1.) Establish initial variables:

$$V_0 = 200, Q_0 = 1, f_{in} = f_{out} = 2, c_{in} = 0, c_{out} = \frac{Q(t)}{V(t)}, V(t) = V_0 = 200$$

Setup differential equation for salt concentration:

$$\frac{dQ}{dt} = f_{in}c_{in} - f_{out}c_{out} = 0 - \frac{2Q}{200} \implies \frac{dQ}{dt} + \frac{Q}{100} = 0$$

Solve for Q:

$$\mu(t) = e^{\int \frac{1}{100} dt} = e^{\frac{t}{100}}$$

$$Q = \frac{1}{e^{\frac{t}{100}}} \left[\int (0) e^{\frac{t}{100}} \; dt \right] = \frac{C}{e^{\frac{t}{100}}}$$

Solve for C given $Q(0) = Q_0$:

$$1 = \frac{C}{e^{\frac{0}{100}}} = C$$

Solve for t given $Q(t) = \frac{Q_0}{100} = \frac{1}{100}$

$$\frac{1}{100} = \frac{1}{e^{\frac{t}{100}}} \implies 100 = e^{\frac{t}{100}} \implies t = 100 \ln(100) \approx 460.52$$

Which is the time (in minutes) before the concentration is 1% of the initial value.

2.) Establish initial variables:

$$V_0 = 120, Q_0 = 0, f_{in} = f_{out} = 2, c_{in} = \gamma, c_{out} = \frac{Q(t)}{V(t)}, V(t) = V_0 = 120$$

Setup differential equation for salt concentration:

$$\frac{dQ}{dt} = 2\gamma - \frac{Q}{60} \implies \frac{dQ}{dt} + \frac{Q}{60} = 2\gamma$$

Solve for Q:

$$\mu(t) = e^{\int \frac{1}{60} dt} = e^{\frac{t}{60}}$$

$$Q = \frac{1}{e^{\frac{t}{60}}} \left[\int 2\gamma e^{\frac{t}{60}} \; dt \right] = \frac{1}{e^{\frac{t}{60}}} \left(120\gamma e^{\frac{t}{60}} + C \right) = 120\gamma + \frac{C}{e^{\frac{t}{60}}}$$

Solve for C given $Q(0) = Q_0 = 0$:

$$0 = 120\gamma + \frac{C}{e^{\frac{0}{60}}} \implies C = -120\gamma$$

$$\therefore \; Q = 120\gamma - \frac{120\gamma}{e^{\frac{t}{60}}} = 120\gamma \left(1 - \frac{1}{e^{\frac{t}{60}}} \right)$$

Which is the explicit solution. To find the limiting amount of salt, take the limit as $t \to \infty$:

$$\lim_{t \to \infty} \left[120\gamma - \frac{120\gamma}{e^{\frac{t}{60}}} \right] = 120\gamma - 0 = 120\gamma$$

Which is the limiting amount.

3.) Establish initial variables:

$$V_0 = 100, Q_0 = 0, f_{in} = f_{out} = 2, c_{in} = \frac{1}{2}, c_{out} = \frac{Q(t)}{V(t)}, V(t) = V_0 = 100$$

Setup differential equation:

$$\frac{dQ}{dt} = 1 - \frac{Q}{50} \implies \frac{dQ}{dt} + \frac{Q}{50} = 1$$

Solve for Q:

$$\mu(t) = e^{\int \frac{1}{50} dt} = e^{\frac{t}{50}}$$

$$Q = \frac{1}{e^{\frac{t}{50}}} \left[\int e^{\frac{t}{50}} \; dt \right] = \frac{1}{e^{\frac{t}{50}}} \left(50 e^{\frac{t}{50}} + C \right) = 50 + \frac{C}{e^{\frac{t}{50}}}$$

Solve for C given $Q(0) = Q_0 = 0$:

$$0 = 50 + \frac{C}{e^{\frac{0}{50}}} \implies C = -50$$

$$\therefore \; Q = 50 \left(1 - \frac{1}{e^{\frac{t}{50}}} \right)$$

$Q(10) \approx 9.06$, and $c_{in} = 0$. Now, setup the equation with new initial values:

$$\frac{dQ}{dt} = 0 - \frac{2Q}{100} \implies \frac{dQ}{dt} + \frac{Q}{50} = 0$$

Solve for Q:

$$\mu(t) = e^{\int \frac{1}{50} dt} = e^{\frac{t}{50}}$$

$$Q = \frac{1}{e^{\frac{t}{50}}} \left[\int (0) e^{\frac{t}{50}} \right] = \frac{C}{e^{\frac{t}{50}}}$$

Solve for C given $Q(10) = 9.06$:

$$9.06 = \frac{C}{e^{\frac{10}{50}}} \implies C = e^{\frac{1}{5}} 9.06 \approx 11.07$$

$$\therefore \; Q(20) \approx 7.42$$

Which is the amount of salt at $t = 20$.

4.) Establish initial variables:

$$V_0 = 200, V_{max} = 500, Q_0 = 100, f_{in} = 3, c_{in} = 1, f_{out} = 2, c_{out} = \frac{Q(t)}{V(t)}$$

$$V_(t) = t(f_{in} - f_{out}) + V_0 = t + 200$$

Setup differential equation:

$$\frac{dQ}{dt} = 3 - \frac{2Q}{t+200} \implies \frac{dQ}{dt} + \frac{2Q}{t+200} = 3$$

Solve for Q:

$$\mu(t) = e^{\int \frac{2}{t+200} dt} = e^{2 \ln |t+200|} = (t+200)^2$$

$$Q = \frac{1}{(t+200)^2} \left[\int 3(t+200)^2 \, dt \right] = \frac{(t+200)^3}{(t+200)^2} + \frac{C}{(t+200)^2} = t + 200 + \frac{C}{(t+200)^2}$$

Solve for C given $Q(0) = Q_0 = 100$:

$$100 = 0 + 200 + \frac{C}{(0+200)^2} \implies -100 = \frac{C}{40000} \implies C = -4000000$$

$$\therefore \; y = t + 200 - \frac{4000000}{(t+200)^2}$$

When $t = 300$, $V(300) = 300 + 200 = 500$, so $t = 300$ is the point at which the tank overflows.

$$Q(300) = 300 + 200 - \frac{4000000}{(300+200)^2} = 500 - \frac{4000000}{500^2}$$

$$= 500 - \frac{4000000}{250000} = 500 - 16 = 484$$

Which is the concentration when the tank overflows. To find the theoretical maximum concentration, take the limit of $Q(t)$ as t approaches infinity:

$$\lim_{t \to \infty} \left[t + 200 - \frac{4000000}{(t+200)^2} \right] = \lim_{t \to \infty} [\infty + 200 - 0] \implies \text{diverges to } \infty$$

Thus the theoretical maximum concentration of salt in the tank is ∞.