Numerical Interpretation of Tensity Structures

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April 21, 2023

Abstract

The simple concept of connecting bars and cables to make a tensegrity structure can make a complex system. The physics behind tensegrity structures offers many interesting problems that can be solved by optimization algorithms. With the BFGS algorithm we have found optimal resting positions for different structures that minimizes the energy in the structures. This ultimately means that we have found the positions where the system is under the least stress. Our results can be applied to model actual physical structures and is a great way to examine how a structure will behave.

1 Introduction

In this project are we going to investigate the properties of tensegrity structures. Tensegrity structures are structures that consists of elastic cables and bars that are connected at joints (nodes). The structure keeps its shape largely due to tensity in the cables and gravity, but are also affected by the compression and stretching in the bars.
We are interested in simulating how a structure would form given different initial conditions. The problem can be described as an optimization problem, where the energy of the system would converge to a local minimum to stay stable.

1.1 Notation and Definitions

We model the structure as a directed graph where the edges is the cables and bars, and the vertexes s are the nodes.
The edges e_{ij} where $i < j$ is represent either a cable or a bar connecting the two joint i, j.
A node is denoted by its positions, $x^{(i)} = (x_1^{(i)}, x_2^{(i)}, x_3^{(i)}) \in R^3$. And we make a large vector $X = (x^{(i)}, ..., x^{(N)}) \in R^{3N}$ that consist of all the nodes.

1.1.1 Energy functions

We are now ready to define the functions we use to calculate the energy.
We assume the weight of the cables are zero, the energy from the cables therefore only depends on its stretching. We introduce the variable l_{ij} that represent the resting length of the cable.

$$E_{\text{elast}}^{\text{cable}}(e_{ij}) = \begin{cases} \frac{k}{2\ell_{ij}^2}\left(\left\|x^{(i)} - x^{(j)}\right\| - \ell_{ij}\right)^2 & \text{if } \left\|x^{(i)} - x^{(j)}\right\| > \ell_{ij} \\ 0 & \text{if } \left\|x^{(i)} - x^{(j)}\right\| \leq \ell_{ij} \end{cases}. \tag{1}$$

Notice that the cable can not be compressed.

The bars can both be compressed and have mass.

$$E^{bar}(e_{ij}) = E^{bar}_{elast}(e_{ij}) + E^{bar}_{grav}(e_{ij}) = \frac{c}{2\ell_{ij}^2}\left(\left\|x^{(i)} - x^{(j)}\right\| - \ell_{ij}\right)^2 + \frac{\rho g \ell_{ij}}{2}\left(x_3^{(i)} + x_3^{(j)}\right). \tag{2}$$

Where ρ and g are the mass density and gravity. $c > 0$ is a parameter depending on the material and the cross section of the bar.

We also include external force in form of weigh on specific nodes. Node mass of node $x^{(i)} = m_i \geq 0$

$$E_{ext}(X) = \sum_{i=1}^{N} m_i g x_3^{(i)}. \tag{3}$$

1.1.2 The Problem

The energy in tensegrity structures can be modeled by equation (4). By optimizing this equation with constraints one can find the resting position for a given structure given initial positions.

$$E(X) = \sum_{e_{ij} \in \mathcal{B}} \left(E_{elast}^{bar}(e_{ij}) + E_{grav}^{bar}(e_{ij}) \right) + \sum_{e_{ij} \in \mathcal{C}} E_{elast}^{cable} + E_{ext}\big(X\big) \tag{4}$$

takes in a connected graph with edges e_{ij} connecting the nodes either as bars ($\mathcal{B}$) or cables ($\mathcal{C}$). For the problem to have a solution a constraint need to be applied.
The following constraint is the case where some of the nodes are fixed denoted by

$$x^{(i)} = p^{(i)} \text{ for } i = 1, ..., M. \tag{5}$$

The second constraint only lets nodes exist above the ground which is restricted in the x_3-direction as follows

