1.) Find all integers $n$ that satisfy the following:

a.) $\phi(n) = n/2$

b.) $\phi(n) = \phi(2n)$

c.) $\phi(n) = 12$

**Solution:**

a.) $n = 2^m$ where $m \in \mathbb{Z}_{\geq 0}$

**Proof:**

a.) Let $n = 2^m$, then using theorem 2.5, we find that

$$\phi(2^m) = 2^m - 2^{m-1} = 2^{m-1} = n/2$$

Next, suppose $n$ is not a power of two and take its prime factorization as $\prod p_i^{a_i}$. Using the fact that $\phi$ is multiplicative, we find that

$$\phi(n) = \phi\left(\prod p_i^{a_i}\right) = \prod \phi(p_i^{a_i})$$

Since $n$ is not a power of two, there exists $i$ where $p_i > 2$ and $a_i \geq 1$, thus $\phi()$ Let $b_i = \phi(p_i^{a_i})$.

$$\phi(n) = b_1 b_2 b_3 \cdots$$

b.) awd

2.) Prove or disprove the following statements:

a.) If $(m, n) = 1$, then