In[ ]:= (T-t) (\frac{\sigma^2}{2r})(\frac{K}{1+\frac{\sigma^2}{2r}})^{\frac{2r+\sigma^2}{\sigma^2}}(S - a t)^{\frac{-2r}{\sigma^2}}

Assuming multiplication | Use a list instead

Assuming multiplication | Use a list instead

Input:

$$(T-t)\times\frac{\sigma^2}{2r}\left(\frac{K}{1+\frac{\sigma^2}{2r}}\right)^{(2r+\sigma^2)/\sigma^2}(S-a\,t)^{(-2r)/\sigma^2}$$

Result:

$$\frac{\sigma^2(T-t)\left(\frac{K}{\frac{\sigma^2}{2r}+1}\right)^{(2r+\sigma^2)/\sigma^2}(S-a\,t)^{-(2r)/\sigma^2}}{2r}$$

Alternate form:

$$-\frac{\sigma^2\,2^{(2r)/\sigma^2}(t-T)\left(\frac{K\,r}{2r+\sigma^2}\right)^{(2r)/\sigma^2+1}(S-a\,t)^{-(2r)/\sigma^2}}{r}$$

Expanded form: Step–by–step solution

$$\frac{\sigma^2\,T\left(\frac{K}{\frac{\sigma^2}{2r}+1}\right)^{(2r+\sigma^2)/\sigma^2}(S-a\,t)^{-(2r)/\sigma^2}}{2r}-\frac{\sigma^2\,t\left(\frac{K}{\frac{\sigma^2}{2r}+1}\right)^{(2r+\sigma^2)/\sigma^2}(S-a\,t)^{-(2r)/\sigma^2}}{2r}$$

Series expansion at r = 0:

$$K(T-t)-\frac{1}{\sigma^2}K\,r\,(t-T)\left(-2\log(S-a\,t)+2\log\left(\frac{K\,r}{\sigma^2}\right)-2+\log(4)\right)+O(r^2)$$

(Puiseux series)

log(x) is the natural logarithm »

Big-O notation »

Series expansion at r = ∞:

$$K^{(2r)/\sigma^2}(S-a\,t)^{-(2r)/\sigma^2}\left(\frac{K(T-t)\sigma^2}{2\,e\,r}+\frac{K(t-T)\sigma^4}{8\,e\,r^2}+\frac{7K(T-t)\sigma^6}{192\,e\,r^3}+\frac{3K(t-T)\sigma^8}{256\,e\,r^4}+\frac{743K(T-t)\sigma^{10}}{184320\,e\,r^5}+O\left(\left(\frac{1}{r}\right)^6\right)\right)$$

Big-O notation »

Derivative: Step–by–step solution

$$\frac{\partial}{\partial r}\left(\frac{(T-t)\sigma^2\left(\frac{K}{1+\frac{\sigma^2}{2r}}\right)^{(2r+\sigma^2)/\sigma^2}(S-a\,t)^{-(2r)/\sigma^2}}{2r}\right)=\frac{1}{2r+\sigma^2}$$

$$2\,K\,(t-T)\left(\frac{K}{\frac{\sigma^2}{2\,r}+1}\right)^{(2\,r)/\sigma^2}(S-a\,t)^{-(2\,r)/\sigma^2}\left(\log(S-a\,t)-\log\left(\frac{K}{\frac{\sigma^2}{2\,r}+1}\right)\right)$$

WolframAlpha

In[ ]:=

d ((T-t) (\frac{\sigma^2}{2r})(\frac{K}{1+\frac{\sigma^2}{2r}})^{\frac{2r+\sigma^2}{\sigma^2}}(S - a t)^{\frac{-2r}{\sigma^2}}) /dt

Derivative:

Step-by-step solution

$$\frac{\partial}{\partial t}\left(\frac{(T-t)\,\sigma^2 \left(\frac{K}{1+\frac{\sigma^2}{2r}}\right)^{(2r+\sigma^2)/\sigma^2} (S-a\,t)^{-(2r)/\sigma^2}}{2\,r}\right) =$$

$$-\frac{1}{2\,r+\sigma^2} K \left(\frac{K}{\frac{\sigma^2}{2r}+1}\right)^{(2r)/\sigma^2} (S-a\,t)^{-(2r)/\sigma^2-1} \left(2\,a\,r\,t - 2\,a\,r\,T - a\,\sigma^2\,t + \sigma^2\,S\right)$$