$$x_3^{(i)} \geq 0 \text{ for all } i = 1, ..., M. \tag{6}$$

2 Theory

2.0.1 Solutions to the optimization problem

To show that (4) admits a solution given by either constraint (5) or (6) we use the fact that if a function is coercive, lower semi continuous and is in $\mathbb{R}^d$ the function has a solution. The function is put together by norms and polynomials which are continuous. A sum of continuous functions is also continuous and therefore lower semi continuous. Since we operate in $\mathbb{R}^3$ the only thing left is to show that the function is coercive by either constraint (5) or (6).
By constraint (5) let one ore more of the nodes in the structure be fixed. When a node is fixed and you let $||x^{(i)}|| \to \infty$ either a bar or a cable will necessarily be stretched to infinity since the graph is connected. This makes the energy in the structure tend to infinity as well. Even if $x_3 \to -\infty$ the second order terms will dominate and still make the function tend to positive infinity.
Constraint (6) will not admit a solution since the function is not coercive by this constraint. As shown in 2.3.2 the function is invariant in the horizontal plane which means you can let $||x^{(i)}|| \to \infty$ but still have finite energy. For this to admit a solution you need another constraint which explained further in 2.3.2.

2.1 Cable Nets

We start with looking at a simpler case. Cable nets is a structure where all members of the structure are cables. We use the constraint (5) were some of the nodes are fixed. We then end up with the optimization problem

$$\min_X E(X) = \sum_{e_{ij} \in \varepsilon} E_{elast}^{cable}(e_{ij}) + E_{ext}(X). \tag{7}$$

2.1.1 Continuous and Differentiable

We can show that $E(X)$ in (7) is C^1 but not C^2.
$E_{ext(X)} \in R^1$ and linear thus continuously differentiable.
Looking at

$$E_{\text{elast}}^{\text{cable}}\ (e_{ij}) = \begin{cases} \frac{k}{2\ell_{ij}^2}\left(\left\|x^{(i)} - x^{(j)}\right\| - \ell_{ij}\right)^2 & \text{if } \left\|x^{(i)} - x^{(j)}\right\| > \ell_{ij} \\ 0 & \text{if } \left\|x^{(i)} - x^{(j)}\right\| \leq \ell_{ij} \end{cases}.$$

If we define a variable $P = ||x^{(i)} - x^{(j)}|| \in R^1$ we can rewrite the equation.

$$E_{\text{elast}}^{\text{cable}}\ (P) = \begin{cases} \frac{k}{2\ell_{ij}^2}\left(P - \ell_{ij}\right)^2 & \text{if } P > \ell_{ij} \\ 0 & \text{if } P \leq \ell_{ij} \end{cases}.$$

$$E_{\text{elast}}^{\text{cable}}\ '(P) = \begin{cases} \frac{k}{\ell_{ij}^2}\left(P - \ell_{ij}\right) & \text{if } P > \ell_{ij} \\ 0 & \text{if } P \leq \ell_{ij} \end{cases}.$$

$$E_{\text{elast}}^{\text{cable}}\ ''(P) = \begin{cases} \frac{k}{\ell_{ij}^2} & \text{if } P > \ell_{ij} \\ 0 & \text{if } P \leq \ell_{ij} \end{cases}.$$

Observe that $\lim\limits_{P\to l_{ij}} E_{\text{elast}}^{\text{cable}}\ (P) = \lim\limits_{P\to l_{ij}} E_{\text{elast}}^{\text{cable}}\ '(P) = 0$,

but $\lim\limits_{P\to l_{ij}^+} E_{\text{elast}}^{\text{cable}}\ ''(P) = \dfrac{k}{\ell_{ij}^2} \neq \lim\limits_{P\to l_{ij}^-} E_{\text{elast}}^{\text{cable}}\ ''(P) = 0$. $\implies E(X) \in C^1$, $E(X) \notin C^2$.

