

2.) Find the critical points of y' :

$$y' = y(y-1)(y-2) \implies y = 0, 1, 2 \text{ are critical points}$$

Fill in the sign chart to determine solution stability:

	y	$(y-1)$	$(y-2)$	result
$y = -\frac{1}{2}$	-	-	-	-
$y = \frac{1}{2}$	+	-	-	+
$y = \frac{3}{2}$	+	+	-	-
$y = \frac{5}{2}$	+	+	+	+

Thus $y = 0, 2$ are unstable solutions and $y = 1$ is a stable solution.

5.) a.) For the sake of establishing a contradiction, suppose $y \neq 1$ is a critical point of y' , thus $k(1-y)^2 = 0$. We can manipulate the equation as follows:

$$\begin{aligned} k(1-y)^2 = 0 &\implies (1-y)^2 = 0 \\ &\implies (1-y) = \pm 0 = 0 \\ &\implies 1 = y \end{aligned}$$

But $y \neq 1 \Rightarrow \Leftarrow$, thus for y to be a critical point of y' , $y = 1$. Q.E.D.

b.) Fill in the sign chart to determine solution stability:

	$k(1-y)^2$	result
$y = \frac{1}{2}$	+	+
$y = \frac{3}{2}$	+	+

Thus $y = 1$ is a semistable solution.

c.) Solve for y :

$$\begin{aligned} \frac{dy}{dt} &= k(1-y)^2 \implies dy = k(1-y)^2 dt \\ \implies \int dy &= \int k(1-y)^2 dt \implies y = k \left[\int 1 - 2y + y^2 dt \right] \\ &\implies y = k [t - 2yt + y^2 t] \end{aligned}$$

Substituting $y(0) = y_0$:

6.) Find the critical points of y' :

$$y' = y^2(y^2 - 1) \implies y = -1, 0, 1 \text{ are critical points}$$

Fill in the sign chart to determine solution stability:

	y^2	$(y^2 - 1)$	result
$y = -\frac{3}{2}$	+	+	+
$y = -\frac{1}{2}$	+	-	-
$y = \frac{1}{2}$	+	-	-
$y = \frac{3}{2}$	+	+	+

Thus $y = -1$ is a stable solution, $y = 0$ is a semistable solution, and $y = 1$ is an unstable solution.

7.) Find the critical points of y' :

$$y' = y(1 - y^2) \implies y = -1, 0, 1 \text{ are critical points}$$

Fill in the sign chart to determine solution stability:

	y	$(1 - y^2)$	result
$y = -\frac{3}{2}$	-	-	+
$y = -\frac{1}{2}$	-	+	-
$y = \frac{1}{2}$	+	+	+
$y = \frac{3}{2}$	+	-	-

Thus $y = -1, 1$ are stable solutions, and $y = 0$ is an unstable solution.

- 14.) Since $f(y) = \frac{dy}{dt}$, $f'(y) = \frac{d^2y}{dt^2}$, thus $f'(y)$ can be used to determine the "acceleration" of the slope. If $f'(y)$ is