Alternate form:

$$-\frac{1}{2\,r+\sigma^2} K\, 2^{(2r)/\sigma^2} \left(\frac{K\,r}{2\,r+\sigma^2}\right)^{(2r)/\sigma^2} (S-a\,t)^{-(2r)/\sigma^2-1} \left(a\left(2\,r\,(t-T) - \sigma^2\,t\right) + \sigma^2\,S\right)$$

Expanded form:

$$-\frac{K\,\sigma^2\,S\left(\frac{K}{\frac{\sigma^2}{2r}+1}\right)^{(2r)/\sigma^2} (S-a\,t)^{-(2r)/\sigma^2-1}}{2\,r+\sigma^2} + \frac{a\,K\,\sigma^2\,t\left(\frac{K}{\frac{\sigma^2}{2r}+1}\right)^{(2r)/\sigma^2} (S-a\,t)^{-(2r)/\sigma^2-1}}{2\,r+\sigma^2} - \frac{2\,a\,K\,r\,t\left(\frac{K}{\frac{\sigma^2}{2r}+1}\right)^{(2r)/\sigma^2} (S-a\,t)^{-(2r)/\sigma^2-1}}{2\,r+\sigma^2} + \frac{2\,a\,K\,r\,T\left(\frac{K}{\frac{\sigma^2}{2r}+1}\right)^{(2r)/\sigma^2} (S-a\,t)^{-(2r)/\sigma^2-1}}{2\,r+\sigma^2}$$

Alternate forms assuming a, K, r, S, t, T, and σ are positive:

$$\frac{1}{2\,r+\sigma^2}\left(\frac{\sigma^2}{2\,r}+1\right)^{-(2r)/\sigma^2} K^{(2r)/\sigma^2+1} (S-a\,t)^{-(2r)/\sigma^2-1} \left(a\left(-2\,r\,t+2\,r\,T+\sigma^2\,t\right) - \sigma^2\,S\right)$$

$$\frac{\left(\frac{\left(\frac{\sigma^2}{2r}+1\right)(S-a\,t)}{K}\right)^{-(2r)/\sigma^2} \left(2\,a\,K\,r\,(T-t) + K\,\sigma^2\,(a\,t-S)\right)}{\left(2\,r+\sigma^2\right)(S-a\,t)}$$

Series expansion at r = 0:

$$-K + \frac{1}{\sigma^2\,(S-a\,t)} 2\,K\,r\left((a\,t-S)\log\left(\frac{2\,K\,r}{\sigma^2}\right) + (S-a\,t)\log(S-a\,t) - 2\,a\,t + a\,T + S\right) + O\left(r^2\right)$$

(Puiseux series)

log(x) is the natural logarithm »

Big-O notation »

WolframAlpha

In[∘]:= d((T-t) (\frac{\sigma^2}{2r})(\frac{K}{1+\frac{\sigma^2}{2r}})^{\frac{2r+\sigma^2}{\sigma^2}}(S - a t)^{\frac{-2r}{\sigma^2}}) / dS

Derivative: Step-by-step solution

$$\frac{\partial}{\partial S}\left(\frac{(T-t)\,\sigma^2\left(\frac{K}{1+\frac{\sigma^2}{2r}}\right)^{(2r+\sigma^2)/\sigma^2}(S-a\,t)^{-(2r)/\sigma^2}}{2r}\right)=(t-T)\left(\frac{K}{\frac{\sigma^2}{2r}+1}\right)^{(2r+\sigma^2)/\sigma^2}(S-a\,t)^{-(2r)/\sigma^2-1}$$

Alternate form:

$$2^{(2r)/\sigma^2+1}\,(t-T)\left(\frac{K\,r}{2r+\sigma^2}\right)^{(2r)/\sigma^2+1}(S-a\,t)^{-(2r)/\sigma^2-1}$$

Expanded form:

$$t\left(\frac{K}{\frac{\sigma^2}{2r}+1}\right)^{(2r+\sigma^2)/\sigma^2}(S-a\,t)^{-(2r)/\sigma^2-1}-T\left(\frac{K}{\frac{\sigma^2}{2r}+1}\right)^{(2r+\sigma^2)/\sigma^2}(S-a\,t)^{-(2r)/\sigma^2-1}$$

