9) Let $f : \mathbb{R}^d \to \mathbb{R}$ be a non-negative integrable function. If $\alpha > 0$ and $E_\alpha = \{x \in \mathbb{R}^d : f(x) > \alpha\}$, then

$$m(E_\alpha) \leq \frac{1}{\alpha} \int_{E_\alpha} f(x)\, dx.$$

Proof: We can see that

$$m(E_\alpha) = \int_{E_\alpha} 1\, dx.$$

Additionally, if $x \in E_\alpha$, then $f(x) > \alpha$, hence $f(x)/\alpha > 1$. Thus, by monotonicity, we have that

$$\int_{E_\alpha} 1\, dx < \int_{E_\alpha} \frac{f(x)}{\alpha}\, dx = \frac{1}{\alpha} \int_{E_\alpha} f(x)\, dx,$$

which proves the inequality. ■

17)