34.) a.) False; let $S = (-\infty, 0]$, thus $\{|x| : x \in S\} = [0, \infty)$, which has no upper bound.

b.) True; let $|S| = \{|x| : x \in S\}$ and let $M \in \mathbb{R}$ be an upper bound of $|S|$, thus $|x| \leq M$ for all $x \in S$, thus $-M \leq x \leq M$ for all $x \in S$, thus $M \geq x$ for all $x \in S$, thus S is bounded above. Q.E.D.

43.) a.) $\lim_{n\to\infty} \frac{10n}{n} = 10$

b.) $\lim_{n\to\infty} \sin n$ diverges

c.) Suppose $x_n \to 15$ and $x_n \to -77$. Since $x_n \to 15$, x_n gets arbitrarily close to 15. Also, since $x_n \to -77$, x_n gets arbitrarily close to -77. However, as x_n gets closer to 15, x_n moves farther from -77, and vice versa, thus x_n cannot get arbitrarily close to both, thus x_n cannot converge to both.

44.) a.) Let $\{x_n\}_{n=1}^{\infty}$ be a sequence such that $x_n \to L$, then by definition for all $n > k$ for some $k \in \mathbb{N}$, $|x_n - L| < \epsilon$ for some $\epsilon > 0$. By the reverse triangle inequality, we know that $\Big||x_n| - |L|\Big| < |x_n - L| < \epsilon$, thus $\Big||x_n| - |L|\Big| < \epsilon$, thus $|x_n| \to |L|$. Q.E.D.

b.) Consider the sequences $x_n = (-1)^n$ and $|x_n|$. Since $|(-1)^n| = 1$ for all $n \in \mathbb{N}$, $|x_n| \to 1$, but x_n does not converge, thus $|x_n| \to |L|$ does not imply that $x_n \to L$. Q.E.D.