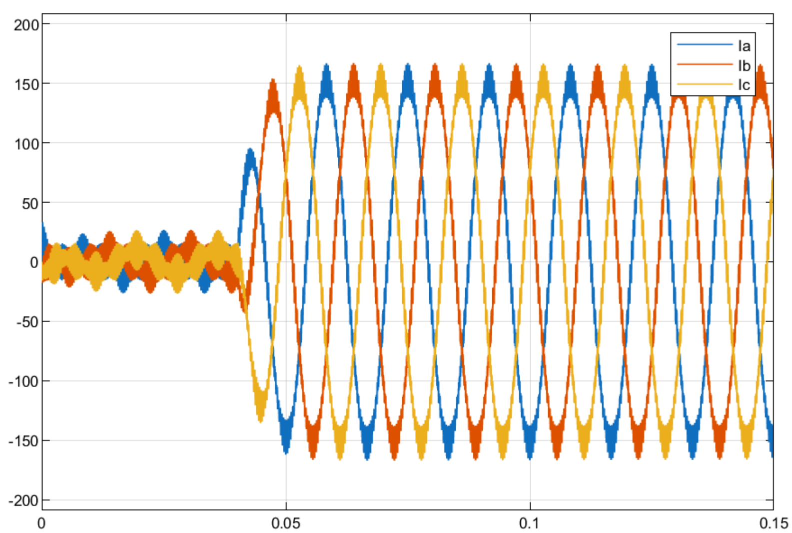
iv. The real power reflects the step up that we are inducing at 0.04s. The Q value stays at zero after the real power steps up, as reactive power should be unaffected.

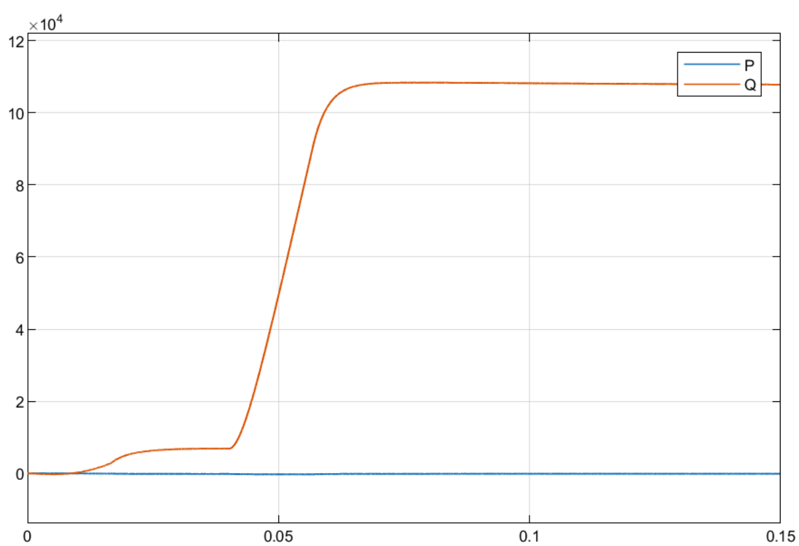
d. Repeat part (c) with a step change of 100 kVAr in reactive power exchange











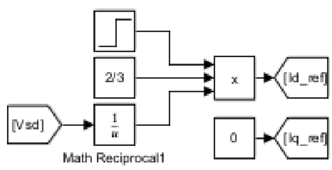
i. Id does not seem to be affected by the disturbance, meaning Id and Iq are well decoupled. Id maintains an average around 0, following Idref.

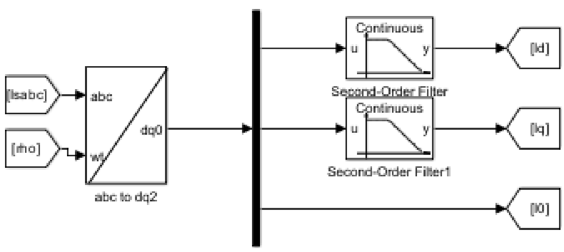
ii. Iqref sees a step down from the increase in reactive power Q. Iq follows Iqref, except that the average of Iq is slightly lower than Iqref.
iii. The three phase abc currents increase in amplitude after the step up of the reactive power.
iv. The reactive power reflects the step up that we are inducing at 0.04s. The P value stays at zero after the reactive power steps up, as real power should be unaffected.

