MA 354: Data Analysis I – Fall 2019 Homework 1:

Complete the following opportunities to use what we've talked about in class. These questions will be graded for correctness, communication and succinctness. Ensure you show your work and explain your logic in a legible and refined submission.

You can complete questions 1-2 now. Split into groups of two and each group will complete the following to do list – you can split into groups and select the question to work on naturally or you can ask R to do it for you as follows. (I often use this functionality to choose where to eat or what movie to watch).

```
> #Denote Group 1
> sample(x = c("Student1", "Student2", "Student3", "Student4"), size=2, replace=F)

[1] "Student1" "Student2"
> #Denotes which question group 1 will do
> sample(x=c(1,2), size=1)

[1] 1
```

- Stage 1 each student in the subgroup will provide a solution
- Stage 2 the subgroup will compare solutions and choose the best aspects of each toward having a better, combined solution.
- Stage 3 each student will read and edit the solution sequentially
- Stage 4 the subgroups will swap solutions for constructive feedback and suggestions
- Stage 5 the subgroups will edit their solutions to address all constructive feedback
- Stage 6 each student will check both solutions for completeness, making any necessary adjustments sequentially until both solutions are agreed upon.

For subsequent questions, students will do the same thing keeping the same subgroups or swapping if desired. While students have assigned jobs for specified questions, I encourage students to help with all parts of each question in collaboration with other students in the group.

- 0. Complete weekly diagnostics.
- 1. (Data Cleaning and Summary) In recent work, psychology researchers investigated how subjective socioeconomic status affects how participants experience negative and positive emotions and whether or not the effect is different by race.

The researchers recruited participants via Amazon MTurk Prime Panels to complete the study. Amazon provides all of the data separated by race, and a separate file denoting which observations yield a representative sample based on age, education, and other control variables. Below, you will work to combine these datasets into one workable file you can use to summarize the data.

- (a) Load Data. There are over 100 columns, so we don't print previews of the data like we did previously.
 - > ###Download data
 - > dat.white<-read.csv("https://cipolli.com/students/data/WhiteParticipants.txt",
 - + header=T, sep=",")
 - > dat.black<-read.csv("https://cipolli.com/students/data/BlackParticipants.txt",</pre>
 - + header=T, sep=",")
 - > dat.matched<-read.csv("https://cipolli.com/students/data/matchedsample.txt",
 - + header=T, sep=",")
 - > colnames(dat.matched)<-c("aid","id","MatchedSample")</pre>
- (b) Remove non-white observations from dat.white; i.e., remove observations from dat.white where Race does not equal 1.
- (c) Create an object called dat.wb that contains all of the observations; i.e., merge the data loaded from WhiteParticipants.csv and BlackParticipants.csv.
- (d) The aid of observations that make up a representative sample have a value of 1; i.e., observation i is in the representative sample if dat.matched\$MatchedSample[i] is 1. Remove observations that are not part of the representative sample from
- (e) Amazon flagged 5 users as a false match for the demographic the researchers were measuring. Remove these users from dat.wb; the aid of each user is listed below.
 - 5c9963bd-8f12-6d35-bafc-cdeee22209ed
 - 5c996fcc-1f89-13c3-50e7-3f7c554aad3c
 - 5c9a3bd8-1cbc-854b-a193-620f5eaee500
 - 5cbcaf2e-3233-62d3-7aa3-ad1c9f4ab5a0
 - \bullet 5c9a68ff-1fd6-4ff1-6bf5-b8a1504075b8
- (f) Create a variable called LadderDiff by subracting the column labeled "LadderSelf" from the column labeled "LadderGroup."
- (g) Create a variable called Posemo by averaging the responses in columns labeled: "Amused", "Awe", "Grateful", "Hopeful", "Inspired", "Interested", "Joy", "Love", "Proud", "Serene."
- (h) Plot LadderDiff versus PosEmo for White participants and for Black participants. Comment on differences in the plot.

