33.) Let $x, y \in \mathbb{R}$, and consider x:

$$x = x + 0 = x - y + y$$

And thus by the triangle inequality:

$$|x - y + y| \leq |x - y| + |y|$$

Manipulating:

$$|x - y + y| \leq |x - y| + |y| \implies |x| - |y| \leq |x - y|$$

$$\implies |x| - |x| - |y| = -|y| \leq |x| - |y| \leq |x - y|$$

Thus the inequality holds. Q.E.D.

37.) awd

40.) Let $\varepsilon > 0$ be given, then we know that

$$\left|\frac{2n-1}{n} - 2\right| < \varepsilon$$

given $n \geq k$ for some $k \in \mathbb{N}$. Manipulating the inequality, we find that

$$\left|\frac{2n-1}{n} - 2\right| = \left|\frac{2n-1-2n}{n}\right| = \left|\frac{-1}{n}\right| = \frac{|-1|}{|n|} = \frac{1}{n} < \varepsilon$$

$$\implies n > \frac{1}{\varepsilon}.$$

Let $k > \frac{1}{\varepsilon}$:

$$\frac{1}{n} < \frac{1}{k} = \frac{1}{\frac{1}{\varepsilon}} = \varepsilon$$

$$\therefore \left|\frac{2n-1}{n} - 2\right| < \varepsilon$$

thus $x_n \to 2$. Q.E.D.

41.) Let $\varepsilon > 0$ be given, then we know that

$$\left|\frac{(-1)^n}{n} - 0\right| < \varepsilon$$

given $n \geq k$ for some $k \in \mathbb{N}$. Manipulating the inequality, we find that

$$\left|\frac{(-1)^n}{n} - 0\right| = \left|\frac{(-1)^n}{n}\right| = \frac{|(-1)^n|}{|n|} = \frac{1}{n} < \varepsilon$$

$$\implies n > \frac{1}{\varepsilon}.$$

Let $k > \frac{1}{\varepsilon}$:

$$\frac{1}{n} < \frac{1}{k} = \frac{1}{\frac{1}{\varepsilon}} = \varepsilon$$

$$\therefore \ \left| \frac{(-1)^n}{n} - 0 \right| < \varepsilon$$

thus $x_n \to 0$. Q.E.D.

42.) Let $\varepsilon > 0$ be given, then we know that

$$\left|\frac{3n+1}{5n-2} - \frac{3}{5}\right| < \varepsilon$$

given $n \geq k$ for some $k \in \mathbb{N}$. Manipulating the inequality, we find that

$$\left|\frac{3n+1}{5n-2} - \frac{3}{5}\right| = \left|\frac{15n+5-15n+6}{25n-10}\right| = \left|\frac{11}{25n-10}\right| = \frac{|11|}{|25n-10|} = \frac{11}{25n-10} < \varepsilon$$

$$\implies 25n - 10 > \frac{11}{\varepsilon} \implies n = \frac{11}{25\varepsilon} + \frac{2}{5}.$$

Let $k > \dfrac{11}{25\varepsilon} + \dfrac{2}{5}$:

$$\frac{11}{25n-10} < \frac{11}{25k-10} = \frac{11}{25\left(\frac{11}{25\varepsilon} + \frac{2}{5}\right) - 10} = \frac{11}{\frac{11}{\varepsilon} + 10 - 10} = \frac{11}{\frac{11}{\varepsilon}} = \varepsilon$$

$$\therefore \left|\frac{3n+1}{5n-2} - \frac{3}{5}\right| < \varepsilon$$

thus $x_n \to \dfrac{3}{5}$. Q.E.D.