

7.11.1.) a.) For f to be continuous at $x = 0$, we must have $f(0) = \lim_{x \rightarrow 0} f(x)$, thus

$$f(0) = \lim_{x \rightarrow 0} \frac{3^x - 2^x}{x} = \lim_{x \rightarrow 0} \frac{3^x \ln(3) - 2^x \ln(2)}{1} = 3^0 \ln(3) - 2^0 \ln(2) = \ln(3) - \ln(2)$$

Thus defining $f(0) = \ln(3) - \ln(2)$ makes f continuous at $x = 0$. ■

b.) To determine if $f'(0)$ exists, consider the following limit:

$$\lim_{x \rightarrow 0} \frac{f(x) - f(0)}{x - 0} = \lim_{x \rightarrow 0} \frac{3^x - 2^x - \ln(3) + \ln(2)}{x} = \frac{\ln(2) - \ln(3)}{0} \implies \text{diverges}$$

Thus $f'(0)$ does not exist given our definition of $f(0)$. ■

c.) As long as the domain of

7.11.5.) awd