10.) We can substitute y_1 and y_2 to validate them:

$$y_1 = t^2,\ y_1' = 2t,\ y_1'' = 2$$
$$\implies t^2y'' - 2y = 2t^2 - 2t^2 = 0$$
$$y_2 = \frac{1}{t},\ y_2' = -\frac{1}{t^2},\ y_2'' = \frac{2}{t^3}$$
$$\implies t^2y'' - 2y = \frac{2t^2}{t^3}y'' - \frac{2}{t} = \frac{2}{t} - \frac{2}{t} = 0$$

Thus y_1 and y_2 are solutions. We can now show that $\phi(t) = c_1y_1 + c_2y_2$ is a solution for all $c_1, c_2 \in \mathbb{R}$.

$$\phi(t) = c_1t^2 + \frac{c_2}{t},\ \phi'(t) = 2c_1t - \frac{c_2}{t^2},\ \phi''(t) = 2c_1 + \frac{2c_2}{t^3}$$
$$\implies t^2y'' - 2y = t^2\left(2c_1 + \frac{2c_2}{t^3}\right) - 2c_1t^2 - \frac{2c_2}{t} = 2c_1t^2 - 2c_1t^2 + \frac{2c_2}{t} - \frac{2c_2}{t} = 0$$

Thus $c_1y_1 + c_2y_2$ is a solution.

11.) Substitute y_1 and y_2:

$$y_1 = 1,\ y_1' = y_1'' = 0$$
$$\implies yy'' + (y')^2 = 0 + 0^2 = 0$$
$$y_2 = \sqrt{t},\ y_2' = \frac{1}{2\sqrt{t}},\ y_2'' = -\frac{1}{4\sqrt{t^3}}$$
$$\implies yy'' + (y')^2 = -\frac{\sqrt{t}}{4\sqrt{t^3}} + \frac{1}{4t} = \frac{1}{4t} - \frac{1}{4t} = 0$$

Thus y_1 and y_2 are solutions. We can show that $\phi(t) = c_1 + c_2y_2$ is not a general solution.

$$\phi(t) = c_1 + c_2\sqrt{t},\ \phi'(t) = \frac{c_2}{2\sqrt{t}},\ \phi''(t) = -\frac{c_2}{4\sqrt{t^3}}$$
$$yy'' + (y')^2 = -\frac{c_1c_2}{4\sqrt{t^3}} - \frac{c_2^2\sqrt{t}}{4\sqrt{t^3}} + \frac{c_2^2}{4t} = \frac{c_2^2}{4t} - \frac{c_2^2}{4t} - \frac{c_1c_2}{4\sqrt{t^3}} = \frac{c_1c_2}{4\sqrt{t^3}} \neq 0$$

Thus $c_1 + c_2y_2$ is not a solution when $t > 0$. This does not contradict theorem 3.2.2 because $c \neq 0$ is not a solution to the equation, thus $c_1 \neq cy$ for some solution y.

14.)

$$W\left[3e^{4t}, g(t)\right] = \begin{vmatrix} 3e^{4t} & g(t) \\ 12e^{4t} & g'(t) \end{vmatrix} = 3e^{4t}g(t) - 12e^{4t}g'(t) = 3e^{4t}$$
$$\implies g(t) = 1$$

to verify:

$$g(t) = 1,\ g'(t) = 0,\ \therefore\ 3e^{4t}g(t) - 12e^{4t}g'(t) = 3e^{4t}(1) - 12e^{4t}(0) = 3e^{4t}$$

19.) Substitute y_1 and y_2:

$$y_1 = \cos(2t),\ y_1' = -2\sin(2t),\ y_1'' = -4\cos(2t)$$
$$\implies y'' + 4y = -4\cos(2t) + 4\cos(2t) = 0$$
$$y_2 = \sin(2t),\ y_2' = 2\cos(2t),\ y_1'' = -4\sin(2t)$$
$$\implies y'' + 4y = -4\sin(2t) + 4\sin(2t) = 0$$

Thus y_1 and y_2 are solutions. Now find $W[y_1, y_2]$:

$$W[\cos(2t), \sin(2t)] = \begin{vmatrix} \cos(2t) & \sin(2t) \\ -2\sin(2t) & 2\cos(2t) \end{vmatrix} = 2\cos^2(2t) + 2\sin^2(2t) = 1$$

Since $W[y_1, y_2] \neq 0$, y_1 and y_2 comprise a fundamental solution set.

21.) Substitute y_1 and y_2:

$$y_1 = x,\ y_1' = 1,\ y_1'' = 0$$
$$\implies x^2y'' - x(x+2)y' + (x+2)y = 0 - x^2 - 2x + x^2 + 2x = 0$$
$$y_2 = xe^x,\ y_2' = e^x + xe^x,\ y_2'' = 2e^x + xe^x$$
$$\implies x^2y'' - x(x+2)y' + (x+2)y = x^2(2e^x + xe^x) - x(x+2)(e^x + xe^x) + (x+2)xe^x$$
$$= 2x^2e^x + x^3e^x - x^2e^x - 2xe^x - x^3e^x - 2x^2e^x + x^2e^x + 2xe^x = 0$$

Thus y_1 and y_2 are solutions. Now find $W[y_1, y_2]$:

$$W[x, xe^x] = \begin{vmatrix} x & xe^x \\ 1 & e^x + xe^x \end{vmatrix} = xe^x + x^2e^x - xe^x = x^2e^x$$

Since $W[y_1, y_2] \neq 0$ for all $t > 0$, y_1 and y_2 comprise a fundamental solution set.