In $\mathbb{R}^3$, we know that a tetrahedron has four 2-dimensional equilateral triangle faces. These faces are all adjacent to one another. Given the definition of Δ^3,

$$\Delta^3 = \left\{ (x_1, x_2, x_3, x_4) \in \mathbb{R}^4 : \sum_{i=1}^{4} x_i = 1 \text{ and } x_i \geq 0 \right\}$$

let $X_i = \left\{ (x_1, x_2, x_3, x_4) \in \Delta^3 : x_i = 0 \right\}$, and consider $f : X_1 \to \Delta^2$ where $(0, x_2, x_3, x_4) \mapsto (x_2, x_3, x_4)$. If f is a bijection, then we know that X_1 is equal to Δ^2, and thus X_1 is an equilateral triangle. First we will prove injectivity.