Section 3.1

1.) Setup and solve the characteristic equation:

$$y'' + 2y' - 3y = 0 \to r^2 + 2r - 3 = (r-1)(r+3) = 0 \implies r = 1, -3$$

Since $r_1 \neq r_2$, the general solution of the equation is given as

$$y(t) = c_1 e^t + c_2 e^{-3t}$$

2.) Setup and solve the characteristic equation:

$$y'' + 3y' + 2y \to r^2 + 3r + 2 = (r+1)(r+2) = 0 \implies r = -1, -2$$

Since $r_1 \neq r_2$, the general solution of the equation is given as

$$y(t) = c_1 e^{-t} + c_2 e^{-2t}$$

9.) Setup and solve the characteristic equation:

$$y'' + 3y' = 0 \to r^2 + 3y = r(r+3) \implies r = 0, -3$$

Since $r_1 \neq r_2$, the general solution of the equation is given as

$$y(t) = c_1 e^0 + c_2 e^{-3t} = c_1 + c_2 e^{-3t}$$

Solving for c_1 and c_2 given $y(0) = -2$, $y'(0) = 3$ we find the particular solution:

$$-2 = c_1 + c_2 e^0 = c_1 + c_2$$

$$3 = -3c_2 e^0 = -3c_2$$

$$\implies \left[\begin{array}{cc|c} 1 & 1 & -2 \\ 0 & -3 & 3 \end{array}\right] \implies \left[\begin{array}{cc|c} 1 & 1 & -2 \\ 0 & -1 & 1 \end{array}\right] \implies \left[\begin{array}{cc|c} 1 & 0 & -1 \\ 0 & -1 & 1 \end{array}\right]$$

$$\implies \left[\begin{array}{cc|c} 1 & 0 & -1 \\ 0 & 1 & -1 \end{array}\right] \implies c_1 = c_2 = -1$$

$$\therefore\ y(t) = -1 - e^{-3t}$$

As $t \to \infty$, $y \to -1$.

13.) Determine r_1 and r_2:

$$y = c_1 e^{2t} + c_2 e^{-3t} \implies r_1 = 2, r_2 = -3$$

$$\therefore\ (r-2)(r+3) = r^2 + t - 6 = 0$$

And derive the equation from its characteristic equation:

$$y'' + y' - 6y = 0$$

Section 3.3

5.) Setup and solve the characteristic equation:

$$y'' - 2y' + 2y \to r^2 - 2r + 2 = 0$$
$$\frac{2 \pm \sqrt{4 - 4(2)}}{2} = \frac{2 \pm \sqrt{-4}}{2} = 1 \pm i$$
$$\therefore\ r_1 = 1 + i, r_2 = 1 - i$$

And find the general solution to the equation:

$$y(t) = c_1 e^t \cos t + c_2 e^t \sin t$$

12.) Setup and solve the characteristic equation:

$$y'' + 4y = 0 \to r^2 + 4 = 0$$
$$\frac{\pm\sqrt{-4(4)}}{2} = \frac{\pm\sqrt{-16}}{2} = \pm 2i$$
$$\therefore\ r_1 = 2i, r_2 = -2i$$

And find the general solution to the equation:

$$y(t) = c_1 e^0 \cos(2t) + c_2 e^0 \sin(2t) = c_1 \cos(2t) + c_2 \sin(2t)$$

Solving for c_1 and c_2 given $y(0) = 0, y'(0) = 1$, we find the particular solution:

$$0 = c_1 \cos(2(0)) + c_2 \sin(2(0)) = c_1$$
$$1 = -2c_1 \sin(2(0)) + 2c_2 \cos(2(0)) = 2c_2 \implies c_2 = \frac{1}{2}$$
$$\therefore\ y(t) = \frac{1}{2}\sin(2t)$$

As $t \to \infty$, $y(t)$ will oscillate.

Section 3.4

1.) Setup and solve the characteristic equation:

$$y'' - 2y' + y = 0 \to r^2 - 2r + 1 = (r-1)^2 = 0$$
$$\therefore\ r_1 = r_2 = 1$$

Since $r_1 = r_2$, the general solution to the equation is given as

$$y(t) = c_1 e^t + c_2 t e^t$$

2.) Setup and solve the characteristic equation:

$$9y'' + 6y' + y = 0 \to 9r^2 + 6r + 1 = (3r+1)^2 = 0$$

$$\therefore\ r_1 = r_2 = -\frac{1}{3}$$

Since $r_1 = r_2$, the general solution to the equation is given as

$$y(t) = c_1 e^{-\frac{1}{3}t} + c_2 t e^{-\frac{1}{3}t}$$

10.) Setup and solve the characteristic equation:

$$y'' - 6y' + 9y = 0 \to r^2 - 6r + 9 = (r-3)^2 = 0$$

$$\therefore\ r_1 = r_2 = 3$$

Since $r_1 = r_2$, the general solution to the equation is given as

$$y(t) = c_1 e^{3t} + c_2 t e^{3t}$$

Solving for c_1 and c_2 given $y(0) = 0, y'(0) = 2$, we find the particular solution:

$$0 = c_1 e^0 + c_2(0)e^0 = c_1$$

$$2 = 3c_1 e^0 + c_2(0 + e^0) \implies c_2 = 2$$

$$\therefore\ y(t) = (0)e^{3t} + 2te^{3t} = 2te^{3t}$$

As $t \to \infty$, $y \to \infty$.