Problem 1

a.) 1.) False

2.) False

3.) True

4.) False

5.) False

6.) True

7.) False

8.) False

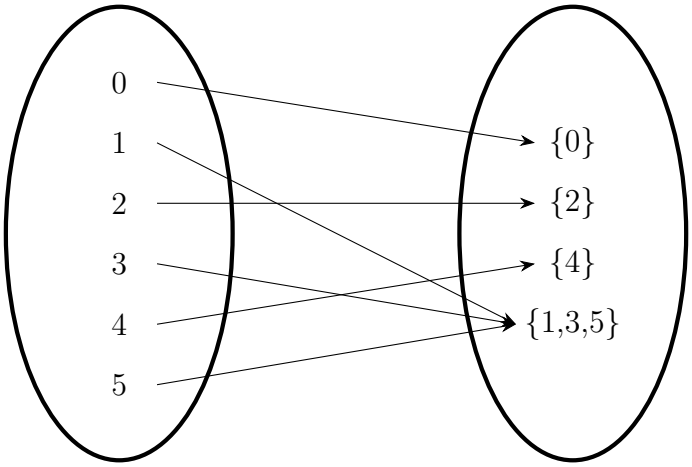
b.) $X/\sim = \{\{1,3,5\},\{0\},\{2\},\{4\}\}$

c.) $[0] = \{0\}$
$[1] = \{1,3,5\}$
$[2] = \{2\}$
$[3] = \{1,3,5\}$
$[4] = \{4\}$
$[5] = \{1,3,5\}$

d.)

$$p : X \to X/\sim$$

$$x \mapsto [x]$$



Problem 2

a.) Given X is equipped with the poset topology, the open subsets of X are $\{5\}$, $\{5,4\}$, $\{5,4,3\}$, $\{5,4,3,2\}$, $\{5,4,3,2,1\}$, $\{5,4,3,2,1,0\}$, and $\varnothing$.

b.) Given $X/\sim$ equipped with the quotient topology, the open subsets of $X/\sim$ are $\{\{0\},\{2\},\{4\},\{1,3,5\}\}$, $\{\{2\},\{4\},\{1,3,5\}\}$, and $\varnothing$.

c.) $p^{-1}(\{\{0\},\{2\},\{4\},\{1,3,5\}\}) = \{5,4,3,2,1,0\}$
$p^{-1}(\{\{2\},\{4\},\{1,3,5\}\}) = \{5,4,3,2,1\}$
$p^{-1}(\varnothing) = \varnothing$

Problem 3

Let $V \subset X/\sim$ be open, then by definiton $p^{-1}(V)$ is open, thus p is continuous. ■