1.)

$$y' - 3y = e^{4t} \implies \mathcal{L}\left[y' - 3y\right] = \mathcal{L}\left[e^{4t}\right] \implies sY(s) - y(0) - 3Y(s) = \frac{1}{s-4}$$

$$\implies (s-3)Y(s) = \frac{1}{s-4} \implies Y(s) = \frac{1}{(s-3)(s-4)}$$

$$\implies y(t) = \mathcal{L}^{-1}\left[\frac{1}{(s-3)(s-4)}\right] = -\mathcal{L}^{-1}\left[\frac{1}{s-3} - \frac{1}{s-4}\right] = e^{4t} - e^{3t}$$

3.)

$$y'' + 4y' + 4y = t^4 e^{-2t} \implies \mathcal{L}\left[y'' + 4y' + 4y\right] = \mathcal{L}\left[t^4 e^{-2t}\right]$$

$$\implies s^2Y(s) - sy(0) - y'(0) + 4(sY(s) - y(0)) + 4Y(s) = \frac{24}{(s+2)^5}$$

$$\implies (s+2)^2Y(s) - s - 6 = \frac{24}{(s+2)^5} \implies Y(s) = \frac{24}{(s+2)^7} + \frac{s}{(s+2)^2} - \frac{6}{(s+2)^2}$$

$$\implies y(t) = \mathcal{L}^{-1}\left[\frac{24}{(s+2)^7}\right] + \mathcal{L}^{-1}\left[\frac{s}{(s+2)^2}\right] + \mathcal{L}^{-1}\left[\frac{6}{(s+2)^2}\right]$$

$$= \frac{1}{30}t^6e^{-2t} + \mathcal{L}^{-1}\left[\frac{1}{s+2} - \frac{2}{(s+2)^2}\right] + 6te^{-2t} = \frac{1}{30}t^6e^{-2t} + (1+4t)e^{-2t}$$

5.)

$$y'' + y = (t-2)u_2(t) \implies \mathcal{L}\left[y'' + y\right] = \mathcal{L}\left[(t-2)u_2(t)\right] \implies s^2Y(s) + Y(s) = \frac{e^{-2s}}{s^2}$$

$$\implies (s^2+1)Y(s) = \frac{e^{-2s}}{s^2} \implies Y(s) = \frac{e^{-2s}}{s^2(s^2+1)} \implies y(t) = \mathcal{L}^{-1}\left[\frac{e^{-2s}}{s^2(s^2+1)}\right]$$

$$\mathcal{L}^{-1}\left[\frac{1}{s^2(s^2+1)}\right] = \mathcal{L}^{-1}\left[\frac{1}{s^2} - \frac{1}{s^2+1}\right] = t - \sin t$$

$$\therefore\ y(t) = u_2(t)[(t-2) - \sin(t-2)]$$