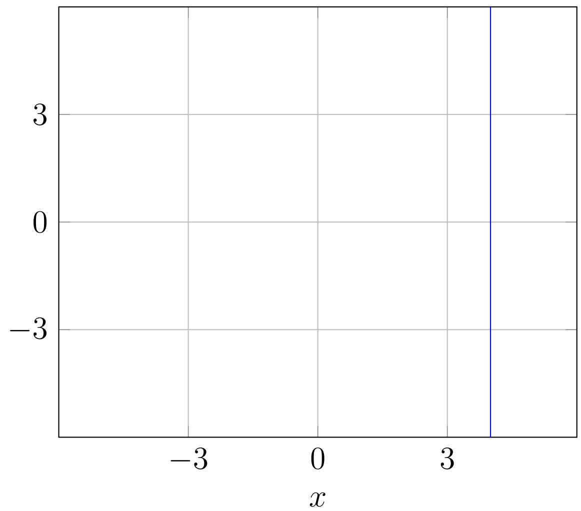
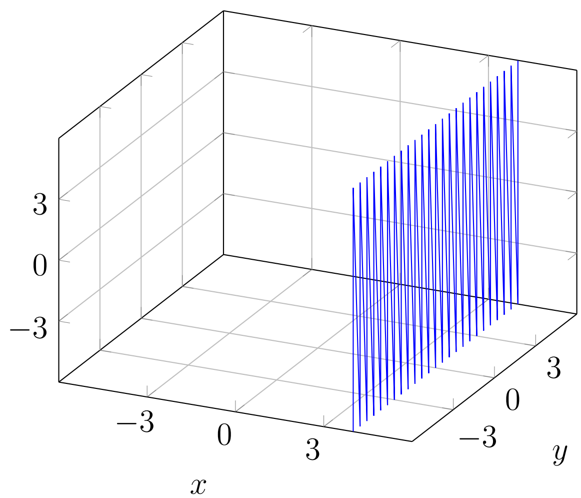
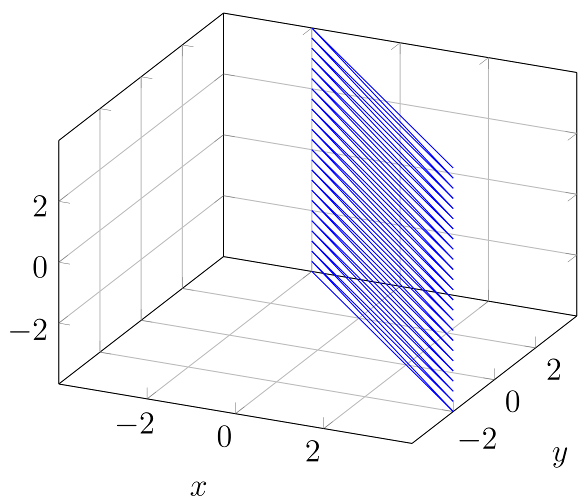
Exercises 12.1

5.) In $\mathbb{R}^2$, $x = 4$ represents a line, while in $\mathbb{R}^3$, $x = 4$ represents a plane.





7.) $x + y = 2$ represents a plane in $\mathbb{R}^3$:



12.) c.) $y = -2$, thus the distance between $(4, -2, 6)$ and the xz-plane is $|y| = 2$.

e.) $x = 4$ and $z = 6$, thus the distance between $(4, -2, 6)$ and the y-axis is $\| \langle x, y \rangle \| = \sqrt{4^2 + 6^2} = \sqrt{16 + 36} = \sqrt{52} = 2\sqrt{13}$.

13.) The equation for the sphere is $(x + 3)^2 + (y - 2)^2 + (z - 5)^2 = 16$; setting $x = 0$ for the intersection of the sphere and the yz-plane, we find it to be $(y - 2)^2 + (z - 5)^2 = 16 - 9 = 7$, or a circle with center $(0, 2, 5)$ and radius $\sqrt{7}$.

