12.) ***

21.) awd

25.) Let $A \subseteq B \subseteq \mathbb{R}$ and $b \in \mathbb{R}$ such that $b = \inf(B)$.

28.) a.) Let $u \in \mathbb{R}$ be an upper bound of S, thus $u \geq x$ for all $x \in S$, thus $-u \leq -x$ for all $x \in S$, thus $-u \leq y$ for all $y \in -S$, thus $-u$ is a lower bound of $-S$, thus $-S$ is bounded below. Q.E.D.

b.) Let $u = \sup(S)$. Since u is an upper bound of S, $-u$ is a lower bound of $-S$. For the sake of establishing a contradiction, suppose there exists $v \in \mathbb{R}$ such that $-u < v$ and v is a lower bound of $-S$, thus $u > -v$. Since v is a lower bound of $-S$, $v \leq y$ for all $y \in -S$, thus $-v \geq -y$ for all $y \in -S$, thus $-v \geq x$ for all $x \in S$, thus $-v$ is an upper bound of S, but since $u > -v$, $u \neq \sup(S)$ $\Rightarrow\!\Leftarrow$, thus $-\sup(S) = -u = \inf(-S)$. Q.E.D.

29.) a.) Since $S \neq \varnothing$ and is bounded below, $\mathscr{L} \neq \varnothing$. In addition, since S is bounded below, there exists $v \in \mathbb{R}$ such that $v = \inf(S)$, thus $v \geq x$ for all $x \in \mathscr{L}$, thus v is an upper bound of $\mathscr{L}$, thus $\mathscr{L}$ is bounded above. Q.E.D.

b.) Let $w = \sup(\mathscr{L})$, thus $w \geq x$ for all $x \in \mathscr{L}$.

c.) ***

30.) a.) For unbounded sets, infinite suprema and infima make sense, as for all $x \in \mathbb{R}$, $-\infty < x < \infty$. In addition, there exists no $y \in \mathbb{R}$ such that $y < -\infty$ or $y > \infty$, thus $\sup(S) = \infty$ and $\inf(S) = -\infty$.

b.) ***

31.) a.) False; let $S = \{x \in \mathbb{Q} : 0 \leq x < \pi\}$. By definition, all $x \in S$ are rational, but $\sup(S) = \pi$ is irrational.

b.) False; let $S = \{x \in \mathbb{R}\backslash\mathbb{Q} : 0 < x < 3\}$. By definition, all $x \in S$ are irrational, but $\sup(S) = 3$ is rational.

33.) Let $x, y \in \mathbb{R}$. From the triangle inequality, we know that

$$|x + y| \leq |x| + |y|$$

34.) a.) False; let $S = (-\infty, 0]$, thus $\{|x| : x \in S\} = [0, \infty)$, which has no upper bound.

b.) True; let $u = \sup(\{|x| : x \in S\})$ ***

35.) a.) Suppose S is bounded, thus there exist u, v such that u is an upper bound of S and v is a lower bound of S, thus $v \leq x \leq u$ for all $x \in S$. Let $w = \max(u, v)$, thus $-w \leq x \leq w$, thus $|x| \leq w$ for all $x \in S$, thus there exists $M \geq 0$ such that $|x| \leq M$ for all $x \in S$. Now, suppose there exists such M, thus $|sxs$

b.) Suppose S is bounded, and let $x \in S$. Let $u, v \in \mathbb{R}$ such that u is an upper bound of S and v is a lower bound of S, thus $v \leq x \leq u$.

36.) When learning the various integration techniques taught in calculus 2, I noticed that some of them involved manipulating the dx term. I initially found this confusing, as I though that $\dfrac{d}{dx}$ was a single operator that could not be seperated. I later found out that this was not the case, and that it can often be treated just like a fraction. What helped me to realize this was when I watched a video series that explained calculus in a more intuitive, less purely algebraic way. In this context, dx being a seperate variable simply made sense.

37.) awed

38.) Let $u = \sup(S)$, thus $u \geq x$ for all $x \in S$. For the sake of establishing a contradiction, suppose $u \notin S$, then for some $\epsilon > 0$, $u = x + \epsilon$ for some $x \in S$. Consider $x + \frac{\epsilon}{2}$. Since $\epsilon > 0$, $\frac{\epsilon}{2} > 0$,

39.) a.) A sequence is defined as a function $x(n)$ such that $x : \mathbb{N} \to \mathbb{R}$.

b.) The sequence $\{x_n\}_{n=1}^{\infty}$ converges to $L \in \mathbb{R}$ if and only if

$$\forall \epsilon > 0 : \exists k \in \mathbb{N} : n \geq k \implies |x_n - L| < \epsilon$$

43.) a.) $\displaystyle\lim_{n \to \infty} \frac{1}{10n} = 10$

b.) $\displaystyle\lim_{n \to \infty} \sin n$ diverges

c.) awd

44.) ***