Stat 461 Programming Assignment 1

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Problem 1

Approach

As discussed in the assignment, we wish to solve a set of equations defined by an affine transformation:

$$\begin{aligned}\hat{P}_1 &= AP_1 + b,\\ \hat{P}_2 &= AP_2 + b,\\ \hat{P}_3 &= AP_3 + b,\\ \hat{P}_4 &= AP_4 + b,\end{aligned}$$

where (P_1, P_2, P_3, P_4) are the original eye, nose, and mouth locations (found by hand) and $(\hat{P}_1, \hat{P}_2, \hat{P}_3, \hat{P}_4)$ are their fixed goal locations in the normalised images. This system can be reduced to an equivalent system

$$\begin{aligned}\hat{p}_x &= Pc_1,\\ \hat{p}_y &= Pc_2,\end{aligned}$$

where

$$\hat{p}_x = \begin{bmatrix}\hat{X}_1\\ \hat{X}_2\\ \hat{X}_3\\ \hat{X}_4\end{bmatrix}, \qquad \hat{p}_y = \begin{bmatrix}\hat{Y}_1\\ \hat{Y}_2\\ \hat{Y}_3\\ \hat{Y}_4\end{bmatrix}, \qquad c_1 = \begin{bmatrix}a_{11}\\ a_{12}\\ b_1\end{bmatrix}, \qquad c_2 = \begin{bmatrix}a_{21}\\ a_{22}\\ b_2\end{bmatrix},$$

and

$$P = \begin{bmatrix}X_1 & Y_1 & 1\\ X_2 & Y_2 & 1\\ X_3 & Y_3 & 1\\ X_4 & Y_4 & 1\end{bmatrix}.$$

Unfortunately, since this an overdetermined system, an exact solution is unlikely to exist. So we find the closest (least squares) solution using Singular Value Decomposition (SVD) and back-substitution. This lets us construct $\tilde{A}$ and $\tilde{b}$ from our least squares solutions $\tilde{c}_1, \tilde{c}_2$ and acquire a close enough affine transformation

$$\hat{P} \approx \tilde{A}P + \tilde{b},$$

for all of our fixed points.