1.)

$$\mathcal{L}\left[e^{at}\right] = \int_0^\infty e^{at}e^{-st}\,dt = \int_0^\infty e^{-(s-a)t}\,dt = \left[-\frac{e^{-(s-a)t}}{(s-a)}\right]_0^\infty$$
$$= -\lim_{t\to\infty}\frac{e^{-(s-a)t}}{(s-a)} + \frac{e^{-(s-a)(0)}}{(s-a)} = -\frac{0}{(s-a)} + \frac{1}{(s-a)} = \frac{1}{s-a}$$

2.)

$$\mathcal{L}\left[t^2\right] = \int_0^\infty t^2e^{-st}\,dt = \left[-\frac{t^2e^{-st}}{s} - \frac{2te^{-st}}{s^2} - \frac{2e^{-st}}{s^3}\right]_0^\infty$$
$$= -\frac{0}{s} - \frac{0}{s^2} - \frac{0}{s^3} + \frac{(0)^2e^{-s(0)}}{s} + \frac{2(0)e^{-s(0)}}{s^2} + \frac{2e^{-s(0)}}{s^3} = 0 + \frac{2}{s^3} = \frac{2}{s^3}$$

3.)

$$\mathcal{L}\left[k\right] = \int_0^\infty ke^{-st}\,dt = \left[-\frac{ke^{-st}}{s}\right]_0^\infty = -\lim_{t\to\infty}\frac{ke^{-st}}{s} + \frac{ke^{-s(0)}}{s} = -\frac{0}{s} + \frac{k}{s} = \frac{k}{s}$$

4.)

$$\mathcal{L}\left[\sinh(t)\right] = \int_0^\infty \sinh(t)e^{-st}\,dt = \int_0^\infty \frac{1}{2}\left(e^t - e^{-t}\right)e^{-st}\,dt$$
$$= \frac{1}{2}\int_0^\infty e^{(1-s)t} - e^{(-1-s)t}\,dt = \frac{1}{2}\left[\frac{e^{(1-s)t}}{(1-s)}\right]_0^\infty - \frac{1}{2}\left[\frac{e^{(-1-s)t}}{(-1-s)}\right]_0^\infty$$
$$= \frac{1}{2}\left(\lim_{t\to\infty}\frac{e^{(1-s)t}}{(1-s)} - \frac{e^{(1-s)(0)}}{(1-s)}\right) - \frac{1}{2}\left(\lim_{t\to\infty}\frac{e^{(-1-s)t}}{(-1-s)} - \frac{e^{(-1-s)(0)}}{(-1-s)}\right)$$
$$= \frac{1}{2}\left(\frac{0}{1-s} - \frac{1}{1-s}\right) - \frac{1}{2}\left(\frac{0}{(-1-s)} - \frac{1}{(-1-s)}\right) = -\frac{1}{2(1-s)} + \frac{1}{2(-1-s)}$$
$$= \frac{(1-s)-(-1-s)}{2(s^2-1)} = \frac{2}{2(s^2-1)} = \frac{1}{s^2-1}$$

5.)

$$\mathcal{L}\left[5 + 3e^{3t} - 5t^3 + 4\sin\left(\sqrt{3}t\right) - 6\cosh(5t)\right] = \frac{5}{s} + \frac{3}{s-3} - \frac{30}{s^4} + \frac{4\sqrt{3}}{s^2+3} - \frac{6s}{s^2-25}$$

6.)

$$\mathcal{L}\left[\sinh(2t) + 5\cos(3t)\right] = \frac{2}{s^2-4} + \frac{5s}{s^2+9}$$

7.)

$$\mathcal{L}\left[(2e^{4t}-5)^2\right] = \mathcal{L}\left[4e^{8t} - 20e^{4t} + 25\right] = \frac{4}{s-8} - \frac{20}{s-4} + \frac{25}{s}$$

8.)

$$\mathcal{L}\left[(\sin(2t) - \cos(2t))^2\right]\,\mathcal{L}\left[\sin^2(2t) - 2\sin(2t)\cos(2t) + \cos^2(2t)\right]$$
$$= -\mathcal{L}\left[2\sin(2t)\cos(2t)\right] = -\mathcal{L}\left[\sin(4t)\right] = -\frac{4}{s^2+16}$$

9.)

$$\mathcal{L}\left[e^{4t}\cos(3t)\right] = F(s-4) = \frac{s-4}{(s-4)^2+9}$$

10.)

$$\mathcal{L}\left[e^{-2t}\sin(7t)\right] = F(s+2) = \frac{7}{(s+2)^2+49}$$

11.)

$$\mathcal{L}\left[2\sinh(t)\sin(2t)\right] = \mathcal{L}\left[\left(e^{t}-e^{-t}\right)\sin(2t)\right] = \mathcal{L}\left[e^{t}\sin(2t) - e^{-t}\sin(2t)\right]$$

$$= F(s-1) - F(s+1) = \frac{2}{(s-1)^2+4} - \frac{2}{(s+1)^2+4}$$

12.)

$$\mathcal{L}\left[e^{-2t}(3\cos(4t) - 6\sin(5t))\right] = \mathcal{L}\left[3e^{-2t}\cos(4t) - 6e^{-2t}\sin(5t)\right]$$

$$= \frac{3(s+2)}{(s+2)^2+16} - \frac{30}{(s+2)^2+25}$$