1.) Manipulate to a function of $\frac{y}{x}$:

$$\frac{dy}{dx} = \frac{x^2 + xy + y^2}{x^2} = 1 + \frac{y}{x} + \frac{y^2}{x^2}$$

Substitute $y = ux$:

$$u + x\frac{du}{dx} = 1 + u + u^2 \implies x\frac{du}{dx} = 1 + u^2$$

Separate:

$$x\frac{du}{dx} = 1 + u^2 \implies \frac{du}{1+u^2} = \frac{dx}{x}$$

Solve for u:

$$\int \frac{du}{1+u^2} = \arctan u = \int \frac{dx}{x} = \ln|x| + C \implies u = \tan(\ln|x| + C)$$

Replace $u = \frac{y}{x}$ and solve for y:

$$\frac{y}{x} = \tan(\ln|x| + C) \implies y = x\tan(\ln|x| + C)$$

Which is the explicit solution.

2.) Manipulate to a function of $\frac{y}{x}$:

$$(x^2 + 3xy + y^2)dx - x^2 dy = 0 \implies \frac{dy}{dx} = \frac{x^2 + 3xy + y^2}{x^2} = 1 + \frac{3y}{x} + \frac{y^2}{x^2}$$

Substitute $y = ux$:

$$u + x\frac{du}{dx} = 1 + 3u + u^2 \implies x\frac{du}{dx} = 1 + 2u + u^2 = (1+u)^2$$

Separate:

$$x\frac{du}{dx} = (1+u)^2 \implies \frac{du}{(1+u)^2} = \frac{dx}{x}$$

Solve for u:

$$\int \frac{du}{(1+u)^2} = -\frac{1}{1+u} = \int \frac{dx}{x} = \ln|x| + C \implies -1 - u = \frac{1}{\ln|x| + C}$$

$$\implies u = -\frac{1}{\ln|x| + C} - 1$$

Replace $u = \frac{y}{x}$ and solve for y:

$$\frac{y}{x} = -\frac{1}{\ln|x| + C} - 1 \implies y = -\frac{x}{\ln|x| + C} - x$$

Which is the explicit solution.

3.) Manipulate to a function of $\frac{y}{x}$:

$$x\frac{dy}{dx} = y + x\tan\left(\frac{y}{x}\right) \implies \frac{dy}{dx} = \frac{y}{x} + \tan\left(\frac{y}{x}\right)$$

Substitute $y = ux$:

$$u + x\frac{du}{dx} = u + \tan(u) \implies x\frac{du}{dx} = \tan(u)$$

Separate:

$$x\frac{du}{dx} = \tan(u) \implies \cot(u)du = \frac{dx}{x}$$

Solve for u:

$$\int \cot(u)du = \ln|\sin(u)| = \int \frac{dx}{x} = \ln|x| + C \implies u = \arcsin\left(xe^C\right)$$

Replace $u = \frac{y}{x}$ and solve for y:

$$\frac{y}{x} = \arcsin\left(xe^C\right) \implies y = x\arcsin\left(xe^C\right)$$

Which is the explicit solution.