12.) a.) Since $0 < \lambda < 1$, $0 < (1 - \lambda) < 1$ and $(1 - \lambda)y < y$.

21.) awdQ

25.) Let $A \subseteq B \subseteq \mathbb{R}$ and $b \in \mathbb{R}$ such that $b = \inf(B)$.

28.) a.) Let $u \in \mathbb{R}$ be an upper bound of S, thus $u \geq x$ for all $x \in S$, thus $-u \leq -x$ for all $x \in S$, thus $-u \leq y$ for all $y \in -S$, thus $-u$ is a lower bound of $-S$, thus $-S$ is bounded below. Q.E.D.

b.) Let $u = \sup(S)$. Since u is an upper bound of S, $-u$ is a lower bound of $-S$. For the sake of establishing a contradiction, suppose there exists $v \in \mathbb{R}$ such that $-u < v$ and v is a lower bound of $-S$, thus $u > -v$. Since v is a lower bound of $-S$, $v \leq y$ for all $y \in -S$, thus $-v \geq -y$ for all $y \in -S$, thus $-v \geq x$ for all $x \in S$, thus $-v$ is an upper bound of S, but since $u > -v$, $u \neq \sup(S) \Rightarrow\Leftarrow$, thus $-\sup(S) = -u = \inf(-S)$. Q.E.D.

29.) a.) Since $S \neq \varnothing$, $\mathscr{L} \neq \varnothing$. In addition, since S is bounded below, there exists $v \in \mathbb{R}$ such that $v = \inf(S)$, thus $v \geq x$ for all $x \in \mathscr{L}$, thus v is an upper bound of $\mathscr{L}$, thus $\mathscr{L}$ is bounded above. Q.E.D.

b.) Let $w = \sup(\mathscr{L})$, thus $w \geq x$ for all $x \in \mathscr{L}$, thus w

c.) ***

30.) a.) For unbounded sets, infinite suprema and infima make sense, as for all $x \in \mathbb{R}$, $-\infty < x < \infty$. In addition, there exists no $y \in \mathbb{R}$ such that $y < -\infty$ or $y > \infty$, thus $\sup(S) = \infty$ and $\inf(S) = -\infty$.

b.) If we constrict the empty set to being a subset of $\mathbb{R}$, then we can reason that ∞ and $-\infty$ are both upper and lower bounds of the empty set. Since $\infty > -\infty$, $\inf(\varnothing) = \infty$, and $\sup(\varnothing) = -\infty$.

31.) a.) False; let $S = \{x \in \mathbb{Q} : 0 \leq x < \pi\}$. By definition, all $x \in S$ are rational, but $\sup(S) = \pi$ is irrational.

b.) False; let $S = \{x \in \mathbb{R} \backslash \mathbb{Q} : 0 < x < 3\}$. By definition, all $x \in S$ are irrational, but $\sup(S) = 3$ is rational.

33.) Let $x, y \in \mathbb{R}$, and consider $|x|$:

$$|x| = |x - y + y| \leq |x - y| + |y|$$

$$\implies |x| \leq |x - y| + |y|$$

$$\implies |x| - |y| \leq |x - y|$$

$$\implies (|x| - |y|) - (|x| - |y|) - |x - y| \leq |x| - |y| \leq |x - y|$$

$$\implies 0 - |x-y| = -|x-y| \leq |x| - |y| \leq |x-y|$$
$$\implies \Big||x| - |y|\Big| \leq |x-y|$$

Thus the inequality holds. Q.E.D.

34.) a.) False; let $S = (-\infty, 0]$, thus $\{|x| : x \in S\} = [0, \infty)$, which has no upper bound.

b.) True; let $u = \sup(\{|x| : x \in S\})$,

35.) a.) Let S be a bounded set, and $u, v \in \mathbb{R}$ be upper and lower bounds of S respectively.

36.) When learning the various integration techniques taught in calculus 2, I noticed that some of them involved manipulating the dx term. I initially found this confusing, as I though that $\dfrac{d}{dx}$ was a single operator that could not be seperated. I later found out that this was not the case, and that it can often be treated just like a fraction. What helped me to realize this was when I watched a video series that explained calculus in a more intuitive, less purely algebraic way. In this context, dx being a seperate variable simply made sense.

38.) Let $u = \sup(S)$, thus $u \geq x$ for all $x \in S$. For the sake of establishing a contradiction, suppose $u \notin S$, then for some $\epsilon > 0$, $u = x + \epsilon$ for some $x \in S$. Consider $x + \frac{\epsilon}{2}$. Since $\epsilon > 0$, $\frac{\epsilon}{2} > 0$,

39.) a.) A sequence is defined as a function $x(n)$ such that $x : \mathbb{N} \to \mathbb{R}$.

b.) The sequence $\{x_n\}_{n=1}^{\infty}$ converges to $L \in \mathbb{R}$ if and only if

$$\forall \epsilon > 0 : \exists k \in \mathbb{N} : n \geq k \implies |x_n - L| < \epsilon$$

40.) awd

41.) awd

42.) We can simplify the fraction to $\dfrac{7}{25n - 10}$.

43.) a.) $\displaystyle\lim_{n\to\infty} \frac{1}{10n} = 10$

b.) $\displaystyle\lim_{n\to\infty} \sin n$ diverges

c.) Suppose $x_n \to 15$ and $x_n \to -77$. Since $x_n \to 15$, x_n gets arbitrarily close to 15. Also, since $x_n \to -77$, x_n gets arbitrarily close to -77. However, as x_n gets closer to 15, x_n moves farther from -77, and vice versa, thus x_n cannot get arbitrarily close to both, thus x_n cannot converge to both.

44.) a.) Let $\{x_n\}_{n=1}^{\infty}$ be a sequence such that $x_n \to L$, then by definition for all $n > k$ for some $k \in \mathbb{N}$, $|x_n - L| < \epsilon$ for some $\epsilon > 0$. By the reverse triangle inequality, we know that $\Big||x_n| - |L|\Big| < |x_n - L| < \epsilon$. Let $\epsilon_1 = |x_n - L|$, thus $\Big||x_n| - |L|\Big| < \epsilon_1$ for some $\epsilon_1 > 0$ given $n > k$, thus $|x_n| \to |L|$. Q.E.D.

b.) Consider the sequence $\{x_n\}_{n=1}^{\infty}$ such that $x_n =$