Degrees of Trust: Temporal Logic and Model Checking

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Abstract. Although plenty of qualitative logical frameworks have been proposed to evaluate and model trust in multi-agent sittings, these approaches generally ignore reasoning about quantitative aspects such as degrees of trust. In this paper, we address this limitation from the logical and model checking perspectives. We start by constructing a logical language to represent the quantitative aspect of trust called TCTL^G. In particular, we extend the logic of trust TCTL to incorporate degrees of trust. Moreover, we develop a symbolic model checking algorithm for quantifying the relationships among the interacting agents.

Keywords: Degrees of Trust · Temporal Logic · Model Checking · Multi-Agent Systems.

1 Introduction

Trust emerged as an essential aspect in evaluating Multi-Agent Systems (MASs) interactions. Qualitative logical frameworks that handle trust in MASs have been widely analyzed in the literature [7,25,13,1]. Trust in these approaches has often been treated as either true or false, i.e., we either trust the behavior of an agent or not. However, such systems have also quantitative temporal properties (such as degrees of trust), which still need further attention from the logical and model checking perspectives. In fact, in many contexts, it is quite difficult to determine with absolute certainty whether a proposition about the behavior of an agent is true or false. For instance, I might trust the agent to a certain degree in relation to such a proposition (i.e., I may have only 50% of trust). That is, although qualitative logical formalisms allow us to reason about various classical properties, their expressiveness is limited in capturing some important aspects that deal with the way of capturing our perception of reality [15]. In this work, we aim to address this issue by introducing a framework for quantifying the relationships among the interacting agents in order to express and evaluate their degrees of trust that can be employed to support decision making.

Indeed, the idea of quantifying trust is not a new topic. It has attracted the attention of several researchers in different domains. Most existing approaches consider trust as a function that is calculated based on multiple opinions through feedback, user ratings, or agent monitoring [28,31,23]. Such approaches represent and quantify the strength level in which an agent trusts another party. In particular, the higher an agent trusts another agent, the more likely the later would be chosen as an interaction partner. Nevertheless, in dynamic MASs where agents may join for a short period of time before leaving the interaction, it might be impossible to collect sufficient information to evaluate the trustworthiness of partners. As a consequence, the well established trust relationships is not guaranteed due to misleading trust results. Considering our approach, in which we introduce a weighted logical formalism, it might be more appealing to enable agents to reason about their degrees of trust in order to make better decisions rather than rely on the external measurable evaluations. In fact, a standard approach of trust quantification involves the use of probability mechanisms accompanied with a representation of agent's beliefs [14,20,24]. However, it is worth noting that this paper presents a different approach that abstracts from the internal mental states and quantifies trust by relying only on accessibility relations inspired by the work proposed in [26]. However, unlike [26] that focuses on the degrees of beliefs by extending the temporal-epistemic logic CTLK, our paper mainly focused on modeling and verifying trust by considering a different logic.

In this paper, we start by constructing TCTL^G, a logical language for reasoning about the qualitative aspect of trust. The concept of trust in our approach is compatible with the definition of trust that has been proposed in our prior research [7]. Specifically, we extend the TCTL logic [7] by assigning a weight to the sets of states that satisfy the trust formula. This allows for counting along a run the number of states where the formula is satisfied according to the ratio condition. By doing so, degrees of trust would be obtained from the possible executions of the giving system. Moreover, we introduce a new symbolic model checking algorithm for the proposed TCTL^G logic that extends the CLT symbolic algorithm [2].

This paper is organized as follows: in Section 2, we briefly summarize the formalism of vector-based interpreted systems introduced in [7] to model MASs. Section 3 introduces the syntax and semantics of TCTL^G. The development of a new BDD-based model checking algorithm for TCTL^G is presented in Section 4. In Section 5, we discuss the related work. Section 6 concludes the paper.

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2 Preliminaries

To describe a MAS, we employ the formalism of Vector-based Interpreted Systems VIS introduced in [7]. A model M_{VIS} with n agents is a tuple:

$$M_{VIS} = (S_v, I_v, R_v, \{\sim_{i\to j} |(i,j) \in Agt^2\}, V_v)$$

where:

- S_v is a non-empty set of reachable global states of the system;
- $I_v \subseteq S_v$ is a set of initial global states;
- $R_v \subseteq S_v \times S_v$ is the transition relation;
- $\sim_{i\to j} \subseteq S_v \times S_v$ is the direct trust accessibility relation for each truster-trustee pair of agents $(i,j) \in Agt^2$ defined by $\sim_{i\to j}$ iff:
  - $l_i(s_v)(\nu^i(j)) = l_i(s'_v)(\nu^i(j))$;
  - s'_v is reachable from s_v using transitions from the transition relation R_v;
- $V_v : S_v \to 2^{AP}$ is a labeling function, where AP is a set of atomic propositions.