Solution:

Part B:

- > #remove obs where dat.white does not equal 1
- > library(dplyr) #download library for data manipulation
- > dat.white1 = data.frame(filter(dat.white, Race==1)) #create a new dataframe where Race != 1
- > dat.black1 = data.frame(filter(dat.black, Race==3)) #create a new dataframe where Race != 0

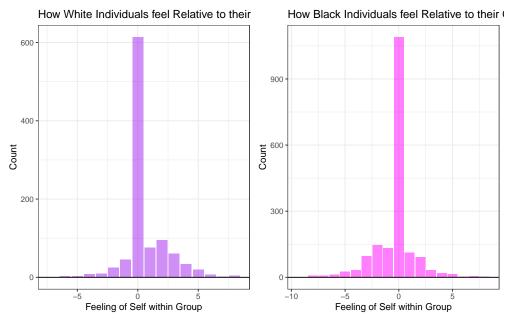
In this failed attempt, we tried to use rm to remove the specified observations, however we got the error that rm "must contain names or character strings"

```
> removeobj = c(NA) #initialize removing the object
> for (i in length(dat.white$Race)) { #loop through each variable in dat.white, column Race
   if (dat.white$Race[i]!=0) { #check if Race doesn't equal 0
       rm(dat.white$Race[i]) #remove if Race doesn't equal 0
+ }
Part C:
> #merge data of white and black participants into new object
> dat.wb <- rbind(dat.white1,dat.black1)</pre>
> #Other attempts
> # dat.wb <- c(dat.white, dat.black, dat.matched) doesn't combine variables
> # column.names <- colnames(dat.white)</pre>
> # dat.wb <- merge(dat.white, dat.black, by = column.names) this merges horizontally- we want vert
> # https://www.statmethods.net/management/merging.html
> #remove observations not in the representative sample
> dat.matched1 = data.frame(filter(dat.matched, MatchedSample==1)) #representative sample
> #Other attempts
> # originally used aid as column to filter on- needed MatchedSample
Part E:
> #remove flagged users as a false match for demographics, filter them out
> dat.wb1 = data.frame(filter(dat.wb,
                             aid!="5c9963bd-8f12-6d35-bafc-cdeee22209ed" &
                             aid!="5c996fcc-1f89-13c3-50e7-3f7c554aad3c" &
                             aid!="5c9a3bd8-1cbc-854b-a193-620f5eaee500" &
                             aid!="5cbcaf2e-3233-62d3-7aa3-ad1c9f4ab5a0" &
                             aid!="5c9a68ff-1fd6-4ff1-6bf5-b8a1504075b8"))
> #Other attempts
> # originally used && when & is correct
Part F:
> #create new variable subtracting LadderSelf from LadderGroup
> dat.wb1$LadderGroup <- as.numeric(dat.wb1$LadderGroup) #changes the var from factor to numeric
> dat.wb1$LadderSelf <- as.numeric(dat.wb1$LadderSelf) #changes the var from factor to numeric
> dat.wb1$LadderDiff <- dat.wb1$LadderGroup - dat.wb1$LadderSelf</pre>
> #Other attempts
> #'-' not meaningful for factors error
> # dat.wb$LadderDiff <- (dat.wb$LadderGroup - dat.wb$LadderSelf) needs to be numeric
Part G:
> #adds all of the values and divides by the number of values w/out the mean function
> dat.wb1$PosEmo <- (as.numeric(dat.wb1$Amused) + as.numeric(dat.wb1$Awe) + as.numeric(dat.wb1$Grat
> #Other attempts
> #create a variable called PosEmo
```

```
> #dat.wb1$PosEmo <- mean("Amused", "Awe", "Grateful", "Hopeful", "Inspired", "Interested",
                            "Joy", "Love", "Proud", "Serene") #didn't account for the column names pr
> #
> # dat.wb$PosEmo <- mean(dat.wb$Amused, dat.wb$Awe, dat.wb$Grateful, dat.wb$Hopeful, dat.wb$Inspin
> #R didn't seem to like when we used the mean function
> #dat.wb1$PosEmo <- mean(as.numeric(dat.wb1$Amused), as.numeric(dat.wb1$Awe), as.numeri(dat.wb1$Gu
> #oops this was inefficient, changing emotions to numerics from factors individually
> # dat.wb1$Amused <- as.numeric(dat.wb1$Amused)</pre>
> # dat.wb1$Awe <- as.numeric(dat.wb1$Awe)</pre>
> # dat.wb1$Grateful <- as.numeric(dat.wb1$Grateful)</pre>
> # dat.wb1$Hopeful <- as.numeric(dat.wb1$Hopeful)</pre>
> # dat.wb1$Inspired <- as.numeric(dat.wb1$Inspired)</pre>
> # dat.wb1$Interested <- as.numeric(dat.wb1$Interested)</pre>
> # dat.wb1$Joy <- as.numeric(dat.wb1$Joy)</pre>
> # dat.wb1$Love <- as.numeric(dat.wb1$Love)</pre>
> # dat.wb1$Proud <- as.numeric(dat.wb1$Proud)</pre>
> # dat.wb1$Serene <- as.numeric(dat.wb1$Serene)</pre>
```

