Exercises 15.8

1.) a.)

$$x = \rho \sin\phi \cos\theta = 6\sin(\pi/6)\cos(\pi/3) = 3/2$$
$$y = \rho \sin\phi \sin\theta = 6\sin(\pi/6)\sin(\pi/3) = 3\sqrt{3}/2$$
$$z = \rho\cos\phi = 6\cos(\pi/6) = 3\sqrt{3}$$
$$\therefore\ (6, \pi/3, \pi/6) \to \left(3/2, 3\sqrt{3}/2, 3\sqrt{3}\right)$$

b.)

$$x = 3\sin(3\pi/4)\cos(\pi/2) = 0$$
$$y = 3\sin(3\pi/4)\sin(\pi/2) = 3\sqrt{2}/2$$
$$z = 3\cos(3\pi/4) = -3\sqrt{2}/2$$
$$\therefore\ (3, \pi/2, 3\pi/4) \to \left(0, 3\sqrt{2}/2, -3\sqrt{2}/2\right)$$

3.) a.)

$$\rho = \sqrt{x^2+y^2+z^2} = \sqrt{(-2)^2} = 2$$
$$0 = 2\cos\phi \implies \phi = \cos^{-1}(0) = \pi/2$$
$$0 = 2\sin(\pi/2)\cos\theta \implies \theta = \cos^{-1}(0) = 3\pi/2$$
$$\therefore\ (0, -2, 0) \to (2, 3\pi/2, \pi/2)$$

b.)

$$\rho = \sqrt{1+1+2} = 2$$
$$-\sqrt{2} = 2\cos\phi \implies \phi = \cos^{-1}\left(-\sqrt{2}/2\right) = 3\pi/4$$
$$1 = \sin(3\pi/4)\sin\theta \implies \theta = \sin^{-1}\left(2/\sqrt{2}\right) = 3\pi/4$$
$$\therefore\ \left(-1, 1, \sqrt{2}\right) \to (2, 3\pi/4, 3\pi/4)$$

9.)

$$x^2+y^2+z^2 = 9 \implies \rho^2 = 9 \implies \rho = 3$$

21.)

$$I = \int_0^{2\pi}\int_0^{\pi}\int_0^5 \rho^6 \sin\phi\, d\rho\, d\phi\, d\theta = \int_0^{2\pi}\int_0^{\pi}\left[\frac{1}{7}\rho^7\sin\phi\right]_0^5 d\phi\, d\theta$$
$$= \frac{5^7}{7}\int_0^{2\pi}\int_0^{\pi}\sin\phi\, d\phi\, d\theta = \frac{5^7}{7}\int_0^{2\pi} 2\, d\theta = \frac{5^7 4\pi}{7}$$

23.)

$$I = 2\int_0^{2\pi}\int_0^{\pi}\int_2^3 \rho^4\sin^3\phi\,d\rho\,d\phi\,d\theta = 2\int_0^{2\pi}\int_0^{\pi}\left[\frac{1}{5}\rho^5\sin^3\phi\right]_2^3 d\phi\,d\theta =$$

$$\frac{422}{5}\int_0^{2\pi}\int_0^{\pi}\sin^3\phi\,d\phi\,d\theta = -\frac{422}{5}\int_0^{2\pi}\left[\cos\phi - \frac{1}{3}\cos^3\phi\right]_0^{\pi} d\theta = \frac{422}{15}\int_0^{2\pi} 2\,d\theta = \frac{1688\pi}{15}$$

27.)

$$V = \int_0^{2\pi}\int_{\pi/6}^{\pi/3}\int_0^a \rho^2\sin\phi\,d\rho\,d\phi\,d\theta = \frac{a^3}{3}\int_0^{2\pi}\int_{\pi/6}^{\pi/3}\sin\phi\,d\phi\,d\theta = \frac{(\sqrt{3}-1)a^3}{6}\int_0^{2\pi} d\theta$$

$$= \frac{1}{3}(\sqrt{3}-1)\pi a^3$$

35.)

41.)