15.) The line segment between points $P(4, 3, -1)$ and $Q(3, 8, 1)$ is a radius of the sphere, thus $\|\vec{p} - \vec{q}\| = \sqrt{(4-3)^2 + (3-8)^2 + (-1-1)^2} = \sqrt{1 + 25 + 4} = \sqrt{30}$ is the length of the radius, thus the equation for the sphere is $(x - 3)^2 + (y - 8)^2 + (z - 1)^2 = 30$.

17.) We can rearrange the equation as follows:

$$
\begin{aligned}
x^2 + y^2 + z^2 - 2x - 4y + 8z &= 15 \\
(x^2 - 2x + 1) + (y^2 - 4y + 4) + (z^2 + 8z + 16) &= 15 + 1 + 4 + 16 \\
(x - 1)^2 + (y - 2)^2 + (z + 4)^2 &= 36 = 6^2
\end{aligned}
$$

Thus the equation represents a sphere with center $(1, 2, -4)$ and radius 6.

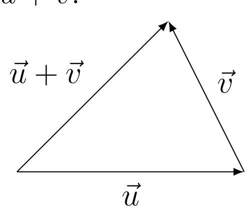
27.) $y < 8$ represents all points behind the plane $y = 8$.

29.) $0 \le z \le 6$ represents all points on or between the planes $z = 0$ and $z = 6$.

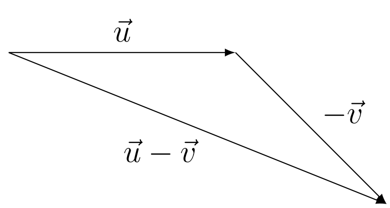
35.) $1 \le x^2 + y^2 + z^2 \le 5$ represents a spherical shell with an inner radius of 1 and an outer radius of $\sqrt{5}$.

Exercises 12.2

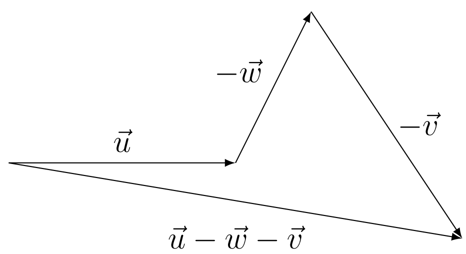
5.) a.) $\vec{u} + \vec{v}$:



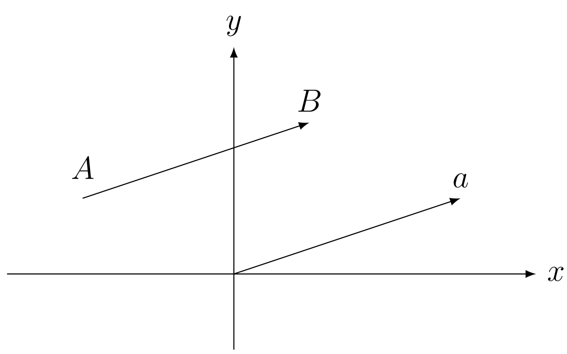
d.) $\vec{u} - \vec{v}$:



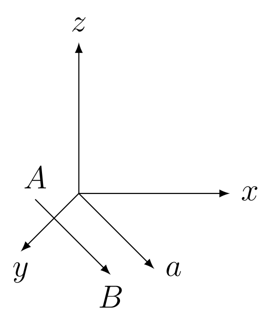
f.) $\vec{u} - \vec{w} - \vec{v}$:



9.) $B - A = \langle 1, 2\rangle - \langle -2, 1\rangle = \langle 3, 1\rangle$



13.) $B - A = \langle 2, 3, -1\rangle - \langle 0, 3, 1\rangle = \langle 2, 0, -2\rangle$



19.) $a + b = \langle -3, 4\rangle + \langle 9, -1\rangle = \langle 6, 3\rangle$
$4a + 2b = 4\langle -3, 4\rangle + 2\langle 9, -1\rangle = \langle -12, 16\rangle + \langle 18, -2\rangle = \langle 6, 14\rangle$
$|a| = \sqrt{(-3)^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$
$|a - b| = \sqrt{(-12)^2 + 5^2} = \sqrt{144 + 25} = \sqrt{169} = 13$

