

Definitions

- 1.) Given a set X , a *topology* on X is a collection of sets $\mathcal{T} \subset \mathcal{P}(X)$ such that
 - i.) $\emptyset, X \in \mathcal{T}$
 - ii.) Given a collection $\{U_\alpha\}_{\alpha \in \mathcal{A}}$ of sets in \mathcal{T} , $\bigcup_{\alpha \in \mathcal{A}} U_\alpha \in \mathcal{T}$
 - iii.) Given a finite collection $\{U_\alpha\}_{\alpha \in \mathcal{A}}$ of sets in \mathcal{T} , $\bigcap_{\alpha \in \mathcal{A}} U_\alpha \in \mathcal{T}$
- 2.) Given a set X and a topology \mathcal{T} on X , (X, \mathcal{T}) is a *topological space*.
- 3.) Given a topological space (X, \mathcal{T}) , an *open subset* of X is a subset $U \subset X$ where $U \in \mathcal{T}$.
- 4.) Given topological spaces (X, \mathcal{T}_X) and (Y, \mathcal{T}_Y) the function $f : X \rightarrow Y$ is *continuous* if for all $U \in \mathcal{T}_Y$, $f^{-1}(U) \in \mathcal{T}_X$.
- 5.) Given a set X , the collection $\{U_\alpha\}_{\alpha \in \mathcal{A}}$ of subsets of X is a *cover* of X if $\bigcup_{\alpha \in \mathcal{A}} U_\alpha = X$
- 6.) Given a cover $\{U_\alpha\}_{\alpha \in \mathcal{A}}$ of X , $\{U_\alpha\}_{\alpha \in \mathcal{A}}$ is an *open cover* if U_α is open in X for all $\alpha \in \mathcal{A}$.
- 7.) Given a cover $\{U_\alpha\}_{\alpha \in \mathcal{A}}$ of X , a *finite subcover* of $\{U_\alpha\}_{\alpha \in \mathcal{A}}$ is a cover $\{U_\beta\}_{\beta \in \mathcal{B}}$ of X where $\mathcal{B} \subset \mathcal{A}$ and \mathcal{B} is finite.
- 8.) Given a topological space (X, \mathcal{T}) , it is *compact* if for all open covers $\{U_\alpha\}_{\alpha \in \mathcal{A}}$ of X , there exists a finite subcover $\{U_\beta\}_{\beta \in \mathcal{B}}$ of $\{U_\alpha\}_{\alpha \in \mathcal{A}}$.

Proof

- a.) Let $m, n \in \mathbb{Z}$ where $m, n \geq 1$.