1.) We can manipulate the equation as follows:

$$\frac{dy}{dx} = e^{x-y} = \frac{e^x}{e^y} \implies e^y dy = e^x dx$$

$$\int e^y dy = e^y$$

$$\int e^x dx = e^x + C$$

$$\therefore \ \ln(e^y) = y = \ln(e^x + C)$$

Which is the explicit solution.

2.) Manipulate:

$$x^2 y^2 \frac{dy}{dx} = (1+x)\csc(2y) \implies y^2 \sin(2y) dy = \frac{1+x}{x^2} dx$$

$$\int y^2 sin(2y) dy = -\frac{y^2 \cos(2y)}{2} + \frac{y \sin(2y)}{2} - \frac{cos(2y)}{4}$$

$$\int \frac{1+x}{x^2} dx = \int \frac{1}{x} + \frac{1}{x^2} dx = \ln|x| - \frac{1}{x} + C$$

$$\therefore \ -\frac{y^2 \cos(2y)}{2} + \frac{y \sin(2y)}{2} - \frac{cos(2y)}{4} = \ln|x| - \frac{1}{x} + C$$

Which is the implicit solution.

3.) Manipulate:

$$(1+e^x)\cos(y)\frac{dy}{dx} = e^x \sin(y) \implies \cot(y) dy = \frac{e^x}{1+e^x} dx$$

$$\int \cot(y) dy = \ln|\sin(y)|$$

$$\int \frac{e^x}{1+e^x} dx = \ln|1+e^x| + C$$

$$\therefore \ y = \sin^{-1}(e^x + C)$$

Which is the explicit solution.

4.) Manipulate:

$$(1+x)dy - ydx = 0 \implies \frac{dy}{y} = \frac{dx}{1+x}$$

$$\int \frac{dy}{y} = \ln|y|$$

$$\int \frac{dx}{1+x} = \ln|1+x| + C$$

$$\therefore\ y = (1+x)e^C$$

Solving given the initial values:

$$1 = (1+0)e^C = e^C \implies \ln(1) = 0 = C$$

$$\therefore\ y = 1+x$$

Which is the particular solution.

5.) Manipulate:

$$\frac{dy}{dx} = \frac{x^2}{y} \implies ydy = x^2dx$$

$$\int ydy = \frac{y^2}{2}$$

$$\int x^2dx = \frac{x^3}{3} + C$$

$$\therefore\ y = \pm\sqrt{\frac{2x^3}{3} + C}$$

Which is the explicit solution.

6.) Manipulate:

$$\frac{dy}{dx} = \frac{x - e^{-x}}{y + e^y} \implies y + e^y dy = x - e^{-x}dx$$

$$\int y + e^y dy = \frac{y^2}{2} + e^y$$

$$\int x - e^{-x}dx = \frac{x^2}{2} + e^{-x} + C$$

$$\therefore\ \frac{y^2}{2} + e^y = \frac{x^2}{2} + e^{-x} + C$$

Which is the implicit solution.

7.) Manipulate:

$$\frac{dy}{dx} = \frac{(1-2x)}{y} \implies ydy = 1 - 2xdx$$

$$\int ydy = \frac{y^2}{2}$$

$$\int 1 - 2xdx = x - x^2 + C$$

$$\therefore\ y = \pm\sqrt{2(x - x^2 + C)}$$

Solving given the initial values:

$$-2 = \sqrt{2(0 - 0^2 + C)} = \sqrt{2C} \implies C = 2i;\ \text{no real solutions}$$

$$-2 = -\sqrt{2(0 - 0^2 + C)} = -\sqrt{2C} \implies C = 2$$

$$y = -\sqrt{2(x - x^2 + 2)}$$

Which is the particular solution given $C \in \mathbb{R}$.

8.) Manipulate:

$$\frac{dy}{dx} = \frac{3x^2 - e^x}{2y - 5} \implies 2y - 5dy = 3x^2 - e^x dx$$

$$\int 2y - 5dy = y^2 - 5y$$

$$\int 3x^2 - e^x dx = x^3 - e^x$$

$$\therefore\ y^2 - 5y = x^3 - e^x + C$$

Solving given the initial values:

$$1^2 - 5(1) = 0^3 - e^0 + C \implies -4 = C - 1 \implies C = 3$$

$$\therefore\ y^2 - 5y = x^3 - e^x + 3$$

Which is the particular implicit solution.