

Abstract Algebra

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Preface

The goal of this book is to present the topics covered in a typical undergraduate-level course on abstract algebra. Beyond the definitions and theorems, however, ideas will be motivated in such a way that the reader may conclude that they could have invented them.

CHAPTER 0

Preliminaries

This book assumes that the reader has had some exposure to proof writing. If this is not the case, I highly recommend working through an introductory proof writing book before continuing, as proofs are one of the primary methods of communicating ideas, not only in abstract algebra, but in most higher level mathematics.

Set Theory

In math, a set is simply a collection of objects. These objects can be literally anything; numbers, functions, abstract concepts, even other sets can be elements of a set. All of the following are examples of sets:

$$\{1, 2, 3\} \quad \{\text{banana}, \text{apple}, \text{pear}\} \quad \{\{1, 3\}, \text{banana}, 1, \{\text{mailbox}, 5\}\}$$

The order of the elements in a set In addition, sets do not consider the “multiplicity” of their elements, that is if an element is included multiple times in a set, we view that as being equivalent to if the element was only included once. For example, the sets $\{1, 1, 2, 2\}$ and $\{1, 2, 2, 2, 2\}$ are both equivalent to $\{1, 2\}$. Sets that *do* take the multiplicity of their elements into consideration are called **multisets**, or less commonly, **bags**. Multisets will not be used in this book, so the reader

Set Builder Notation

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