Subject: Physics

Title: Portal "Bouncing" and Oscillations, Lesson 2

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School / Organization, City and State / Province:
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Grade level: 10th – 12th grade

Standards met:
Frameworks PS2.C: Stability and instability in systems (5th through 12th)
Frameworks PS4.A: Properties of waves— Amplitude (by end of grade 8)

Time needed for lesson:
45 minutes

Objective(s) / Overarching question:

- What is simple harmonic motion?
- Does Hooke's law apply in Portal 2?
- How does height of a drop into a portal affect the amplitude of an oscillation?

Summary
Students learn about simple harmonic motion (SHM) and Hooke's law. Then, they determine that the force of gravity rather than Hooke's law better applies in explaining the SHM observed in Portal 2. Students test how height affects the amplitude of SHM in Portal 2 by creating a test chamber that allows for SHM to be set up from different heights.

Vocabulary:
Simple Harmonic Motion (SHM), Simple Harmonic Oscillations (SHO), Small oscillations, Period, Amplitude, Equilibrium, Hooke's law, Restoring force, Linearly proportional

Student Prerequisites:
Experience with harmonic motion, algebra, and working with the value for the acceleration/force of gravity (*g*)

Teacher Materials needed:
None

Student Materials needed:
Stopwatch

Lesson Plan

Simple harmonic motion (SHM)

A common type of motion is Simple Harmonic Motion (SHM) or Simple Harmonic Oscillations (SHO), or more generally, Small Oscillations. Systems that exhibit this motion are pendulums, swings, systems that vibrate, and systems that have repeated cycles of motion (like the Earth-moon system).

For nearly every system in nature that oscillates, oscillations with small enough amplitude (the size of the oscillation) will follow SHM.

Systems that have SHM have a **period** and an **amplitude**. The period is the time it takes for one full cycle of motion. The amplitude is the maximum height or distance the system achieves relative to **equilibrium**. A force called a **restoring force** returns harmonic motion systems back to equilibrium. However, in these systems, the object that is oscillating overshoots equilibrium so that the system keeps oscillating!

Oscillations in Portal 2

If you drop from rest at a height h, the <u>time</u> it takes to reach the ground is found (using the formula:

$$h = \frac{1}{2} g t^2$$

where g equals the acceleration of gravity (9.8 m/s^2) and t equals time in seconds.

$$t = \sqrt{\frac{2h}{g}}$$

This time value is related to the period of the oscillation.

The time it takes to fall from height h and go through, say, a blue portal in the floor, come out the orange portal in the floor, and reach a height h again is thus 2 t_{Fall}. But this is only *half* an oscillation cycle. In other words, it is just half of the period of the cycle.

The full cycle is when you fall *back* through the orange portal, come out the blue side, and reach height h. Therefore, the period of oscillation for jumping-through-portal oscillations is:

$$T_P = 4\sqrt{\frac{2h}{g}}$$

a. Calculate the period for one cycle of oscillation through two portals if you begin at a height of 2 meters.

b. What will happen to the period if you decrease your height? Increase your height? Investigate with some values.

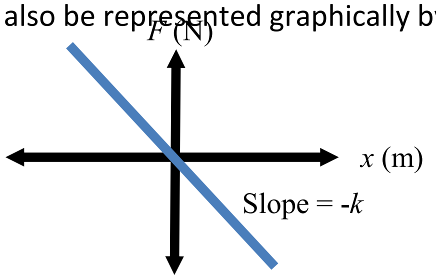
The height *h* is the *amplitude* of the oscillation. See how the period depends on the amplitude? This is one way in which these oscillations in Portal 2 are different from the “simple harmonic motion” we describe below. (In simple harmonic oscillations, period is independent of amplitude.)

Learning about Hooke’s law

Simple harmonic motion is a consequence of **Hooke’s law**. Hooke’s law describes a restoring force*, F*, which is **linearly proportional** to the displacement from equilibrium*.*

$$F = - kx$$

The minus sign in this formula accounts for the “restoring” motion that returns the system to equilibrium. Hooke’s law can also be represented graphically by a line with a slope of -*k*:



(Hooke’s Law)

The basic system for understanding Hooke’s Law and SHM is a mass attached to a spring. If the mass is *m* and the spring constant or “springiness” of the spring is *k* (in newtons/meter), then the period of oscillation (*Tp*) is given by:

$$T_{\text{SHM}} = 2p\sqrt{\frac{m}{k}}$$

This formula illustrates that smaller masses (*m*) and stiffer (larger *k)* springs produce shorter periods of oscillation.

Note that in the above formula, the period of oscillation <u>does not</u> depend on the <u>amplitude</u>*,* but it <u>does</u> depend on the <u>mass</u>.

a. What kinds of conditions would produce a long period of oscillation?
b. Would a spring made out of plastic have a higher or lower *k* value than a spring made out of metal?

Does Hooke's law apply to Portal 2?

In Portal 2, the value of *h* is equivalent to the amplitude of the oscillation. It is important to also note that the period increases as the amplitude of the oscillation increases.

a. How can you increase the period of oscillation in Portal 2? Investigate in a test chamber.
b. Does Hooke's law apply in Portal 2?

Making a big jump

The period of the oscillations in Portal 2 depend on the amplitude not mass.

In Portal 2, jumping off a higher point produces a longer period of oscillation. Furthermore, we see that the mass of whatever is oscillating (e.g., a cube or your body) is not a factor in the oscillation and does not affect the period. This is because, instead of Hooke's Law, the force of gravity on your body is constant with respect to how your position changes (displacement) and scales with the mass.

$F = - mg$

Another way of saying this is that all objects accelerate at the same rate in a constant gravitational field, and this is quite different from Hooke's law. (We see this from Newton's Second Law, which says *F = ma where a is the acceleration. Thus a = -g for all objects in a uniform gravitational field, regardless of mass.)*

If your body were experiencing Hooke's law would be like the acceleration your body experiences being greater the higher up you are.

Activities to try

a. Test for yourself that two different objects of different mass will fall at the same time when dropped from the same height.
b. As a mathematical exercise, show that $T_P = \sqrt{h_{\text{ft}}}$ where h_{ft} is the initial height measured in feet. (Hint: g = 32 ft/s^2.)
c. Estimate the height from which you are about to drop into a portal, and use that to find the time it will take to reach the floor. Then, multiply that by 4 and see if the time you get is the same as the time you measure by playing the game with a stopwatch in hand!

Extension

Create your own test chamber that allows you to drop a cube through two portals from three different heights. Time their periods and show that these scale like the square root of the heights.