

a.) Given $p : S^3 \rightarrow \mathbb{C} \times \mathbb{R}$, consider $\mathcal{I} = \text{Im}(p)$ and S^2 :

$$\mathcal{I} = \{(z, x) \in \mathbb{C} \times \mathbb{R} : \exists (w_1, w_2) \in S^3 \text{ where } p(w_1, w_2) = (z, x)\}$$

$$S^2 = \left\{ (x_1, x_2, x_3) \in \mathbb{R}^3 : \sum_{i=1}^3 x_i^2 = 1 \right\}$$

If we show that $a \in \mathcal{I} \iff a \in S^2$, then we will have shown that $\mathcal{I} = S^2$.

First, let $a \in \mathcal{I} = (z, x) = (2w_1\overline{w_2}, w_1\overline{w_1} - w_2\overline{w_2})$ for some $w_1, w_2 \in S^3$.

b.) awd

c.) awd