1.) Manipulate to standard form:

$$x^2\frac{dy}{dx} + x(x+2)y = x^2\frac{dy}{dx} + x^2y + 2xy \implies \frac{dy}{dx} + y\left(1+\frac{2}{x}\right) = \frac{e^x}{x^2}$$

Find $\mu(x)$ then y:

$$\mu(x) = e^{\int p(x)dx} = e^{\int 1+\frac{2}{x}dx} = e^{x+2\ln|x|} = e^x e^{2\ln|x|} = e^x x^2$$

$$y = \frac{1}{\mu(x)}\int \mu(x)g(x)\ dx = \frac{1}{e^x x^2}\left[\int e^{2x}\ dx\right] = \frac{e^x}{2x^2} + \frac{C}{e^x x^2}$$

Which is the explicit solution.

2.) Manipulate to standard form:

$$\cos x dy + y\sin x dx - dx = 0 \implies \cos x\frac{dy}{dx} + y\sin x - 1 = 0$$

$$\implies \frac{dy}{dx} + y\tan x = \sec x$$

Find $\mu(x)$ then y:

$$\mu(x) = e^{\int \tan x dx} = e^{-\ln|\cos x|} = \sec x$$

$$y = \frac{1}{\sec x}\left[\int \sec^2 x\ dx\right] = \frac{\tan x}{\sec x} + \frac{C}{\sec x}$$

Which is the explicit solution.

3.) $y = \dfrac{\tan x}{\sec x} + \dfrac{C}{\sec x}$ as per problem 2.

4.) Find $\mu(x)$ then y:

$$\mu(x) = e^{\int \sin x dx} = e^{-\cos x} = \frac{1}{e^{\cos x}}$$

$$y = e^{\cos x}\left[\int \frac{e^{\cos x}}{e^{\cos x}}\ dx\right] = e^{\cos x}(x+C)$$

Solving given the initial values:

$$-2.5 = e^{\cos(0)}(0+C) = Ce^1 \implies C = \frac{-2.5}{e}$$

$$\therefore y = e^{\cos x}\left(x + \frac{-2.5}{e}\right)$$

Which is the particular solution.

5.) Manipulate to standard form:

$$\frac{dy}{dx} = x - 4xy \implies \frac{dy}{dx} + 4xy = x$$

Find $\mu(x)$ then y:

$$\mu(x) = e^{\int 4x dx} = e^{2x^2}$$

$$y = \frac{1}{e^{2x^2}} \left[\int xe^{2x^2} \ dx \right] = \frac{1}{e^{2x^2}} \left(\frac{e^{2x^2}}{4} + C \right) = \frac{1}{4} + \frac{C}{e^{2x^2}}$$

Which is the explicit solution.

6.) awd