2.1.2 Convexity

We want to show that (7) is convex. We use that a function h is convex if for all $0 \leq t \leq 1$ and all $x, y \in X$

$$h(tx + (1-t)y) \leq th(x) + (1-t)h(y) \tag{8}$$

holds.
For (7) we have that:
$E_{ext}(X)$ is a linear function, therefore convex.
If $||x^i - x^j|| \leq l_{ij}$ then

$$E_{elast}^{cable}(e_{ij}) = 0$$

which is convex.
We now want to show that $E_{elast}^{cable}(e_{ij})$ is convex for $||x^i - x^j|| > 0$.
We define a function

$$f(s) = \begin{cases} (s-\mu)^2, \text{ if } s > \mu \\ 0, \text{ if } s \leq \mu \end{cases}$$

f is a squared function, therefore convex. We also see that $f'(s) \geq 0$ thus f is an increasing function. The norm is convex by the triangle inequality.

$$\Rightarrow ||x^{(i)} - x^{(j)}|| = L(e_{ij})$$

where e_{ij} is an arbitrary cable and $x^{(i)}, x^{(j)}$ are the nodes connecting the cable is convex.
For two arbitrary cables c_1, c_2 we can use the property of a convex function.

$$L(tc_1 + (1-t)c_2) \leq tL(c_1) + (1-t)L(c_2).$$

f is an increasing function and convex thus

$$f(L(tc_1 + (1-t)c_2) \leq f(tL(c_1) + (1-t)L(c_2)) \leq tf(L(c_1)) + (1-t)f(L(c_2)).$$

We can now show that

$$E_{elast}^{cable} = \underbrace{\frac{k}{l_{ij}^2}}_{constant} \underbrace{f(L(e_{ij}))}_{convex}, \ L(e_{ij}) > l_{ij}.$$

is convex because it is the product of a constant and a convex function.
therefore (7)

$$\min_X E(X) = \sum_{e_{ij} \in \varepsilon} \underbrace{E_{elast}^{cable}(e_{ij})}_{convex} + \underbrace{E_{ext}(X)}_{convex},$$

as a sum of convex function is convex.

The function is not necessarily strictly convex. If we look at a system consisting of just one cable e_{ij}. Let the masses of the nodes $x^{(i)}$, $x^{(j)}$ to the corresponding cable be zero. Then $E_{ext}(X) = 0$ for the system.
If we fix the node $x^{(i)} = P$ then $E(X) = E_{elast}^{cable}(e_{ij}) = 0$ for all $x^{(j)}$ such that $||x^{(i)} - x^{(j)}|| \leq l_{ij}$
Therefore we do not have a unique solution for all systems.

2.1.3 Necessary and Sufficient Conditions

When there are multiple solutions to an optimization problem, we need optimality conditions to identify the best one. Necessary conditions are what any optimal solution must satisfy, while sufficient conditions guarantee that a solution is optimal. Establishing these conditions is essential even when the solution is not unique.
The first order necessary optimality conditions states that if x^* is a local solution to a optimisation problem then the gradient at x^* is zero. Therefore we get the equation:

$$\nabla E = \nabla(\sum_{e_{ij} \in \mathcal{C}} E_{elast}^{cable}(e_{ij}) + E_{ext}(X)) = 0$$

Replacing the terms, and calculate change order of operations, since gradient is a linear operation.

$$\sum_{e_{ij} \in \mathcal{C}} \frac{k}{2l_{ij}} \nabla(||x^{(i)} - x^{(j)}|| - l_{ij})^2 + \sum_{i=1}^{N} \nabla m_i g x_3^{(i)}$$

Then:

$$\sum_{e_{ij} \in \mathcal{C}} \frac{k}{2l_{ij}} \frac{d}{dx}(||x^{(i)} - x^{(j)}|| - l_{ij})^2 + \sum_{i=1}^{N} \begin{bmatrix} 0 \\ 0 \\ m_i g \end{bmatrix}$$