Part H:

- > #plot LadderDiff v. PosEmo for white and black participants
- > #download necessary packages
- > library(ggplot2)
- > library(graphics)
- > library(gridExtra)



Figures 1 and 2: How Individuals Perceive Themselves Relative to their Group by Race

Ladder Difference in this data set is defined as the difference between an individual's perceived status of their group and the same individual's perceived status of themselves as an individual. By finding the difference between these two values, we are able to gauge how an individual feels relative to their group. While the predominant individual perception is that the individual perceives themselves to be an average member of their own group, there is slight variation between white and black individuals.

White individuals in the sample have more observations that are positive than negative, meaning that white individuals perceive themselves more often to be lower on the social ladder compared to their group. Black individuals in the sample, however, have more negative observations relative to white individuals. This shows that black individuals more often view themselves as being better than their social group.

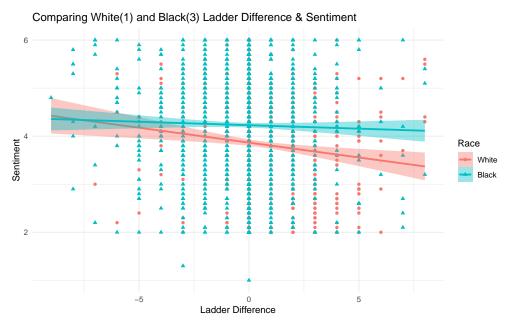


Figure 3: Difference in General Sentiment by Difference in Perceived Individual and Group Status and Race

This graph compares the Black and White sample in their individual sentiments and their Ladder Differences. As stated before, Ladder Difference is how individuals gauge themselves relative to their group, with positive values indicating individuals perceive themselves to be lower on the social ladder relative to their group and the opposite for negative values. Sentiment can be seen as an individual's emotions when taking the survey. Greater values indicate more positive emotions. As we can see from the graph, white individuals have a downward trend, indicating that positive emotions while taking the survey are correlated with individuals perceiving themselves to be higher on the social ladder relative to their group. Black individuals have a horizontal trend, indicating that individual feelings were the same when taking the survey regardless of how a Black person perceived themselves relative to their group. It's also important to note that Black individuals, on average, had a higher sentiment score relative to their white peers for any ladder difference greater than -7.5.

Frequencies of Positive Emotion Across White and Black Races

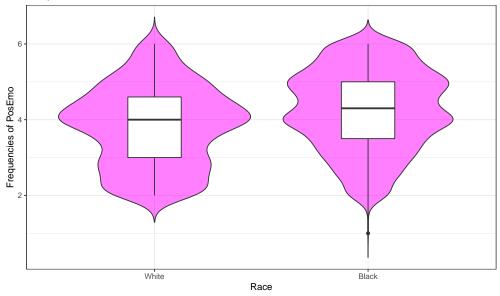


Figure 4: Comparing Individual Sentiment Across Race

This plot shows us the difference in distribution of positive emotions among white individuals and black individuals, respectively. We used a violin plot because positive emotion, in this case, is a continuous variable which is the average of many positive sentiment numerical metrics. The overlaid boxplots allows us to compare the 25th percentile, the median, and the 75th percentile easily between racial groups. As we can see, black individuals tend to feel more positive emotions, where a higher PosEmo value corresponds to more positive feelings, since the first quartile, median, and third quartile values are all higher. There is a slight left skew in the distribution of positive emotions for black individuals, with the median between two local peaks, and there is only one clear maximum for white individuals.