The formalism of VIS includes the notion of agents' vector ν. That is, for all states $s_v, s'_v \in S_v$ and $i, j \in Agt$, a vector of size n is associated with each local state $l_i \in L_i$ for each agent. The vector ν is used to define the trust accessibility relation $\sim_{i\to j}$. Intuitively, the relation $\sim_{i\to j}$ relates the states that are considered to be trustful from the vision of agent i with regard to agent j. Specifically, this is obtained by comparing the element $\nu^i(j)$ in the local state l_i at the global state s_v (denoted by $l_i(s_v)(\nu^i(j))$) with $\nu^i(j)$ in the local state l_i at the global state s'_v (denoted by $l_i(s'_v)(\nu^i(j))$). Thus, the trust accessibility of agent i towards agent j (i.e., $\sim_{i\to j}$) does exist only if the element value that we have for agent j in the vector of the local states of agent i for both global states is the same, i.e., $l_i(s_v)(\nu^i(j)) = l_i(s'_v)(\nu^i(j))$ (we refer to [7] for more details about TCTL model). Finally, infinite sequences of states linked by transitions define paths. If π is a path, then $\pi(i)$ is the $(i + 1)$th state in π.

3 Graded Trust Temporal Logic: TCTL^G

In this section, we present the syntax and semantics of the Graded Trust Temporal Logic (TCTL^G), which extends TCTL introduced in [7].

Definition 1 *Syntax of TCTL^G*

The syntax of TCTL^G is defined recursively as follows:

$$\varphi ::= \rho \mid \neg\varphi \mid \varphi \vee \varphi \mid EX\varphi \mid EG\varphi \mid E(\varphi \cup \varphi) \mid T$$
$$T ::= T_p^{\Delta k}(i, j, \varphi, \varphi) \mid T_c^{\Delta k}(i, j, \varphi, \varphi)$$

The formula φ is one of the following: an atomic proposition, a negated formula, or two formulas connected by the connective $\vee$. Moreover, the formula $EX\varphi$ stands for "φ holds in the next state in at least one path"; $EG\varphi$ stands for "there exists a path in which φ holds globally", and the formula $E(\varphi \cup \psi)$ holds at the current state if there is some future moment for which ψ holds and φ holds at all moments until that future moment. the universal quantifier over paths, can be defined in terms of the above as usual: $AX\varphi = \neg EX\neg\varphi$; $AG\varphi = \neg EF\neg\varphi$; and $A(\varphi \cup \psi) = \neg(E(\neg\psi \cup (\neg\varphi \wedge \neg\psi)) \vee EG\neg\psi)$. The trust operator T represents the trust relationship between two agents. There are two trust modalities: $T_p^{\Delta k}$ and $T_c^{\Delta k}$, that represent respectively preconditional and conditional graded trust. From the syntax perspective, $T_p^{\Delta k}(i, j, \psi, \varphi)$ expresses that "the truster i trusts the trustee j to bring about φ given that the precondition ψ holds with a degree of trust Δk", where k is a rational number in [0, 1], and Δ is a relation symbol in the set $\{\leq, \geq, <, >, =\}$. While the formula $T_c^{\Delta k}(i, j, \psi, \varphi)$ reads as "agent i trusts agent j about the consequent φ when the antecedent ψ holds with a degree of trust Δk". It is worth pointing that the advantage of representing the trustworthiness of an agent by a single real number format is that it is obvious for an agent to estimate her degrees of trust and to distinguish between certain agents in order to choose the one satisfying their personal expectations. In fact, we can say that when $k = 0$, it means the trust has not been achieved, however, when $k = 1$, the trust has been perfectly fulfilled. Moreover, when the degree of trust $k = 1$, the standard trust operators $T_p(i, j, \psi, \varphi)$ and $T_c(i, j, \psi, \varphi)$ can be obtained as abbreviations:
$T_p(i, j, \psi, \varphi) \triangleq T_p^{\geq 1}(i, j, \psi, \varphi)$ and $T_c(i, j, \psi, \varphi) \triangleq T_c^{\geq 1}(i, j, \psi, \varphi)$.

Definition 2 *Semantics of TCTL^G*

Given the model M_{VIS}, the satisfaction for a TCTL^G formula φ in a global state s_v, denoted as $(M_{VIS}, s_v) \models \varphi$, is recursively defined as follows:

$s_v \models \rho$ iff $\rho \in V(s_v)$;

$s_v \models \neg\varphi$ iff $s_v \not\models \varphi$;

$s_v \models \varphi_1 \vee \varphi_2$ iff $s \models \varphi_1$ or $s_v \models \varphi_2$;

$s_v \models EX\varphi$ iff there exists a path π starting at s_v such that $\pi(1)) \models \varphi$;

$s_v \models EG\varphi$ iff there exists a path π starting at s_v such that $\pi(k)) \models \varphi, \forall k \geq 0$;

$s_v \models E(\varphi_1 \cup \varphi_2)$ iff there exists a path π starting at s_v such that for some $k \geq 0,\ \pi(k)) \models \varphi_2$ and $\forall 0 \leq i < k, \pi(i)) \models \varphi_1$;

$s_v \models T_p^{\Delta k}(i, j, \psi, \varphi)$ iff $s_v \models \psi \wedge \neg\varphi$ and $\exists s'_v \neq s_v$ such that $s_v \sim_{i \to j} s'_v$, and $\dfrac{|s_v \sim_{i \to j} s'_v \colon s'_v \neq s_v \ \ \& \ \ s'_v \models \varphi|}{|s_v \sim_{i \to j} s'_v \colon s'_v \neq s_v|} \ \Delta k$;

$s_v \models T_c^{\Delta k}(i, j, \psi, \varphi)$ iff $s_v \models \neg\varphi$ and $\exists s'_v \neq s_v$ such that $s_v \sim_{i \to j} s'_v$ and $s'_v \models \psi$, and $\dfrac{|s_v \sim_{i \to j} s'_v \colon s'_v \neq s_v \ \ \& \ \ s'_v \models \psi \to \varphi|}{|s_v \sim_{i \to j} s'_v \colon s'_v \neq s_v|} \ \Delta k$.

For the atomic propositions, Boolean connectives, and temporal modalities, the relation $\models$ is defined in the standard manner (see for example [2]). The intuition behind the semantics of $T_p^{\Delta k}(i, j, \psi, \varphi)$ and $T_c^{\Delta k}(i, j, \psi, \varphi)$ is: the degrees of trust that an agent associates to a formula φ in a global state s_v is the ratio between the number of states s'_v distinguishable and accessible from s_v and satisfying φ (i.e., $|s_v \sim_{i \to j} s'_v \colon s'_v \neq s_v \ \ \& \ \ s'_v \models \varphi|$), and the total number of distinguishable and accessible states from s_v (i.e., $|s_v \sim_{i \to j} s'_v \colon s'_v \neq s_v|$).

Example 1 *Figure 1 illustrates the model of the following preconditional graded trust formula: $T_p^{\geq 0.75}(i, j, payment, deliver)$. It states the buyer (represented as agent's i) trusts that the seller (agent's j) will deliver the requested items with degrees at least 0.75% under the precondition that the latter has received the payment.*

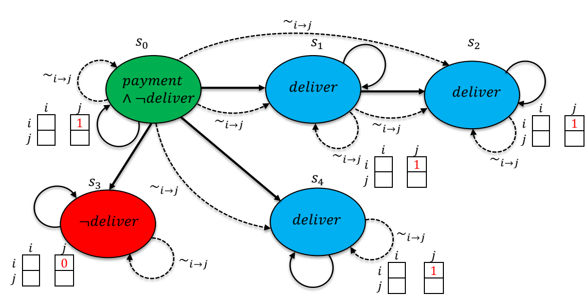

Fig. 1: An example of preconditional graded trust formula:$T_p^{\geq 0.75}(buyer, seller, payment, deliver)$

4 Model Checking TCTL^G

Model checking is the problem of automatically establishing whether or not a formula ϕ is satisfied on a given model M_{VIS}. In this section, we present an efficient algorithm for the TCTL^G model-checking problem. We start by presenting the main algorithm (**Algorithm** 1) that extends the standard symbolic model checking algorithm for CTL [2]. **Algorithm** 1 works as follows. First, it takes as input the model M_{VIS} and the TCTL^G formula ϕ and returns the set $[[\phi]]$ of states that satisfy ϕ in M_{VIS}. By giving the model M_{VIS}, the algorithm recursively go through the structure of ϕ and constructs the set $[[\phi]]$ with respect to a set of Boolean operations applied to sets. In **Algorithm** 1, the lines 1 to 6 invoke the standard algorithms used in CTL to compute the set of states that satisfy regular CTL formulas. Line 7 and 8 call our procedures which compute the set of states that satisfy the graded trust formula.

Algorithm 1 $SMC(\phi, M_{VIS})$: the set $[[\phi]]$ of states satisfying the TCTL^G formula ϕ

1: ϕ is an atomic formula: return $V_v(\phi)$;
2: ϕ is $\neg\phi_1$: return $S_v - SMC(\phi_1, M_{VIS})$;
3: ϕ is $\phi_1 \vee \phi_2$: return $SMC(\phi_1, M_{VIS}) \cup SMC(\phi_2, M_{VIS})$;
4: ϕ is $EX\phi_1$: return $SMC_{EX}(\phi_1, M_{VIS})$;
5: ϕ is $EG\phi_1$: return $SMC_{EG}(\phi_1, M_{VIS})$;
6: ϕ is $E(\phi_1 \cup \phi_2)$: return $SMC_{E\cup}(\phi_1, \phi_2, M_{VIS})$;
7: ϕ is $T_p^{\Delta k}(i, j, \phi_1, \phi_2)$: return $SMC_{T_p}(i, j, \phi_1, \phi_2, \Delta k, M_{VIS})$;
8: ϕ is $T_c^{\Delta k}(i, j, \phi_1, \phi_2)$: return $SMC_{T_c}(i, j, \phi_1, \phi_2, \Delta k, M_{VIS})$;

4.1 BDD-based Algorithm of Graded Trust

This section introduces a model checking algorithms for both the $T_p^{\Delta k}$ and $T_c^{\Delta k}$ operators. **Algorithm** 2 describes the procedure $T_p(i, j, \psi, \phi, \Delta k, M_{VIS})$. This procedure returns the set of states in which the preconditional graded trust formula holds. First, the algorithm starts by computing the set Y of states in which the negation of the formula ϕ holds. Afterwards, the procedure calculates the set **X** (the set of states satisfying $\psi \wedge \neg\phi$). Thereafter, it initializes the variable $n = 0$, and then the sets $Z = \emptyset$. Also, it instantiates the set $V'' = \emptyset$. The algorithm then iterates using *for each . . . do* to go over all the states of **X** to construct the set **V** of those states that are reachable from the states in **X** without considering the state itself to avoid any non-trivial loop to the same state. Thereafter, the algorithm proceeds to build the set **V**$'$ of all the states that are reachable and accessible through the accessibility relation $i \sim j$ (i.e., where their global states have identical local states for agent i with regard to the element $\nu^i(j)$ of the vector ν^i). Precisely, in each iteration, the algorithm first checks if from a given state in **X** there exists an accessible state different from that state (**V**$' \neq \emptyset$). In this case, it counts the number of states in **V**$'-$**V**$''$ and adds them to the variable n (Line 8 and 9). By doing so, we eliminate the case of counting the states that could be reachable and accessible from several states in the set X more than one. Then, it proceeds to check if those accessible

Algorithm 2 $SMC_{T_p}(i, j, \psi, \phi, \Delta k, M_{VIS})$

1: $\mathbf{Y} \leftarrow SMC(\neg\phi)$;
2: $\mathbf{X} \leftarrow SMC(\psi) \cap \mathbf{Y}$;
3: Initialize $n = 0, V'' = \emptyset, Z = \emptyset$
4: **for each** $x \in \mathbf{X}$ **do**
5: $\mathbf{V} \leftarrow \{s \in S \mid s$ is reachable from $x\} \setminus \{x\}$;
6: $\mathbf{V}' \leftarrow \{s \in \mathbf{V} \mid l_i(s)(\nu^i(j)) = l_i(x)(\nu^i(j))\}$;
7: **if** $\mathbf{V}' \neq \emptyset$ **then**
8: $\quad n \leftarrow n + |V' - V''|$
9: $\quad V'' \leftarrow V'' \cup V'$
10: $\quad$ **if** $\mathbf{V}' \cap \mathbf{Y} = \emptyset$ **then**
11: $\quad\quad \mathbf{Z} \leftarrow \mathbf{Z} \cup \{x\}$
12: $\quad$ **end if**
13: **end if**
14: **end for**
15: **if** $n \neq 0$ **and** $\dfrac{|Z|}{n} \Delta\, k$ **then**
16: $\quad$ **return Z**
17: **else**
18: $\quad$ **return** $\emptyset$
19: **end if**

states from that state satisfy ϕ ($\mathbf{V}' \cap \mathbf{Y} = \emptyset$), then that particular state will be added to the set $\mathbf{Z}$. Finally, the algorithm checks whether the ratio between the number of states in the set $\mathbf{Z}$ in which the formula is true (i.e., $|\mathbf{Z}|$) over the total number of the actual accessible states $|n|$ satisfies the appropriate relation Δk. If this is the case, the procedure returns the set $\mathbf{Z}$ of the states that satisfy the formula $T_p^{\Delta k}(i, j, \psi, \phi)$, otherwise, it returns $\emptyset$ if the formula is false.

Algorithm 3 that computes the formula $T_c^{\Delta k}(i, j, \psi, \phi)$ is very similar to **Algorithm** 2, except line 2 which assigns to the set **X** the set of states satisfying $\neg\psi \vee \phi$. Indeed, this is based on our proposed semantics of conditional graded trust where the set of global states satisfying the formula $T_c^{\Delta k}(i, j, \psi, \phi)$ in a given model M is computed by calculating and checking if the ratio between the number of states satisfying $\psi \rightarrow \phi$ over the total number of all states that can reach and see such states through the accessibility relation $\sim_{i \rightarrow j}$ satisfies the appropriate relation Δk.

4.2 Model Checking Complexity

Our automated verification framework accepts as inputs a VIS model (i.e., M_{VIS}) and a TCTL^G specification formula φ, and determines whether $M_{VIS} \models \varphi$.

Theorem 1. *The explicit model checking problem for TCTL^G can be solved in time $O(|M_{VIS}| \times |\varphi|)$ where $|M_{VIS}|$ and $|\varphi|$ are the size of the vector-extended model and length of the TCTL^G formula, respectively.*

Proof. TCTL^G extends CTL, and it is known from [2] that the CTL model checking problem for explicit models is linear in the size of the model and length of the formula.

Algorithm 3 $SMC_{T_c}(i, j, \psi, \phi, \Delta k, M)$

1: **Y** $\leftarrow SMC(\neg\phi)$;
2: **X** $\leftarrow SMC(\neg\psi) \cup (S - Y)$;
3: Initialize $n = 0, Z = \emptyset$
4: **for each** $y \in$ **Y** **do**
5: **V** $\leftarrow \{s \in S \mid s$ is reachable from $y\} \setminus \{y\}$;
6: **V**$' \leftarrow \{s \in$ **V** $\mid l_i(s)(\nu^i(j)) = l_i(y)(\nu^i(j))\}$;
7: **if V**$' \neq \emptyset$ **then**
8:     $n \leftarrow n + |V' - V''|$
9:     $V'' \leftarrow V'' \cup V'$
10:     **if V**$' \cap$ **X** $\neq \emptyset$ **then**
11:         **Z** $\leftarrow$ **Z** $\cup \{y\}$
12:     **end if**
13: **end if**
14: **end for**
15: **if** $n \neq 0$ **and** $\frac{|Z|}{n} \Delta\, k$ **then**
16:     **return Z**
17: **else**
18:     **return** $\emptyset$
19: **end if**

Thus, we just need to analyze the time complexity of both Algorithms 2 and 3. The steps 1 and 2 in these algorithms are simple and it is clear that they can be done in linear running time in the size of the model as they are simply constructing sets by performing comparison operations on states. Moreover, the same argument is valid for the other steps since all the operations require at most polynomial time: iterating over the states, calculating the set of reachable and accessible states, and computing the intersection of given states. For instance, Algorithm 2 is recursively called by Algorithm 1 till the CTL subformulae ϕ_1 and ϕ_2 are encountered. Thus, the depth of the recursion is bounded by the length of the formula ϕ, measured as the number of elements in the closure of ϕ. As again model checking CTL is linear in both the size of the model and length of the formula, we conclude that those algorithms remains in the same polynomial complexity class of the standard CTL model checking algorithm.

Theorem 2. *The model checking problem for TCTL^G is P-complete for explicit models.*

Proof. Membership (i.e., upper bound) in P follows from Theorem 1. Hardness (i.e., lower bound) in P follows directly from the P-completeness of the CTL model checking problem [29].

