28.) a.) Let $u \in \mathbb{R}$ be an upper bound of S, thus $u \geq x$ for all $x \in S$, thus $-u \leq -x$ for all $x \in S$, thus $-u \leq y$ for all $y \in -S$, thus $-u$ is a lower bound of $-S$, thus $-S$ is bounded below. Q.E.D.

b.) Let $u = \sup(S)$. Since u is an upper bound of S, $-u$ is a lower bound of $-S$. For the sake of establishing a contradiction, suppose there exists $v \in \mathbb{R}$ such that $-u < v$ and v is a lower bound of $-S$, thus $u > -v$. Since v is a lower bound of $-S$, $v \leq y$ for all $y \in -S$, thus $-v \geq -y$ for all $y \in -S$, thus $-v \geq x$ for all $x \in S$, thus $-v$ is an upper bound of S, but since $u > -v$, $u \neq \sup(S) \Rightarrow\!\Leftarrow$, thus $-\sup(S) = -u = \inf(-S)$. Q.E.D.

30.) a.) For unbounded sets, infinite suprema and infima make sense, as for all $x \in \mathbb{R}$, $-\infty < x < \infty$. In addition, there exists no $y \in \mathbb{R}$ such that $y < -\infty$ or $y > \infty$, thus $\sup(S) = \infty$ and $\inf(S) = -\infty$.

b.) ***

33.) Let $x, y \in \mathbb{R}$, and consider $|x|$:

$$|x| = |x - y + y| \leq |x - y| + |y|$$

$$\implies |x| \leq |x - y| + |y|$$

$$\implies |x| - |y| \leq |x - y|$$

$$\implies (|x| - |y|) - (|x| - |y|) - |x - y| \leq |x| - |y| \leq |x - y|$$

$$\implies 0 - |x - y| = -|x - y| \leq |x| - |y| \leq |x - y|$$

$$\implies \Big||x| - |y|\Big| \leq |x - y|$$

Thus the inequality holds. Q.E.D.

36.) When learning the various integration techniques taught in calculus 2, I noticed that some of them involved manipulating the dx term. I initially found this confusing, as I though that $\dfrac{d}{dx}$ was a single operator that could not be seperated. I later found out that this was not the case, and that it can often be treated just like a fraction. What helped me to realize this was when I watched a video series that explained calculus in a more intuitive, less purely algebraic way. In this context, dx being a seperate variable simply made sense.

39.) a.) A sequence is defined as a function $x(n)$ such that $x : \mathbb{N} \to \mathbb{R}$.

b.) The sequence $\{x_n\}_{n=1}^{\infty}$ converges to $L \in \mathbb{R}$ if and only if

$$\forall \epsilon > 0 : \exists k \in \mathbb{N} : n \geq k \implies |x_n - L| < \epsilon$$