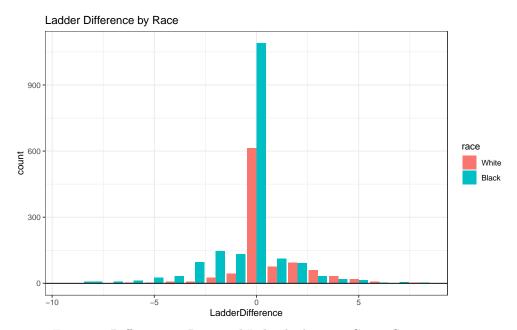


Figure 5: Difference in Perceived Individual versus Group Status

This plot shows us a side by side comparison of the Ladder Differences for two races: white, and black. Ladder difference, in this case, shows us how each individual feels they are perceived relative to their racial group. As we can see in the large blue bar at LD=0, many black individuals tend to think of themselves as perceived in the same way as their group, thus yielding a difference of zero. A negative value, in this case, would indicate that the individual feels they are perceived better than their group; looking at values for LD that are less than zero, we see that black individuals tend to feel this way more than white individuals. Beyond LD=0, which signfies that an individual feels they are perceived more poorly than their group, there is no obvious trend between black individuals and white individuals.

Other Attempts:

```
> #sentiment of individual taking the survey
> ggdat<-data.frame(y=dat.wb1$PosEmo[which(dat.wb1$Race==1)]) #sentiment of white people
> p1<-ggplot(data=ggdat,aes(x='',y=y))+
    geom_violin(fill="green",
                #trim=FALSE,
                alpha=0.5,
                show.legend = FALSE) +
    geom_boxplot(width=.25, fill="white") +
    xlab('') +
    ylab("Emotion Level") +
    ggtitle("Sentiment of White Surveyors") +
 ggdat<-data.frame(y=dat.wb1$PosEmo[which(dat.wb1$Race==3)]) #sentiment of black people
 p2<-ggplot(data=ggdat,aes(x='',y=y))+
    geom_violin(fill="lightgreen",
                #trim=FALSE,
+
                alpha=0.5,
                show.legend = FALSE) +
    geom_boxplot(width=.25, fill="white") +
    xlab('') +
    ylab("Emotion Level") +
```

```
ggtitle("Sentiment of Black Surveyors") +
  theme_bw()
> grid.arrange(p1,p2,ncol=2)
> #comparing white and black ladderdiff and posemo
> ggdat<-data.frame(x=dat.wb1$LadderDiff[which(dat.wb1$Race==1)], y=dat.wb1$PosEmo[which(dat.wb1$Ra
> t1<-ggplot(data=ggdat, aes(x=x,y=y)) +</pre>
   geom_point(size=2,shape=18,color='violet') + #shape, size and color of points
   geom_smooth(method = lm, fill="lightblue") + #adds regression line, confidence interval color
   xlab("Ladder Difference") +
   ylab("Sentiment") +
   ggtitle("White Ladder Difference versus Individual Sentiment")
> ggdat<-data.frame(x=dat.wb1$LadderDiff[which(dat.wb1$Race==3)], y=dat.wb1$PosEmo[which(dat.wb1$Ra
> t2<-ggplot(data=ggdat, aes(x=x,y=y)) +</pre>
   geom_point(size=2,shape=18,color="purple") + #shape, size and color of points
   geom_smooth(method = lm, fill="lightblue") + #adds regression line, CI color
  xlab("Ladder Difference") +
  ylab("Sentiment") +
   ggtitle("Black Ladder Difference versus Individual Sentiment")
> grid.arrange(t1,t2,ncol=2)
> #what do ladderdiff and posemo mean
> #laddergroup - where would you place your group of people
> #ladderself - where would put yourself
> #posemo - how people feel when taking the survey
    #relationship between how people feel when taking the survey
> #what we got from office hours
> ggdat<-data.frame(x=dat.wb1$LadderDiff[which(dat.wb1$Race==3)])</pre>
> ggplot(data=ggdat, aes(x=x,y=..count..))+
   geom_bar()
> ggdat<-data.frame(LD=dat.wb1$LadderDiff,race=dat.wb1$Race)
> ggplot(data=ggdat,aes(x=LD,fill=race))+
   geom_bar(position=position_dodge()) #we unstacked
> ggdat<-data.frame(PE=dat.wb1$PosEmo,race=dat.wb1$Race)</pre>
> #failed attempt number 2 where I didn't know what data to extract
> ggdat<-data.frame((dat.wb1$LadderDiff))</pre>
> colnames(ggdat)=c("LadderDiff", "PosEmo")
> p1<-ggplot(data=ggdat, aes(x=LadderDiff)) +</pre>
   geom_bar(stat = "identity",
      color= "black",
             fill= "lightblue") +
   xlab("LadderDiff") +
   ylab("Frequency") +
   ggtitle("Frequency of LadderDiff for Black and White Surveyors") +
   geom_hline(yintercept=0) +
   theme_bw()
> p1
> #ggdat2<-data.frame(PosEmo=dat.wb$PosEmo,race=dat.wb$Race)
> #ggplot(data=ggdat2,aes(x=PosEmo,fill=race))+
> # geom_bar(position = position_dodge()) #don't leave it stacked- put side by side
> #ggdat_white<-data.frame(table(data.frame(filter(dat.wb$LadderDiff, Race == 1))))
> #colnames(ggdat_white)=c("LadderDiff", "PosEmo")
```

```
\verb| > \#p\_white < -ggplot(data = ggdat\_white, aes(x = Ladder Diff, y = PosEmo)) + \#tell \ ggplot \ which \ data \ to \ use
> # geom_bar(stat="identity",
                                 #plot the count (no transformation needed)
             color="black",
                                       #bar outline color
> #
> #
              fill="lightblue")
                                      + #bar colors
                                            + #x axis label
> # xlab("Ladder Difference")
                                              + #y axis label
> # ylab("Positive Emotion")
> # ggtitle("Positive Emotion vs Ladder Difference") + #add title to plot
> # geom_hline(yintercept=0)
                                       + #adds a line for the x-axis
> # theme_bw()
                                    #removes grey background
> #p_white
> # tried to plot all of the information in one graph
> # not the prettiest way to do that
> # colnames(ggdat)=c("LadderDiff", "PosEmo")
> # p1<-ggplot(ggdat, aes(x=LadderDiff)) +</pre>
      # geom_histogram(bins=10, #how many bins to use
> #
                        fill = "lightblue",
      #
                        color="black") +
> #
     geom_bar()+
> #
     xlab("Difference between the Group and Individual") +
> #
> # ylab("Emotion") +
> # ggtitle("Emotions with respect to Ladder Difference") +
> # theme_bw() +
> # geom_hline(yintercept=0)
> # p1
```

