Let X and Y be compact topological spaces, and consider an open cover $\{W_\alpha\}_{\alpha\in\mathcal{A}}$ of $X \times Y$. For each $\alpha \in \mathcal{A}$, there exist collections $\{U_\beta\}_{\beta\in\mathcal{B}_\alpha}$ and $\{V_\beta\}_{\beta\in\mathcal{B}_\alpha}$ of open sets in X and Y respectively, where

$$W_\alpha = \bigcup_{\beta\in\mathcal{B}_\alpha} U_\beta \times V_\beta \,.$$

In addition, let $\{U_\alpha\}_{\alpha\in\mathcal{A}}$ and $\{V_\alpha\}_{\alpha\in\mathcal{A}}$ be collections where

$$U_\alpha = \bigcup_{\beta\in\mathcal{B}_\alpha} U_\beta \quad \text{and} \quad V_\alpha = \bigcup_{\beta\in\mathcal{B}_\alpha} V_\beta \,,$$

thus $W_\alpha = U_\alpha \times V_\alpha$. It is easy to see that U_α is open for all α. We can also see that $(x, y) \in W_\alpha \implies x \in U_\alpha$, so $\{U_\alpha\}_{\alpha\in\mathcal{A}}$ is an open cover of X. A similar argument shows that $\{V_\alpha\}_{\alpha\in\mathcal{A}}$ is an open cover of Y. Now, using the compactness of X and Y, we obtain respective finite subcovers $\{U'_\alpha\}_{\alpha\in\mathcal{A}'}$ and $\{V'_\alpha\}_{\alpha\in\mathcal{A}'}$, where $\mathcal{A}' \subseteq \mathcal{A}$ and is finite. Finally, let $\{W'_\alpha\}_{\alpha\in\mathcal{A}'}$ be a collection where $W'_\alpha = U'_\alpha \times V'_\alpha$. We can see that $(x, y) \in W_\alpha \implies (x, y) \in U_\alpha \times V_\alpha \implies \exists \alpha' \in \mathcal{A}' : (x, y) \in U'_{\alpha'} \times V'_{\alpha'} \implies (x, y) \in W'_{\alpha'}$, thus $\{W'_\alpha\}_{\alpha\in\mathcal{A}'}$ is a cover of $X \times Y$, and since $\mathcal{A}' \subseteq \mathcal{A}$ and is finite, we know $\{W'_\alpha\}_{\alpha\in\mathcal{A}'}$ is a finite subcover of $\{W_\alpha\}_{\alpha\in\mathcal{A}}$, thus $X \times Y$ is compact. $\blacksquare$