1.

a.

$$\begin{bmatrix} 2 & 1 & -3 & 6 & 13 \\ -1 & 1 & 1 & -8 & -4 \\ 1 & 0 & 3 & -4 & -3 \\ 1 & 2 & 1 & -8 & 3 \end{bmatrix} \quad R_1 = R_3, \; R_3 = R_1$$

$$\begin{bmatrix} 1 & 0 & 3 & -4 & -3 \\ -1 & 1 & 1 & -8 & -4 \\ 2 & 1 & -3 & 6 & 13 \\ 1 & 2 & 1 & -8 & 3 \end{bmatrix} \begin{matrix} \\ +R_1 \\ -2R_1 \\ -R_1 \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & 3 & -4 & -3 \\ 0 & 1 & 4 & -12 & -7 \\ 0 & 1 & -9 & 14 & 19 \\ 0 & 2 & -2 & -4 & 6 \end{bmatrix} \begin{matrix} \\ \\ -R_2 \\ -2R_2 \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & 3 & -4 & -3 \\ 0 & 1 & 4 & -12 & -7 \\ 0 & 0 & -13 & 26 & 26 \\ 0 & 0 & -10 & 20 & 20 \end{bmatrix} \quad R_3 = -\frac{1}{13} R_3$$

$$\begin{bmatrix} 1 & 0 & 3 & -4 & -3 \\ 0 & 1 & 4 & -12 & -7 \\ 0 & 0 & 1 & -2 & -2 \\ 0 & 0 & -10 & 20 & 20 \end{bmatrix} \begin{matrix} \\ \\ \\ +10R_3 \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & 3 & -4 & -3 \\ 0 & 1 & 4 & -12 & -7 \\ 0 & 0 & 1 & -2 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} -3R_3 \\ -4R_3 \\ \\ \\ \end{matrix}$$

$$\begin{array}{c} \begin{matrix} w & x & y & z & \end{matrix} \\ \begin{bmatrix} 1 & 0 & 0 & 2 & 3 \\ 0 & 1 & 0 & -4 & 1 \\ 0 & 0 & 1 & -2 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$x = 4z + 1$ $\quad w = -2z + 3$

$y = 2z - 2$

$z =$ free var

b. $w + 2z = 3$

$x - 4z = 1$

$y - 2z = -2$

$$\begin{bmatrix} w \\ x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix} + z \begin{bmatrix} -2 \\ 4 \\ 2 \end{bmatrix}$$

this basically = in math

a. Our chemical equation is $Ba_3N_2 + H_2O \rightarrow Ba(OH)_2 + NH_3$

Each element in the equation is a variable so we convert each combination of chemical equations into the element vector $\begin{bmatrix} Ba \\ N \\ H \\ O \end{bmatrix}$ like so

$$x_1 \begin{bmatrix} 3 \\ 2 \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 0 \\ 2 \\ 1 \end{bmatrix} = x_3 \begin{bmatrix} 1 \\ 0 \\ 2 \\ 2 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ 1 \\ 3 \\ 0 \end{bmatrix}$$

then we want to move everything to one side to create an augmented matrix we can solve!

$$\begin{bmatrix} 3 \\ 2 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 2 \\ 1 \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \\ -2 \\ -2 \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \\ -3 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

which gives us the linear system of equations

$$\begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array}$$

$3Ba + 0Ba - 1Ba + 0Ba = 0$

$2N + 0N + 0N - N = 0$

$0H + 2H - 2H - 3H = 0$ or

$00 + 10 - 20 + 00 = 0$

$$\begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & \\ \end{array}$$
$$\begin{bmatrix} 3 & 0 & -1 & 0 & 0 \\ 2 & 0 & 0 & -1 & 0 \\ 0 & 2 & -2 & -3 & 0 \\ 0 & 1 & -2 & 0 & 0 \end{bmatrix}$$

3. b. $Ba_3N_2 + H_2O \rightarrow Ba(OH)_2 + NH_3$

$$\begin{bmatrix} Ba \\ N \\ H \\ O \end{bmatrix} \quad \begin{bmatrix} 3 \\ 2 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 2 \\ 2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 3 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 3 \\ 2 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 2 \\ 1 \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \\ -2 \\ -2 \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \\ -3 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 0 & -1 & 0 & 0 \\ 2 & 0 & 0 & -1 & 0 \\ 0 & 2 & -2 & -3 & 0 \\ 0 & 1 & -2 & 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 2 & -2 & -3 & 0 \\ 0 & 0 & -1 & \frac{3}{2} & 0 \end{bmatrix} -2R_2$$

$$\begin{bmatrix} 2 & 0 & 0 & -1 & 0 \\ 3 & 0 & -1 & 0 & 0 \\ 0 & 2 & -2 & -3 & 0 \\ 0 & 1 & -2 & 0 & 0 \end{bmatrix} \tfrac{1}{2}R_1 \qquad \begin{bmatrix} 1 & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 2 & -3 & 0 \\ 0 & 0 & -1 & \frac{3}{2} & 0 \end{bmatrix} R_3 = \tfrac{1}{2}R_3$$

$$\begin{bmatrix} 1 & 0 & 0 & -\frac{1}{2} & 0 \\ 3 & 0 & -1 & 0 & 0 \\ 0 & 2 & -2 & -3 & 0 \\ 0 & 1 & -2 & 0 & 0 \end{bmatrix} -3R_1 \qquad \begin{bmatrix} 1 & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 1 & -\frac{3}{2} & 0 \\ 0 & 0 & -1 & \frac{3}{2} & 0 \end{bmatrix} +R_3$$

$$\begin{bmatrix} 1 & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & -1 & \frac{3}{2} & 0 \\ 0 & 2 & -2 & -3 & 0 \\ 0 & 1 & -2 & 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 1 & -\frac{3}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} +2R_3 \qquad \begin{bmatrix} 1 & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 1 & 0 & -3 & 0 \\ 0 & 0 & 1 & -\frac{3}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

C. $x_1 - \frac{1}{2}x_4 = 0$ $\quad x_2 - 3x_4 = 0$ $\quad x_3 - \frac{3}{2}x_4 = 0$ $\quad x_4 =$ free var

$x_1 = \frac{1}{2}x_4$ $\quad x_2 = 3x_4$ $\quad x_3 = \frac{3}{2}x_4$

$$\boxed{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_4 \begin{bmatrix} \frac{1}{2} \\ 3 \\ \frac{3}{2} \end{bmatrix}}$$

$x_4 = 2$ $\quad$ ~~$x_4=1$~~ $\quad x_1 = 1,\ x_2 = 6,\ x_3 = 3$

$$Ba_3N_2 + 6H_2O \rightarrow 3Ba(OH)_2 + 2NH_3$$

Basically this means that the equation is balanced for all x_4 as long as x_1, x_2, x_3 follow those proportions described in the matrix where x_1 corresponds to Ba_3N_2, x_2 to H_2O, x_3 to $Ba(OH)_2$, and x_4 to NH_3.

5. a.

$$A = \begin{bmatrix} -6 & 3 & 4 \\ 3 & -1 & -2 \\ 2 & -1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 & -1 \\ 3 & -1 & -2 \\ -6 & 3 & 4 \end{bmatrix} R_1 = \tfrac{1}{2} R_1$$

$\frac{4}{2}$ $\quad \frac{3}{2} - \frac{2}{2}$

$$\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 3 & -1 & -2 \\ -6 & 3 & 4 \end{bmatrix} \begin{matrix} \\ -3R_1 \\ +6R_1 \end{matrix}$$

$-\frac{6}{3}+3$ $\quad -\frac{6}{3} - 3 + 4 = 1$ $\quad 0$ $\quad -\frac{5}{2}$

$$\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & 1 \end{bmatrix} \cdot 2 R_2$$

$$\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{matrix} +\frac{1}{2}R_3 \\ +1R_3 \\ \end{matrix}$$

$$\begin{bmatrix} 1 & -\frac{1}{2} & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \tfrac{1}{2} R_2$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ its invertable!

1 2 3 4 5 6 7 8 9 10 11 12

$$\left[\begin{array}{ccc|ccc} -6 & 3 & 4 & 1 & 0 & 0 \\ 3 & -1 & -2 & 0 & 1 & 0 \\ 2 & -1 & -1 & 0 & 0 & 1 \end{array}\right] R_1 = -\tfrac{1}{6} R_1$$

8 rows - 6 rank
2

$\frac{6}{3}$

$$\left[\begin{array}{ccc|ccc} 1 & -\frac{1}{2} & -\frac{2}{3} & -\frac{1}{6} & 0 & 0 \\ 3 & -1 & -2 & 0 & 1 & 0 \\ 2 & -1 & -1 & 0 & 0 & 1 \end{array}\right] \begin{matrix} \\ -3R_1 \\ -2R_1 \end{matrix}$$

$\frac{4}{3} - \frac{3}{3}$

$$\left[\begin{array}{ccc|ccc} 1 & -\frac{1}{2} & -\frac{2}{3} & -\frac{1}{6} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 1 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 & 1 \end{array}\right] R_2 = 2R_2$$

$$\left[\begin{array}{ccc|ccc} 1 & -\frac{1}{2} & -\frac{2}{3} & -\frac{1}{6} & 0 & 0 \\ 0 & 1 & 0 & 1 & 2 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 & 1 \end{array}\right] R_3 = 3R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & -\frac{1}{2} & -\frac{2}{3} & -\frac{1}{6} & 0 & 0 \\ 0 & 1 & 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 1 & 0 & 3 \end{array}\right] + \tfrac{2}{3} R_3$$ $\quad \frac{2}{3} \quad \frac{4}{6} - \frac{1}{6}$

$$\left[\begin{array}{ccc|ccc} 1 & -\frac{1}{2} & 0 & \frac{1}{2} & 0 & 2 \\ 0 & 1 & 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 1 & 0 & 3 \end{array}\right] + \tfrac{1}{2} R_2$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 2 \\ 0 & 1 & 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 1 & 0 & 3 \end{array}\right] \quad A^{-1} = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 0 \\ 1 & 0 & 3 \end{bmatrix}$$