

1.)

$$W[e^{2t}, e^{-3t/2}] = \begin{vmatrix} e^{2t} & e^{-3t/2} \\ 2e^{2t} & -\frac{3}{2}e^{-3t/2} \end{vmatrix} = -\frac{3}{2}e^{t/2} - 2e^{t/2} = -\frac{7}{2}e^{t/2}$$

2.)

$$W[\cos t, \sin t] = \begin{vmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{vmatrix} = \cos^2 t + \sin^2 t = 1$$

3.)

$$W[e^{-2t}, te^{-2t}] = \begin{vmatrix} e^{-2t} & te^{-2t} \\ -2e^{-2t} & e^{-2t} - 2te^{-2t} \end{vmatrix} = e^{-4t} - 2te^{-4t} + 2te^{-4t} = e^{-4t}$$

6.) Determine $p(t)$, $q(t)$, and $g(t)$:

$$ty'' + 3y = t \implies y'' + \frac{3}{t}y = 1$$

$$\therefore p(t) = 0, \quad q(t) = \frac{3}{t}, \quad g(t) = 1$$

Since the only discontinuity point is $t = 0$, and since $t_0 = 1$, the largest interval is $(0, \infty)$.

7.) Determine $p(t)$, $q(t)$, and $g(t)$:

$$t(t-4)y'' + 3ty' + 4y = 2 \implies y'' + \frac{3}{t-4}y' + \frac{4}{t(t-4)}y = \frac{2}{t(t-4)}$$

$$\therefore p(t) = \frac{3}{t-4}, \quad q(t) = \frac{4}{t(t-4)}, \quad g(t) = \frac{2}{t(t-4)}$$

Since $t_0 = 3$, and there is discontinuity when $t = 0$ and $t = 4$, the largest interval is $(0, 4)$.

8.) Determine $p(t)$, $q(t)$, and $g(t)$:

$$y'' + (\cos t)y' + 3(\ln |t|)y = 0$$

$$\therefore p(t) = \cos t, \quad q(t) = 3 \ln |t|, \quad g(t) = 0$$

Since $t_0 = 2$, and since $q(t)$ is only continuous when $t > 0$, the largest interval is $(0, \infty)$.

9.) Determine $p(x)$, $q(x)$, and $g(x)$:

$$(x-2)y'' + y' + (x-2)(\tan x)y = 0 \implies y'' + \frac{1}{x-2}y' + (\tan x)y = 0$$

$$\therefore p(x) = \frac{1}{x-2}, \quad q(x) = \tan x, \quad g(x) = 0$$

Since $q(x)$ is continuous when $\frac{\pi}{2} < x < \frac{3\pi}{2}$, and since $p(x)$ is discontinuous at $x = 2$, the largest interval is $(2, \frac{3\pi}{2})$.