Chain rule gives:

$$\sum_{e_{ij} \in \mathcal{C}} \frac{2k}{2l_{ij}} (||x^{(i)} - x^{(j)}|| - l_{ij}) \cdot \frac{d}{dx}(||x^{(i)} - x^{(j)}|| - l_{ij}) + \sum_{i=1}^{N} \begin{bmatrix} 0 \\ 0 \\ m_i g \end{bmatrix}$$

$$\sum_{e_{ij} \in \mathcal{C}} \frac{2k}{2l_{ij}} (||x^{(i)} - x^{(j)}|| - l_{ij}) \cdot \frac{(x^{(i)} - x^{(j)})}{||x^{(i)} - x^{(j)}||} + \sum_{i=1}^{N} \begin{bmatrix} 0 \\ 0 \\ m_i g \end{bmatrix} \tag{9}$$

For the second order necessary conditions we need the Hessian to be positive semi-definite. Since E is convex its Hessian will be positive semi-definite, and we have second order necessary conditions.
From (2.1.2) we have shown that the function is convex. In the preliminaries from Markus Grasmair [1] it is stated that the first order necessary conditions are also sufficient for convex functions.

2.2 Bars and cables (Tensegrity-domes)

We are now ready to include bars to our analysis. Bars acts similar to cables, but adds tensity when the bar is under compression as well as extension. We still optimize with respect to constraint (5). The optimisation problems reads;

$$\min_X E(X) = \sum_{e_{ij}\in\mathcal{B}} \left(E_{elast}^{bar}(e_{ij}) + E_{grav}^{bar}(e_{ij})\right) + \sum_{e_{ij}\in\mathcal{C}} E_{elast}^{cable} + E_{ext}(X). \tag{10}$$

2.2.1 Differentiable

We now work with problem (10) where bars are added to the system. Here you will encounter a problem when you differentiate because of the term $E_{elast}^{bar}(e_{ij}) = \frac{c}{1l_{ij}^2}(||x^{(i)} - x^{(j)}|| - l_{ij})^2$. When writing this out the term $2l_{ij}||x^{(i)} - x^{(j)}||$ will appear. This is not differentiable when $x^{(i)} = x^{(j)}$. What this means in physical terms is that the bar is compressed to zero length which in practice will not happen.

2.2.2 Necessary and sufficient conditions

To find necessary optimality conditions we compute $\nabla E = 0$ with the function (10)

$$\nabla E(X) = \nabla \sum_{e_{ij}\in\mathcal{B}} (\frac{c}{2l_{ij}^2}(||x^{(i)} - x^{(j)}|| - l_{ij})^2 + \sum_{e_{ij}\in\mathcal{B}} \frac{\rho g l_{ij}}{2}(x_3^{(i)} - x_3^{(j)}) + (9) = 0$$

Gradient is a linear operation, so the terms ∇E_{elast}^{cable} and ∇E_{grav} will just be the same as in (9).

$$= \sum_{e_{ij}\in\mathcal{B}} \frac{2c}{2l_{ij}}(||x^{(i)} - x^{(j)}|| - l_{ij}) \cdot \frac{d}{dx}(||x^{(i)} - x^{(j)}|| - l_{ij}) + \sum_{i=1}^{N} \begin{bmatrix} 0 \\ 0 \\ \sum_{e_{ij}} \frac{\rho g l_{ij}}{2} \end{bmatrix} + (9)$$

Altogether this gives us the expression:

$$\sum_{e_{ij}\in\mathcal{B}} \frac{c}{l_{ij}^2}(||x^{(i)} - x^{(j)}|| - l_{ij}) \cdot \frac{(x^{(i)} - x^{(j)})}{||x^{(i)} - x^{(j)}||} + \sum_{i=1}^{N} \begin{bmatrix} 0 \\ 0 \\ \sum_{e_{ij}} \frac{\rho g l_{ij}}{2} \end{bmatrix} + \sum_{e_{ij}\in\mathcal{C}} \frac{2k}{2l_{ij}}(||x^{(i)} - x^{(j)}|| - l_{ij}) \cdot \frac{(x^{(i)} - x^{(j)})}{||x^{(i)} - x^{(j)}||} + \sum_{i=1}^{N} \begin{bmatrix} 0 \\ 0 \\ m_i g \end{bmatrix} \tag{11}$$

To find out if the necessary conditions is also sufficient conditions, we need to figure out if the function is convex.

