88.) We can use induction to show that $x_n > 0$ for all $n \in \mathbb{N}$. Since $x_1 = 2 > 0$, the base case holds. For the induction step, assume $x_n > 0$:

$$x_n > 0 \implies \frac{1}{x_n} > 0 \text{ and } \frac{x_n}{2} > 0$$
$$\implies \frac{1}{x_n} + \frac{x_n}{2} = x_{n+1} > 0$$

Thus $x_n > 0 \implies x_{n+1} > 0$, thus x_n is bounded below by 0.

107.) a.) awd

b.) awd

c.) Consider y_n when $n = 2^k - 1$ for some $k \in \mathbb{N}$:

$$y_n = 1 + \left(\frac{1}{2} + \frac{1}{2}\right) + \left(\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}\right) + \cdots + \left(\frac{1}{2^k} + \cdots + \frac{1}{2^k}\right)$$

We can see that the terms in each set of parentheses sum to 1, thus $y_n = 1 + 1 + \cdots + 1$. As $n \to \infty$, this sum diverges, thus y_n is not bounded, convergent, nor cauchy. However, y_n is nondecreasing, and thus monotone.

140.) Let $f : \mathbb{R} \to \mathbb{R}$ be defined as follows:

$$f(x) = \begin{cases} \dfrac{1}{x-1} + 1 & x < 0 \\ \dfrac{1}{x+1} - 1 & x > 0 \\ 0 & x = 0 \end{cases}$$

Since $-1 < f(x) < 1$ for all $x \in \mathbb{R}$, $f(x)$ is bounded, but since there are horizontal asymptotes at $y = -1$ and $y = 1$, $f(x)$ has no maximal nor minimal value.

152.) a