Derivatives

1.) $\dfrac{d}{dx}\left[4x^5 - 5\sqrt{x} + \cos^4 x\right] = 20x^4 + \frac{5}{2\sqrt{x}} - 4\cos^3 x \sin x$

2.) $\dfrac{d}{dx}\left[e^x \sin(6x)\right] = e^x \sin(6x) + 6e^x \cos(6x)$

3.) $\dfrac{d}{dx}\left[\dfrac{1}{x^4+3x}\right] = \dfrac{0(x^4+3x) - 1(4x^3+3)}{(x^4+3x)^2} = -\dfrac{4x^3+3}{(x^4+3x)^2}$

4.) $\dfrac{d}{dx}\left[\dfrac{x}{1+x^2}\right] = \dfrac{1(1+x^2) - x(2x)}{(1+x^2)^2} = \dfrac{1+x^2-2x^2}{(1+x^2)^2} = \dfrac{1-x^2}{(1+x^2)^2}$

5.) $\dfrac{d}{dx}\left[\ln(xe^{7x}) + \dfrac{1}{x^3}\right] = \dfrac{d}{dx}\left[\ln x + 7x\right] - \dfrac{3}{x^4} = \dfrac{1}{x} + 7 - \dfrac{3}{x^4} = \dfrac{7x^4 + x^3 - 3}{x^4}$

6.) $\dfrac{d}{dx}\left[\sqrt{e^{2x}+e} - \dfrac{4}{\sqrt{x}}\right] = \dfrac{2e^{2x}}{2\sqrt{e^{2x}+e}} + \dfrac{2}{\sqrt{x^3}}$

7.) $\dfrac{d}{dx}\left[\dfrac{5x^2-7x}{x^2+2}\right] = \dfrac{(x^2+2)(10x-7) - (5x^2-7x)(2x)}{(x^2+2)^2}$

8.) $\dfrac{d}{dx}\left[\sec^2 x \tan x\right] = 2\sec^2 x \tan^2 x + \sec^4 x$

9.) $\dfrac{d}{dx}\left[(\tan^{-1} x)^{10}\right] = \dfrac{10(\tan^{-1} x)^9}{x^2+1}$

10.) $\dfrac{d}{dx}\left[\sqrt[3]{x^2} - \dfrac{1}{\sqrt{x^3}}\right] = \dfrac{2}{3\sqrt[3]{x}} + \dfrac{3}{2\sqrt{x^5}}$

First Order Partial Derivatives

11.) $f_x = \dfrac{\partial}{\partial x}\left[x\sin(xy)\right] = \sin(xy) + xy\cos(xy)$

$f_y = \dfrac{\partial}{\partial y}\left[x\sin(xy)\right] = x^2\cos(xy)$

12.) $f_x = \dfrac{\partial}{\partial x}\left[xy^2e^{-xz}\right] = y^2e^{-xz} - xy^2ze^{-xz}$

$f_y = \dfrac{\partial}{\partial y}\left[xy^2e^{-xz}\right] = 2xye^{-xz}$

$f_z = \dfrac{\partial}{\partial z}\left[xy^2e^{-xz}\right] = -x^2y^2e^{-xz}$

13.) $f_x = \dfrac{\partial}{\partial x}\left[(x^2-y^2+5)^3\right] = 2x[3(x^2-y^2+5)^2] = 6x(x^2-y^2+5)^3$

$f_y = \dfrac{\partial}{\partial y}\left[(x^2-y^2+5)^3\right] = -2y[3(x^2-y^2+5)^2] = -6y(x^2-y^2+5)^2$

Second Order Partial Derivatives

14.) $f_{xx} = \dfrac{\partial^2}{\partial x^2}[x^4y - 2x^3y^2] = \dfrac{\partial}{\partial x}[4x^3y - 6x^2y^2] = 9x^2y - 12xy^2$

$f_{xy} = f_{yx} = \dfrac{\partial^2}{\partial x \partial y}[x^4y - 2x^3y^2] = \dfrac{\partial}{\partial y}[4x^3y - 6x^2y^2] = 4x^3 - 12x^2y$

$f_{yy} = \dfrac{\partial^2}{\partial y^2}[x^4y - 2x^3y^2] = \dfrac{\partial}{\partial y}[x^4 - 4x^3y] = -4x^3$

