1. $$\hat{y}_i = x_i \hat{\beta}$$

$$= x_i \cdot \frac{\sum_{i=1}^{n} \overset{\text{빠져}\downarrow}{x_i} y_i}{\sum_{i'=1}^{n} x_{i'}^2} = \frac{x_i}{\sum_{i'=1}^{n} x_{i'}^2} \cdot \sum_{i=1}^{n} x_i y_i = \frac{\sum_{i=1}^{n} x_i \cdot x_{i'} \cdot y_{i'}}{\sum_{i'=1}^{n} x_{i'}^2}$$

$$\therefore a_{i'} = \frac{x_i \cdot x_{i'}}{\sum_{i'=1}^{n} x_{i'}^2}$$

2. $$\hat{\beta} = (X^T X)^{-1} X^T y, \qquad X = QR$$

$$= (R^T Q^T Q R)^{-1} (R^T Q^T) y$$

$$= (R^T R)^{-1} (R^T Q^T) y \qquad (\because Q^T Q = I : \text{orthonormal})$$

$$= R^{-1} (R^T)^{-1} R^T Q^T y = R^{-1} Q^T y, \qquad y = \hat{\beta} \cdot R \cdot Q = X \cdot \hat{\beta}$$

$$Q = Z \cdot D^{-1}$$

$$= \begin{bmatrix} z_{11} & \cdots & z_{p1} \\ \vdots & & \\ z_{1p} & \cdots & z_{pp} \end{bmatrix} \begin{bmatrix} \frac{1}{\|z_{11}\|} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \frac{1}{\|z_{pp}\|} \end{bmatrix} = \begin{bmatrix} \frac{z_{11}}{\|z_{11}\|} & & 0 \\ & \ddots & \\ 0 & & \frac{z_{pp}}{\|z_{pp}\|} \end{bmatrix} = [q_1 \cdots q_p]$$

$$X = QR \quad (R = D\Gamma)$$

$$= [q_1 \cdots q_p] \begin{bmatrix} (x_1 \cdot q_1) & (x_2 \cdot q_1) & \cdots & (x_p \cdot q_1) \\ 0 & (x_2 \cdot q_2) & \cdots & (x_p \cdot q_2) \\ \vdots & 0 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & & (x_p \cdot q_p) \end{bmatrix}$$

$$\left(\begin{bmatrix} (x_1 \cdot q_1) & 0 & \cdots & 0 \\ (x_2 \cdot q_1) & (x_2 \cdot q_2) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ (x_p \cdot q_1) & (x_p \cdot q_2) & & (x_p \cdot q_p) \end{bmatrix} \begin{bmatrix} (x_1 q_1) & \cdots & (x_p q_1) \\ 0 & & (x_p q_2) \\ \vdots & & \vdots \\ 0 & & (x_p \cdot q_p) \end{bmatrix} \right)^{-1} \begin{bmatrix} (x_1 \cdot q_1) & 0 & \cdots & 0 \\ (x_2 \cdot q_1) & (x_2 \cdot q_2) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ (x_p \cdot q_1) & (x_p \cdot q_2) & & (x_p \cdot q_p) \end{bmatrix} \cdot Q^T y$$

$$= \begin{bmatrix} \frac{1}{(x_1 q_1)^2} & & 0 \\ & \ddots & \\ 0 & & \frac{1}{(x_p q_p)^2} \end{bmatrix} \begin{bmatrix} (x_1 q_1) & & 0 \\ & \ddots & \\ 0 & & (x_p q_p) \end{bmatrix} Q^T y = \begin{bmatrix} \frac{1}{(x_1 q_1)} & & 0 \\ & \ddots & \\ 0 & & \frac{1}{(x_p q_p)} \end{bmatrix} Q^T y = R^{-1} Q^T y$$

```
3.
```

In [23]:
```
import ssl
import pandas as pd
ssl._create_default_https_context = ssl._create_unverified_context #Github에서 데이터를 바로 불러오도록 하는 세팅입니다. 해당 코드 목
data = pd.read_csv('https://github.com/YonseiESC/ESC-21SUMMER/blob/main/week1/HW/week1_data.csv?raw=True')

y = data['mpg']
x = dataQ1. 자연수를 입력 받아서 해당 숫자 미만의 자연수에서 3, 7 의 배수의 총합을 구하는 함수를 만들어라! 반환의 결과는 int형으로 반환해야 한다. [2

import numpy as np

index = np.where(x['horsepower'] == '?')
```

In [24]:
```
index
```

Out[24]:
```
(array([ 32, 126, 330, 336, 354]),)
```

In [25]:
```
index[0]
```

Out[25]:
```
array([ 32, 126, 330, 336, 354])
```

In [26]:
```
data1= data.drop(index[0])
```

In [27]:
```
data1 = data1.astype({'horsepower': 'float'})
```

In [28]:
```
y = data1['mpg']
x = data1.drop(['mpg'],axis=1)

def YourOwnRegression(x,y):
    # beta와 y의 추정값을 반환하는 함수를 만들어주세요.
    beta_hat = np.dot(np.dot(np.linalg.inv(np.dot(x.T,x)),x.T),y)
    y_hat= np.dot(x,beta_hat)
    return beta_hat,y_hat

# 결과물 반환
YourOwnRegression(x,y)
```

Out[28]:
```
(array([-0.5226089 ,  0.01022108, -0.020873  , -0.00639456, -0.05202195,
         0.61025869]),
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```

```
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