2.2.3 Convexity

In problem (10) let $\mathcal{B} \neq \emptyset$. We now need to consider the term $E_{elast}^{bar}(e_{ij}) = \frac{c}{2l_{ij}^2}(||x^{(i)} - x^{(j)}|| - l_{ij})^2$. Assume an edge $X = (x^{(i)}, x^{(j)})$ such that $||x^{(i)} - x^{(j)}|| = l_{ij}$ and an opposite edge $Y = -X = (-x^{(i)}, -x^{(j)})$. This will result in $E(X) = E(Y) = 0$. Now we calculate one side of the convex inequality

$$E(\lambda X + (1-\lambda)Y) = \frac{c}{2l_{ij}^2}(||\lambda(x^{(i)} - x^{(j)}) + (1-\lambda)(-x^{(i)} + x^{(j)})|| - l_{ij})^2$$

$$= \frac{c}{2l_{ij}^2}(||\lambda x^{(i)} - \lambda x^{(j)} - x^{(i)} + \lambda x^{(i)} + x^{(j)} - \lambda x^{(j)}|| - l_{ij})^2 = \frac{c}{2l_{ij}^2}((2\lambda - 1)||x^{(i)} - x^{(j)}|| - l_{ij})^2.$$

We can see that for all $0 \leq \lambda < 1$ we get $((2\lambda - 1)||x^{(i)} - x^{(j)}|| - l_{ij})^2 > 0$. It follows from this that there exist a $0 \leq \lambda \leq 1$ such that $E(\lambda X + (1-\lambda)Y) > \lambda E(X) + (1-\lambda)E(Y)$ and thus problem (10) is non-convex. As the function is non-convex the optimality conditions we found was only necessary conditions, not sufficient.

A simple example of system with a local minima is a standing pyramid of bars with mass, and with fixed nodes at the base. Standing in position will be a local minima, but the global minima will be a flipped structure with the peak below the base. This is illustrated in figure (1) with local minima on the left and global on the right.

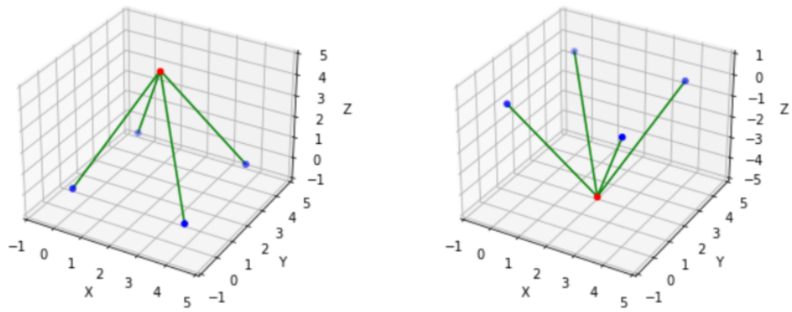

Figure 1: The blue nodes are fixed while the red is free

2.3 Free-Standing Structures

Finally we will focus on a tensegrity structure that is not fixed to anything, and all its nodes are unrestricted except for the requirement that it stays elevated off the ground.
Meaning we are looking at the optimization problem

$$\min_X E(X) = \sum_{e_{ij}\in\mathcal{B}} \left(E^{bar}_{elast}(e_{ij}) + E^{bar}_{grav}(e_{ij})\right) + \sum_{e_{ij}\in\mathcal{C}} E^{cable}_{elast} + E_{ext}(X) \tag{12}$$

with constraint (6).

