1.) Manipulate to standard form and find $p(t)$ and $g(t)$:

$$(t-3)y' + (\ln t)y = 2t \implies y' + \frac{\ln t}{t-3}y = \frac{2t}{t-3}$$

$$p(t) = \frac{\ln t}{t-3},\ g(t) = \frac{2t}{t-3}$$

Determine continuity of p and g around initial value $t = 1$:

$$p(1) = \frac{\ln 1}{1-3} = \frac{0}{-2} = 0,\ p(t) \text{ is continuous over } (0,3) \cup (3,\infty)$$

$$g(1) = \frac{2(1)}{1-3} = \frac{2}{-2} = -1,\ g(t) \text{ is continuous over } (-\infty,3) \cup (3,\infty)$$

Thus the equation has a unique solution when $t \in (-\infty, 3)$.

2.) Find $p(t)$ and $g(t)$:

$$p(t) = \tan t,\ g(t) = \sin t$$

Determine continuity of p and g around intitial value $t = \pi$:

$$p(\pi) = \tan \pi = 0,\ p(t) \text{ is continuous over } \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$$

$$g(\pi) = \sin \pi = -1,\ g(t) \text{ is continuous over all reals}$$

Thus the equation has a unique solution when $t \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$.

3.) Manipulate to standard form and find $p(t)$ and $g(t)$:

$$(4-t^2)y' + 2ty = 3t^2 \implies y' + \frac{2t}{4-t^2}y = \frac{3t^2}{4-t^2}$$

$$p(t) = \frac{2t}{4-t^2},\ g(t) = \frac{3t^2}{4-t^2}$$

Determine continuity of p and g around initial value $t = -3$:

$$p(-3) = \frac{2(-3)}{4-(-3)^2} = \frac{-6}{-5} = \frac{6}{5},\ p(t) \text{ is continuous over } (-\infty,-2) \cup (-2,2) \cup (2,\infty)$$

$$g(-3) = \frac{3(-3)^2}{4-(-3)^2} = \frac{27}{-5} = -\frac{27}{5},\ \text{ is continuous over } (-\infty,-2) \cup (-2,2) \cup (2,\infty)$$

Thus the equation has a unique solution when $t \in (-\infty, -2)$.

4.) Manipulate to standard form and find $p(t)$ and $g(t)$:

$$(\ln t)y' + y = \cot t \implies y' + \frac{y}{\ln t} = \frac{\cot t}{\ln t}$$

$$p(t) = \frac{1}{\ln t},\ g(t) = \frac{\cot t}{\ln t}$$

Determine continuity of p and g around initial value $t = 2$:

$$p(2) = \frac{1}{\ln 2},\ p(t) \text{ is continuous over } (0,1) \cup (1,\infty)$$

$$g(2) = \frac{\cot 2}{\ln 2} \approx -0.660,\ g(t) \text{ is continuous over } (1,\pi)$$

Thus the equation has a unique solution when $t \in (1, \pi)$.

5.) Find $\partial f/\partial y$ from $f(t,y)$:

$$\frac{\partial}{\partial y}\left[\sqrt{1-t^2-y^2}\right] = \frac{-2y}{2\sqrt{1-t^2-y^2}} = -\frac{y}{\sqrt{1-t^2-y^2}}$$

Determine continuity of f and $\partial f/\partial y$ on the ty-plane:

$$f(t,y) = \sqrt{1-t^2-y^2} = \sqrt{1-(t^2+y^2)} \text{ is continuous when } t^2+y^2 < 1$$

$$\frac{\partial f}{\partial y} = -\frac{y}{\sqrt{1-t^2-y^2}} = -\frac{y}{\sqrt{1-(t^2+y^2)}} \text{ is continuous when } t^2+y^2 < 1$$

Thus the equation has a unique solution when $t^2 + y^2 < 1$.

6.) Find $\partial f/\partial y$ from $f(t,y)$:

$$\frac{\partial}{\partial y}\left[\frac{\ln|ty|}{1-t^2+y^2}\right] = \frac{\frac{1-t^2-y^2}{y} - 2y\ln|ty|}{(1-t^2+y^2)^2}$$

Determine continuity of f and $\partial f/\partial y$ on the ty-plane:

$$f(t,y) = \frac{\ln|ty|}{1-t^2+y^2} = \frac{\ln|ty|}{1-(t^2-y^2)} \text{ is continuous when } t^2-y^2 \neq 1 \text{ and } t, y \neq 0$$

$$\frac{\partial f}{\partial y} = \frac{\frac{1-t^2-y^2}{y} - 2y\ln|ty|}{(1-t^2+y^2)^2} \text{ is continuous when } t^2-y^2 \neq 1 \text{ and } t, y \neq 0$$

Thus the equation has a unique solution when $t^2 - y^2 \neq 1$ and $t, y \neq 0$.

7.) Find $\partial f/\partial y$ from $f(t, y)$:

$$\frac{\partial}{\partial y}\left[\sqrt{(t^2+y^2)^3}\right] = 3y\sqrt{t^2+y^2}$$

Determine continuity of f and $\partial f/\partial y$ on the ty-plane:

$$f(t, y) = \sqrt{(t^2+y^2)^3} \text{ is continuous when } t^2+y^2 > 0$$

$$\frac{\partial f}{\partial y} = 3y\sqrt{t^2+y^2} \text{ is continuous when } t^2+y^2 > 0$$

Thus the equation has a unique solution when $t^2 + y^2 > 0$.

8.) Find $\partial f/\partial y$ from $f(t, y)$:

$$\frac{\partial}{\partial y}\left[\frac{1+t^2}{3y-y^2}\right] = -\frac{1+t^2}{(3y-y^2)^2}$$

Determine continuity of f and $\partial f/\partial y$ on the ty-plane:

$$f(t, y) = \frac{1+t^2}{3y-y^2} = \frac{1+t^2}{y(3-y)} \text{ is continuous when } y \neq 0, y \neq 3$$

$$\frac{\partial f}{\partial y} = -\frac{1+t^2}{(3y-y^2)^2} = -\frac{1+t^2}{(y(3-y))^2} \text{ is continuous when } y \neq 0, y \neq 3$$

Thus the equation has a unique solution when $y \neq 0, y \neq 3$.