Analyze of Part One: Pascal Matrix

Why is it justified to use the LU or QR factorizations as opposed of calculating an inverse matrix?

A small disturbance when solving Ax = b by inverse matrices in high dimension matrices may results in a big difference in the result. However, LU and QR factorizations are using different ways to minimum the error difference in matrix A so it is better.

What is the benefit of using LU or QR factorization in this way?

The LU factorization makes Ax=b into two individual steps. One is Ly=b and the other is Ux=y.

Considering the condition numbers for L, U and A, we know that cond(A) is equal to cond(LU), since A=LU.

But cond(A) must be equal or less than cond(L)\*cond(U).

The QR factorization using an even better way to split A as an orthogonal matrix Q and an upper diagonal matrix R.

cond(Q) = 1 since its orthogonal so cond(A) is equals to cond(QR), which equals to cond(R).

So when we are using LU or even QR, we are basically calculating matrices with less error disturbance(minimized the error amplification), and that’s the reason why we prefer solving Ax=b by decompositions.