Exercises 16.3

3.)

$$M_y = x + 2y, \neq N_x = 2x + 2y$$

Thus F is not a conservative field.

7.)

$$M_y = 2y\cos x - \sin y = N_x = 2y\cos x - \sin y$$

Thus F is a conservative field.

$$F = \nabla f \implies f = \int y^2\cos x + \cos y\,dx = y^2\sin x + x\cos y + C$$

13.)

$$f = \int x^2y^3\,dx = \frac{1}{3}x^3y^3 + C$$

$$I = \int_C \nabla f\,dr = f(r(1)) - f(r(0)) = -9 - 0 = -9$$

19.)

$$M_y = -2xe^{-y} = N_x = -2xe^{-y}$$

Thus F is a conservative field.

$$r(t) = \langle t+1, t\rangle$$

$$f = \int 2xe^{-y}\,dx + \int 2y\,dy = x^2e^{-y} + y^2 + C$$

$$\implies I = f(r(1)) - f(r(0)) = \frac{4}{e} + 1 - 1 = \frac{4}{e}$$

23.)

$$M_y = 0 = N_x = 0$$

$$r(t) = \langle t+1, 2t\rangle$$

$$f = \int x^3\,dx + \int y^3\,dy = \frac{1}{4}x^4 + \frac{1}{4}y^4$$

$$\implies W = f(r(1)) - f(r(0)) = 8 - \frac{1}{4} = \frac{31}{4}$$

Exercises 16.4

1.)

$$I = \oint_C y^2\,dx + x^2y\,dy = \int_0^5 \int_0^4 2xy - 2y\,dy\,dx = \int_0^5 \left[xy^2 - y^2\right]_0^4 = 16\int_0^5 x - 1\,dx$$

$$= 16\left[\frac{1}{2}x^2 - x\right]_0^5 = 200 - 80 = 120$$

5.)

$$I = \oint_C ye^x\,dx + 2e^x\,dy = \int_0^4 \int_0^3 e^x\,dx\,dy = \int_0^4 e^3 - 1\,dy = \left[y(e^3-1)\right]_0^4 = 4(e^3-1)$$

7.)

$$I = \oint_C y + e^{\sqrt{x}}\,dx + 2x + \cos y^2\,dy = \int_0^1 \int_{x^2}^{\sqrt{x}} dy\,dx = \int_0^1 \sqrt{x} - x^2\,dx = \left[\frac{2}{3}x^{3/2} - \frac{1}{3}x^3\right]_0^1$$

$$= \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$$

11.)