

- 2.) a.)  $\gcd(2, 10) = 2$ ,  $\text{lcm}(2, 10) = 10$   
 b.)  $\gcd(20, 8) = 4$ ,  $\text{lcm}(20, 8) = 40$   
 c.)  $\gcd(12, 40) = 4$ ,  $\text{lcm}(12, 40) = 120$   
 d.)  $\gcd(21, 50) = 1$ ,  $\text{lcm}(21, 50) = 1050$

e.)  $\gcd(p^2q^2, pq^3) = pq^2$ ,  $\text{lcm}(p^2q^2, pq^3) = p^2q^3$

- 7.) Let  $a, b \in \mathbb{Z}$  and  $n \in \mathbb{N}$ . Assume  $a \equiv b \pmod{n}$ , thus  $a = b + kn$  for some  $k \in \mathbb{Z}$ .  
 $a = b + kn \implies a - b = kn \implies n \mid a - b$ , thus  $a \equiv b \pmod{n} \implies n \mid a - b$ .

Next, assume  $n \mid a - b$ , thus  $ln = a - b$  for some  $l \in \mathbb{Z}$ .  $ln = a - b \implies a = b + ln$   
 $\implies a \equiv b \pmod{n}$ , thus  $n \mid a - b \implies a \equiv b \pmod{n}$ ,

Thus  $a \equiv b \pmod{n} \iff n \mid a - b$ . Q.E.D.

- 18.)  $8^1 \equiv 3 \pmod{5}$ ,  $8^2 \equiv 4 \pmod{5}$ ,  $8^3 \equiv 2 \pmod{5}$ ,  $8^4 \equiv 1 \pmod{5}$ ,  $8^5 \equiv 3 \pmod{5}$ ,  
 thus  $8^{402} \equiv (8^4)^{100} \times 8^2 \equiv 1^{100} \times 4 \equiv 4 \pmod{5}$

- 58.) Define a relation  $R$  on  $\mathbb{R}$  where  $a \sim b \implies a - b \in \mathbb{Z}$  for  $a, b \in \mathbb{R}$ . Since for all  $a \in \mathbb{R}$ ,  
 $a - a = 0$ , we know that  $a \sim a$ , thus  $R$  is reflexive.

Next, assume  $a \sim b$ , thus  $a - b \in \mathbb{Z}$ . Let  $n = a - b$ ;  $n = a - b \implies -n = -(a - b) = b - a$ .  
 Since  $n \in \mathbb{Z} \implies -n \in \mathbb{Z}$ , we know that  $b \sim a$ , thus  $R$  is symmetric. \*\*\*

- 60.) awd