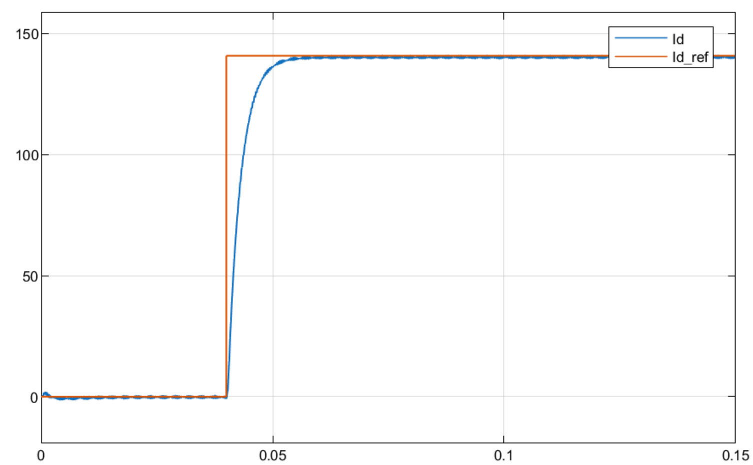
3. [**40 points**] Repeat question (2) by placing second order low-pass filters on the i_{dq} measurements. The filter has a unity gain, $\omega_n = 5000\ rad/s$, $\zeta = 0.8$, with an expression of

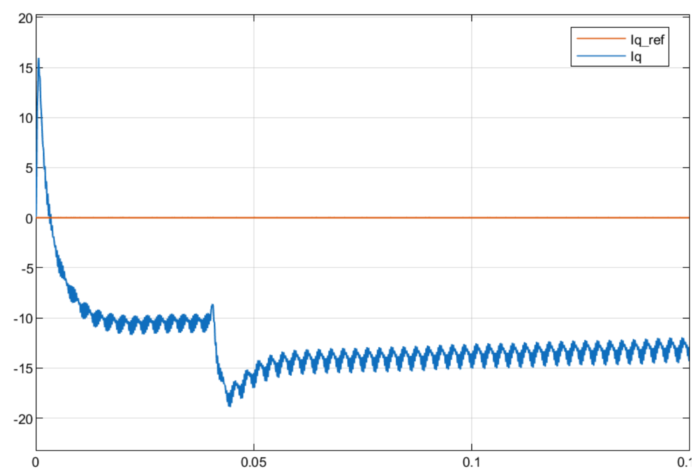
$$L(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

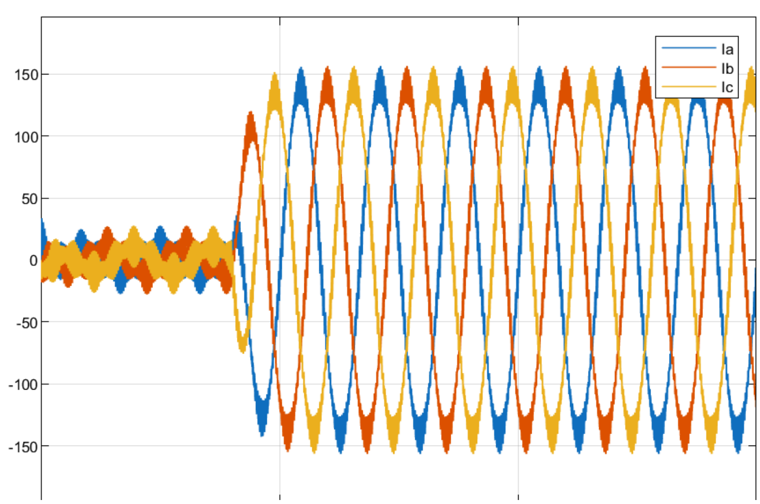
Keep in mind that there is no need to (i) re-design LR filter and (ii) re-design PLL controller again for this question.