Series expansion at r = 0:

$$\frac{2\,K\,r\,(t-T)}{\sigma^2\,(S-a\,t)}-\frac{4\,r^2\left(K\,(t-T)\left(\log(S-a\,t)-\log\left(\frac{2Kr}{\sigma^2}\right)+1\right)\right)}{\sigma^4\,(S-a\,t)}+\frac{1}{\sigma^6\,(S-a\,t)}$$
$$4\,K\,r^3\,(t-T)\left(-2\log(S-a\,t)\left(\log\left(\frac{2\,K\,r}{\sigma^2}\right)-1\right)+\log^2(S-a\,t)+\left(\log\left(\frac{2\,K\,r}{\sigma^2}\right)-2\right)\log\left(\frac{2\,K\,r}{\sigma^2}\right)\right)-$$
$$\left(8\,r^4\left(K\,(t-T)\left(-3\log^2(S-a\,t)\left(\log\left(\frac{2\,K\,r}{\sigma^2}\right)-1\right)+3\log(S-a\,t)\left(\log\left(\frac{2\,K\,r}{\sigma^2}\right)-2\right)\log\left(\frac{2\,K\,r}{\sigma^2}\right)+\right.\right.\right.$$
$$\left.\left.\left.\log^3(S-a\,t)-\log^3\left(\frac{2\,K\,r}{\sigma^2}\right)+3\log^2\left(\frac{2\,K\,r}{\sigma^2}\right)-3\right)\right)\right)\Big/\left(3\left(\sigma^8\,(S-a\,t)\right)\right)+$$
$$\left(4\,K\,r^5\,(t-T)\left(-4\log^3(S-a\,t)\left(\log\left(\frac{2\,K\,r}{\sigma^2}\right)-1\right)+6\log^2(S-a\,t)\left(\log\left(\frac{2\,K\,r}{\sigma^2}\right)-2\right)\log\left(\frac{2\,K\,r}{\sigma^2}\right)-\right.\right.$$
$$4\log(S-a\,t)\left(\log^3\left(\frac{2\,K\,r}{\sigma^2}\right)-3\log^2\left(\frac{2\,K\,r}{\sigma^2}\right)+3\right)+\log^4(S-a\,t)+$$
$$\left.\left.\log^4\left(\frac{2\,K\,r}{\sigma^2}\right)-4\log^3\left(\frac{2\,K\,r}{\sigma^2}\right)+12\log\left(\frac{2\,K\,r}{\sigma^2}\right)-8\right)\right)\Big/\left(3\,\sigma^{10}\,(S-a\,t)\right)+O\left(r^6\right)$$

(generalized Puiseux series)

log(x) is the natural logarithm »

Big-O notation »

WolframAlpha

In[∘]:= d^2 ((T-t) (\frac{\sigma^2}{2r})(\frac{K}{1+\frac{\sigma^2}{2r}})^{\frac{2r+\sigma^2}{\sigma^2}}(S - a t)^{\frac{-2r}{\sigma^2}}) / dS^2

Derivative: Step-by-step solution

$$\frac{\partial^2}{\partial S^2}\left(\frac{(T-t)\,\sigma^2\left(\frac{K}{1+\frac{\sigma^2}{2r}}\right)^{(2r+\sigma^2)/\sigma^2}(S-a\,t)^{-(2r)/\sigma^2}}{2\,r}\right)=-\frac{2\,K\,r\,(t-T)\left(\frac{K}{\frac{\sigma^2}{2r}+1}\right)^{(2r)/\sigma^2}(S-a\,t)^{-(2r)/\sigma^2-2}}{\sigma^2}$$

Alternate form:

$$K\,r\,2^{(2 r)/\sigma^2+1}\left(\frac{K\,r}{2\,r+\sigma^2}\right)^{(2 r)/\sigma^2}\left(\frac{T}{\sigma^2}-\frac{t}{\sigma^2}\right)(S-a\,t)^{-(2 r)/\sigma^2-2}$$

Alternate form assuming a, K, r, S, t, T, and σ are positive:

$$\frac{2\,K\,r\,(T-t)\left(\frac{\left(\frac{\sigma^2}{2 r}+1\right)(S-a\,t)}{K}\right)^{-(2 r)/\sigma^2}}{\sigma^2\,(S-a\,t)^2}$$

Expanded form:

$$\frac{2\,K\,r\,T\left(\frac{K}{\frac{\sigma^2}{2 r}+1}\right)^{(2 r)/\sigma^2}(S-a\,t)^{-(2 r)/\sigma^2-2}}{\sigma^2}-\frac{2\,K\,r\,t\left(\frac{K}{\frac{\sigma^2}{2 r}+1}\right)^{(2 r)/\sigma^2}(S-a\,t)^{-(2 r)/\sigma^2-2}}{\sigma^2}$$

