1.)

$$\mathcal{L}\left[t\cos(3t)\right] = -\frac{d}{ds}\,\frac{s}{s^2+9} = -\frac{s^2+9-2s^2}{(s^2+9)^2} = -\frac{9-s^2}{(s^2+9)^2}$$

2.)

$$\mathcal{L}\left[t\sin(5t)\right] = -\frac{d}{ds}\,\frac{5}{s^2+25} = -\frac{-10s}{(s^2+25)^2} = \frac{10s}{(s^2+25)^2}$$

3.)

$$\mathcal{L}\left[t^2\sin t\right] = (-1)^2\frac{d^2}{ds^2}\,\frac{1}{s^2+1} = \frac{d}{ds}\frac{-2s}{(s^2+1)^2}$$

$$= \frac{-2(s^2+1)^2+2s(2s)(s^2+1)+2s(s^2+1)(2s)}{(s^2+1)^4} = \frac{-2(s^2+1)^2+4s+4s}{(s^2+1)^3}$$

$$= -\frac{2(s^2+1)^2+8s}{(s^2+1)^3}$$

4.)

$$\mathcal{L}\left[te^{4t}\cos(3t)\right] = -\frac{d}{ds}F(s-4) = -\frac{d}{ds}\,\frac{s-4}{(s-4)^2+9} = -\frac{(s-4)^2-(s-4)(2s-8)}{(s^2-8s+25)^2}$$

$$= -\frac{s^2-8s+25-2s^2+8s+8s-32}{(s^2-8s+25)^2} = \frac{s^2-8s+7}{(s^2-8s+25)^2}$$

5.)

$$\mathcal{L}\left[te^{-3t}\sin t\right] = -\frac{d}{ds}F(s+3) = -\frac{d}{ds}\,\frac{1}{(s+3)^2+1} = -\frac{2s+6}{(s^2+6s+10)^2}$$

6.)

$$\mathcal{L}\left[(t^2+2)^2e^{-2t}\right] = \mathcal{L}\left[(t^4+4t^2+4)e^{-2t}\right] = \mathcal{L}\left[t^4e^{-2t}+4t^2e^{-2t}+4e^{-2t}\right]$$

$$\frac{d^4}{ds^4}\frac{1}{s+2}+4\frac{d^2}{ds^2}\frac{1}{s+2}+\frac{4}{s+2} = \frac{24}{(s+2)^5}+\frac{8}{(s+2)^3}+\frac{4}{s+2}$$

7.)

$$\mathcal{L}\left[t^9e^{-3t}\right] = -\frac{d^9}{ds^9}\frac{1}{s+3} = \frac{9!}{(s+3)^{10}}$$

8.)

$$\mathcal{L}\left[t\sinh(2t)\sin(3t)\right] = -\frac{1}{2}\frac{d}{ds}\mathcal{L}\left[(e^{3t}+e^{-3t})\sin(3t)\right]$$

$$= -\frac{1}{2}\frac{d}{ds}\left(\frac{3}{(s-3)^2+9}+\frac{3}{(s+3)^2+9}\right) = \frac{1}{2}\left(\frac{2s-6}{\left((s-3)^2+9\right)^2}+\frac{2s+6}{\left((s+3)^2+9\right)^2}\right)$$

9.)

$$\mathcal{L}\left[(t+2)e^{-2t}\sin t\right] = \mathcal{L}\left[te^{-2t}\sin t+2e^{-2t}\sin t\right]$$

$$= -\frac{d}{ds}\frac{1}{(s+2)^2+1}+\frac{2}{(s+2)^2+1} = \frac{2s+4}{\left((s+2)^2+1\right)^2}+\frac{2}{(s+2)^2+1}$$

10.)
$$\mathcal{L}\left[\int_0^t x^2 + \cos x\, dx\right] = \frac{1}{s}\left(\frac{2}{s^3} + \frac{s}{s^2+1}\right) = \frac{2}{s^4} + \frac{1}{s^2+1}$$

11.)
$$\mathcal{L}\left[\int_0^t xe^{-3x} \sin x\, dx\right] = \frac{1}{s}\mathcal{L}\left[te^{-3t} \sin t\right] = -\frac{2s+6}{s(s^2+6s+10)^2}$$

12.)
$$\mathcal{L}\left[e^t \int_0^t \cos x\, dx\right] = \frac{1}{s-1}F(s-1) = \frac{s-1}{(s-1)((s-1)^2+1)} = \frac{1}{(s-1)^2+1}$$