2.) Find the critical points of y':

$$y' = y(y-1)(y-2) \implies y = 0, 1, 2 \text{ are critical points}$$

Fill in the sign chart to determine solution stability:

	y	$(y-1)$	$(y-2)$	result
$y = -\frac{1}{2}$	$-$	$-$	$-$	$-$
$y = \frac{1}{2}$	$+$	$-$	$-$	$+$
$y = \frac{3}{2}$	$+$	$+$	$-$	$-$
$y = \frac{5}{2}$	$+$	$+$	$+$	$+$

Thus $y = 0, 2$ are unstable solutions and $y = 1$ is a stable solution.

5.) a.) For the sake of establishing a contradiction, suppose $y \neq 1$ is a critical point of y', thus $k(1-y)^2 = 0$. We can manipulate the equation as follows:

$$\begin{aligned} k(1-y)^2 = 0 &\implies (1-y)^2 = 0 \\ &\implies (1-y) = \pm 0 = 0 \\ &\implies 1 = y \end{aligned}$$

But $y \neq 1 \Rightarrow\Leftarrow$, thus for y to be a critical point of y', $y = 1$. Q.E.D.

b.) Fill in the sign chart to determine solution stability:

	$k(1-y)^2$	result
$y = \frac{1}{2}$	$+$	$+$
$y = \frac{3}{2}$	$+$	$+$

Thus $y = 1$ is a semistable solution.

c.) Solve for y:

$$\frac{dy}{dt} = k(1-y)^2 \implies \frac{dy}{k(1-y)^2} = dt$$

$$\implies \int \frac{dy}{k(1-y)^2} = \int dt \implies \frac{1}{k(1-y)} = t + C$$

$$\implies 1 - y = \frac{1}{k(t+C)} \implies y = 1 - \frac{1}{k(t+C)} = \frac{k(t+C)-1}{k(t+C)}$$

Substitute $y(0) = y_0$:

$$y_0 = \frac{k(0+C)-1}{k(0+C)} = \frac{kC-1}{kC} = 1 - \frac{1}{kC} \implies 1 - y_0 = \frac{1}{kC}$$

$$\implies k(1-y_0) = \frac{1}{C} \implies C = \frac{1}{k(1-y_0)}$$

$$\therefore\ y = 1 - \frac{1}{k\left(t + \frac{1}{k(1-y_0)}\right)} = 1 - \frac{1-y_0}{kt(1-y_0)+1} = \frac{kt(1-y_0)+y_0}{kt(1-y_0)+1}$$

Which is the particular solution.

6.) Find the critical points of y':

$$y' = y^2(y^2 - 1) \implies y = -1, 0, 1 \text{ are critical points}$$

Fill in the sign chart to determine solution stability:

	y^2	$(y^2 - 1)$	result
$y = -\frac{3}{2}$	$+$	$+$	$+$
$y = -\frac{1}{2}$	$+$	$-$	$-$
$y = \frac{1}{2}$	$+$	$-$	$-$
$y = \frac{3}{2}$	$+$	$+$	$+$

Thus $y = -1$ is a stable solution, $y = 0$ is a semistable solution, and $y = 1$ is an unstable solution.

7.) Find the critical points of y':

$$y' = y(1 - y^2) \implies y = -1, 0, 1 \text{ are critical points}$$

Fill in the sign chart to determine solution stability:

	y	$(1 - y^2)$	result
$y = -\frac{3}{2}$	$-$	$-$	$+$
$y = -\frac{1}{2}$	$-$	$+$	$-$
$y = \frac{1}{2}$	$+$	$+$	$+$
$y = \frac{3}{2}$	$+$	$-$	$-$

Thus $y = -1, 1$ are stable solutions, and $y = 0$ is an unstable solution.

14.) $f'(y) = \dfrac{d^2y}{dy^2}$, thus $f'(y)$ can be used to determine the concavity of a given solution $\phi(t)$. Suppose y_1 is a critical point; when $f'(y_1) < 0$, then $\phi(t) = y_1$ is concave with respect to the y-axis, thus the solution converges to y_1, making it stable. When $f'(y_1) > 0$, the solution is convex, thus the solution does not converge to y_1, making it unstable.