- 2. (Data Cleaning and Summary) Recreate the supplementary materials posted under this homework on Moodle. You can omit the statistical tests until we get there.
- 3. (Probability I) The zinc-phosphate coating on the threads of steel tubes used in oil and gas wells is critical to their performance. However, 12 percent of all tubes receive an improper amount of coating (either much too low or much too high). That is, about 12 percent of the tubes are defective. Assume that the tubes are independent.
 - (a) If we take a sample of 10 tubes, what is the probability at least 2 are defective?
 - (b) If we continually observe tubes until we find the first defective tube, what is the probability we will observe no more than 3 tubes?
 - (c) If we continually observe tubes until we find the second defective tube, what is the probability we will observe more than 4 tubes?

(Probability II) Sowbugs are primarily nocturnal, thrive in a moist environment, and they eat decaying leaf litter and vegetable matter. Suppose that Y, the number of sowbugs on a square-foot plot, follows a Poisson distribution with $\lambda=15$.

- (a) Find the probability that a square-foot plot contains exactly 10 sowbugs.
- (b) In terms of the Poisson PMF, write an expression for P(Y >= 100).

(Probability III) Explore all the plots for continuous data to compare the two vectors of data x1 and x2 loaded using the code above. Reflect on the importance of creating several graphs. Compare the likelihood of each data vector being from the Gaussian($\mu = 0, \sigma = 1$) distribution and compare this to your visual interpretation.

```
> dat.x<-read.csv("https://cipolli.com/students/data/HW1-twoDatasets.txt",
+ header=T,sep=",")</pre>
```

4. Estimation Consider data originally from a study of the nesting horseshoe crabs (Brockmann, 1996). Each female crab in the study had a male crab attached to her in her nest. The study investigated factors that affect whether the female crab had any other males, called satellites, residing nearby her. Explanatory variables thought possibly to affect this included the female crab's color, spine condition, weight, and carapace width. The response outcome for each female crab is her number of satellites. The sample is

Number of Satellites	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Number of Observations	62	16	9	19	19	15	13	4	6	3	3	1	1	1	1	1

It is believed that the distribution of the number of satellites for a female crab is distributed $Poisson(\lambda)$ where the parameter λ is of interest.

