1.) I am currently double majoring in math and computer science. I first started as just a computer science major, but added a math major when I started getting into studying math as a hobby. In my free time I enjoy programming, especially making video games, and playing/listening to music. I was actually introduced to studying math through youtube channels like numberphile.

2.) a.) $4! = 4 \times 3 \times 2 \times 1 = 12 \times 2 = 24$

b.) $\binom{8}{5} = \frac{8!}{5!(8-5)!} = \frac{8\times 7\times 6}{6} = 8 \times 7 = 56$

3.) a.) Suppose $k = n + 1$, then

$$\begin{aligned}\binom{n}{k-1} + \binom{n}{k} &= \binom{n}{n+1-1} + \binom{n}{n+1} \\ &= \binom{n}{n} + 0 = 1 \\ &= \binom{n+1}{n+1} \\ &= \binom{n+1}{k}\end{aligned}$$

Thus $k = n + 1$ satisfies the equation. Now suppose $k < 0$. According to the definition, $\binom{n}{k-1} = \binom{n}{k} = \binom{n+1}{k} = 0$, thus

$$\binom{n}{k-1} + \binom{n}{k} = 0 + 0 = \binom{n+1}{k} = 0$$

Thus the equality holds for all $k \leq n + 1$. Q.E.D.

b.) We have not considered $k > n+1$. However, this case is similar to proving for all $k < 0$, as we can leverage the binomial definition to show that all terms equal 0 given $k > n + 1$.

4.) According to the binomial theorem, we know that

$$(1 + x)^n = \sum_{k=0}^{n} \binom{n}{k} x^k$$

We can manipulate this as follows:

$$\begin{aligned}(1 + x)^n &= \sum_{k=0}^{n} \binom{n}{k} x^k = \binom{n}{0} x^0 + \binom{n}{1} x^1 + \ldots + \binom{n}{k} x^k \\ (1 + 1)^n &= \binom{n}{0} 1^0 + \binom{n}{1} 1^1 + \ldots + \binom{n}{k} 1^k = \binom{n}{0} + \binom{n}{1} + \ldots + \binom{n}{k} \\ 2^n &= \sum_{k=0}^{n} \binom{n}{k}\end{aligned}$$

Thus the equality holds. Q.E.D.

5.) Consider the base case where $n = 1$:

$$\sum_{k=1}^{1}(2k-1) = 2(1) - 1 = 1 = 1^2$$

Thus the base case holds. Next, suppose n satisfies the equation, and consider $n+1$:

$$\begin{aligned}\sum_{k=1}^{n+1}(2k-1) &= \sum_{k=1}^{n}(2k-1) + 2(n+1) - 1 \\ &= n^2 + 2n + 2 - 1 \\ &= n^2 + 2n + 1 \\ &= (n+1)^2\end{aligned}$$

Thus the equality holds. Q.E.D.

6.) Consider the base case where $n = 1$:

$$\sum_{k=1}^{1} k^3 = 1^3 = 1 = \frac{1^2(1+1)^2}{4} = \frac{4}{4} = 1$$

Thus the base case holds. Next, suppose n satisfies the equation, and consider $n+1$:

$$\begin{aligned}\sum_{k=1}^{n+1} k^3 &= \sum_{k=1}^{n} k^3 + (n+1)^3 \\ &= \frac{n^2(n+1)^2}{4} + \frac{4(n+1)^3}{4} \\ &= \frac{n^2(n^2+2n+1) + 4(n^3+3n^2+3n+1)}{4} \\ &= \frac{n^4 + 6n^3 + 13n^2 + 12n + 4}{4} \\ \text{(substituting } n+1\text{)} &= \frac{(n+1)^2((n+1)+1)^2}{4} \\ &= \frac{(n+1)^2(n+2)^2}{4} \\ &= \frac{(n^2+2n+1)(n^2+4n+4)}{4} \\ &= \frac{(n^4+4n^3+4n^2) + (2n^3+8n^2+8n) + (n^2+4n+4)}{4} \\ &= \frac{n^4 + 6n^3 + 13n^2 + 12n + 4}{4}\end{aligned}$$

Thus the equality holds. Q.E.D.

7.) a.) Let $a = 0$ and $b = 1$, thus $ax = 0x = 0$, thus $ax + b = 0 + 1 = 1 = 0 \Rightarrow\Leftarrow$, thus the statement is false. Q.E.D.

b.) The proof does not restrict a to the nonzero reals. When $a = 0$, $-b/a$ is undefined, making the equation invalid.

8.) a.) Consider $\sqrt[3]{y}$, $(\sqrt[3]{y})^3 = y$, thus for all y we can construct x such that $x^3 = y$, thus the statement is true. Q.E.D.

b.) Suppose you have $x, y \in \mathbb{R}$ such that $x^3 = y$. For the statement $\exists x \in \mathbb{R}$ $[\forall y \in \mathbb{R}(x^3 = y)]$ to be true, $x^3 = y + 1$ must hold, thus $x^3 = x^3 + 1$ but there are no real solutions x for this equation $\Rightarrow\Leftarrow$, thus the original statement is false. Q.E.D.

9.) a.) Consider $n + 1$: $n + 1 \in \mathbb{N}$ and $n + 1 > n$, thus the statement is true. Q.E.D.

b.) Consider $j + 1$: $j + 1 \in \mathbb{N}$, but $j \ngeq j + 1$, thus the statement is false. Q.E.D.

10.) a.) Let $x = 0$, $xy = 0y = 0$ for all real y, thus the statement is true. Q.E.D.

b.) Let $y = 1$, $xy = 1x = x$ for all real x, thus the statement is true. Q.E.D.