21.) $a + b = 4i - 3j + 2k + 2i - 4k = 6i - 3j - 2k$
$4a + 2b = 4(4i - 3j + 2k) + 2(2i - 4k) = 16i - 12j + 8k + 4i - 8k = 20i - 12j$
$|a| = |4i - 3j + 2k| = \sqrt{4^2 + (-3)^2 + 2^2} = \sqrt{16 + 9 + 4} = \sqrt{29}$
$|a - b| = |2i - 3j - 6k| = \sqrt{2^2 + (-3)^2 + (-6)^2} = \sqrt{4 + 9 + 36} = \sqrt{49} = 7$

25.) Let $\vec{v} = 8i - j + 4k$. We can find the unit vector of $\vec{v}$ by evaluating $\frac{\vec{v}}{\|\vec{v}\|}$.

$$\|\vec{v}\| = \sqrt{8^2 + (-1)^2 + 4^k} = \sqrt{64 + 1 + 16} = \sqrt{81} = 9$$

$$\frac{\vec{v}}{\|\vec{v}\|} = \frac{1}{9}\vec{v} = \frac{8}{9}i - \frac{1}{9}j + \frac{4}{9}k$$

Which is the unit vector of $\vec{v}$.

31.) $x = r\cos\theta = 60\cos(40°) \approx 45.96$
$y = r\sin\theta = 60\sin(40°) \approx 38.57$

Exercises 12.3

3.) $a \cdot b = [1.5(-4) + 0.4(6)] = -6 + 2.4 = -3.6$

7.) $a \cdot b = [2(1) + 1(-1) + 0(1)] = 2 - 1 + 0 = 1$

9.) $a \cdot b = \|a\|\|b\|\cos\theta = (7)(4)\cos(30°) = 28\frac{\sqrt{3}}{2} = 14\sqrt{3}$

19.) $\|a\| = \sqrt{4^2 + (-3)^2 + 1^2} = \sqrt{16 + 9 + 1} = \sqrt{26}$
$\|b\| = \sqrt{2^2 + 0^2 + (-1)^2} = \sqrt{4 + 1} = \sqrt{5}$
$a \cdot b = [4(2) - 3(0) + 1(-1)] = (8 - 1) = 7$
$\frac{a \cdot b}{\|a\|\|b\|} = \frac{7}{\sqrt{130}}$
$\theta = \cos^{-1}(\frac{7}{\sqrt{130}}) \approx 52.13°$

27.) Let $\vec{v} = \langle x, y, z\rangle$. For $\vec{v}$ to be orthogonal to both $i+j$ and $i+k$, $\vec{v}\cdot\langle 1, 1, 0\rangle = \vec{v}\cdot\langle 1, 0, 1\rangle = 0$, thus $x + y = x + z = 0$. Let $\vec{v} = \langle 1, -1, -1\rangle$, thus $\vec{v} \cdot \langle 1, 1, 0\rangle = [1(1) - 1(1) - 1(0)] = 1 - 1 = 0$, and $\vec{v} \cdot \langle 1, 0, 1\rangle = [1(1) - 1(0) - 1(1)] = 1 - 1 = 0$, thus $\vec{v}$ is orthogonal to both vectors. To find the unit vector of $\vec{v}$, evaluate $\frac{\vec{v}}{\|v\|} = \frac{\vec{v}}{\sqrt{3}} = \left\langle \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}} \right\rangle$.

29.) $2x - y = 3 \implies y = 2x - 3$, thus $\langle 1, 2\rangle$ represents this equation.
$3x + y = 7 \implies y = 7 - 3x$, thus $\langle 1, -3\rangle$ represents this equation.
$\|\langle 1, 2\rangle\| = \sqrt{1^2 + 2^2} = \sqrt{5}$, $\|\langle 1, -3\rangle\| = \sqrt{1^2 + (-3)^2} = \sqrt{10}$
$\langle 1, 2\rangle \cdot \langle 1, -3\rangle = (1(1) + 2(-3)) = 1 - 6 = -5$
$\theta = \cos^{-1}\left(\frac{-5}{\sqrt{5}\sqrt{10}}\right) = 135°$, thus the acute angle is $45°$.