

Exercises 15.8

1.) a.)

$$\begin{aligned}
 x &= \rho \sin \phi \cos \theta = 6 \sin(\pi/6) \cos(\pi/3) = 3/2 \\
 y &= \rho \sin \phi \sin \theta = 6 \sin(\pi/6) \sin(\pi/3) = 3\sqrt{3}/2 \\
 z &= \rho \cos \phi = 6 \cos(\pi/6) = 3\sqrt{3} \\
 \therefore (6, \pi/3, \pi/6) &\rightarrow (3/2, 3\sqrt{3}/2, 3\sqrt{3})
 \end{aligned}$$

b.)

$$\begin{aligned}
 x &= 3 \sin(3\pi/4) \cos(\pi/2) = 0 \\
 y &= 3 \sin(3\pi/4) \sin(\pi/2) = 3\sqrt{2}/2 \\
 z &= 3 \cos(3\pi/4) = -3\sqrt{2}/2 \\
 \therefore (3, \pi/2, 3\pi/4) &\rightarrow (0, 3\sqrt{2}/2, -3\sqrt{2}/2)
 \end{aligned}$$

3.) a.)

$$\begin{aligned}
 \rho &= \sqrt{x^2 + y^2 + z^2} = \sqrt{(-2)^2} = 2 \\
 0 &= 2 \cos \phi \implies \phi = \cos^{-1}(0) = \pi/2 \\
 0 &= 2 \sin(\pi/2) \cos \theta \implies \theta = \cos^{-1}(0) = 3\pi/2 \\
 \therefore (0, -2, 0) &\rightarrow (2, 3\pi/2, \pi/2)
 \end{aligned}$$

b.)

$$\begin{aligned}
 \rho &= \sqrt{1 + 1 + 2} = 2 \\
 -\sqrt{2} &= 2 \cos \phi \implies \phi = \cos^{-1}(-\sqrt{2}/2) = 3\pi/4 \\
 1 &= \sin(3\pi/4) \sin \theta \implies \theta = \sin^{-1}(2/\sqrt{2}) = 3\pi/4 \\
 \therefore (-1, 1, \sqrt{2}) &\rightarrow (2, 3\pi/4, 3\pi/4)
 \end{aligned}$$

9.)

$$x^2 + y^2 + z^2 = 9 \implies \rho^2 = 9 \implies \rho = 3$$

21.)

$$\begin{aligned}
 I &= \int_0^{2\pi} \int_0^\pi \int_0^5 \rho^6 \sin \phi \, d\rho \, d\phi \, d\theta = \int_0^{2\pi} \int_0^\pi \left[\frac{1}{7} \rho^7 \sin \phi \right]_0^5 \, d\phi \, d\theta \\
 &= \frac{5^7}{7} \int_0^{2\pi} \int_0^\pi \sin \phi \, d\phi \, d\theta = \frac{5^7}{7} \int_0^{2\pi} 2 \, d\theta = \frac{5^7 4\pi}{7}
 \end{aligned}$$

23.)

$$\begin{aligned}
 I &= 2 \int_0^{2\pi} \int_0^\pi \int_2^3 \rho^4 \sin^3 \phi \, d\rho \, d\phi \, d\theta = 2 \int_0^{2\pi} \int_0^\pi \left[\frac{1}{5} \rho^5 \sin^3 \phi \right]_2^3 d\phi \, d\theta = \\
 &= \frac{422}{5} \int_0^{2\pi} \int_0^\pi \sin^3 \phi \, d\phi \, d\theta = -\frac{422}{5} \int_0^{2\pi} \left[\cos \phi - \frac{1}{3} \cos^3 \phi \right]_0^\pi d\theta = \frac{422}{15} \int_0^{2\pi} 2 \, d\theta = \frac{1688\pi}{15}
 \end{aligned}$$

27.)

$$\begin{aligned}
 V &= \int_0^{2\pi} \int_{\pi/6}^{\pi/3} \int_0^a \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta = \frac{a^3}{3} \int_0^{2\pi} \int_{\pi/6}^{\pi/3} \sin \phi \, d\phi \, d\theta = \frac{(\sqrt{3}-1)a^3}{6} \int_0^{2\pi} d\theta \\
 &= \frac{1}{3}(\sqrt{3}-1)\pi a^3
 \end{aligned}$$

35.)

41.)

