In $\mathbb{R}^3$, we can see that a tetrahedron consists of four triangular faces, each adjacent to eachother, six edges, each adjacent to two other lines, and four vertices. First, we will show that Δ^3 contains the previously mentioned shapes, and then show that these shapes are adjacent, and thus Δ^3 is a tetrahedron.

Given the following definition of Δ^3

$$\Delta^3 = \left\{ (x_1, x_2, x_3, x_4) \in \mathbb{R}^4 : \sum_{i=1}^{4} x_i = 1 \text{ and } x_i \geq 0 \right\}$$

Let $S \subset \{1, 2, 3, 4\}$ where $S \neq \varnothing$ and define $\Delta^3_S = \left\{ x \in \Delta^3 : i \in S \implies x_i = 0 \right\}$. We can show that for each S, $\Delta^3_S \cong \Delta^{3-|S|}$. (Am I using $\cong$ right here?)

Proof: let $S \subset \{1, 2, 3, 4\}$ where $S \neq \varnothing$. Consider the case where $|S| = 1$. Without loss of generality, let $S = \{1\}$, then every point in Δ^3_S is $(0, x_1, x_2, x_3)$ for $a, b, c \in \mathbb{R}$. Consider the mapping $f : \Delta^3_S \to \Delta^2$ where $(0, x_1, x_2, x_3) \mapsto (x_1, x_2, x_3)$. We can show that f is bijective, and thus $\Delta^3_S \cong \Delta^2$. Suppose $a, b \in \Delta^3_S$ where $a = (0, a_1, a_2, a_3)$ and $b = (0, b_1, b_2, b_3)$, and where $f(a) = f(b)$, thus $(a_1, a_2, a_3) = (b_1, b_2, b_3)$, and thus $(0, a_1, a_2, a_3) = (0, b_1, b_2, b_3)$, thus f is injective. Next, suppose $a \in \Delta^2$ where $a = (a_1, a_2, a_3)$ and consider $b = (0, b_1, b_2, b_3)$,