Section 3.7

1.) Since $c_1 = 3$ and $c_2 = 4$, we can find δ and R:

$$\delta = \tan^{-1}\left(\frac{c_2}{c_1}\right) = \tan^{-1}\left(\frac{4}{3}\right)$$

$$R = \sqrt{c_1^2 + c_2^2} = \sqrt{3^2 + 4^2} = 5$$

By the original equation, $\omega_0 = 2$, thus the solution can be written as

$$5\cos[2t - \tan^{-1}(4/3)]$$

2.) Since $c_1 = -2$ and $c_2 = -3$, we can find δ and R:

$$\delta = \tan^{-1}\left(\frac{3}{2}\right)$$

$$R = \sqrt{(-2)^2 + (-3)^2} = \sqrt{4+9} = \sqrt{13}$$

By the original equation, $\omega_0 = \pi$, thus the solution can be written as

$$\sqrt{13}\cos[\pi t - \tan^{-1}(3/2)]$$

3.) Since $L = 0.05$ and $m = 0.1$, $k = \dfrac{mg}{L} = \dfrac{0.98}{0.05} = 19.6$. We can solve the resulting initial value problem:

$$0.1y'' + 19.6y = 0;\ y(0) = 0,\ y'(0) = 0.1$$

$$\implies y'' + 196y = 0 \implies r = \pm 14i$$

$$\therefore\ y = c_1\sin(14t) + c_2\cos(14t)$$

Solving for c_1 and c_2:

$$0 = c_1\sin(0) + c_2\cos(0) = c_2$$

$$0.1 = 14c_1\cos(0) + 14c_2\sin(0) = 14c_1 \implies c_1 = \frac{0.1}{14} = \frac{1}{140}$$

$$\therefore\ y = \frac{1}{140}\sin(14t)$$

Since $\sin(\pi) = 0$, we know that when $t = \dfrac{\pi}{14}$, $y(t) = 0$, thus the spring returns to equilibrium after $\dfrac{\pi}{14}$ seconds.

4.) Since $L = 0.25$ and $m = 3$, $k = \dfrac{96}{0.25} = 384$. We can solve the resulting initial value problem:

$$3y'' + 384y = 0 \implies y'' + 128y = 0;\; y(0) = -\frac{1}{12},\; y'(0) = 2$$

$$\implies r = \pm 8\sqrt{2}i$$

$$\therefore\; y = c_1 \sin\left(8\sqrt{2}t\right) + c_2 \cos\left(8\sqrt{2}t\right)$$

Solving for c_1 and c_2:

$$-\frac{1}{12} = c_1 \sin(0) + c_2 \cos(0) = c_2$$

$$2 = 8\sqrt{2}c_1 \cos(0) + 8\sqrt{2}c_2 \sin(0) = 8\sqrt{2}c_1 \implies c_1 = \frac{1}{4\sqrt{2}}$$

$$\therefore\; y = \frac{1}{4\sqrt{2}} \sin\left(8\sqrt{2}t\right) - \frac{1}{12} \cos\left(8\sqrt{2}t\right)$$

With c_1 and c_2, we can find R and δ:

$$R = \sqrt{\left(-\frac{1}{12}\right)^2 + \left(\frac{1}{4\sqrt{2}}\right)^2} = \sqrt{\frac{1}{144} + \frac{1}{32}} = \sqrt{\frac{11}{288}}$$

$$\delta = \tan^{-1}\left(\frac{c_2}{c_1}\right) \approx -25.24$$

And since $\omega = 8\sqrt{2}$, the period T is $\dfrac{1}{8\sqrt{2}}$.

Section 3.8

4.) Since $L = 0.1$ and $m = 5$, $k = \dfrac{49}{0.1} = 490$. In addition, $\gamma = \dfrac{2}{0.04} = 50$. This can be modeled with the following initial value problem:

$$5y'' + 50y' + 490y = 10\sin(t/2) \implies y'' + 10y' + 98y'' = 2\sin(t/2)$$

$$\text{where } y(0) = 0,\; y'(0) = 0.03$$

7a.) Since $L = 0.5$ and $m = 8$, $k = \dfrac{256}{0.5} = 512$. In addition, $\gamma = \dfrac{1}{4}$. We can solve the resulting initial value problem:

$$8y'' + \frac{1}{4}y' + 512y = 4\cos(2t) \implies y'' + \frac{1}{32}y' + 64y = \frac{1}{2}\cos(2t)$$

Which gives us some $y_h + Y$, where Y is the particular (steady state) solution.