Exercises 12.4

5.) $\left\langle \frac{1}{2}, \frac{1}{3}, \frac{1}{4} \right\rangle \times \langle 1,2,-3\rangle = \left(-1-\frac{1}{2}\right) i - \left(-\frac{3}{2}-\frac{1}{4}\right) j + \left(1-\frac{1}{3}\right) k = \left\langle -\frac{3}{2}, \frac{7}{4}, \frac{2}{3} \right\rangle$

$\left\langle \frac{1}{2}, \frac{1}{3}, \frac{1}{4} \right\rangle \cdot \left\langle -\frac{3}{2}, \frac{7}{4}, \frac{2}{3} \right\rangle = -\frac{3}{4} + \frac{7}{12} + \frac{2}{12} = 0$

$\langle 1,2,-3\rangle \cdot \left\langle -\frac{3}{2}, \frac{7}{4}, \frac{2}{3} \right\rangle = -\frac{3}{2} + \frac{14}{4} - \frac{6}{3} = 0$

7.) $\left\langle t, 1, \frac{1}{t} \right\rangle \times \langle t^2, t^2, 1\rangle = (1-t)\, i - (t-t)j + (t^3-t^2)k = \langle 1-t, 0, t^3-t^2\rangle$

$\left\langle t, 1, \frac{1}{t} \right\rangle \cdot \langle 1-t, 0, t^3-t^2\rangle = (t-t^2) + (t^2-t) = 0$

$\langle t^2, t^2, 1\rangle \cdot \langle 1-t, 0, t^3-t^2\rangle = (t^2-t^3) + (t^3-t^2) = 0$

17.) $\langle 2,-1,3\rangle \times \langle 4,2,1\rangle = (-1-6)i - (2-12)j + (4+4)k = \langle -7, 10, 8\rangle$

$\langle 4,2,1\rangle \times \langle 2,-1,3\rangle = -\langle 2,-1,3\rangle \times \langle 4,2,1\rangle = \langle 7,-10,-8\rangle.$

19.) $\langle 3,2,1\rangle \times \langle -1,1,0\rangle = \langle -1,-1,5\rangle\,; \frac{\langle -1,-1,5\rangle}{\|\langle -1,-1,5\rangle\|} = \frac{1}{3\sqrt{3}} \langle -1,-1,5\rangle$

Thus $\pm\frac{1}{3\sqrt{3}} \langle -1,-1,5\rangle$ are orthogonal unit vectors.

27.) $\overline{AB} = \langle -1+3, 3-0\rangle = \langle 2,3\rangle;\ \overline{BC} = \langle 5+1, 2-3\rangle = \langle 6,-1\rangle.$

$\begin{vmatrix} 2 & 3 \\ 6 & -1 \end{vmatrix} = -2-18 = -20, |-20| = 20$

29.) a.) $\overline{PQ} = \langle -2-1, 1-0, 3-1\rangle\ \langle -3,1,2\rangle$

$\overline{PR} = \langle 4-1, 2-0, 5-1\rangle = \langle 3,2,4\rangle$

$$\overline{PQ} \times \overline{PR} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ -3 & 1 & 2 \\ 3 & 2 & 4 \end{vmatrix} = \bar{i}(4-4) - \bar{j}(-12-6) + \bar{k}(-6-3) = \langle 0, 18, -9\rangle$$

b.) $A = \frac{\|\overline{PQ}\times\overline{PR}\|}{2} = \frac{\sqrt{0^2+18^2+(-9)^2}}{2} = \frac{\sqrt{405}}{2} = \frac{9\sqrt{5}}{2}$

33.) The volume of the parallelepiped formed by 3 vectors is equal to their determinant:

$$\begin{vmatrix} 1 & -1 & 2 \\ 2 & 1 & 1 \\ 3 & 2 & 4 \end{vmatrix} = 1(4-2) + 1(8-3) + 2(4-3) = 2+5+2 = 9$$

Which is the volume of the parallelepiped.