- (a) Calculate the method of moments estimator for λ .
- (b) Find the maximum likelihood estimator for λ .
- (c) Plot the data with the Poisson distribution fit with the MLE estimates. How well does the distribution fit the data?
- (d) Let's try another distribution the zero-inflated Poisson distribution. Now, it is believed that the distribution of the number of satellites for a female crab is distributed $Poisson_0(\lambda, \sigma)$ where the parameters λ and σ are of interest. Find the method of moments estimators for both σ and λ .

$$f_X(x|\lambda,\sigma) = (1-\sigma)\frac{\lambda^x e^{-\lambda}}{x!}I(x \ge 1) + (\sigma + (1-\sigma)e^{-\lambda})I(x=0)$$

- (e) Find the maximum likelihood estimator for λ and σ .
- (f) Plot a histogram of the data with the zero-inflated Poisson distribution fit with the MLE estimates. How well does the distribution fit the data?

Solution: Setting Up the Data:

```
> #formatting the data in different ways
> numofsatellites <- c(0:15)
> numofobs <- c(62, 16, 9, 19, 19, 15, 13, 4, 6, 3, 3, 1, 1, 1, 1, 1)
> crabby_data1 <- data.frame(numofobs,numofsatellites)</pre>
> #create a vector of crab data
  crabby_data \leftarrow c(rep(0, 62),
                    rep(1,16),
                    rep(2, 9),
                    rep(3, 19),
                    rep(4, 19),
                    rep(5,15),
                    rep(6, 13),
                    rep(7, 4),
                    rep(8, 6),
                    rep(9, 3),
                    rep(10,3),
                    rep(11, 1),
                    rep(12, 1),
                    rep(13, 1),
                    rep(14, 1),
                    rep(15, 1))
> #Other Attempts
> #sumofobs <- sum(numofobs)</pre>
> #sample_mean=sum(numofobs * numofsatellites)/sumofobs
```

Part A:

```
> #install.packages("gmm") #install packages
> library(gmm) #load packages
> set.seed(69) #makes data comparable to others
> g<- function(x,theta) { #(data, theta)
    #set the sample and population moments to be equal
   b <- theta
   m1 \leftarrow b - mean(x) \#moment
   return(m1)
+ }
> gmm(g=g,#function
     x=crabby_data,#data
      t0=mean(crabby_data), #inital guess
     method="Brent",#univariate
      lower=0, #reasonable lower
      upper=1.5*max(crabby_data)) #reasonable upper
Method
twoStep
Objective function value: 3.983414e-30
Theta[1]
   2.977
Convergence code = 0
                                       Part B:
> #Part B - find maximum likelihood estimator for lambda
> LL <- function(lambda) { #function inputting lambda
   return(-1*sum(dpois(crabby_data, lambda = lambda, log = TRUE))) #likelihood of distribution
+ }
> a <- optim(fn = LL, #function
       par=mean(crabby_data), #best guess
       method="Brent", #
        lower=0,
        upper=1.5*max(crabby_data))
> a
$par
[1] 2.977011
$value
[1] 505.4901
$counts
function gradient
     NA
$convergence
[1] 0
$message
NULL
```

Part C:

- > library(ggplot2) #load library
- > crabby_data1 <- tibble(crabby_data) #reformatting for histogram

