Chapter 1

For all problems in this chapter, lowercase latin letters $a, b, \ldots, y, z$ represent integers.

1.) *Claim:* If $(a,b) = 1$, and $c \mid a$ and $d \mid b$, then $(c,d) = 1$.

Proof: Since $(a,b) = 1$, there exist x and y where $ax + by = 1$. We also know that $c \mid a$ and $b \mid d$, so there exists m and n where $a = cm$ and $b = dn$, thus $ax + by = c(mx) + d(ny) = 1$, thus $(c,d) = 1$. ■

2.) *Claim:* If $(a,b) = (a,c) = 1$, then $(a,bc) = 1$.

Proof: Since $(a,b) = (a,c) = 1$, there exist x_1, x_2, y_1, and y_2 where $ax_1 + by_1 = 1$ and $ax_2 + cy_2 = 1$. We can see that

$$1 = (ax_1 + by_1)(ax_2 + cy_2) = (a^2 x_1 x_2 + abx_2 y_1 + acx_1 y_2 + bcy_1 y_2)$$

$$= a(ax_1 x_2 + bx_2 y_1 + cx_1 y_2) + bc(y_1 y_2)\,,$$

so $(a,bc) = 1$. ■

3.) *Claim:* If $(a,b) = 1$, then $(a^n, b^k) = 1$ for all n and k.

Proof: We can take the prime factorizations of a and b:

$$a = \prod p_i^{x_i} \quad \text{and} \quad b = \prod p_i^{y_i}\,,$$

where p_i are the primes, and x_i and y_i are integers that depend on p_i. Further, the prime factorizations for a^n and b^k are

$$a^n = \left(\prod p_i^{x_i}\right)^n = \prod p_i^{x_i^n}$$

and

$$b^k = \left(\prod p_i^{y_i}\right)^k = \prod p_i^{y_i^k}$$

Since $(a,b) = 1$, we know that $\min\{x_i, y_i\} = 0$ for all i, thus $\min\{x_i^n, y_i^k\} = 0$, thus $(a^n, b^k) = 1$. ■

4.) *Claim:* If $(a,b) = 1$, then $(a+b, a-b)$ is either 1 or 2.

Proof: Let $d = (a+b, a-b)$, then $d \mid a+b$ and $d \mid a-b$, thus $a+b = dm$ and $a-b = dn$ for some m and n. From

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