Path Finding Star Power 🤩

An Analysis of the A^* Algorithm

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Table of Contents 🖋️

Object of Study 📚

Definition: δ Graph

A δ **graph** is a directed, weighted graph where the cost of each edge is is strictly greater than δ where δ is a positive real number.

The Problem at Hand 🤔

Give a δ graph G, we would like to find a *minimum-cost* path from some starting vertex to a *preferred goal vertex* of s.

Definition: Preferred Goal Vertex

Given some vertex s and a set of goal vertices T in a graph G, a **preferred goal vertex** of s is a vertex t in T such that the cost of a path from s to t is minimal.

Introducing... A^* ⭐

Definition: Admissibility

We say an algorithm is **admissible** if it is guaranteed to return a minimum-cost path for *any* δ graph.

The $A*$ algorithm is the **most optimal** algorithm for solving the minimum-cost paths problem in the sense that it expands the least amount of vertices possible (given the information it has).

A Key Ingredient for an Optimal Algorithm 🧑‍🍳

A^* would like to expand the least amount of vertices possible.

Helpful Tool: The Evaluation Function

The **evaluation function** $\hat{f}$ helps $A*$ decide which vertex to expand next. For a vertex v,

$$\hat{f}(v) = \hat{h}(v) + \hat{g}(v)$$

Where $\hat{g}(v)$ is the cost of the path from the starting vertex s to v with the minimum cost found so far and $\hat{h}(v)$ is a *heuristic*.

Wait, What's a Heuristic? 😵‍💫

A **heuristic** is a function that *estimates* a particular quantity. In $A*$, our heuristic will often use information from the problem domain. $\hat{h}$ must satisfy two constraints for all vertices v and w:

- $\hat{h}(v) \leq h(v)$
- $\hat{h}(v) \leq \hat{h}(v, w) + \hat{h}(w)$ (consistency)

Some example choices of $\hat{h}$:

- $\hat{h}(v) = 0$ for all v
- "Taylor Swift Distance"

The *Star* of the Show 🤸

define `A*`(s, T)

Mark s as "open" and calculate $\hat{f}(s)$

let *pred* $= []$

while True

let $n =$ "open" vertex whose value of $\hat{f}$ is minimal. Resolve ties arbitrarily but always in favor of a vertices in T.

Mark n as "closed".

if $n \in T$

Reconstruct an optimal path using *pred* starting with n and terminate.

let *succs* $= \Gamma(n)$

for $v \in$ *succs*

pred$[v] = n$

Calculate $\hat{f}(v)$.

if v is not marked "closed" or $\hat{f}$(v) is smaller than before

Mark $v[0]$ as "open".

Let's see some examples of A^* in action! 💃

Thank you! ❤️
Questions?

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fin.