

Let $f(x, y, z) = 5xy + 2xz + 2yz$ represent the cost of the materials and $g(x, y, z) = xyz = 540$ be the volume of the container. We can solve for x and y given the following equation:

$$\nabla f = \lambda \nabla g \implies \langle 5y + 2z, 5x + 2z, 2x + 2y \rangle = \lambda \langle yz, xz, xy \rangle$$

Find the critical points:

$$5y + 2z = \lambda yz, \quad 5x + 2z = \lambda xz, \quad 2x + 2y = \lambda xy, \quad xyz = 540$$

$$\lambda xyz = x(5y + 2z) = y(5x + 2z) = z(2x + 2y)$$

$$x(5y + 2z) = y(5x + 2z) \implies 5xy + 2xz = 5xy + 2yz \implies x = y$$

$$y(5x + 2z) = z(2x + 2y) \implies 5xy + 2yz = 2xz + 2yz \implies z = \frac{5}{2}y = \frac{5}{2}x$$

$$\implies xyz = \frac{5x^3}{2} = 540 \implies x^3 = 216 \implies x = 6$$

$$\implies 6yz = 6y \cdot \frac{5}{2}y = \frac{30y^2}{2} = 15y^2 = 540 \implies y^2 = 36 \implies y = 6$$

$$\implies 36z = 540 \implies z = 15$$

Thus the dimensions of the tank that minimize cost of materials are $6 \times 6 \times 15$.