Exercises 16.2

1.)

$$r(t) = \left\langle t^2, 2t\right\rangle,\ r'(t) = \langle 2t, 2\rangle,\ \|r'(t)\| = 2\sqrt{t^2+1}$$

$$\implies I = \int_C y\,ds = \int_0^3 4t\sqrt{t^2+1}\,dt = 2\int_0^3\sqrt{u}\,du = 2\left[\frac{2}{3}(t^2+1)^{3/2}\right]_0^3 = \frac{4}{3}(10^{3/2}-1)$$

3.)

$$r(t) = \langle 4\cos t, 4\sin t\rangle,\ r'(t) = \langle -4\sin t, 4\cos t\rangle,\ \|r'(t)\| = \sqrt{16\sin^2 t + 16\cos^2 t} = 4$$

$$\implies I = \int_C xy^4\,ds = 4^5\int_{-\pi/2}^{\pi/2}\cos t\sin^4 t\,dt = 4^5\left[\frac{1}{5}\sin^5 t\right]_{-\pi/2}^{\pi/2} = \frac{2\cdot 4^5}{5}$$

7.)

$$r_1(t) = \langle 2t, t\rangle,\ r_1'(t) = \langle 2, 1\rangle$$

$$\implies I_1 = 2\int_0^1 4t\,dt + \int_0^1 4t^2 = 4 + \frac{4}{3} = \frac{16}{3}$$

$$r_2(t) = \langle t+2, 1-t\rangle,\ r_2'(t) = \langle 1, -1\rangle$$

$$\implies I_2 = \int_0^1 4 - t\,dt - \int_0^1 t^2 + 4t + 4\,dt = \frac{7}{2} - \frac{19}{3}$$

$$I = I_1 + I_2 = \frac{16}{3} + \frac{7}{2} - \frac{19}{3} = \frac{7}{2} - 1 = \frac{5}{2}$$

9.)

$$r(t) = \langle \cos t, \sin t, t \rangle\,,\ r'(t) = \langle -\sin t, \cos t, 1 \rangle\,,\ \|r'(t)\| = \sqrt{\sin^2 t + \cos^2 t + 1} = \sqrt{2}$$

$$\implies I = \int_C x^2 y\, ds = \sqrt{2} \int_0^{\pi/2} \cos^2 t \sin t\, dt = -\sqrt{2} \int_0^{\pi/2} u^2\, du$$

$$= -\sqrt{2} \left[\frac{1}{3} \cos^3 t \right]_0^{\pi/2} = \frac{\sqrt{2}}{3}$$

15.)

$$r(t) = \langle 3t + 1, t, 2t \rangle\,,\ r'(t) = \langle 3, 1, 2 \rangle\,,\ \|r'(t)\| = \sqrt{9 + 1 + 4} = \sqrt{14}$$

$$\implies I = 3 \int_0^1 4t^2\, dt + \int_0^1 9t^2 + 6t + 1\, dt + 2 \int_0^1 t^2\, dt = 4 + 7 + \frac{2}{3} = \frac{35}{3}$$

19.)

$$r(t) = \left\langle t^3, t^2 \right\rangle\,,\ r'(t) = \left\langle 3t^2, 2t \right\rangle$$

$$\implies I = \int_0^1 F(r(t)) \cdot r'(t)\, dt = \int_0^1 \left\langle t^7, -t^6 \right\rangle \cdot \left\langle 3t^2, 2t \right\rangle\, dt = \int_0^1 3t^9 - 2t^7\, dt = \frac{1}{20}$$

21.)

$$r(t) = \left\langle t^3, -t^2, t \right\rangle\,,\ r'(t) = \left\langle 3t^2, -2t, 1 \right\rangle$$

$$\implies I = \int_0^{\pi} \left\langle \sin(t^3) + \cos(-t^2) + t^4 \right\rangle \cdot \left\langle 3t^2, -2t, 1 \right\rangle\, dt$$

$$= \int_0^{\pi} 3t^2 \sin(t^3) - 2t \cos(-t^2) + t^4\, dt = \left[-\cos(t^3) - \sin(-t^2) + \frac{1}{5} t^5 \right]_0^{\pi}$$

41.)