

# CS 485 Programming Assignment 1

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## Problem 1

### Approach

As discussed in the assignment, we wish to solve a set of equations defined by an affine transformation:

$$\begin{aligned}\hat{P}_1 &= AP_1 + b, \\ \hat{P}_2 &= AP_2 + b, \\ \hat{P}_3 &= AP_3 + b, \\ \hat{P}_4 &= AP_4 + b,\end{aligned}$$

where  $(P_1, P_2, P_3, P_4)$  are the original eye, nose, and mouth locations (found by hand) and  $(\hat{P}_1, \hat{P}_2, \hat{P}_3, \hat{P}_4)$  are their fixed goal locations in the normalised images. This system can be reduced to an equivalent system

$$\begin{aligned}\hat{p}_x &= Pc_1, \\ \hat{p}_y &= Pc_2,\end{aligned}$$

where

$$\hat{p}_x = \begin{bmatrix} \hat{X}_1 \\ \hat{X}_2 \\ \hat{X}_3 \\ \hat{X}_4 \end{bmatrix}, \quad \hat{p}_y = \begin{bmatrix} \hat{Y}_1 \\ \hat{Y}_2 \\ \hat{Y}_3 \\ \hat{Y}_4 \end{bmatrix}, \quad c_1 = \begin{bmatrix} a_{11} \\ a_{12} \\ b_1 \end{bmatrix}, \quad c_2 = \begin{bmatrix} a_{21} \\ a_{22} \\ b_2 \end{bmatrix},$$

and

$$P = \begin{bmatrix} X_1 & Y_1 & 1 \\ X_2 & Y_2 & 1 \\ X_3 & Y_3 & 1 \\ X_4 & Y_4 & 1 \end{bmatrix}.$$

Unfortunately, since this an overdetermined system, an exact solution is unlikely to exist. So we find the closest (least squares) solution using Singular Value Decomposition (SVD) and back-substitution. This lets us construct  $\tilde{A}$  and  $\tilde{b}$  from our least squares solutions  $\tilde{c}_1, \tilde{c}_2$  and acquire a close enough affine transformation

$$\hat{P} \approx \tilde{A}P + \tilde{b},$$

for all of our fixed points.

Unfortunately, this transformation goes from our original image to our normalised image, which is smaller, so multiple pixels in the original images might map to the same pixel in our normalised image. To fix this, we instead examine the transformation

$$P \approx \tilde{A}^{-1}(\hat{P} - \tilde{b}),$$

which allows us to go from pixels in our normalised image to pixels in our original image.

## Recovered parameters

Listing 1: Recovered affine transformation parameters

Image: S1/1.pgm	0.02333908, 0.7495932]	A:
A:	b:	[0.9046151, -0.042434029;
[0.81106955, 0.011512009;	[-13.981894;	0.010883292, 0.77987325]
0.018788472, 0.70804304]	-22.861431]	b:
b:	Image: S1/8.pgm	[-22.009628;
[-17.449177;	A:	-30.925449]
-26.372976]	[0.82870156, -0.044768382;	Image: S2/5.pgm
Image: S1/2.pgm	0.057225708, 0.66983867]	A:
A:	b:	[0.8753379, -0.051854614;
[0.63765675, -0.12793434;	[-17.102766;	0.033892509, 0.74633247]
0.012262158, 0.63261092]	-22.269697]	b:
b:	Image: S1/9.pgm	[-27.604916;
[-9.308032;	A:	-30.172432]
-18.16786]	[0.77405214, -0.054874368;	Image: S2/6.pgm
Image: S1/3.pgm	0.050093923, 0.68491095]	A:
A:	b:	[0.83532917, 0.087086655;
[0.80458605, -0.052049682;	[-16.856405;	-0.0606854, 0.73299879]
0.063097544, 0.60443187]	-14.759286]	b:
b:	Image: S1/10.pgm	[-10.929342;
[-12.687053;	A:	-25.309187]
-20.856146]	[0.7020334, -0.091534093;	Image: S2/7.pgm
Image: S1/4.pgm	0.036999591, 0.63424462]	A:
A:	b:	[0.9323281, -0.0066212923;
[0.71355957, 0.096051939;	[-14.211249;	-0.015280746, 0.74975789]
0.037234824, 0.67909944]	-24.280432]	b:
b:	Image: S2/1.pgm	[-21.985582;
[-9.3448973;	A:	-28.101305]
-20.465378]	[0.85121626, 0.061459769;	Image: S2/8.pgm
Image: S1/5.pgm	-0.051318996, 0.75441742]	A:
A:	b:	[0.81688845, 0.096094571;
[0.66885239, -0.17847067;	[-19.138283;	-0.058720399, 0.75161552]
0.033128645, 0.65050197]	-26.797432]	b:
b:	Image: S2/2.pgm	[-17.035641;
[-9.9804173;	A:	-25.762505]
-19.721277]	[0.9054355, -0.037551839;	Image: S2/9.pgm
Image: S1/6.pgm	-0.0076464284, 0.72428685]	A:
A:	b:	[0.87887299, 0.021145072;
[0.75792283, 0.10522006;	[-25.122662;	-0.03594663, 0.73742545]
0.04734626, 0.69487804]	-25.633995]	b:
b:	Image: S2/3.pgm	[-26.239241;
[-10.240383;	A:	-27.074223]
-23.031525]	[0.83396959, 0.13415296;	Image: S2/10.pgm
Image: S1/7.pgm	-0.08302597, 0.75544268]	A:
A:	b:	[0.87253439, -0.16093141;
[0.7886039, -0.0022163868;	[-18.500818;	0.064573579, 0.7818585]
Image: S2/4.pgm	-25.446737]	b:
		[-26.569763;
		-35.136772]

**Normalised images**

Figure 1: S1/1.pgm



Figure 8: S1/8.pgm



Figure 15: S2/5.pgm



Figure 2: S1/2.pgm



Figure 9: S1/9.pgm



Figure 16: S2/6.pgm



Figure 3: S1/3.pgm



Figure 10: S1/10.pgm



Figure 17: S2/7.pgm



Figure 4: S1/4.pgm



Figure 11: S2/1.pgm



Figure 18: S2/8.pgm



Figure 5: S1/5.pgm



Figure 12: S2/2.pgm



Figure 19: S2/9.pgm



Figure 6: S1/6.pgm

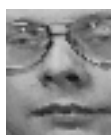


Figure 13: S2/3.pgm



Figure 20: S2/10.pgm



Figure 7: S1/7.pgm



Figure 14: S2/4.pgm

## Results

As seen in the next section, the transformation does an exceedingly good job at transforming the facial features to be exactly where I want them to be. The average error is always much less than 1.

## Average error

Listing 2: Average transformation error for fixed facial features

Image: S1/1.pgm  
Avg. Error:  
[1.4305115e-006;  
-4.7683716e-007]

Image: S1/2.pgm  
Avg. Error:  
[-1.6689301e-006;  
4.7683716e-007]

Image: S1/3.pgm  
Avg. Error:  
[5.4836273e-006;  
2.3841858e-006]

Image: S1/4.pgm  
Avg. Error:  
[-3.0994415e-006;  
-2.8610229e-006]

Image: S1/5.pgm  
Avg. Error:  
[-2.3841858e-007;  
-4.7683716e-006]

Image: S1/6.pgm  
Avg. Error:  
[4.7683716e-007;  
3.8146973e-006]

Image: S1/7.pgm  
Avg. Error:  
[-1.9073486e-006;  
0]

Image: S1/8.pgm  
Avg. Error:  
[-3.3378601e-006;  
-4.2915344e-006]

Image: S1/9.pgm  
Avg. Error:  
[2.8610229e-006;  
2.8610229e-006]

Image: S1/10.pgm  
Avg. Error:  
[1.6689301e-006;

-4.2915344e-006]

Image: S2/1.pgm  
Avg. Error:  
[2.8610229e-006;  
9.5367432e-007]

Image: S2/2.pgm  
Avg. Error:  
[-2.3841858e-006;  
-9.5367432e-007]

Image: S2/3.pgm  
Avg. Error:  
[-9.5367432e-007;  
-1.4305115e-006]

Image: S2/4.pgm  
Avg. Error:  
[-1.4305115e-006;  
2.8610229e-006]

Image: S2/5.pgm  
Avg. Error:  
[2.8610229e-006;  
9.5367432e-007]

Image: S2/6.pgm  
Avg. Error:  
[-9.5367432e-007;  
1.4305115e-006]

Image: S2/7.pgm  
Avg. Error:  
[-4.7683716e-006;  
2.3841858e-006]

Image: S2/8.pgm  
Avg. Error:  
[-4.7683716e-007;  
9.5367432e-007]

Image: S2/9.pgm  
Avg. Error:  
[2.8610229e-006;  
2.8610229e-006]

Image: S2/10.pgm  
Avg. Error:  
[-9.5367432e-007;  
-6.6757202e-006]