41.) $\theta = \tan^{-1}\left(\frac{4}{3}\right)$

$\tau = 100 = Fr\sin\theta = F(0.3)(0.8) \implies F = \frac{100}{0.24} \approx 417N$

Exercises 12.5

3.) $P_0(2, 2.4, 3.5)$, $\overline{v} = \langle 3, 1, -1\rangle$
$L = P_0 + t\overline{v} = \langle 2, 2.4, 3.5\rangle + t\langle 3, 1, -1\rangle = \langle 2+3t, 2.4+t, 3.5-t\rangle$

Parametric:
$x = 2 + 3t,\ y = 2.4 + t,\ z = 3.5 - t$

5.) $P_0(1, 0, 6)$, $P = x + 3y + z = 5 \implies x + 3y + z - 5 = (x-5) + 3(y-0) + (z-0) = 0$
$L = P_0 + t\hat{n} = \langle 1, 0, 6\rangle + t\langle 1, 3, 1\rangle = \langle 1+t, 3t, 6+t\rangle$

9.) $P_0(-8, 1, 4)$, $\overline{v} = \langle 3+8, -2-1, 4-4\rangle = \langle 11, -3, 0\rangle$
$L = \langle -8, 1, 4\rangle + t\langle 11, -3, 0\rangle = \langle 11t-8, 1-3t, 4\rangle$

Parametric:
$x = 11t - 8,\ y = 1 - 3t,\ z = 4$

Symmetric:
$t = \dfrac{x+8}{11} = -\dfrac{y-1}{3},\ z = 4$

17.) $P_0(6, -1, 9)$, $\overline{v} = \langle 7-6, 6+1, 0-9\rangle = \langle 1, 7, -9\rangle$
$L = \langle 6, -1, 9\rangle + t\langle 1, 7, -9\rangle = \langle 6+t, 7t-1, 9-9t\rangle$, $t \in [0, 1]$

23.) $P_0(0, 0, 0),\ \hat{n} = \langle 1, -2, 5\rangle$
$P = \hat{n} \cdot (\overline{v} - P_0) = (x-0) - 2(y-0) + 5(z-0) = 0 \implies P = x - 2y + 5z = 0$

27.) $P_0(1, -1, -1),\ P|_{P_0} = 5(x-1) - (y+1) - (z+1) = 5x - y - z = 7$

33.) $\overline{AB} = \langle 3-2, -8-1, 6-2\rangle = \langle 1, -9, 4\rangle$
$\overline{AC} = \langle -2-2, -3-1, 1-2\rangle = \langle -4, -4, -1\rangle$

$$\overline{AB} \times \overline{AC} = \begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ 1 & -9 & 4 \\ -4 & -4 & -1 \end{vmatrix} = i(9+16) - j(-1+16) + k(-4-36) = 25i - 15j - 40k$$

$P = 25(x-2) - 15(y-1) - 40(z-2) = 0 \implies 5(x-2) - 3(y-1) - 8(z-2)$
$\implies P = 5x - 3y - 8z = -9$

45.) $x + 2y - z = 7 \implies 2 - 2t + 6t - 1 - t = 1 + 3t = 7 \implies t = 2$
$L|_t = \langle 2-2t, 3t, 1+t\rangle = \langle -2, 6, 3\rangle$

69.) $\overline{A} = \overline{P_0L_0} = \langle 1-4, 3-1, 4+2\rangle = \langle -3, 2, 6\rangle$
$\overline{B} = \overline{L_1L_0} = \langle 1-2, 3-1, 4-1\rangle = \langle -1, 2, 3\rangle$

$$A \times B = \begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ -3 & 2 & 6 \\ -1 & 2 & 3 \end{vmatrix} = i(6-12) - j(-9+6) + k(-6+2) = -6i + 3j - 4k$$

$\|A \times B\| = \sqrt{6^2 + 3^2 + 4^2} = \sqrt{36 + 9 + 16} = \sqrt{61}$
$\|B\| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14}$

$$d = \frac{\sqrt{61}}{\sqrt{14}} = \sqrt{\frac{61}{14}}$$

71.) $P = \langle 1, 1, 0 \rangle$ is e point on the plane

$\overline{PS} = \langle 1 - 1, -2 - 1, 4 - 0 \rangle = \langle 0, -3, 4 \rangle$

$\hat{n} = \langle 3, 2, 6 \rangle$

$$D = \text{comp}_{\hat{n}} \overline{PS} = \frac{\overline{PS} \cdot \hat{n}}{\|\hat{n}\|} = \frac{-6 + 24}{\sqrt{9 + 4 + 36}} = \frac{18}{7}$$