5 Related Work

Trust logics provide an important basis for the development of MASs applications. Although many trust modeling frameworks can be found in the literature, most of these approaches focus solely on qualitative reasoning tasks. For instance, in [7], the authors proposed a new logical framework for specifying and model checking preconditional trust in MASs. In this work, a TCTL logic, a branching temporal logic of preconditional trust, which extended the Computation Tree Logic (CTL) is introduced. In another work [6], Drawel and her colleagues extended this logic by introducing a new modality for conditional trust and described the logical relationship between preconditional and conditional trust along with the model checking techniques. [30] provided a formal semantics for trust with various logical postulates used to reason about trust from an architectural perspective. Some other approaches formalized the theory of cognitive trust as proposed in [9] using modal logic. In [12], the authors proposed a logical framework for the concept of trust where trust is basically expressed as a combination of different modalities based on the logic of action and time [11] and the BDI logic [3]. In [16], a modal logic for reasoning about the interaction between belief, evidence and trust is presented. Moreover, some proposals have addressed trust in the context of computer security [10,18]. [27] present a logic called Certain Logic to evaluate trust under uncertainty by evaluating a number of Boolean expressions in terms of real values. However, none of these approaches tend to focus on quantitative trust.

Indeed, there are relatively small amount of directly related work. For instance, Demolombe [5] has proposed several approaches that addresses the graded trust. He developed a logic by combining a dynamic logic [11] with BDI-like logic [3]. In particular, [4] defined a logical framework to represent graded trust in terms of two independent components: graded beliefs and graded regularities. In this work, trust is reduced to graded beliefs, so the graded trust is defined as the strength level of truster agent belief about the trustee agent sincerity. In other proposal, Demolombe and Lorini [17] have focused on analyzing the trust that can be associated with information sources. The authors have integrated graded beliefs into a logical framework that defines different kinds of trust. In another work [19], Lorini et al. considered the quantitative aspects of trust in a dynamic epistemic logic setting, where the relationship between trust and belief change is presented. The authors proposed a modal logic called DL-BT, which stands for Dynamic Logic of graded Belief and Trust, that supports reasoning about agents changing their beliefs based on the degree of trust the receiver agent has in the information source. The proposed logic combines modal operators of knowledge, graded belief and trust with dynamic operators of trust-based belief change. The graded trust operator is interpreted using a neighborhood semantics [21], whose model checking is still an open problem [8]. In this work, two kinds of trust-based belief change policies have been considered: additive and compensatory policies, along with the detailed analysis of their logical properties. Moreover, the authors provided a sound and complete axiomatization. In [22], the authors introduced a logical framework that combines a formal logic based on a logic of belief, a logic of time, and a dynamic logic to cognitively represent the concept of trust (qualitative aspects), and a fuzzy logic to represent the degree (qualitative aspect) of trust. However, the proposed logics are mostly focused on agents with mental states where the trusting entity is normally capable of exhibiting beliefs, desires, and intentions, which makes them difficult to be applicable of model checking implementation. Moreover, the trust from quantitative approach is not considered as the grades are embedded in the modal operators. Following the path of cognitive trust, [14] considered the setting of stochastic multi-agent systems, where an automated verification framework for quantifying and reasoning about cognitive trust is proposed. The authors focus on (quantitative) cognitive notion of trust defined as a subjective evaluation in order to capture the social notation of trust. In this work, a probabilistic rational temporal logic PRTL*, which extends the logic PCTL* with reasoning about agents' mental states is introduced. Thus, new operators that can express trust in terms of belief in which probabilistically quantifies the degree of trust as a function of subjective certainty. Yet, our approach considers trust from a high-level abstraction without having to depend on agent's internal mental states, and moreover, quantifies trust by relying only on the accessibility relations.

6 Conclusion

In this paper we have introduced a logical language called TCTL^G, an extension of Trust Computation Tree Logic TCTL to allows us to formally represent and reason about the quantitative aspect of trust in multi-agent systems. In particular, we assigned a weight to the sets of states that satisfy the trust formula. Thus, the degrees of trust will be obtained from the possible executions of the giving system. Moreover, we have presented model checking algorithms for TCTL^G that extended the CTL symbolic algorithm. For future work, many issues are left open, in particular, we are currently working at the implementation of the model checking algorithms in order to investigate its scalability in dealing with practical applications.

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