Poisson Distribution

PMF Distribution for $\lambda = 2.977011$

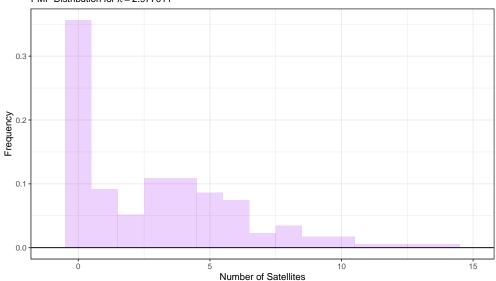


Figure: The Poisson Distribution fit with MLE Estimation

This plot shows the PMF of a discrete random variable X, given by bquote (fx) = P(X=x) for all x. In the context of this problem, the discrete random variable X is the number of satellites found around the observed female horseshoe crabs. Assuming our observations have an underlying Poisson distribution, which is used to represent the number of times a specific event (where an event is an observation of X satellites around a female horseshoe crab) occurs in a given time or space, we are able to obtain a Maximum Likelihood Estimate for lambda, the parameter of the model. The MLE maximizes the likelihood of having a specific value for lambda given the underlying Poisson distribution (i.e. Given the distribution, the likelihood of its true lambda value being 1,000,000 is essentially zero, whereas having a lambda value near the sample mean, is likely the most probable because the sample mean is usually a good indicator of the theoretical mean); in our case, the MLE gives us 2.98, which we then use in the below graph of the PMF of our Poisson distribution. As we can see, the Poisson distribution fit with the MLE estimate fits the data well, and we can see this because the lambda values are most likely between 2 and 4, which is also observable in the data given to us. When we compare the MLE estimate for lambda fits the data much better than the MoM estimate for lambda. It is important to note that while the distribution fits the data well, it does not properly account for the number of zero observations in our dataset.

Part D:

```
> #Part D - zero-inflated poisson distribution
> set.seed(69)
> g_1<- function(x,theta) { #(data, theta)
+    #set the sample and population moments equal
+    lambda = theta[1]
+    sigma = theta[2]
+    EX = (1-sigma)*lambda
+    varX = (1-sigma) * (lambda + lambda^2) - ((1-sigma)*lambda)^2
+    m1 <- EX - x
+    m2 <- varX - (x-EX)^2
+    return(c(m1, m2)) #return moments</pre>
```

```
+ }
> gmm(t0=c(0,60), \#best guess)
                         g=g_1,#function
                         x=crabby_data) #data
Method
   twoStep
Objective function value: 0.4674955
    Theta[1]
                                                    Theta[2]
    0.049805 -0.246094
Convergence code = 0
                                                                                                                                                                          Part E:
> #Part E - find the maximum likelihood estimator for lambda and sigma
> alexa <-function(x, lambda, sigma) { #function given, probability of x given lambda and sigma
                 \log(((1-\operatorname{sigma})*(((\operatorname{lambda^x})*(\operatorname{exp}(-\operatorname{lambda}))/(\operatorname{factorial}(x))))*(x>=1)) + ((\operatorname{sigma} + (1-\operatorname{sigma})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{lambda^x})*(\operatorname{
+ }
> LL1 <- function(x, theta, neg=FALSE) {#takes the data and theta
                 sigma<-theta[1]
                lambda<-theta[2]
                #applies a function to a vector and sums the values
                11 <-sum(sapply(X=crabby_data, FUN = alexa, sigma = sigma, lambda = lambda))</pre>
                ifelse(!neg,11,-11)
+ }
> c \leftarrow optim(fn = LL1, \#function)
                                                        x = crabby_data, #data
                                                        par=c(min(crabby_data), max(crabby_data)), #best guess
                                                       neg=TRUE)
> c
$par
[1] 0.3494643 4.5775109
$value
[1] 389.4734
$counts
function gradient
                         63
                                                                NA
$convergence
 [1] 0
$message
NULL
                                                                                                                                                                          Part F:
> #Part F - plot a histogram of zero-inflated poisson distribution fit
> library(ggplot2)
> alexa_non_log <-function(x, lambda, sigma) { #nonlog function to use for plotting
```

Zero–Inflated Poisson Distribution Fit with MLE Estimates 0.3 0.1

Figure: The Zero-Inflated Poisson Distribution

As we can see, the Zero-Inflated Poisson Distribution actually fits better than the Poisson Distribution because the Poisson Distribution did not fit to the large number of zero values. However, we can see in this plot that the Zero-Inflated Poisson Distribution almost perfectly accounts for the amount of zeros we have in our data set. In fact, it is almost the same distribution as the Poisson Distribution besides the fact that it also accounts for the zero observations.

Number of Satellites

10

15

References

0.0

0

Brockmann, H. J. (1996). Satellite male groups in horseshoe crabs, limulus polyphemus. *Ethology*, 102(1):1–21.