Series expansion at r = 0:

$$-\frac{2\,r\,(K\,(t-T))}{\sigma^2\,(S-a\,t)^2}+\frac{4\,K\,r^2\,(t-T)\left(\log(S-a\,t)-\log\left(\frac{2\,K\,r}{\sigma^2}\right)\right)}{\sigma^4\,(S-a\,t)^2}-\frac{4\,r^3\left(K\,(t-T)\left(-2\log(S-a\,t)\log\left(\frac{2\,K\,r}{\sigma^2}\right)+\log^2(S-a\,t)+\log^2\left(\frac{2\,K\,r}{\sigma^2}\right)-2\right)\right)}{\sigma^6\,(S-a\,t)^2}+\frac{1}{3\,\sigma^8\,(S-a\,t)^2}$$
$$8\,K\,r^4\,(t-T)\left(-3\log^2(S-a\,t)\log\left(\frac{2\,K\,r}{\sigma^2}\right)+3\log(S-a\,t)\left(\log^2\left(\frac{2\,K\,r}{\sigma^2}\right)-2\right)+\log^3(S-a\,t)-\log^3\left(\frac{2\,K\,r}{\sigma^2}\right)+6\log\left(\frac{2\,K\,r}{\sigma^2}\right)-3\right)-\frac{1}{3\left(\sigma^{10}\,(S-a\,t)^2\right)}$$
$$4\,r^5\left(K\,(t-T)\left(-4\log^3(S-a\,t)\log\left(\frac{2\,K\,r}{\sigma^2}\right)-4\log(S-a\,t)\left(\log^3\left(\frac{2\,K\,r}{\sigma^2}\right)-6\log\left(\frac{2\,K\,r}{\sigma^2}\right)+3\right)+6\log^2(S-a\,t)\left(\log^2\left(\frac{2\,K\,r}{\sigma^2}\right)-2\right)+\log^4(S-a\,t)+\log^4\left(\frac{2\,K\,r}{\sigma^2}\right)-12\log^2\left(\frac{2\,K\,r}{\sigma^2}\right)+12\log\left(\frac{2\,K\,r}{\sigma^2}\right)+4\right)\right)+O\left(r^6\right)$$

(generalized Puiseux series)

log(x) is the natural logarithm »

Big-O notation »

Series expansion at r = ∞:

$$K^{(2 r)/\sigma^2}(S-a\,t)^{-(2 r)/\sigma^2-2}\left(-\frac{2\,(K\,(t-T))\,r}{e\,\sigma^2}+\frac{K\,(T-t)}{2\,e}+\frac{5\,K\,(t-T)\,\sigma^2}{48\,e\,r}-\frac{5\left(K\,(t-T)\,\sigma^4\right)}{192\,e\,r^2}+\frac{337\,K\,(t-T)\,\sigma^6}{46\,080\,e\,r^3}+\frac{503\,K\,(t-T)\,\sigma^8}{61\,440\,e\,r^4}-\frac{15\,089\left(K\,(t-T)\,\sigma^{10}\right)}{13\,271\,040\,e\,r^5}+O\left(\left(\frac{1}{r}\right)^6\right)\right)$$

Big-O notation »

Here we have the right part f :

$$-\frac{K\left(\frac{K}{\frac{\sigma^2}{2r}+1}\right)^{\frac{2r}{\sigma^2}}(S-a\,t)^{-\frac{2r}{\sigma^2}-1}\left(2\,a\,r\,t-2\,a\,r\,T-a\,\sigma^2\,t+\sigma^2\,S\right)}{2\,r+\sigma^2}+\frac{\sigma^2}{2}S^2\left(-\frac{2\,K\,r\,(t-T)\left(\frac{K}{\frac{\sigma^2}{2r}+1}\right)^{\frac{2r}{\sigma^2}}(S-a\,t)^{-\frac{2r}{\sigma^2}-2}}{\sigma^2}\right)+$$

$$\mathrm{rS}(t-T)\left(\frac{K}{\frac{\sigma^2}{2\,r}+1}\right)^{\frac{2\,r+\sigma^2}{\sigma^2}}(S-a\,t)^{-\frac{2r}{\sigma^2}-1}-r(T-t)\,\frac{\sigma^2}{2\,r}\left(\frac{K}{1+\frac{\sigma^2}{2\,r}}\right)^{\frac{2\,r+\sigma^2}{\sigma^2}}(S-a\,t)^{\frac{-2\,r}{\sigma^2}}$$