14.) $f_{xx} = \dfrac{\partial^2}{\partial x^2}[\ln(3x+2y)] = \dfrac{\partial}{\partial x}\left[\dfrac{3}{3x+2y}\right] = -\dfrac{9}{(3x+2y)^2}$

$f_{xy} = f_{yx} = \dfrac{\partial^2}{\partial x \partial y}[\ln(3x+2y)] = \dfrac{\partial}{\partial y}\left[\dfrac{3}{3x+2y}\right] = -\dfrac{6}{(3x+2y)^2}$

$f_{yy} = \dfrac{\partial^2}{\partial y^2}[\ln(3x+2y)] = \dfrac{\partial}{\partial y}\left[\dfrac{2}{3x+2y}\right] = -\dfrac{4}{(3x+2y)^2}$

14.) $f_{xx} = \dfrac{\partial^2}{\partial x^2}\left[x^3\sqrt{y} + \sin(y^2) + x\cos y\right] = \dfrac{\partial}{\partial x}\left[3x^2\sqrt{y} + \cos y\right] = 6x\sqrt{y}$

$f_{xy} = f_{yx} = \dfrac{\partial^2}{\partial x \partial y}\left[x^3\sqrt{y} + \sin(y^2) + x\cos y\right] = \dfrac{\partial}{\partial y}\left[3x^2\sqrt{y} + \cos y\right] = -\dfrac{3x^2}{\sqrt{y}} - \sin y$

$f_{yy} = \dfrac{\partial^2}{\partial y^2}\left[x^3\sqrt{y} + \sin(y^2) + x\cos y\right] = \dfrac{\partial}{\partial y}\left[\dfrac{x^3}{2\sqrt{y}} + 2y\cos(y^2) - x\sin y\right]$

$= -\dfrac{x^3}{4\sqrt{y^3}} - 4y^2\sin(y^2) + 2\cos(y^2) - x\cos y$

Integrals

17.) $\displaystyle\int x^2 + 2x - 4 + \frac{1}{x^2} + \sin x \; dx = \frac{x^3}{3} + x^2 - 4x - \frac{1}{x} - \cos x + C$

18.) $\displaystyle\int_0^4 (4-t)\sqrt{t}\; dt = \int_0^4 4\sqrt{t} - t\sqrt{t}\; dt = \left[\frac{8\sqrt{t^3}}{3} - \frac{2\sqrt{t^5}}{5}\right]_0^4 = \frac{8\sqrt{4^3}}{3} - \frac{2\sqrt{4^5}}{5} = \frac{8(2^3)}{3} - \frac{2(2^5)}{5} = \frac{64}{3} - \frac{64}{5} = \frac{128}{15}$

19.) $\displaystyle\int_0^1 \cos(\pi t/2)\; dt = \left[\frac{2\sin(\pi t/2)}{\pi}\right]_0^1 = \frac{2(1)}{\pi} - \frac{2(0)}{\pi} = \frac{2}{\pi}$

20.) $\displaystyle\int (1-2x)^9\; dx = -\frac{1}{2}\int u^9\; du = -\frac{u^{10}}{20} + C = -\frac{(1-2x)^{10}}{20} + C$

21.) $\displaystyle\int x\sqrt{1-x^2}\; dx = -\frac{1}{2}\int \sqrt{u}\; du = -\frac{\sqrt{u^3}}{3} + C = -\frac{\sqrt{(1-x^2)^3}}{3} + C$

22.) $\displaystyle\int \sin x\cos^2 x\; dx = -\int u^2\; du = -\frac{u^3}{3} + C = -\frac{cos^3 x}{3} + C$

23.) $\displaystyle\int e^{2x}x\ dx = \frac{xe^{2x}}{2} - \frac{e^{2x}}{4} + C$

24.) $\displaystyle\int e^{2x}x^2\ dx = \frac{x^2e^{2x}}{2} - \frac{xe^{2x}}{2} + \frac{e^{2x}}{4} + C$

25.) $\displaystyle\int \frac{1}{4x+3}\ dx = \frac{1}{4}\int \frac{1}{u}\ du = \frac{\ln|u|}{4} + C = \frac{\ln|4x+3|}{4} + C$

26.) $\displaystyle\int \frac{1}{(2x+5)^3}\ dx = \frac{1}{2}\int \frac{1}{u^3}\ du = -\frac{1}{4u^2} + C = -\frac{1}{4(2x+5)^2} + C$