Stat 461 Programming Assignment 1

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Problem 1

Approach

As discussed in the assignment, we wish to solve a set of equations defined by an affine transformation:

$$\begin{aligned}\hat{P}_1 &= AP_1 + b,\\ \hat{P}_2 &= AP_2 + b,\\ \hat{P}_3 &= AP_3 + b,\\ \hat{P}_4 &= AP_4 + b,\end{aligned}$$

where (P_1, P_2, P_3, P_4) are the original eye, nose, and mouth locations (found by hand) and $(\hat{P}_1, \hat{P}_2, \hat{P}_3, \hat{P}_4)$ are their fixed goal locations in the normalised images. This system can be reduced to an equivalent system

$$\begin{aligned}\hat{p}_x &= Pc_1,\\ \hat{p}_y &= Pc_2,\end{aligned}$$

where

$$\hat{p}_x = \begin{bmatrix}\hat{X}_1\\ \hat{X}_2\\ \hat{X}_3\\ \hat{X}_4\end{bmatrix}, \qquad \hat{p}_y = \begin{bmatrix}\hat{Y}_1\\ \hat{Y}_2\\ \hat{Y}_3\\ \hat{Y}_4\end{bmatrix}, \qquad c_1 = \begin{bmatrix}a_{11}\\ a_{12}\\ b_1\end{bmatrix}, \qquad c_2 = \begin{bmatrix}a_{21}\\ a_{22}\\ b_2\end{bmatrix},$$

and

$$P = \begin{bmatrix}X_1 & Y_1 & 1\\ X_2 & Y_2 & 1\\ X_3 & Y_3 & 1\\ X_4 & Y_4 & 1\end{bmatrix}.$$

Unfortunately, since this an overdetermined system, an exact solution is unlikely to exist. So we find the closest (least squares) solution using Singular Value Decomposition (SVD) and back-substitution. This lets us construct $\tilde{A}$ and $\tilde{b}$ from our least squares solutions $\tilde{c}_1, \tilde{c}_2$ and acquire a close enough affine transformation

$$\hat{P} \approx \tilde{A}P + \tilde{b},$$

for all of our fixed points.
Unfortunately, this transformation goes from our original image to our normalised image, which is smaller, so multiple pixels in the original images might map to the same pixel in our normalised image. To fix this, we instead examine the transformation

$$P \approx \tilde{A}^{-1}(\hat{P} - \tilde{b}),$$

which allows us to go from pixels in our normalised image to pixels in our original image.

Recovered parameters

Listing 1: Recovered affine transformation parameters

```
Image: S1/1.pgm
A:
[0.81106955, 0.011512009;
 0.018788472, 0.70804304]
b:
[-17.449177;
 -26.372976]

Image: S1/2.pgm
A:
[0.63765675, -0.12793434;
 0.012262158, 0.63261092]
b:
[-9.308032;
 -18.16786]

Image: S1/3.pgm
A:
[0.80458605, -0.052049682;
 0.063097544, 0.60443187]
b:
[-12.687053;
 -20.856146]

Image: S1/4.pgm
A:
[0.71355957, 0.096051939;
 0.037234824, 0.67909944]
b:
[-9.3448973;
 -20.465378]

Image: S1/5.pgm
A:
[0.66885239, -0.17847067;
 0.033128645, 0.65050197]
b:
[-9.9804173;
 -19.721277]

Image: S1/6.pgm
A:
[0.75792283, 0.10522006;
 0.04734626, 0.69487804]
b:
[-10.240383;
 -23.031525]

Image: S1/7.pgm
A:
[0.7886039, -0.0022163868;
 0.02333908, 0.7495932]
b:
[-13.981894;
 -22.861431]

Image: S1/8.pgm
A:
[0.82870156, -0.044768382;
 0.057225708, 0.66983867]
b:
[-17.102766;
 -22.269697]

Image: S1/9.pgm
A:
[0.77405214, -0.054874368;
 0.050093923, 0.68491095]
b:
[-16.856405;
 -14.759286]

Image: S1/10.pgm
A:
[0.7020334, -0.091534093;
 0.036999591, 0.63424462]
b:
[-14.211249;
 -24.280432]

Image: S2/1.pgm
A:
[0.85121626, 0.061459769;
 -0.051318996, 0.75441742]
b:
[-19.138283;
 -26.797432]

Image: S2/2.pgm
A:
[0.9054355, -0.037551839;
 -0.0076464284, 0.72428685]
b:
[-25.122662;
 -25.633995]

Image: S2/3.pgm
A:
[0.83396959, 0.13415296;
 -0.08302597, 0.75544268]
b:
[-18.500818;
 -25.446737]

Image: S2/4.pgm
A:
[0.9046151, -0.042434029;
 0.010883292, 0.77987325]
b:
[-22.009628;
 -30.925449]

Image: S2/5.pgm
A:
[0.8753379, -0.051854614;
 0.033892509, 0.74633247]
b:
[-27.604916;
 -30.172432]

Image: S2/6.pgm
A:
[0.83532917, 0.087086655;
 -0.0606854, 0.73299879]
b:
[-10.929342;
 -25.309187]

Image: S2/7.pgm
A:
[0.9323281, -0.0066212923;
 -0.015280746, 0.74975789]
b:
[-21.985582;
 -28.101305]

Image: S2/8.pgm
A:
[0.81688845, 0.096094571;
 -0.058720399, 0.75161552]
b:
[-17.035641;
 -25.762505]

Image: S2/9.pgm
A:
[0.87887299, 0.021145072;
 -0.03594663, 0.73742545]
b:
[-26.239241;
 -27.074223]

Image: S2/10.pgm
A:
[0.87253439, -0.16093141;
 0.064573579, 0.7818585]
b:
[-26.569763;
 -35.136772]
```

