1.) Manipulate to standard form:

$$x^2\frac{dy}{dx} + x(x+2)y = x^2\frac{dy}{dx} + x^2y + 2xy \implies \frac{dy}{dx} + y\left(1+\frac{2}{x}\right) = \frac{e^x}{x^2}$$

Find $\mu(x)$ then y:

$$\mu(x) = e^{\int p(x)dx} = e^{\int 1+\frac{2}{x}dx} = e^{x+2\ln|x|} = e^x e^{2\ln|x|} = e^x x^2$$

$$y = \frac{1}{\mu(x)}\int \mu(x)g(x)\ dx = \frac{1}{e^x x^2}\left[\int e^{2x}\ dx\right] = \frac{e^x}{2x^2} + \frac{C}{e^x x^2}$$

Which is the explicit solution.

2.) Manipulate to standard form:

$$\cos x dy + y\sin x dx - dx = 0 \implies \cos x\frac{dy}{dx} + y\sin x - 1 = 0$$

$$\implies \frac{dy}{dx} + y\tan x = \sec x$$

Find $\mu(x)$ then y:

$$\mu(x) = e^{\int \tan x dx} = e^{-\ln|\cos x|} = \sec x$$

$$y = \frac{1}{\sec x}\left[\int \sec^2 x\ dx\right] = \frac{\tan x}{\sec x} + \frac{C}{\sec x}$$

Which is the explicit solution.

3.) $y = \dfrac{\tan x}{\sec x} + \dfrac{C}{\sec x}$ as per problem 2.

4.) Find $\mu(x)$ then y:

$$\mu(x) = e^{\int \sin x dx} = e^{-\cos x} = \frac{1}{e^{\cos x}}$$

$$y = e^{\cos x}\left[\int \frac{e^{\cos x}}{e^{\cos x}}\ dx\right] = e^{\cos x}(x+C)$$

Solving given the initial values:

$$-2.5 = e^{\cos(0)}(0+C) = Ce^1 \implies C = \frac{-2.5}{e}$$

$$\therefore y = e^{\cos x}\left(x + \frac{-2.5}{e}\right)$$

Which is the particular solution.

5.) Manipulate to standard form:

$$\frac{dy}{dx} = x - 4xy \implies \frac{dy}{dx} + 4xy = x$$

Find $\mu(x)$ then y:

$$\mu(x) = e^{\int 4x dx} = e^{2x^2}$$

$$y = \frac{1}{e^{2x^2}} \left[\int x e^{2x^2} \, dx \right] = \frac{1}{e^{2x^2}} \left(\frac{e^{2x^2}}{4} + C \right) = \frac{1}{4} + \frac{C}{e^{2x^2}}$$

Which is the explicit solution.

6.) Find $\mu(t)$ then y:

$$\mu(t) = e^{\int -dt} = e^{-t}$$

$$y = e^t \left[\int 2te^t \, dt \right] = e^t(2te^t - 2e^t + C) = 2te^{2t} - 2e^{2t} + e^t C$$

Solving given the initial values:

$$1 = 2(0)e^{2(0)} - 2e^{2(0)} + e^0 C = C - 2 \implies C = 3$$

$$\therefore y = 2te^{2t} - 2e^{2t} + 3e^t$$

Which is the particular solution.

7.) Manipulate to standard form:

$$t\frac{dy}{dx} + (t+1)y = t \implies \frac{dy}{dx} + \left(\frac{t+1}{t} \right) y = 1$$

Find $\mu(t)$ then y:

$$e^{\int \left(\frac{t+1}{t} \right) dt} = e^{t + \ln|t|} = te^t$$

$$y = \frac{1}{te^t} \left[\int te^t \, dt \right] = \frac{1}{te^t} \left(te^t - e^t + C \right) = 1 - \frac{1}{t} + \frac{C}{te^t}$$

Solving given the initial values:

$$1 = 1 - \frac{1}{\ln 2} + \frac{C}{\ln 2 e^{\ln 2}} \implies \frac{1}{\ln 2} = \frac{C}{2 \ln 2} \implies C = 2$$

$$\therefore y = 1 - \frac{1}{t} + \frac{1}{\ln 2}$$

Which is the particular solution.

8.) Find $\mu(x)$ then y:

$$\mu(x) = e^{\int 2dt} = e^{2t}$$

$$y = \frac{1}{e^{2t}}\left[\int t\ dt\right] = \frac{1}{e^{2t}}\left(\frac{t^2}{2} + C\right) = \frac{t^2}{2e^{2t}} + \frac{C}{e^{2t}}$$

Solving given the initial values:

$$0 = \frac{(1)^2}{2e^{2(1)}} + \frac{C}{e^{2(1)}} = \frac{1}{2e^2} + \frac{C}{e^2} \implies C = -\frac{1}{2}$$

$$\therefore y = \frac{t^2}{2e^{2t}} - \frac{1}{2e^{2t}} = \frac{t^2 - 1}{2e^{2t}}$$

Which is the particular solution.