Chapter 8

3.) Let $(G, *)$ and $(H, \circ)$ be groups with respective identity elements e_G and e_H. Consider $f : G \to G \bigoplus \{e_H\}$ where $g \mapsto (g, e_H)$. We can show that f is bijective. Let $g_1, g_2 \in G$ where $f(g_1) = f(g_2)$, thus $(g_1, e_H) = (g_2, e_H)$, thus $g_1 = g_2$, thus f is injective. Next, for all $(g, e_H) \in G \bigoplus \{e_H\}$, $f(g) = (g, e_H)$, thus f is injective, and thus bijective, and thus is an isomorphism from G to $G \bigoplus \{e_H\}$, thus $G \cong G \bigoplus \{e_H\}$. A similar argument shows that $h \mapsto (h, e_G)$ is an isomorphism from H to $H \bigoplus \{e_G\}$, thus $H \cong H \bigoplus \{e_G\}$. ■

6.) awd

14.) awd

20.) awd

55.) awd