Definitions

1.) Given $x \in \mathbb{R}^n$ and $r > 0$, the open ball of radius r centered at x is defined as follows:

$$\text{Ball}(x, r) := \left\{ y \in \mathbb{R}^n : \sum_{i=1}^{n} (y_i - x_i)^2 < r^2 \right\}$$

2.) A subset $U \subset \mathbb{R}^n$ is open if it is a union of open balls

3.) Given $A \subset X$, the complement A' of A is the set defined as follows:

$$A' = \{x \in X : x \notin A\}$$

4.) A subset $U \subset \mathbb{R}^n$ is closed if its complement is open.

Proof

Given a subset $K \subset \mathbb{R}^n$,