Chapter 19

7.) Suppose $a_1u + a_2(u+v) + a_3(u+v+w) = 0$, thus we have that

$$(a_1 + a_2 + a_3)u + (a_2 + a_3)v + a_3w = 0$$

Since u, v, and w are linearly independent, we have $a_1 + a_2 + a_3 = 0$, $a_2 + a_3 = 0$, and $a_3 = 0$, so $a_2 + a_3 = a_2 = 0$, so $a_1 + a_2 + a_3 = a_1 = 0$, thus $a_1 = a_2 = a_3 = 0$, thus u, v, and w are linearly independent. ■

8.) Suppose $a_1v_1 + a_2v_2 + \cdots + a_nv_n = 0$. Since the vectors are linearly dependent, we can set $a_i \neq 0$ for some i, so $a_1v_1 + \cdots + a_{i-1}v_{i-1} + a_{i+1}v_{i+1} + \cdots + a_nv_n = -a_iv_i$. Dividing out by $-a_i$, we obtain v_i as a linear combination of the other vectors. ■

9.)

10.)

22.) Each vector has n elements and each element has p choices, so the total number of elements is p^n. ■