a.) Let $x \in S^3 \subset \mathbb{R}^4$. Since $\mathbb{R}^4 \cong \mathbb{C}^2$, we know that $(x_1, x_2, x_3, x_4) \mapsto (w_1, w_2)$ where $w_1, w_2 \in \mathbb{C}$. Let $w_1 = a + bi$ and $w_2 = c + di$ for $a, b, c, d \in \mathbb{R}$. By definition of S^3, $x_1^2 + x_2^2 + x_3^2 + x_4^2 = 1$, thus $a^2 + b^2 + c^2 + d^2 = 1$. Consider $2w_1\overline{w_2}$ and $w_1\overline{w_1} - w_2\overline{w_2}$:

$$2w_1\overline{w_2} = 2(a+bi)(c-di) = 2(ac + bci - adi + bd) = 2(ac+bd) + 2i(bc - ad)$$

$$w_1\overline{w_1} - w_2\overline{w_2} = (a+bi)(a-bi) - (c+di)(c-di) = a^2 + b^2 - \left(c^2 + d^2\right)$$

Thus $2w_1\overline{w_2} \in \mathbb{C}$ and $(w_1\overline{w_1} - w_2\overline{w_2}) \in \mathbb{R}$. Since $\mathbb{C} \times \mathbb{R} \cong \mathbb{R}^3$, we know that $(z, r) \mapsto y \in \mathbb{R}^3$. Let $y_1 = 2(ac+bd)$, $y_2 = 2(bc - ad)$, and $y_3 = a^2 + b^2 - \left(c^2 + d^2\right)$. Consider y_1^2, y_2^2, and y_3^2:

$$y_1^2 = 2^2(ac+bd)^2 = 4\left(a^2c^2 + 2abcd + b^2d^2\right)$$

$$y_2^2 = 2^2(bc-ad)^2 = 4\left(b^2c^2 - 2abcd + a^2d^2\right)$$

$$y_3^2 = \left(\left(a^2+b^2\right) - \left(c^2+d^2\right)\right)^2 = \left(a^2+b^2\right)^2 - 2\left(a^2+b^2\right)\left(c^2+d^2\right) + \left(c^2+d^2\right)^2$$

$$=$$

b.) awd

c.) awd