1.3 Symmetric Groups

2) We have that

$$\sigma = (1\ 13\ 5\ 10)(3\ 15\ 8)(4\ 14\ 11\ 7\ 12\ 9)$$

and

$$\tau = (1\ 14)(2\ 9\ 15\ 13\ 4)(3\ 10)(5\ 12\ 7)(8\ 11),$$

thus

$$\begin{aligned}\sigma\tau &= (1\ 13\ 5\ 10)(3\ 15\ 8)(4\ 14\ 11\ 7\ 12\ 9)(1\ 14)(2\ 9\ 15\ 13\ 4)(3\ 10)(5\ 12\ 7)(8\ 11),\\ &= (1\ 11\ 3)(2\ 4)(5\ 9\ 8\ 7\ 10\ 15)(13\ 14).\end{aligned}$$

10) Let σ be an n-cycle and assume indices are taken as their lowest positive residue mod n. We will show by induction that $\sigma^i(a_k) = a_{k+i}$. For $i = 1$, we have by definition that $\sigma(a_k) = a_{k+1}$. Now, assume the induction hypothesis $\sigma^{i-1}(a_k) = a_{k+i-1}$, then $\sigma^i(a_k) = \sigma(\sigma^{i-1}(a_k)) = \sigma(a_{k+i-1}) = a_{k+i}$. ■

13) Let $\sigma \in S_n$ and consider its cycle decomposition $\sigma = \sigma_1\sigma_2 \cdots \sigma_m$. To establish a contradiction, assume that one of the cycles σ_i has length ≥ 3,

15)

18)

1.4 Matrix Groups

7)

8)

9)

10a)

10b)

1.6 Homomorphisms and Isomorphisms

3)

5)

6)

9)

11)

14)

20)

22)

25)