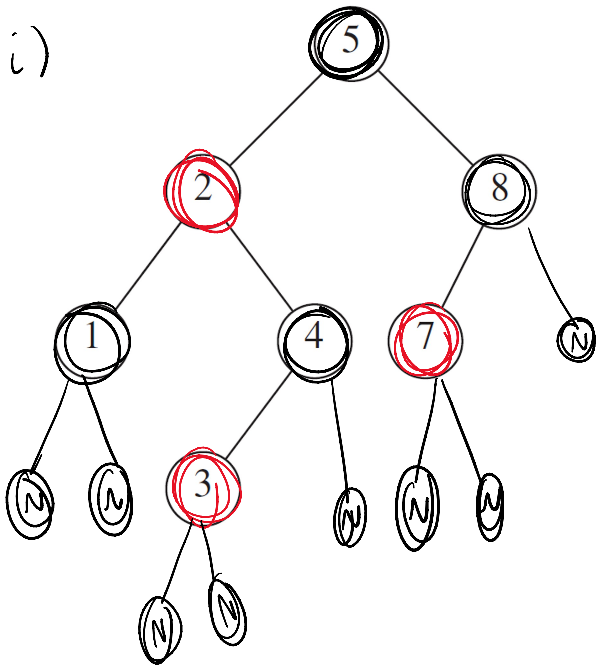
HW 3 Scratch

Sunday, March 14, 2021 2:38 PM

A(i)



This is a red-black tree because the root & leaves are black, there are no two adjacent red nodes, and every path from a node has the same number of black nodes.

A(ii)

a) The root is always black as the function colors it black before recursing.

b) The sentinel nodes are defaulted to NULL and black and serve as the leaf nodes.

c) No red node can have a red child

c) No red node can have a red child because the function only colors nodes of alternating heights red.

d) The black depth of a node is the number of black nodes from the root to that node (ancestors).

Strong induction: prove black depth $=$ ceil $(h/2)+1$

Basis Step

Inductive Step

↳ Induction Hypothesis

Conclusion

IH: At least half of all nodes in a path must be black (no two red nodes in a row, leaf and root are black).

Basis Step: Tree with one node.

Height $= 0$ and root is black, so black depth is 1.

Supported by ceil $(0/2)+1 = 1$ ✓

Inductive Step: Tree with two nodes.

Height $= 1$ and leaf node is black, so black depth is 2.

Supported by ceil $(1/2)+1 = 2$ ✓

$P(n+1)$ implies height $= h+1$

$$\text{ceil}\left(\frac{h+1}{2}\right) + 1 = \text{ceil}\left(h/2\right) + \text{ceil}\left(1/2\right) + 1$$

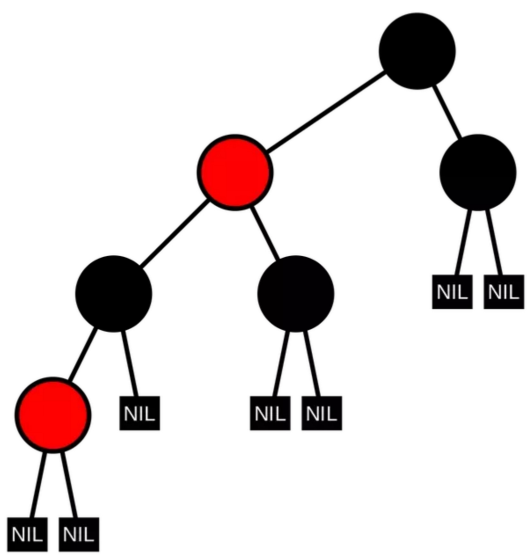
$$= \text{ceil}\left(h/2\right) + 2$$

black depth $= \text{ceil}(h/2) + 1$ by IH, so

black depth $+1 = \text{ceil}(h/2) + 2 = \text{ceil}(h/2)$ ✓

∴ The black depth of a tree of height h can be determined by $\text{ceil}(h/2) + 1$, as proved by strong induction.

B)



C)

Find the index values for all the keys using the hash function $h(i)$ as shown below:

$h(12) = (3\times 12 + 5)$ mod $11 = 41$ mod $11 = 8$

$h(44) = (3\times 44 + 5)$ mod $11 = 137$ mod $11 = 5$

$h(13) = (3\times 13 + 5)$ mod $11 = 44$ mod $11 = 0$

$h(88) = (3\times 88 + 5)$ mod $11 = 269$ mod $11 = 5$

$h(23) = (3\times 23 + 5)$ mod $11 = 74$ mod $11 = 8$

$h(94) = (3\times 94 + 5)$ mod $11 = 287$ mod $11 = 1$

$h(11) = (3\times 11 + 5)$ mod $11 = 38$ mod $11 = 5$

$h(39) = (3\times 39 + 5)$ mod $11 = 122$ mod $11 = 1$

$h(20) = (3\times 20 + 5)$ mod $11 = 65$ mod $11 = 10$

$h(16) = (3\times 16 + 5)$ mod $11 = 53$ mod $11 = 9$

c)

Find the index values for all the keys using the hash function $h(i)$ as shown below:

$h(12)=(3\times 12+5) \bmod 11=41 \bmod 11=8$

$h(44)=(3\times 44+5) \bmod 11=137 \bmod 11=5$

$h(13)=(3\times 13+5) \bmod 11=44 \bmod 11=0$

$h(88)=(3\times 88+5) \bmod 11=269 \bmod 11=5$

$h(23)=(3\times 23+5) \bmod 11=74 \bmod 11=8$

$h(94)=(3\times 94+5) \bmod 11=287 \bmod 11=1$

$h(11)=(3\times 11+5) \bmod 11=38 \bmod 11=5$

$h(39)=(3\times 39+5) \bmod 11=122 \bmod 11=1$

$h(20)=(3\times 20+5) \bmod 11=65 \bmod 11=10$

$h(16)=(3\times 16+5) \bmod 11=53 \bmod 11=9$

$h(5)=(3\times 5+5) \bmod 11=20 \bmod 11=9$

The hash table after inserting all the keys using the above index values is shown as below: