Slide 1: Project Overview

Detailed Notes:

- **Problem Statement:** The ball-and-beam system is a fundamental example of an unstable system in control theory:
  - Small disturbances can cause significant deviations in the ball's position.
  - A feedback controller is necessary to stabilize the system and return the ball to its desired position.
- **Objective:**
  - Design a feedback controller to adjust the beam angle and stabilize the ball at the center.
  - Use a 3D simulation model (Simscape Multibody) to analyze the system's real-world behavior.
- **Technical Approach:**
  - Use **Newtonian mechanics** to derive force-based equations of motion.
  - Use **Lagrangian mechanics** to represent energy-based system dynamics.
  - Linearize the equations around the equilibrium point ($x = 0, \theta = 0$) for control design.
  - Implement a Linear Quadratic Regulator (LQR) to optimize control performance.

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Slide 2: Challenges and Control Goals

Detailed Notes:

- **System Challenges:**
  - The system is inherently unstable. Small angular changes (θ) amplify into large displacements (x).
  - Dynamics of the ball and beam are coupled, meaning the motion of one affects the other.
  - The nonlinear nature of the dynamics complicates control design.
- **Control Goals:**
  - Stabilize the ball at the center position ($x = 0$).
  - Minimize transient response metrics:
    - **Overshoot:** Limit the extent to which the ball overshoots its target position.
    - **Settling Time:** Reduce the time required for the ball to reach and remain near the target.
    - **Steady-State Error:** Ensure the ball remains as close to the center as possible over time.
- The diagram shows how forces interact within the system, highlighting the instability and complexity of control.

Slide 3: Degrees of Freedom and Modeling Approach

Detailed Notes:

- **Degrees of Freedom:**
  - **Translational motion (x):** Describes the ball's position along the beam axis.
  - **Rotational motion (θ):** Refers to the beam's angle relative to the horizontal.
- **Modeling Approach:**
  - **Newtonian Mechanics:**
    * $F = ma$**:** Governs the translational motion of the ball, where F is the net force, m is the ball's mass, and a is the acceleration.
    * $\tau = I\alpha$**:** Describes the beam's rotational motion, where τ is the torque, I is the moment of inertia, and α is the angular acceleration.
  - **Lagrangian Mechanics:**
    * The system dynamics are formulated based on energy:
      · **Kinetic Energy (T):** Represents energy due to motion of the ball and the beam.
      · **Potential Energy (V):** Accounts for gravitational effects acting on the ball.
    * The Lagrangian is defined as:

$$\mathcal{L} = T - V$$

    * Equations of motion are derived using:

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}}\right) - \frac{\partial \mathcal{L}}{\partial q} = 0$$

      where q represents generalized coordinates (e.g., x and θ).

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Slide 4: System Parameters and State-Space Representation

Detailed Notes:

- **System Parameters:**
  - **Beam:**
    * Length ($L = 1.0\,\text{m}$).
    * Width ($w = 0.05\,\text{m}$).
    * Height ($h = 0.1\,\text{m}$).
  - **Ball:**
    * Mass ($m = 0.5\,\text{kg}$).
    * Radius ($r = 0.05\,\text{m}$).
    * Moment of inertia ($I = 0.02\,\text{kg}\cdot\text{m}^2$).
  - **Gravity:** $g = 9.81\,\text{m/s}^2$, which influences the potential energy and torque acting on the system.
- **State-Space Representation:**
  - The linearized equations of motion are expressed in state-space form:

$$\dot{x} = Ax + Bu, \quad y = Cx + Du$$

  - Matrices A and B capture the system's dynamics:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{g}{L} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\frac{mgr}{I} & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{I} \end{bmatrix}.$$

  - Linearization is necessary to simplify control design while ensuring accuracy for small deviations.

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Slide 5: Controller Design

Detailed Notes:

- **Objective:** Design a controller to balance system performance and control effort by minimizing the cost function:

$$J = \int_0^\infty \left(x^T Q x + u^T R u\right) dt$$

  - The first term ($x^T Q x$) penalizes state deviations (e.g., ball position and beam angle).
  - The second term ($u^T R u$) penalizes excessive control input to ensure energy efficiency and practicality.

- **Design Parameters:**
  - $Q = \text{diag}([200, 10, 10, 10])$:
    - Emphasizes the importance of precise ball position (x).
    - Penalizes deviations in beam angle (θ).
  - $R = 1$: Ensures smooth and efficient control inputs.

- **MATLAB Implementation:**
  - Feedback gain (K) is computed using MATLAB's LQR function:

$$K = \texttt{lqr}(A, B, Q, R)$$

  - The control law is defined as:

$$u = -Kx$$

    where u is the control input, and x represents the system states.
  - This approach ensures optimal trade-offs between control precision and energy usage.

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Slide 6: MATLAB Integration

Detailed Notes:

- MATLAB was used for the numerical computation, design of the controller, and performance evaluation.
- **Purpose:**
  - Define system parameters, including the ball and beam properties.
  - Derive and implement the state-space representation of the system.
  - Design the controller using the `lqr` function to calculate feedback gains (K).
- **MATLAB Features:**
  - Performance metrics:
    * **IAE (Integral Absolute Error):** Measures the cumulative error magnitude to ensure precise control.
    * **ISE (Integral Square Error):** Emphasizes larger errors to penalize instability.
    * **ITAE (Integral Time-weighted Absolute Error):** Focuses on reducing sustained errors, improving settling time.
  - Assess the transient response:
    * Overshoot quantifies how much the ball exceeds its desired position.
    * Settling time measures how quickly the system stabilizes.
- **Outputs:**
  - Plots for:
    * Ball position (x) and beam angle (θ).
    * Control input (u) over time to evaluate controller performance.
  - Metrics provide a quantitative way to compare the performance of different controllers.

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Slide 7: Simulation Results

Detailed Notes:

- **Objective:** Evaluate the performance of the LQR controller using simulations.
- **State Trajectories:**
  - The ball's position stabilizes within 5 seconds after minor disturbances.
  - The beam angle (θ) adjusts dynamically to counteract disturbances and stabilize the ball.
  - Both plots demonstrate the controller's ability to achieve stability with minimal overshoot.
- **Control Input:**
  - Smooth control input (u) indicates the controller does not demand excessive effort, ensuring practical implementation.
  - Oscillation-free behavior validates the LQR controller's effectiveness over alternatives like pole placement.
- **Performance Metrics:**
  - **IAE (0.06657):** Indicates minimal cumulative error, ensuring precise stabilization.
  - **ISE (0.00494):** Reflects small deviations and penalizes larger errors effectively.
  - **ITAE (0.02813):** Demonstrates efficient settling and minimal sustained errors.
- Visual representations, including trajectories and control input, illustrate the system's dynamic response under LQR control.

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Slide 8: Simulink System Diagram

Detailed Notes:

- **Purpose:** Simulate the real-world behavior of the ball-and-beam system using Simulink and the Simscape Multibody toolbox.
- **System Components:**
  - **Global Configuration:** Includes world frame and solver configuration for numerical simulations.
  - **Ball and Beam Dynamics:**
    - Models the interaction between the ball's motion and the beam's rotation.

    * Captures nonlinear dynamics realistically.
  - **Integral Compensation:** Corrects for steady-state errors by integrating position errors over time.
  - **LQR Controller:** Provides feedback gains (K) for stabilizing the system.
  - **States Subsystem:** Tracks key variables, including $x, \dot{x}, \theta, \dot{\theta}$.

- **Benefits:**
  - Realistic visualization of the ball and beam motion.
  - Validates MATLAB-based controller design in a 3D environment.

- **Outputs:** Includes dynamic animations and plots that align with experimental results, enabling intuitive analysis.

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Slide 9: Challenges and Future Work

Detailed Notes:

- **Challenges:**
  - Parameter sensitivity: Variations in ball mass or beam length can destabilize the system.
  - Linearization limitations: The model's accuracy decreases for large beam angles (θ).
  - Pole placement was ineffective:
    * Led to oscillations and high control effort.
    * LQR provided a more balanced approach.
- **Future Work:**
  - Incorporate external disturbances (e.g., friction) for robust testing.
  - Use Kalman filtering for noise reduction in state estimation.
  - Extend the control system for dynamic reference tracking.
  - Investigate nonlinear control techniques to improve large-angle stability.

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