Exercises 16.2

1.)

$$\begin{aligned}
 r(t) &= \langle t^2, 2t \rangle, \quad r'(t) = \langle 2t, 2 \rangle, \quad \|r'(t)\| = 2\sqrt{t^2 + 1} \\
 \implies I &= \int_C y \, ds = \int_0^3 4t\sqrt{t^2 + 1} \, dt = 2 \int_0^3 \sqrt{u} \, du = 2 \left[\frac{2}{3}(t^2 + 1)^{3/2} \right]_0^3 = \frac{4}{3}(10^{3/2} - 1)
 \end{aligned}$$

3.)

$$\begin{aligned}
 r(t) &= \langle 4 \cos t, 4 \sin t \rangle, \quad r'(t) = \langle -4 \sin t, 4 \cos t \rangle, \quad \|r'(t)\| = \sqrt{16 \sin^2 t + 16 \cos^2 t} = 4 \\
 \implies I &= \int_C xy^4 \, ds = 4^5 \int_{-\pi/2}^{\pi/2} \cos t \sin^4 t \, dt = 4^5 \left[\frac{1}{5} \sin^5 t \right]_{-\pi/2}^{\pi/2} = \frac{2 \cdot 4^5}{5}
 \end{aligned}$$

7.)

$$\begin{aligned}
 r_1(t) &= \langle 2t, t \rangle, \quad r'_1(t) = \langle 2, 1 \rangle \\
 \implies I_1 &= 2 \int_0^1 4t \, dt + \int_0^1 4t^2 \, dt = 4 + \frac{4}{3} = \frac{16}{3} \\
 r_2(t) &= \langle t + 2, 1 - t \rangle, \quad r'_2(t) = \langle 1, -1 \rangle \\
 \implies I_2 &= \int_0^1 4 - t \, dt - \int_0^1 t^2 + 4t + 4 \, dt = \frac{7}{2} - \frac{19}{3} \\
 I &= I_1 + I_2 = \frac{16}{3} + \frac{7}{2} - \frac{19}{3} = \frac{7}{2} - 1 = \frac{5}{2}
 \end{aligned}$$

9.)

$$\begin{aligned}
 r(t) &= \langle \cos t, \sin t, t \rangle, \quad r'(t) = \langle -\sin t, \cos t, 1 \rangle, \quad \|r'(t)\| = \sqrt{\sin^2 t + \cos^2 t + 1} = \sqrt{2} \\
 \implies I &= \int_C x^2 y \, ds = \sqrt{2} \int_0^{\pi/2} \cos^2 t \sin t \, dt = -\sqrt{2} \int_0^{\pi/2} u^2 \, du \\
 &= -\sqrt{2} \left[\frac{1}{3} \cos^3 t \right]_0^{\pi/2} = \frac{\sqrt{2}}{3}
 \end{aligned}$$

15.)

$$\begin{aligned}
 r(t) &= \langle 3t + 1, t, 2t \rangle, \quad r'(t) = \langle 3, 1, 2 \rangle, \quad \|r'(t)\| = \sqrt{9 + 1 + 4} = \sqrt{14} \\
 \implies I &= 3 \int_0^1 4t^2 \, dt + \int_0^1 9t^2 + 6t + 1 \, dt + 2 \int_0^1 t^2 \, dt = 4 + 7 + \frac{2}{3} = \frac{35}{3}
 \end{aligned}$$

19.)

$$\begin{aligned}
 r(t) &= \langle t^3, t^2 \rangle, \quad r'(t) = \langle 3t^2, 2t \rangle \\
 \implies I &= \int_0^1 F(r(t)) \cdot r'(t) \, dt = \int_0^1 \langle t^7, -t^6 \rangle \cdot \langle 3t^2, 2t \rangle \, dt = \int_0^1 3t^9 - 2t^7 \, dt = \frac{1}{20}
 \end{aligned}$$

21.)

$$\begin{aligned}
 r(t) &= \langle t^3, -t^2, t \rangle, \quad r'(t) = \langle 3t^2, -2t, 1 \rangle \\
 \implies I &= \int_0^\pi \langle \sin(t^3) + \cos(-t^2) + t^4 \rangle \cdot \langle 3t^2, -2t, 1 \rangle \, dt \\
 &= \int_0^\pi 3t^2 \sin(t^3) - 2t \cos(-t^2) + t^4 \, dt = \left[-\cos(t^3) - \sin(-t^2) + \frac{1}{5}t^5 \right]_0^\pi
 \end{aligned}$$

41.)