2.3.1 KKT-Conditions

First we need to state the first order optimality conditions [2].
For the first condition we have

$$\nabla E(X^*) - \sum_{i\in A(x^*)} \lambda_i^* \nabla c_i(X^*) = 0.$$

Where $A(x^*)$ denote the active constraints.
The gradient for the constraints can be expressed as:

$$\nabla c_i(X^*) = \underbrace{[(0,0,0), ...(0.0.0), \underbrace{(0,0,1)}_{\text{i-th element}}, (0,0,0), ..., (0.0.0)]}_{N} \quad \text{for } i = 1, 2, ..., N. \tag{13}$$

Which gives us the conditions:

$$\begin{align} \nabla E(X^*) - [(0,0,\lambda_1), (0,0,\lambda_2), ..., (0,0,\lambda_N)] = 0 \tag{14a}\\ c_i(X^*) \geq 0 \quad \text{for all } i \in I \tag{14b}\\ \lambda_i^* \geq 0 \quad \text{for all } i \in I \tag{14c}\\ \lambda^* c_i(X^*) = 0 \quad \text{for all } i \in \epsilon \bigcup I \tag{14d} \end{align}$$

Where $\lambda_i = 0$ for the non active constraints in (14a). Also observe from (13) that the LICQ condition for the active constraints is satisfied.

From (2.2.3) we have that the system is not convex. Therefore the conditions are only necessary, but not sufficient. There could be a case with a global maxima or saddle point that can satisfy the conditions.

2.3.2 Horizontal shifts

In the optimization problem (10) horizontal coordinates x_1 and x_2 only appears in the normed differences
$||x^{(i)} - x^{(j)}||$. If all nodes where shifted horizontally by some shift length $s = (\alpha, \beta, 0)^T$ the normed differences would be the following

$$||(x_1^{(i)}+\alpha, x_2^{(i)}+\beta, x_3^{(i)})^T-(x_1^{(j)}+\alpha, x_2^{(j)}+\beta, x_3^{(j)})^T|| = ||(x_1^{(i)}+\alpha-(x_1^{(j)}+\alpha), x_2^{(i)}+\beta-(x_2^{(j)}+\beta), x_3^{(i)}-x_3^{(j)})^T||$$
$$= ||(x_1^{(i)} - x_1^{(j)}, x_2^{(i)} - x_2^{(j)}, x_3^{(i)} - x_3^{(j)})^T|| = ||x^{(i)} - x^{(j)}||.$$

As we can see the value remains unchanged by the shift and thus will not change the value of the energy E. The problem is then invariant under simultaneous horizontal shifts of the nodes. This will admit an infinite set of solutions as long as the ground is the $z = 0$ plane.
This can be solved by adding a constraint to one of the nodes such that the node cannot move in x_1 and x_2 directions. This is simply just a choice of reference and will not affect the solution since it is horizontally invariant.

3 Numerical interpretation

We are now ready to implement algorithms to solve the different optimization problems.

3.1 Implementation

We have chosen to use BFGS with strong Wolfe conditions to solve the problems with quadratic penalization.

$$\min_{x\in R^n} Q(x;\mu) := E(X) + \frac{\mu}{2}\sum_{i\in\varepsilon} c_i^2(x) + \frac{\mu}{2}\sum_{i\in I}((c_i(x))^-)^2 \quad \mu > 0 \tag{15}$$

The BFGS method has the parameters: $f, \nabla f, X_0, edges, l, p, mg, k, c, \rho g, \mu = False, maxiter = 1000, \varepsilon = 10^{-14}$. f and ∇f is the objective function and its gradient one wants to optimize. X_0 is a $N \times 3$ matrix, which holds the initial position of the N nodes. $edges$, is a tuple of two lists containing which node is connected to each other with cable and bars. l is a $N \times N$ neighbour matrix which holds all the initial cable and bar lengths. At column i, row j in the matrix, one finds the cable/bar length l_{ij} from i to j.

$$l = \begin{bmatrix} 0 & & \cdots & & \\ & \ddots & l_{ij} & \ddots & \\ & & \cdots & & 0 \end{bmatrix} \tag{16}$$

$mg, k, c, \rho g$ is used in the energy functions. μ is $False$ if one uses no penalties/constraints, otherwise μ is put into the functions and used as a penalty constant. $X_{analytic}$ is used to calculate the error at iteration i by $||X^i - X_{analytic}||_2$. If $X_{analytic}$ is not specified, then the convergence plot is calculated using $||\nabla E(X^i)||_2$.

