76.)

i_n							
j_n			X		X	X	
k_n	X						
l_n			X				X

96.) Let $S \subseteq \mathbb{N}$, and consider the cases of S:

Case $S = \mathbb{N}$: Let $x_n = 1$. Since $1 \geq 1$, the friends of x_n are $\mathbb{N}$, and thus S.

Case $S \subset \mathbb{N}$: Let the sequence x_n be defined as follows:

$$x_n = \begin{cases} 1 & n \in S \\ -\frac{1}{n} & n \notin S \end{cases}$$

Since $1 > -\dfrac{1}{n}$ for all $n \in \mathbb{N}$, all n for which $x_n = 1$ are friends of x_n. In addition, for all $m, n \in \mathbb{N}$:

$$m \geq n \implies \frac{1}{n} \geq \frac{1}{m} \implies -\frac{1}{n} \leq -\frac{1}{m}$$

Thus all n for which $x_n = -\dfrac{1}{n}$ cannot be friends of x_n, thus $x_n = 1$ for all friends n of x_n, thusly all $n \in S$. Finally, we know that S is the set of all friends of x_n, thus for all $S \subseteq \mathbb{N}$, there exists a sequence such that S is the set of friends of that sequence. Q.E.D.

97.) Let $x_n \to L$, then by theorem 19, $y_k \to L$ for all subsequences y_k of x_n. Similarly, since

99.) Let $x_n = n - (-1)^n n$. x_n is unbounded, but $y_k = x_{2k} = 2k - (-1)^{2k} 2k = 2k - 2k = 0$, thus $y_k \preceq x_n$ and $y_k \to 0$.

100.) Every cauchy sequence is convergent according to theorem 23, and no convergent sequence can be unbounded.

103.) Let $x_n = \dfrac{1}{n^2}$. Since $x_n \to 0$, x_n is convergent and thus cauchy. Q.E.D.

105.) Since x_n and y_n are cauchy, there exist $A, B \in \mathbb{R}$ where $x_n \to A$ and $y_n \to B$. Since $y_n \neq 0$ for all $n \in \mathbb{N}$, $B \neq 0$. Let $z_n = x_n \,/\, y_n$. According to theorem 14, $z_n = x_n \,/\, y_n \implies z_n \to A \,/\, B$, thus z_n is convergent and thus cauchy. Q.E.D.