9.3.3 Since $f_n \to f$ pointwise on D, thus for all $x \in D$, we know that $f(x) = \lim_{x\to\infty} f_n(x)$, thus for all $\varepsilon > 0$, there exists $N_x \in \mathbb{N}$ that depends on x where

$$n > N_x \implies |f_n(x) - f(x)| < \varepsilon$$

Since D is finite, let $N = \max_{x\in D} \{N_x\}$, thus for all $x \in D$, we have

$$n > N \implies |f_n(x) - f(x)| < \varepsilon\,,$$

, thus $f_n \to f$ uniformly on D. ■

9.3.4 Since $f_n \to f$ uniformly on E_1 and E_2, thus for all $\varepsilon > 0$, there exist $N_1, N_2 \in \mathbb{N}$ where

$$x \in E_1 \text{ and } n > N_1 \implies |f_n(x) - f(x)| < \varepsilon$$

and

$$x \in E_2 \text{ and } n > N_2 \implies |f_n(x) - f(x)| < \varepsilon\,.$$

Let $N = \max\{N_1, N_2\}$, thus we have

$$x \in E_1 \cup E_2 \text{ and } n > N \implies |f_n(x) - f(x)| < \varepsilon\,,$$

thus $f_n \to f$ uniformly on $E_1 \cup E_2$. ■