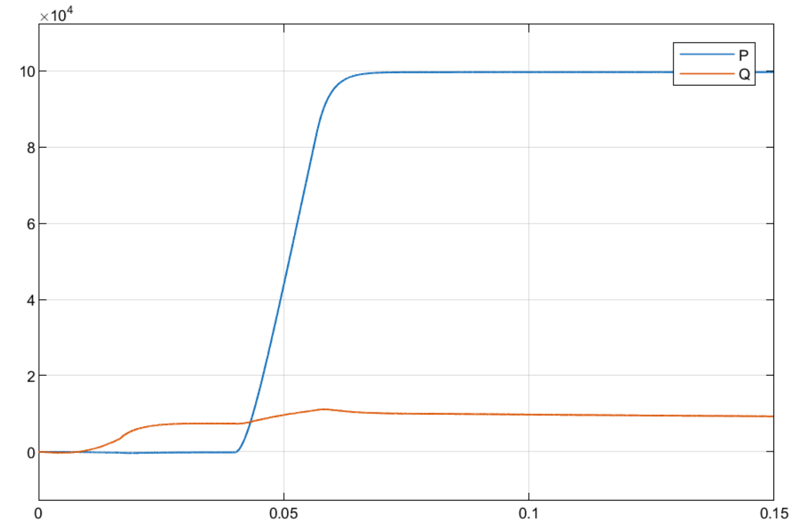




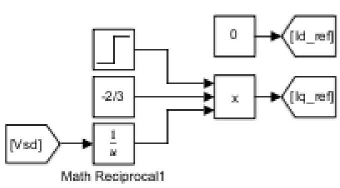


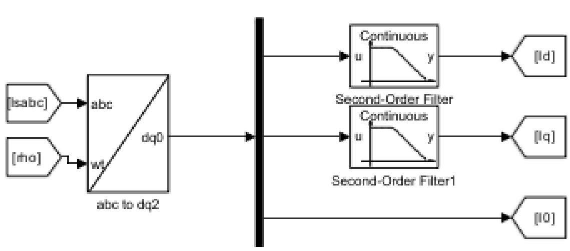


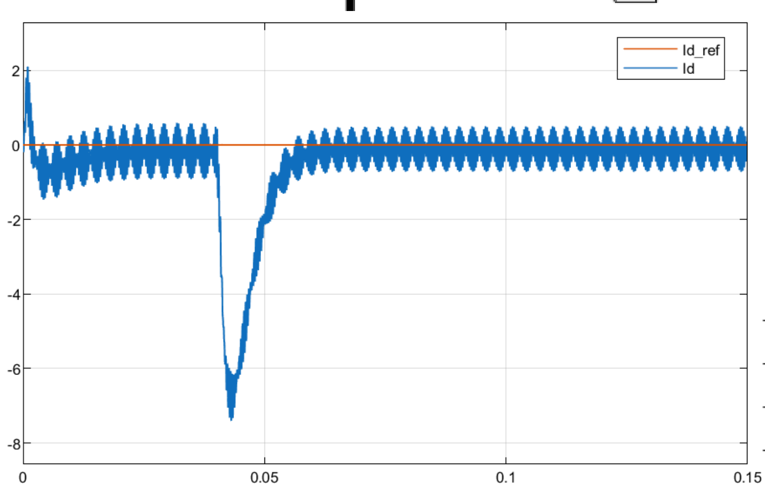


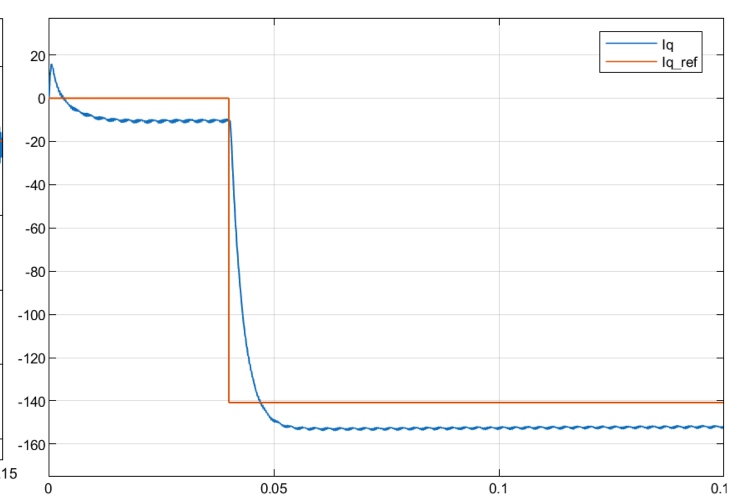


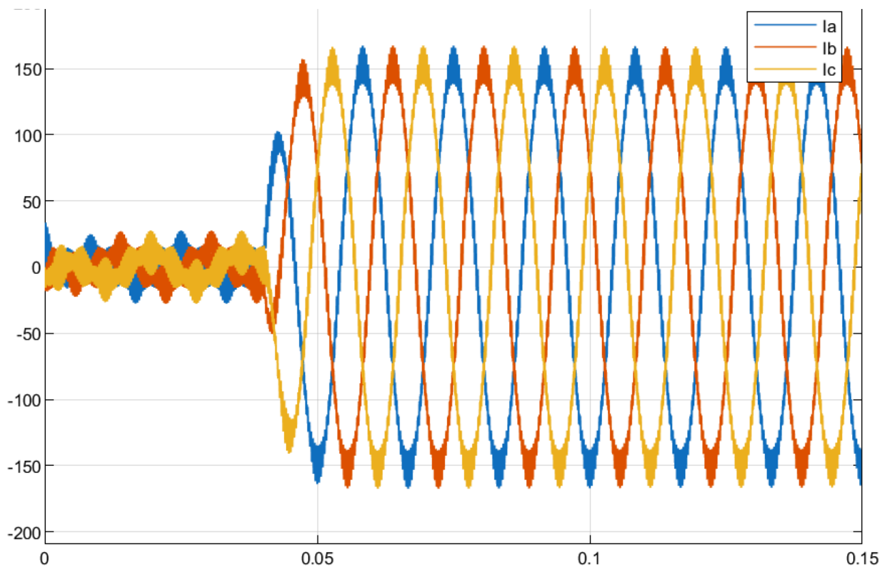
i. Id follows Idref well and the addition of the low-pass filter smooths out the noise in Id well.
ii. With the addition of the low-pass filter, Iq does not follow Iqref as well and is slightly lower than Iqref. However, it is constant after the disturbance and with less noise than without the filter.
iii. There is not any noticeable difference in Iabc after adding the low-pass filter. This makes sense as Id and Iq are filtered, not Iabc.
iv. The real power reflects the step up that we are inducing at 0.04s, however, with the addition of the filter, the reactive power Q increases above 0 and stays constant after the disturbance. This could indicate some filter delay that is causing incorrect decoupling between Id and Iq.

