

40.) Let  $\varepsilon > 0$ , and  $n \geq k$  for some  $k \in \mathbb{N}$ , then

$$\left| \frac{2n-1}{n} - 2 \right| < \varepsilon$$

Solving for  $n$ , we can find a sufficiently large value for  $k$ :

$$\begin{aligned} \left| \frac{2n-1}{n} - 2 \right| &= \left| \frac{2n-1-2n}{n} \right| = \left| \frac{-1}{n} \right| = \frac{1}{n} < \varepsilon \\ \implies n &> \frac{1}{\varepsilon} \end{aligned}$$

Now, let  $k > \frac{1}{\varepsilon}$ , then

$$\begin{aligned} \left| \frac{2n-1}{n} - 2 \right| &= \frac{1}{n} \leq \frac{1}{k} < \frac{1}{1/\varepsilon} = \varepsilon \\ \therefore \left| \frac{2n-1}{n} - 2 \right| &< \varepsilon \end{aligned}$$

Thus  $x_n \rightarrow 2$ . Q.E.D.

41.) Let  $\varepsilon > 0$ , and  $n \geq k$  for some  $k \in \mathbb{N}$ , then

$$\left| \frac{(-1)^n}{n} - 0 \right| < \varepsilon$$

Solving for  $n$ , we can find a sufficiently large value for  $k$ :

$$\begin{aligned} \left| \frac{(-1)^n}{n} - 0 \right| &= \left| \frac{(-1)^n}{n} \right| = \frac{|(-1)^n|}{|n|} = \frac{1}{n} < \varepsilon \\ \implies n &> \frac{1}{\varepsilon} \end{aligned}$$

Now, let  $k > \frac{1}{\varepsilon}$ , then

$$\begin{aligned} \left| \frac{(-1)^n}{n} - 0 \right| &= \frac{1}{n} \leq \frac{1}{k} < \frac{1}{1/\varepsilon} = \varepsilon \\ \therefore \left| \frac{(-1)^n}{n} - 0 \right| &< \varepsilon \end{aligned}$$

Thus  $x_n \rightarrow 0$ . Q.E.D.

42.) Let  $\varepsilon > 0$  be given, then we know that

$$\left| \frac{3n+1}{5n-2} - \frac{3}{5} \right| < \varepsilon$$

given  $n \geq k$  for some  $k \in \mathbb{N}$ . Manipulating the inequality, we find that

$$\left| \frac{3n+1}{5n-2} - \frac{3}{5} \right| = \left| \frac{15n+5-15n+6}{25n-10} \right| = \left| \frac{11}{25n-10} \right| = \frac{|11|}{|25n-10|} = \frac{11}{25n-10} < \varepsilon$$

$$\implies 25n - 10 > \frac{11}{\varepsilon} \implies n = \frac{11}{25\varepsilon} + \frac{2}{5}.$$

Let  $k > \frac{11}{25\varepsilon} + \frac{2}{5}$ :

$$\frac{11}{25n - 10} < \frac{11}{25k - 10} = \frac{11}{25\left(\frac{11}{25\varepsilon} + \frac{2}{5}\right) - 10} = \frac{11}{\frac{11}{\varepsilon} + 10 - 10} = \frac{11}{\frac{11}{\varepsilon}} = \varepsilon$$

$$\therefore \left| \frac{3n + 1}{5n - 2} - \frac{3}{5} \right| < \varepsilon$$

thus  $x_n \rightarrow \frac{3}{5}$ . Q.E.D.