3.2 Cable net

We are now going to solve (7). Since the function is convex, and we use BFGS with strong Wolfe conditions the algorithm will converge towards the global minima.
We have used the following parameters:

- $k = 3$.
- $l_{ij} = 3$ for all edges (i, j).
- Fixed points: $p^1 = (5, 5, 0)$, $p^2 = (-5, 5, 0)$, $p^3 = (-5, -5, 0)$, $p^4 = (5, -5, 0)$.
- $m_i g = 1/6$ for $i = 5, 6, 7, 8$.

In figure (2) we observe from the rightmost plot that we get seemingly quadratic convergence. The initial position of the nodes are randomly generated and as we can see the optimized structure converges towards the analytical solution.

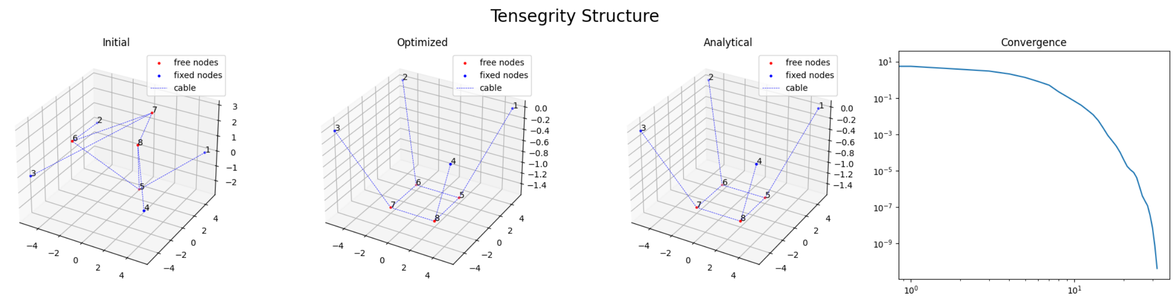

Figure 2: Cable net and Energy convergence plot

3.3 Bars and Cables Structures

As stated earlier we will now include bars and solve (10). We saw that bars add a new complexity to the structures, this is also illustrated in figure (1). Since bars add tension when compressed, the smallest energy on a bar is when its length is equal to its resting length. That can result in standing structures. These structures illustrates local minimas of the Energy-function. The parameters used is:

- $\ell_{15} = \ell_{26} = \ell_{37} = \ell_{48} = 10$
- $\ell_{18} = \ell_{25} = \ell_{36} = \ell_{47} = 8$
- $\ell_{56} = \ell_{67} = \ell_{78} = \ell_{58} = 2$
- $c = 1$
- $k = 0.1$
- $g\rho = 0$
- $p^{(1)} = (1, 1, 0), p^{(2)} = (-1, 1, 0), p^{(3)} = (-1, -1, 0), p^{(4)} = (1, -1, 0)$
- $m_i g = 0$ for $i = 5, 6, 7, 8$

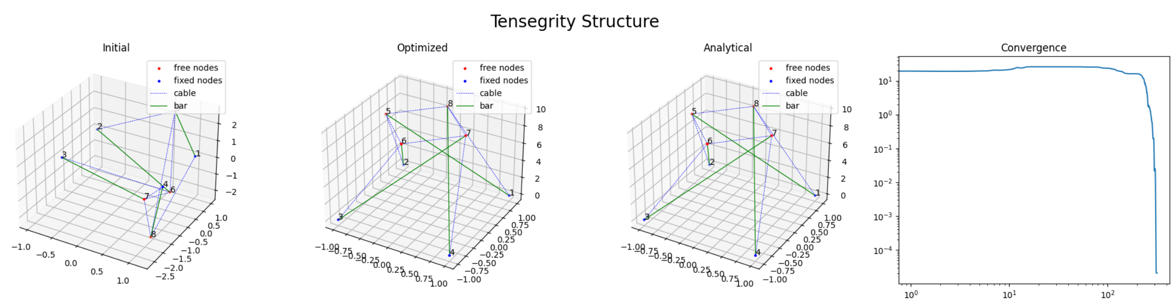

Figure 3: Bars and Cables Structure and Energy convergence plot

Figure (3) shows a solution to a problem with 4 fixed points and 4 randomly placed free points. As seen by the convergence it converges superlinear, and converges to the analytical solution. The interesting thing about this solution is that all the free nodes are actually higher up in z-direction at the optimal solution than it is at the initial conditions. This indicates that the energy from bars and cables in this example is much bigger than the gravitational energy.

3.4 Free-Standing Structures

Optimizing the problem (12).
To handle the shifting problem stated in (2.3.2) we have a implemented the constraints

$$c_i(x) = x_3^{(i)} \geq 0, \quad i = 1, \cdots, N, \quad \mathcal{I} = \{1, \cdots, N\} \tag{17}$$

$$c_{N+1}(x) = x_1^{(1)} - 1 = 0 \tag{18}$$

$$c_{N+2}(x) = x_2^{(1)} - 1 = 0 \tag{19}$$

$$c_{N+3}(x) = x_1^{(1)} - x_2^{(2)} = 0, \quad \mathcal{E} = \{N+1, N+2, N+3\}. \tag{20}$$

This penalizes the model for moving and rotating around the reference point set by two arbitrary nodes. In our case node 1 and 2.

We have used the following parameters and the same cables and bars as in figure (3), additionally there is added four more cables between the nodes that was fixed in the previous example:

- $\ell_{12} = \ell_{23} = \ell_{34} = \ell_{41} = 2$.
- $c = 1$.
- $k = 0.1$.
- $g\rho = 10^{-10}$.
- $m_i g = 10^{-3}$.

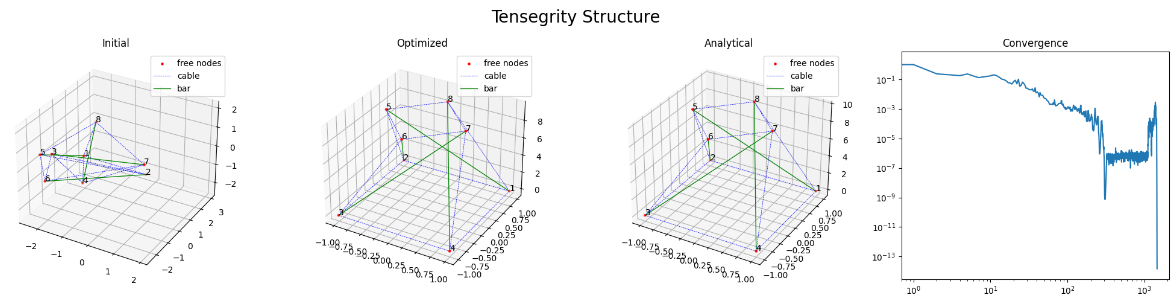

Figure 4: Free-Standing Structures and Energy convergence plot

From the convergence plot it seems like the energy converges towards a minima. And we see that the optimized solution is similar to the analytical. It is interesting that the solution becomes similar as figure (3) just by adding cables between the nodes that was fixed in the previous example.

4 Conclusion

In this project we have looked at the behaviour of tensity structures. Unsurprisingly, the convex case in (7) converged towards the analytical solution. We also found for the not convex cases (10) (12) that the initial conditions in figure (3) and figure (4) converged towards the analytical solution.

Tensegrity is a Mystery, But We've Got the Key: BFGS Optimization is Our Cup of Tea.

References

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[2] J. Nocedal and S. J. Wright, *Numerical optimization*, 2nd ed., ser. Springer Series in Operations Research and Financial Engineering. Springer, New York, 2006, no. 2244940.