

7.8.6.) Define an interval  $[a, b]$ , and let  $f$  be continuous on  $[a, b]$  and differentiable on  $(a, b)$ . In addition, suppose  $f$  satisfies the Lipschitz condition, thus for all  $x, y \in [a, b]$ , there exists  $M \in \mathbb{R}$  where

$$|f(x) - f(y)| \leq M |x - y|$$

thus

$$\frac{|f(x) - f(y)|}{|x - y|} = \left| \frac{f(x) - f(y)}{x - y} \right| \leq M$$

We can take the limit to find  $f'(y)$ :

$$f'(y) = \lim_{x \rightarrow y} \left| \frac{f(x) - f(y)}{x - y} \right|$$