Supplementary material: Proofs

Proof of Proposition 3.1. The proof is an immediate consequence of equation (19) and the fact that, for all $A \in \mathcal{P}(\mathcal{A}_1)$, it holds that

$$\widehat{\nu}^{\alpha}_{n+1}(S_{n+1} \in As_n | S_n = s_n) = \widehat{\beta}^{\alpha}_1(A).$$

Indeed, by the linearity of the Choquet integral with respect to the integrating capacity (see, e.g., [30]) it holds that

$$\oint_{\mathcal{A}_1} \varphi^{\alpha}_{n+1}(as_n)\mathrm{d}\widehat{\beta}^{\alpha}_1(a) = \alpha \oint_{\mathcal{A}_1} \varphi^{\alpha}_{n+1}(as_n)\mathrm{d}\widehat{\beta}_1(a) + (1-\alpha)\oint_{\mathcal{A}_1} \varphi^{\alpha}_{n+1}(as_n)\mathrm{d}\overline{\widehat{\beta}}_1(a),$$

where by Proposition 3 in [16] we have that

$$\oint_{\mathcal{A}_1} \varphi^{\alpha}_{n+1}(as_n)\mathrm{d}\widehat{\beta}_1(a) = \widehat{b_u}\varphi^{\alpha}_{n+1}(us_n) + \widehat{b_d}\varphi^{\alpha}_{n+1}(ds_n) + (1-(\widehat{b_u}+\widehat{b_d}))\min_{a\in\mathcal{A}_1}\varphi^{\alpha}_{n+1}(as_n),$$

$$\oint_{\mathcal{A}_1} \varphi^{\alpha}_{n+1}(as_n)\mathrm{d}\overline{\widehat{\beta}}_1(a) = \widehat{b_u}\varphi^{\alpha}_{n+1}(us_n) + \widehat{b_d}\varphi^{\alpha}_{n+1}(ds_n) + (1-(\widehat{b_u}+\widehat{b_d}))\max_{a\in\mathcal{A}_1}\varphi^{\alpha}_{n+1}(as_n).$$

□

Proof of Proposition 3.2. We prove the statement by backward induction, focusing only on the case φ is non-decreasing, as the non-increasing case has a similar proof.

Define the function $\varphi^{\alpha}_T = \varphi$ on $\mathcal{S}_T$, which is non-decreasing since φ is non-decreasing by hypothesis. For $n = T-1$, define $\varphi_{T-1}, \overline{\varphi}_{T-1} : \mathcal{S}_{T-1} \to \mathbf{R}$ by setting, for every $s_{T-1} \in \mathcal{S}_{T-1}$,

$$\begin{aligned}\varphi_{T-1}(s_{T-1}) &= \tfrac{1}{1+r}\widehat{\mathbf{C}}[\varphi_T(S_T)|S_{T-1} = s_{T-1}] \\ &= \tfrac{1}{1+r}\left(\widehat{b_u}\varphi^{\alpha}_T(us_{T-1}) + (1-\widehat{b_u})\varphi^{\alpha}_T(ds_{T-1})\right),\\ \overline{\varphi}_T(s_{T-1}) &= -\tfrac{1}{1+r}\widehat{\mathbf{C}}[-\overline{\varphi}_T(S_T)|S_{T-1} = s_{T-1}] \\ &= \tfrac{1}{1+r}\left((1-\widehat{b_d})\varphi^{\alpha}_T(us_{T-1}) + \widehat{b_d}\varphi^{\alpha}_T(ds_{T-1})\right),\end{aligned}$$

where the last equality of both equations follows from Proposition 3 in [16]. We have that both $\varphi_{T-1}, \overline{\varphi}_{T-1}$ are non-decreasing since, for every $s^i_{T-1}, s^j_{T-1} \in \mathcal{S}_{T-1}$ with $s^i_{T-1} < s^j_{T-1}$ it holds $ds^i_{T-1} < us^i_{T-1} \le ds^j_{T-1} < us^j_{T-1}$, which implies $\varphi_{T-1}(s^i_{T-1}) \le \varphi_{T-1}(s^j_{T-1})$ and $\overline{\varphi}_{T-1}(s^i_{T-1}) \le \overline{\varphi}_{T-1}(s^j_{T-1})$. In turn, the function $\varphi^{\alpha}_{T-1} : \mathcal{S}_{T-1} \to \mathbf{R}$ defined, for every $s_{T-1} \in \mathcal{S}_{T-1}$, as

$$\varphi^{\alpha}_{T-1}(s_{T-1}) = \alpha\varphi_{T-1}(s_{T-1}) + (1-\alpha)\overline{\varphi}_{T-1}(s_{T-1}),$$

is non-decreasing since it is the α-mixture of two non-decreasing functions.

Now, for every $n = 0, \dots, T-1$, assuming that $\varphi^{\alpha}_{n+1} : \mathcal{S}_{n+1} \to \mathbf{R}$ is non-decreasing, define $\varphi_n, \overline{\varphi}_n : \mathcal{S}_n \to \mathbf{R}$ by setting, for every $s_n \in \mathcal{S}_n$,

$$\begin{aligned}\varphi_n(s_n) &= \tfrac{1}{1+r}\widehat{\mathbf{C}}[\varphi^{\alpha}_{n+1}(S_{n+1})|S_n = s_n] \\ &= \tfrac{1}{1+r}\left(\widehat{b_u}\varphi^{\alpha}_{n+1}(us_n) + (1-\widehat{b_u})\varphi^{\alpha}_{n+1}(ds_n)\right),\\ \overline{\varphi}_n(s_n) &= -\tfrac{1}{1+r}\widehat{\mathbf{C}}[-\overline{\varphi}_{n+1}(S_{n+1})|S_n = s_n] \\ &= \tfrac{1}{1+r}\left((1-\widehat{b_d})\varphi^{\alpha}_{n+1}(us_n) + \widehat{b_d}\varphi^{\alpha}_{n+1}(ds_n)\right),\end{aligned}$$

where the last equality of both equations follows again from Proposition 3 in [16]. We have that both $\varphi_n, \overline{\varphi}_n$ are non-decreasing since, for every $s^i_n, s^j_n \in \mathcal{S}_n$ with $s^i_n < s^j_n$ it holds $ds^i_n < us^i_n \le ds^j_n < us^j_n$, which implies $\varphi_n(s^i_n) \le \varphi_n(s^j_n)$ and $\overline{\varphi}_n(s^i_n) \le \overline{\varphi}_n(s^j_n)$. Again, the function $\varphi^{\alpha}_n : \mathcal{S}_n \to \mathbf{R}$ defined, for every $s_n \in \mathcal{S}_n$, as

$$\varphi^{\alpha}_n(s_n) = \alpha\varphi_n(s_n) + (1-\alpha)\overline{\varphi}_n(s_n),$$

is non-decreasing since it is the α-mixture of two non-decreasing functions.

Finally, substituting the expression of φ^{α}_{n+1} in that of φ^{α}_n and proceeding backward from $n = T-1$, we get the claim after setting $\gamma^{\alpha} = \alpha\widehat{b_u} + (1-\alpha)(1-\widehat{b_d})$ and grouping similar terms. □