CSCE 221 HW 3

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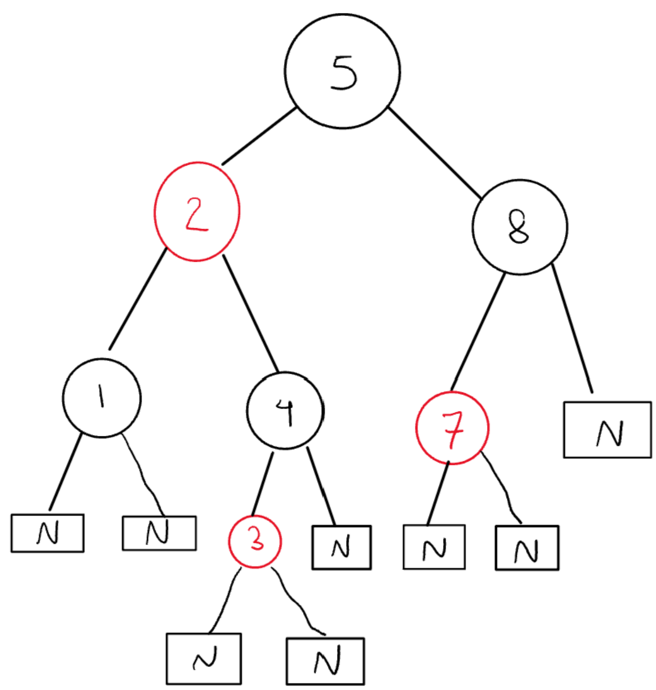
1 Exercise A

A(i)



This is a red-black tree because the root and leaves are black, there are no two adjacent red nodes, and every path from a node has the same number of black nodes (uniform black depth from each leaf).

A(ii)
a) The root is always black as the function colors it black before beginning recursion.

b) The augmented sentinel nodes are defaulted to NULL and black and serve as leaf nodes.

c) No red node can have a red child because the function only colors nodes of alternating heights red at most.

d) The black depth of a node is the number of black nodes from the root to that node (the number of ancestors).

Strong induction: prove that black depth = ceil(h/2) + 1.
Inductive hypothesis: At least half of all nodes in a path must be black (no two red nodes in a row, leaf and root nodes are black).

Basis Step: Tree with one node (P(1)).
Height = 0
Black depth = 1 (root is black).
Supported by ceil (0/2) + 1 = 1.

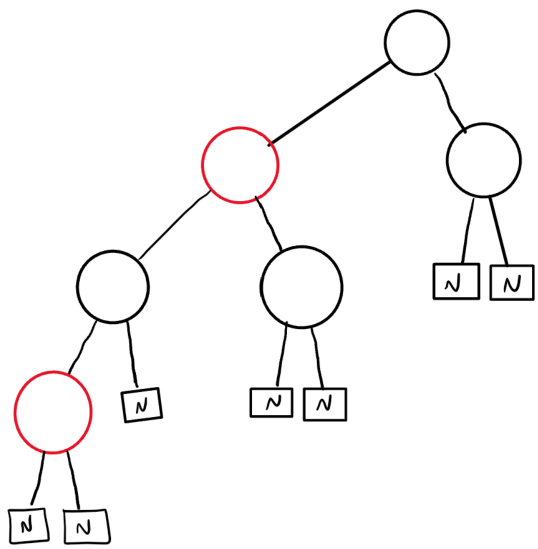
Inductive Step: Tree with two nodes (P(2)).
Height = 1
Black depth = 2 (root and leaf are black).
Supported by ceil (1/2) + 1 = 2.

P(n+1) implies height = h + 1
and that black depth = black depth + 1
ceil((h+1)/2) + 1 = ceil(h/2) + ceil(1/2) + 1
=ceil(h/2) + 2
black depth = ceil(h/2) + 1 by IH, so black depth + 1 = ceil(h/2) + 2 = ceil(h/2) + 2.

In conclusion, the black depth of a tree of height h can be determined by ceil(h/2) + 1, as proved by strong induction.

2 Exercise B

Non-AVL Red-Black Tree:



3 Exercise C

h(12)=(3x12+5) mod 11 = 41 mod 11 = 8
h(44)=(3x44+5) mod 11 = 137 mod 11 = 5
h(13)=(3x13+5) mod 11 = 44 mod 11 = 0
h(88)=(3x88+5) mod 11 = 269 mod 11 = 5
h(23)=(3x23+5) mod 11 = 74 mod 11 = 8
h(94)=(3x94+5) mod 11 = 287 mod 11 = 1
h(11)=(3x11+5) mod 11 = 38 mod 11 = 5
h(39)=(3x39+5) mod 11 = 122 mod 11 = 1
h(20)=(3x20+5) mod 11 = 65 mod 11 = 10
h(16)=(3x16+5) mod 11 = 53 mod 11 = 9
h(5)=(3x5+5) mod 11 = 20 mod 11 = 9

The hash table after inserting all keys using the above indices is: