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Performance Analysis of a Ball-on-Sphere System using Linear Quadratic Regulator Controller

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Abstract— **This paper presents the performance analysis on a benchmark control problem, the ball on sphere (BoS) system using, linear quadratic regulator (LQR) controller. Mathematical modelling was done by the use of Euler-Lagrange formulation. Linear quadratic regulator controller was used for stabilizing the ball on the sphere. Simulation was done in MATLAB 2022a environment, and the simulated results showed that the state weighting matrix Q had a higher penalty on the ball's position on the x-axis and on the ball's velocity on the y-axis. Also, as the diagonal element of the state weighting matrix increases, the optimal controller K becomes better for smaller values, the reduced ricatti matrix P is better for higher values and the estimated Eigen values E is better also for smaller values of Q.**

Keywords— Ball on sphere system, linear quadratic regulator, optimal control law, weighting matrices.

I. Introduction

This Ball-on-Sphere (BoS) system is a standard test benchmark control system, which belongs to a class of multiple-input multiple-output (MIMO) nonlinear system that is designed to control the ball on a sphere [1]. It is used for stabilization in the field of control, teaching and learning, research and for industrial purposes. It comprises of a sphere, two servo motors and a frictional wheel [2]. The servo motors drive the frictional wheels, which in turn controls the rolling of the sphere along the two horizontal axes of the system [3]. The control objective of the BoS system is stabilization, and this is achieved by balancing the ball on the sphere. Due to its under-actuated, nonlinear and unstable nature, finding a controller to perform this task has become a challenge [4]. Other test benchmark control systems include the ball on ball system, ball and plate system, ball on wheel system amongst others [5].

BoS finds application in robotics, aerial systems and transportation in the field of humanoid robot, launching of the rockets, self-transportation machines, robotic upper limb modeling and missile guidance [2]. The BoS system is shown in Fig. 1.

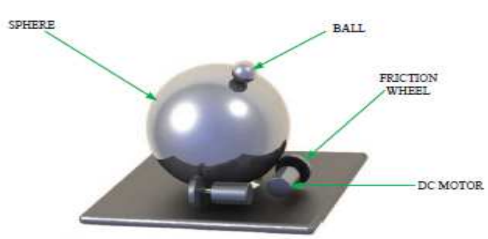

Fig. 1. Ball on Sphere (BoS) System [6]

Some research have been carried out with regards to the BoS. [7] used feedback linearization to transform the nonlinear model of the BoS system to its linear model, and stabilized the ball on the sphere using sliding mode controller (SMC). In the work of [8], they designed a dynamical model of the BoS system using feedback linearization to update the parameters of the system for stabilization purpose. In the work of [9], the design, implementation and validation of the stabilization of BoS system was achieved using SMC, and was compared with linear quadratic regulator (LQR) on the BoS system based on stabilization and robustness of the system. [10] used adaptive feedback linearization technique to stabilize the ball on the sphere with uncertain parameters to a certain position. Also, fuzzy logic controller was used in [11] for point stabilization of the ball on the sphere, the results showed good performance on the regulation of the ball on the sphere. Bond graph technique was adopted in [6] to investigate the effect of friction on the BoS system in terms of stabilization of the ball on the sphere. Also, a fuzzy sliding mode controller was proposed by [12] in which the membership function were optimized using artificial bee colony (ABC) algorithm.

However, some research applied some optimal controllers to stabilize BoS system. [4] designed an optimal LQR controller for the stabilization of the BoS system, and linear quadratic Gaussian (LQG) controller was also proposed for the stabilization of the BoS system, while in the work of [5], LQR controller was compared with LQG controller, in which it was found that LQG performed better than LQR controller.

This paper proposes to investigate the performance of the Q and R matrix on the BoS system, in order to find which of the weighting matrix has penalty effect on the minimization of the performance index of the LQR controller.

The rest of the paper is structured as follows: in section 1, the BoS is introduced, while section 2 presents the mathematical model of the system. Controller design is given in section 3, sections 4 discusses the simulated results, while conclusion is presented in section 5.

II. Mathematical Moel of the BoS System

The dynamical equations of the BoS was derived using the Euler-Lagrange formulation. Like the ball and wheel system, the system of BoS can be simplified and linearized under the following assumptions [6]:

i. The ball rolls on the sphere without slippage
ii. The ball is always in contact with the sphere
iii. All frictional forces are neglected

Then the system can be considered as two independent ball on wheel system which can be represented schematically as shown in Fig. 2.

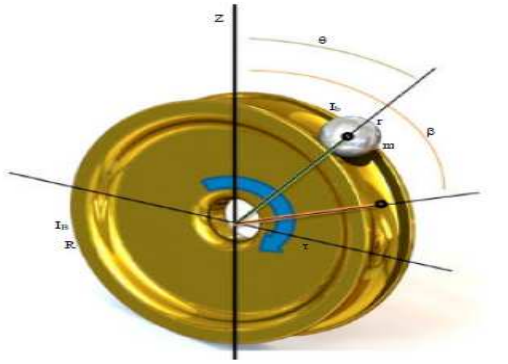

Fig. 2. Schematic Diagram of a BoS System [10]

The generalized coordinates of the BoS system are θ_x and β_x which represents the ball and sphere angles in the x-direction. θ_y and β_y represents the ball and sphere angles in the y-direction. I_B , I_b , m , R and r are the moments of inertia of the sphere and ball, the mass of the ball, the radius of the sphere and ball respectively [10].

The system equation is derived by the use of Euler-Lagrange formulation as [13]:

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{q}}\right)-\frac{\partial L}{\partial q}=Q_i \quad i=1,2,3,4 \tag{1}$$

The Lagragian $L=T-V$ where T and V are the kinetic and potential energies of the system, q is the generalized coordinate and i an integer.

$$Q_1=0 \tag{2}$$

$$Q_2=T_x \tag{3}$$

$$Q_3=0 \tag{4}$$

$$Q_4=T_y \tag{5}$$

$$\left(\frac{R+r}{r^2}I_b+(R+r)m\right)\ddot{\theta}_x-\left(\frac{R}{r^2}I_b\right)\ddot{\beta}_x-mg\sin\theta_x=0 \tag{6}$$

$$\left(-I_b\frac{R(R+r)}{r^2}\right)\ddot{\theta}_x+\left(I_B+I_b\frac{R^2}{r^2}\right)\ddot{\beta}_x=T_x \tag{7}$$

$$\left((R+r)m+I_b\frac{R+r}{r^2}\right)\ddot{\theta}_y-\left(I_b\frac{R}{r^2}\right)\ddot{\beta}_y-mg\sin\theta_y=0 \tag{8}$$

$$\left(-I_b\frac{R(R+r)}{r^2}\right)\ddot{\theta}_y+\left(I_B+I_b\frac{R^2}{r^2}\right)\ddot{\beta}_y=T_y \tag{9}$$

$$q=\begin{bmatrix}\theta_x & \beta_x & \theta_y & \beta_y\end{bmatrix} \tag{10}$$

The variables and constants used for the BoS was adopted from [13].

Then, the state space representation of the BoS is given as:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\\ \dot{x}_5\\ \dot{x}_6\\ \dot{x}_7\\ \dot{x}_8\end{bmatrix}=\begin{bmatrix}0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1\\ v&0&0&0&0&0&0&0\\ z&0&0&0&0&0&0&0\\ 0&0&v&0&0&0&0&0\\ 0&0&z&0&0&0&0&0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\\ x_5\\ x_6\\ x_7\\ x_8\end{bmatrix}+\begin{bmatrix}0&0\\ 0&0\\ 0&0\\ 0&0\\ -w&0\\ -e&0\\ 0&-w\\ 0&-e\end{bmatrix}\begin{bmatrix}T_x\\ T_y\end{bmatrix} \tag{11}$$

$$\begin{bmatrix}\theta_x\\ \beta_x\\ \theta_y\\ \beta_y\end{bmatrix}=\begin{bmatrix}1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\\ x_5\\ x_6\\ x_7\\ x_8\end{bmatrix}+\begin{bmatrix}0&0\\ 0&0\\ 0&0\\ 0&0\end{bmatrix}\begin{bmatrix}T_x\\ T_y\end{bmatrix} \tag{12}$$

