

When I took number theory, we never got the chance to cover quadratic reciprocity. I vaguely knew what the concept was but didn't have a full understanding. Despite this, I was interested in learning more about it. My current understanding of quadratic reciprocity is that it has something to do with the integers mod  $n$  or mod  $p$ , and an identity involving them.

After looking up what quadratic reciprocity was, I was given the following identity from wolfram.com:

Let  $p$  and  $q$  be distinct odd primes, and define the legendre symbol as

$$\left(\frac{p}{q}\right) = \begin{cases} 1 & \text{if } x^2 \equiv p \pmod{q} \text{ for some } x \in \mathbb{Z}_p \\ -1 & \text{otherwise} \end{cases}$$

Then the following identity is true:

$$\left(\frac{p}{q}\right) \left(\frac{q}{p}\right) = (-1)^{\frac{p-1}{2} \frac{q-1}{2}}$$

I had forgotten that this identity uses the Legendre symbol. I understand the definition of the Legendre symbol, but I don't see the motivation for it, other than it being related to primes. My best guess is something to do with generators mod  $p$ , as I know my number theory class talked a lot about them. Despite this gap in my understanding, I do generally have a better grasp of what quadratic reciprocity means after looking it up.