Normalised images

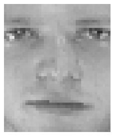
Figure 1: S1/1.pgm

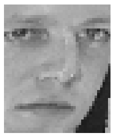
Figure 2: S1/2.pgm


Figure 3: S1/3.pgm

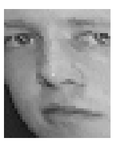
Figure 4: S1/4.pgm


Figure 5: S1/5.pgm


Figure 6: S1/6.pgm

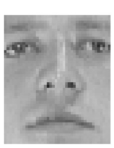
Figure 7: S1/7.pgm


Figure 8: S1/8.pgm

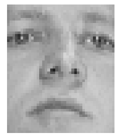
Figure 9: S1/9.pgm


Figure 10: S1/10.pgm


Figure 11: S2/1.pgm


Figure 12: S2/2.pgm


Figure 13: S2/3.pgm


Figure 14: S2/4.pgm


Figure 15: S2/5.pgm


Figure 16: S2/6.pgm


Figure 17: S2/7.pgm


Figure 18: S2/8.pgm


Figure 19: S2/9.pgm


Figure 20: S2/10.pgm

Results

As seen in the next section, the transformation does an exceedingly good job at transforming the facial features to be exactly where I want them to be. The average error is always much less than 1.

Average error

Listing 2: Average transformation error for fixed facial features

```
Image: S1/1.pgm
Avg. Error:
[1.4305115e-006;
 -4.7683716e-007]

Image: S1/2.pgm
Avg. Error:
[-1.6689301e-006;
 4.7683716e-007]

Image: S1/3.pgm
Avg. Error:
[5.4836273e-006;
 2.3841858e-006]

Image: S1/4.pgm
Avg. Error:
[-3.0994415e-006;
 -2.8610229e-006]

Image: S1/5.pgm
Avg. Error:
[-2.3841858e-007;
 -4.7683716e-006]

Image: S1/6.pgm
Avg. Error:
[4.7683716e-007;
 3.8146973e-006]

Image: S1/7.pgm
Avg. Error:
[-1.9073486e-006;
 0]

Image: S1/8.pgm
Avg. Error:
[-3.3378601e-006;
 -4.2915344e-006]

Image: S1/9.pgm
Avg. Error:
[2.8610229e-006;
 2.8610229e-006]

Image: S1/10.pgm
Avg. Error:
[1.6689301e-006;
 -4.2915344e-006]

Image: S2/1.pgm
Avg. Error:
[2.8610229e-006;
 9.5367432e-007]

Image: S2/2.pgm
Avg. Error:
[-2.3841858e-006;
 -9.5367432e-007]

Image: S2/3.pgm
Avg. Error:
[-9.5367432e-007;
 -1.4305115e-006]

Image: S2/4.pgm
Avg. Error:
[-1.4305115e-006;
 2.8610229e-006]

Image: S2/5.pgm
Avg. Error:
[2.8610229e-006;
 9.5367432e-007]

Image: S2/6.pgm
Avg. Error:
[-9.5367432e-007;
 1.4305115e-006]

Image: S2/7.pgm
Avg. Error:
[-4.7683716e-006;
 2.3841858e-006]

Image: S2/8.pgm
Avg. Error:
[-4.7683716e-007;
 9.5367432e-007]

Image: S2/9.pgm
Avg. Error:
[2.8610229e-006;
 2.8610229e-006]

Image: S2/10.pgm
Avg. Error:
[-9.5367432e-007;
 -6.6757202e-006]
```