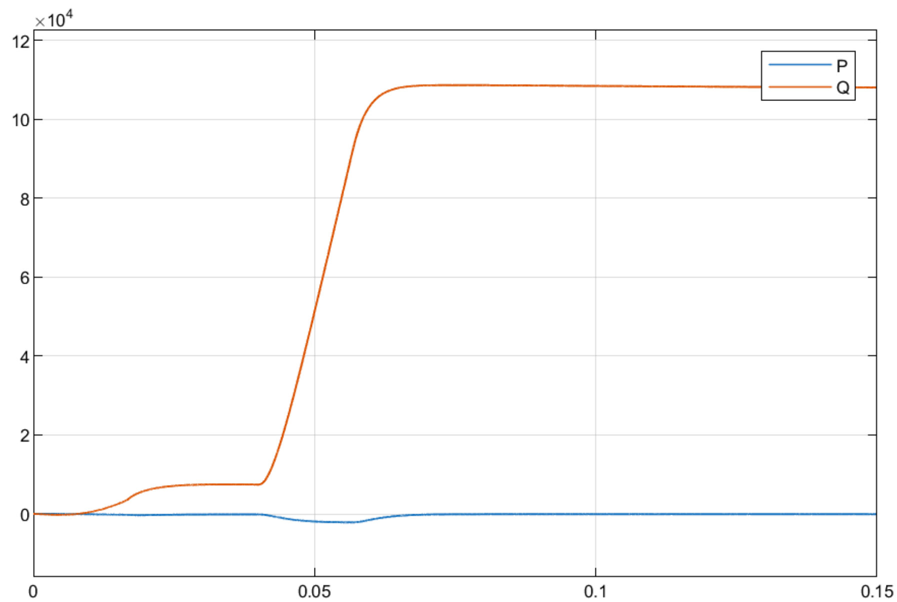












i. Id follows Idref before and after the step up in P, but there is a noticeable dip when the step up occurs. This indicates incorrect decoupling in the current controller.
ii. With the addition of the low-pass filter, Iq is much less noisy, however, the value of Iq is slighly off from Iqref.
iii. There is not any noticeable difference in Iabc after adding the low-pass filter. This makes sense as Id and Iq are filtered, not Iabc.
iv. The rective power reflects the step up that we are inducing at 0.04s and the real power maintains a constant zero.

4. **[10 points]** For the 3-phase system shown in class (gnd—V_t—L—R—V_s—gnd), we showed that $[V_{s\alpha\beta0}] = R[i_{\alpha\beta0}] + L\frac{d}{dt}[i_{\alpha\beta0}] + [V_{t\alpha\beta0}]$, where R and L are scalars. How, if at all, does this formulation change if V_s is unbalanced?

If V_s is unbalanced, negative sequence and possibly zero sequence exist.

However, $\begin{bmatrix} V_\alpha \\ V_\beta \\ V_0 \end{bmatrix} = [C] \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$ is still maintained, it's just that V_0 has a nonzero value.

so in a balanced system, $[V_{s\alpha\beta 0}] = [V_{s\alpha\beta}]$, but in the unbalanced system,

$[V_{s\alpha\beta 0}] \neq [V_{s\alpha\beta}]$ and so we cannot get rid of the 0 term.

$$V_{s\alpha} = Ri_\alpha + Li_\alpha + V_{t\alpha}$$

$$V_{s\beta} = Ri_\beta + Li_\beta + V_{t\beta}$$

$$V_{s0} = Ri_0 + Li_0 + V_t$$