III. LQR Controller

The LQR controller design takes the system states equation as its feedback and generates a feedback error signal [14]. This is shown in Fig. 3.

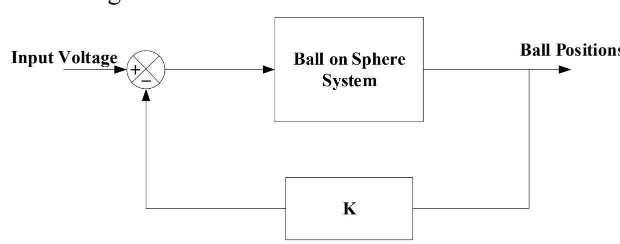

Fig. 3. LQR Control Structure

Given the system dynamics as [5]:

$$\begin{aligned}\dot{x}&=Ax+Bu\\ y&=Cx+Du\end{aligned} \tag{13}$$

The optimization procedure is to find the optimal control law which minimizes the performance index J which is given as [15]:

$$J=\int_0^{t_1}\left(x^TQx+u^TRu\right)dt \tag{14}$$

The optimal control law is given as [16]:

$$u_{opt}=-R^{-1}B^TPx \tag{15}$$

The optimal controller is given as [16]:

$$K=R^{-1}B^TP$$

Then, the optimal control law is written as [16]:

$$u_{opt}=-Kx \tag{16}$$

The matrix P must satisfy the reduced matrix equation given as:

$$A^TP+PA-PBR^{-1}B^TP+Q=0 \tag{17}$$

A. Choosing Q and R matrices

From equation (14), matrix R penalizes the cost of the energy consumed, while matrix Q the performance of the states that controls the response of the system. In order to improve the performance of the system, the Q matrix is been focused on, while to improve on the cost, R matrix is been focused on. Q and R matrices are chosen to be diagonal matrix, while considering the effect of increasing or decreasing their values.

The values of Q and R matrices used are:

$$Q_0 = C^T C \tag{18}$$

$$Q_1 = diag\left(0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1\right) \tag{19}$$

$$Q_2 = diag\left(10, 10, 10, 10, 10, 10, 10, 10\right) \tag{20}$$

$$Q_3 = diag\left(50, 50, 50, 50, 50, 50, 50, 50\right) \tag{21}$$

$$R_0 = diag\left(1, 1\right) \tag{22}$$

The parameters of the BoS used for this article was adopted from [17], while the initial conditions of the system was adopted from [5].

IV. Results of the Simulation

The results were obtained from MATLAB 2022a environment. The various values of the states weighting matrices was simulated against the control weighting matrix. For Q_0, R_0, the values of the optimal controller, reduced ricatti matrix and estimated Eigen values are:

$$K_0 = \begin{bmatrix} -110.1762 & -1.0000 & -0.0000 & 0.0000 & -16.8048 & -2.0314 & -0.0000 & -0.0000 \\ -0.0000 & 0.0000 & -110.1762 & -1.0000 & -0.0000 & -0.0000 & -16.8048 & -2.0314 \end{bmatrix}$$

$$P_0 = \begin{bmatrix} 926.6467 & 16.8048 & 0.0000 & -0.0000 & 140.7010 & 31.5840 & 0.0000 & 0.0000 \\ 16.8048 & 2.0314 & -0.0000 & -0.0000 & 2.5537 & 1.5633 & -0.0000 & -0.0000 \\ 0.0000 & -0.0000 & 926.6467 & 16.8048 & 0.0000 & -0.0000 & 140.7010 & 31.5840 \\ -0.0000 & -0.0000 & 16.8048 & 2.0314 & -0.0000 & -0.0000 & 2.5537 & 1.5633 \\ 140.7010 & 2.5537 & 0.0000 & -0.0000 & 21.4416 & 4.7983 & 0.0000 & 0.0000 \\ 31.5840 & 1.5633 & -0.0000 & -0.0000 & 4.7983 & 2.7864 & -0.0000 & 0.0000 \\ 0.0000 & -0.0000 & 140.7010 & 2.5537 & 0.0000 & -0.0000 & 21.4416 & 4.7983 \\ 0.0000 & -0.0000 & 31.5840 & 1.5633 & 0.0000 & 0.0000 & 4.7983 & 2.7864 \end{bmatrix}$$

$$E_0 = \begin{bmatrix} -0.8719 + 0.5001i \\ -0.8719 - 0.5001i \\ -0.8719 + 0.5001i \\ -0.8719 - 0.5001i \\ -6.0817 + 0.0000i \\ -6.0817 + 0.0000i \\ -7.0913 + 0.0000i \\ -7.0913 + 0.0000i \end{bmatrix}$$

The effect on the angular position and angular velocity of the ball is shown in Fig. 4.

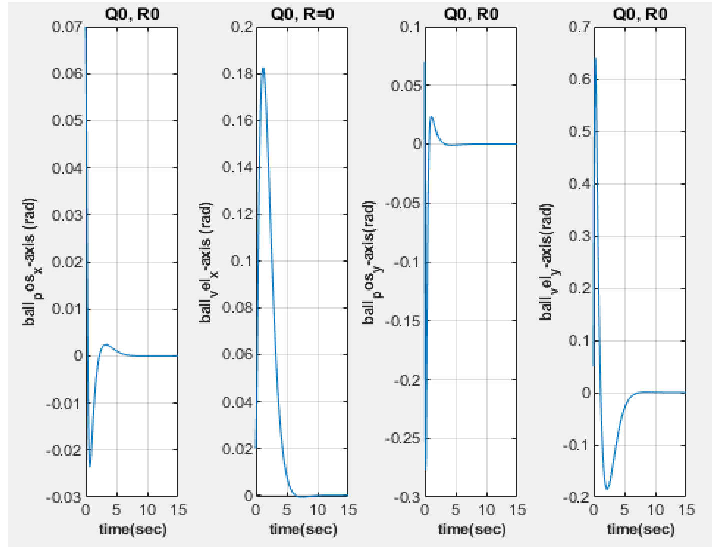

Fig. 4. Effect of Q_0, R_0 weighting matrices

From Fig. 4, it can be seen that the ball's position on the x-axis stabilizes at about 9 secs, while on the y-axis, it stabilizes at about 5 secs. The velocity of the ball on the sphere has a higher energy on the y-axis with respect to the x-axis, which made it settles at about 12 secs on the y-axis, while its settles at about 10 secs on the x-axis. This shows that the state weighting function Q has higher penalty on ball's position on the x-axis and ball's velocity on the y-axis.

Also, for Q_1, R_0, the values of the optimal controller, reduced ricatti matrix and estimated Eigen values are:

$$K_1 = \begin{bmatrix} -97.2605 & -0.3162 & 0.0000 & 0.0000 & -14.8129 & -0.9484 & 0.0000 & 0.0000 \\ 0.0000 & -0.0000 & -97.2605 & -0.3162 & 0.0000 & 0.0000 & -14.8129 & -0.9484 \end{bmatrix}$$

$$P_1 = \begin{bmatrix} 720.4268 & 4.6843 & -0.0000 & -0.0000 & 109.6610 & 13.3356 & -0.0000 & -0.0000 \\ 4.6843 & 0.2999 & 0.0000 & 0.0000 & 0.7129 & 0.3997 & 0.0000 & 0.0000 \\ -0.0000 & 0.0000 & 720.4268 & 4.6843 & -0.0000 & -0.0000 & 109.6610 & 13.3356 \\ -0.0000 & 0.0000 & 4.6843 & 0.2999 & -0.0000 & -0.0000 & 0.7129 & 0.3997 \\ 109.6610 & 0.7129 & -0.0000 & -0.0000 & 16.7000 & 2.0295 & -0.0000 & -0.0000 \\ 13.3356 & 0.3997 & -0.0000 & -0.0000 & 2.0295 & 1.0903 & -0.0000 & -0.0000 \\ -0.0000 & 0.0000 & 109.6610 & 0.7129 & -0.0000 & -0.0000 & 16.7000 & 2.0295 \\ -0.0000 & 0.0000 & 13.3356 & 0.3997 & -0.0000 & -0.0000 & 2.0295 & 1.0903 \end{bmatrix}$$

$$E_1 = \begin{bmatrix} -0.4304 + 0.3664i \\ -0.4304 - 0.3664i \\ -0.4304 + 0.3664i \\ -0.4304 - 0.3664i \\ -6.4112 + 0.0000i \\ -6.4112 + 0.0000i \\ -6.7271 + 0.0000i \\ -6.7271 + 0.0000i \end{bmatrix}$$

The effect on the position and angular position of the ball is shown in Fig. 5.

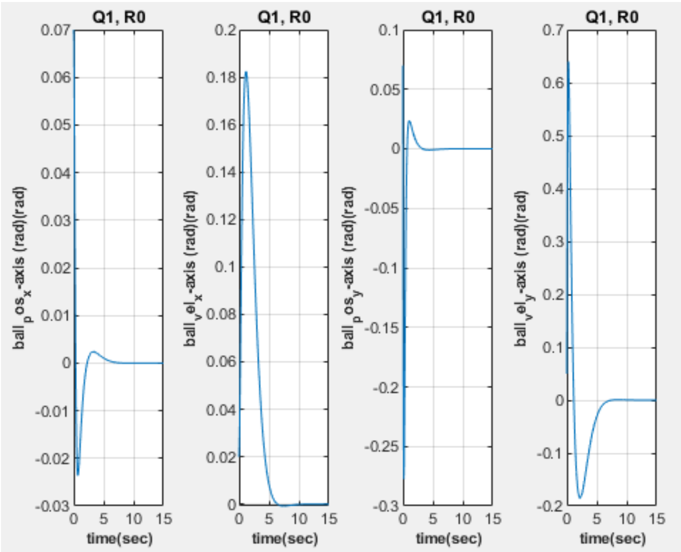

Fig. 5. Effect of Q_1, R_0 weighting matrices

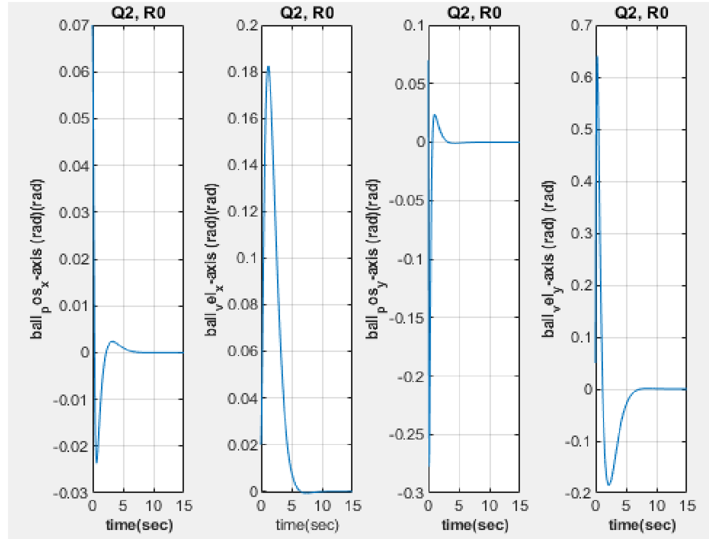

Fig. 6. Effect of Q_2, R_0 weighting matrices

From Fig. 5, it can be seen that the ball's position on the x-axis stabilizes at about 10 secs, while on the y-axis, it stabilizes at about 6 secs. The velocity of the ball on the sphere has a higher energy on the y-axis with respect to the x-axis, which made it settles at about 12 secs on the y-axis, while its settles at about 10 secs on the x-axis. This shows that the state weighting function Q has higher penalty on ball's position on the x-axis and ball's velocity on the y-axis.

Also, for Q_2, R_0, the values of the optimal controller, reduced ricatti matrix and estimated Eigen values are:

$$K_2=\begin{bmatrix} -145.6321 & -3.1623 & 0.0000 & 0.0000 & -22.3923 & -5.0084 & 0.0000 & 0.0000 \\ -0.0000 & -0.0000 & -145.6321 & -3.1623 & -0.0000 & 0.0000 & -22.3923 & -5.0084 \end{bmatrix}$$

$$P_2=10^3\begin{bmatrix} 1.6459 & 0.0708 & 0.0000 & 0.0000 & 0.2457 & 0.1015 & 0.0000 & 0.0000 \\ 0.0708 & 0.0158 & 0.0000 & 0.0000 & 0.0107 & 0.0075 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 1.6459 & 0.0708 & 0.0000 & 0.0000 & 0.2457 & 0.0105 \\ 0.0000 & 0.0000 & 0.0708 & 0.0158 & 0.0000 & 0.0000 & 0.0107 & 0.0075 \\ 0.2457 & 0.0107 & 0.0000 & 0.0000 & 0.0374 & 0.0153 & 0.0000 & 0.0000 \\ 0.1015 & 0.0075 & 0.0000 & 0.0000 & 0.0153 & 0.0103 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.2457 & 0.0107 & 0.0000 & 0.0000 & 0.0374 & 0.0153 \\ 0.0000 & 0.0000 & 0.1015 & 0.0075 & 0.0000 & 0.0000 & 0.0153 & 0.0103 \end{bmatrix}$$

$$E_2=\begin{bmatrix} -1.0600 \\ -1.0600 \\ -3.1684 \\ -3.1684 \\ -4.8298 \\ -4.8298 \\ -8.4945 \\ -8.4945 \end{bmatrix}$$

The effect on the position and angular position of the ball is shown in Fig. 6.

From Fig. 6, it can be seen that the ball's position on the x-axis stabilizes at about 10 secs, while on the y-axis, it stabilizes at about 7 secs. The velocity of the ball on the sphere has a higher energy on the y-axis with respect to the x-axis, which made it settles at about 12 secs on the y-axis, while its settles at about 11 secs on the x-axis. This shows that the state weighting function Q has higher penalty on ball's position on the x-axis and ball's velocity on the y-axis.

Also, for Q_3, R_0, the values of the optimal controller, reduced ricatti matrix and estimated Eigen values are:

$$K_3=\begin{bmatrix} -209.0513 & -7.0711 & -0.0000 & -0.0000 & -32.5855 & -10.2231 & -0.0000 & -0.0000 \\ -0.0000 & -0.0000 & -209.0513 & -7.0711 & -0.0000 & -0.0000 & -32.5855 & -10.2231 \end{bmatrix}$$

$$P_3=10^3\begin{bmatrix} 3.5132 & 0.2304 & 0.0000 & 0.0000 & 0.5059 & 0.2989 & 0.0000 & 0.0000 \\ 0.2304 & 0.0723 & 0.0000 & 0.0000 & 0.0343 & 0.0273 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 3.5132 & 0.2304 & 0.0000 & 0.0000 & 0.5059 & 0.2989 \\ 0.0000 & 0.0000 & 0.2304 & 0.0723 & 0.0000 & 0.0000 & 0.0343 & 0.0273 \\ 0.5059 & 0.0343 & 0.0000 & 0.0000 & 0.0765 & 0.0442 & 0.0000 & 0.0000 \\ 0.2989 & 0.0273 & 0.0000 & 0.0000 & 0.0442 & 0.0341 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.5059 & 0.0343 & 0.0000 & 0.0000 & 0.0765 & 0.0442 \\ 0.0000 & 0.0000 & 0.2989 & 0.0273 & 0.0000 & 0.0000 & 0.0442 & 0.0341 \end{bmatrix}$$

$$E_3=\begin{bmatrix} -1.0101+0.0000i \\ -1.0101+0.0000i \\ -4.6561+1.7901i \\ -4.6561-1.7901i \\ -4.6561+1.7901i \\ -4.6561-1.7901i \\ -12.2571+0.0000i \\ -12.2571+0.0000i \end{bmatrix}$$

The effect on the position and angular position of the ball is shown in Fig. 7.

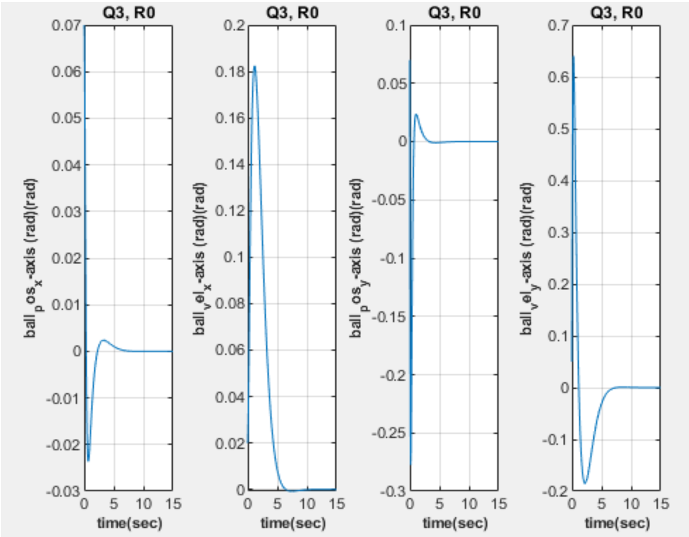

Fig. 7. Effect of Q_3, R_0 weighting matrices

From Fig. 7, it can be seen that the ball's position on the x-axis stabilizes at about 10 secs, while on the y-axis, it stabilizes at about 7 secs. The velocity of the ball on the sphere has a higher energy on the y-axis with respect to the x-axis, which made it settles at about 12 secs on the y-axis, while its settles at about 10 secs on the x-axis. This shows that the state weighting function Q has higher penalty on ball's position on the x-axis and ball's velocity on the y-axis.

However, it can be deduced that as the diagonal elements of Q increases from 0.1 to 50, the optimal controllers K_0, K_1, K_2, K_3, becomes better for lower values of Q, while the reduced ricatti matrix P_0, P_1, P_2, P_3 is better for higher values of Q, and the estimated Eigen values E_0, E_1, E_2, E_3 is better for lower values of Q.

V. CONCLUSION

An investigation of the effect of the state and control weighting matrices on the bench mark control problem, the ball on sphere (BoS) system has been proposed. The BoS is an under-actuated system with a multiple input multiple output characteristics. The system was modeled using Euler-Lagrange formulation technique. Linear quadratic regulator was used to stabilize the ball on the sphere. Simulation was done in MATLAB 2022a environment, and it showed that the states weighting matrix had a higher penalty on the ball's position on the x-axis and on the ball's velocity on the y-axis. However, future research will consider the effect of varying the control weighting matrix over some range of values on the BoS system.

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