So far, the concept that has given me the most trouble in this class has been the product topology and product spaces. I feel like I understand the definition well enough, but when it comes to extracting properties of open sets in a product space, I feel that I tend to make assumptions based on my intuition of these spaces that turn out to be false. This in turn leads to me doubting my own intuition of how these spaces behave. Because of this, I am aiming to establish a better intuition of product spaces to accompany my understanding of their definition.

The two examples I chose to work were the past two extra credit problems, as they were both challenging problems that involved product spaces. On submission, extra credit problem 9 had an error. I falsely assumed that

$$W_\alpha = U_\alpha \times V_\alpha$$

where W_α was an open set in some product topology, and where U_α and V_α were unions of the respective collections of opens set that comprised W_α. It turns out this equality does not hold, as the relationships between the individual sets in the collections are destroyed when the union is taken, so certain elements that did not exist in any pair of sets before might exist in the union. As a result, my proof was invalid, but this new insight into a property of sets in product topologies has led to a more accurate intuition of how they behave. As for extra credit problem 10, I didn't make any errors, but it did serve as a good exercise in juggling different types of topologies, including the product topology, at the same time.

Overall, this exposure to working with product spaces has strengthened not only my understanding of the definition, but has also corrected some errors in my previous intuition. The error I made in problem 9 was effective in highlighting a gap in my intuition that I was then able to fix, and problem 10 served as a good exercise to strengthen my knowledge of how the product topology interacts with other topologies.