Beam and Ball Controller Design

Simulating and Controlling a Ball-and-Beam System Using State Feedback Controller

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EE486 - Final Project

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Project Overview

Problem Statement: The ball-and-beam system is inherently unstable, requiring a controller to manage coupled dynamics, stabilize the system, and minimize error and control effort.

Objective:

- Design & implement feedback controller for a ball-and-beam system.
  - Adjust the beam angle to return the ball to center position.
- Simulate the system using a 3D model in Simscape Multibody.

Technical Approach:

- Derive system dynamics using Newtonian and Lagrangian methods.
- Linearize nonlinear dynamics for control design.
- Use state-space methods to design and implement feedback control.

Challenges and Control Goals

System Challenges:

- Amplifies small angular changes into large ball displacements.
- Coupled ball and beam dynamics complicate control.

Control Goals:

- Stabilize the ball at the beam's center position.
- Minimize overshoot, settling time, and steady-state error.

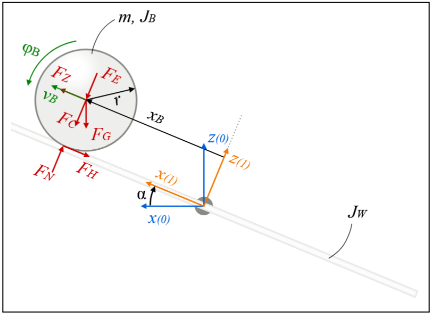


Figure: Ball-and-Beam Dynamics. Source: Wikipedia

Degrees of Freedom and Modeling Approach

Degrees of Freedom:

- Ball position (x) along the beam axis.
- Beam angle (θ) relative to the horizontal.

Modeling Approach:

- Newtonian Mechanics used to derive equations of motion:
  - $\boldsymbol{F = ma}$: Describes the translational motion of the ball.
  - $\boldsymbol{\tau = I\alpha}$: Describes the rotational motion of the beam.
- Lagrangian Mechanics used to formulate system dynamics:
  - **Kinetic Energy (T):** The energy due to motion.
  - **Potential Energy (V):** The energy due to the ball's position in a gravitational field.

System Parameters and State-Space Representation

Assumed Parameters:

- Beam:
  - Length: $L = 1.0\ \mathrm{m}$
  - Width: $w = 0.05\ \mathrm{m}$
  - Height: $h = 0.1\ \mathrm{m}$
- Ball:
  - Mass: $m = 0.5\ \mathrm{kg}$
  - Radius: $r = 0.05\ \mathrm{m}$
  - Moment of inertia: $I = 0.02\ \mathrm{kg{\cdot}m^2}$
- Gravitational acceleration: $g = 9.81\ \mathrm{m/s^2}$

State-Space Representation:

- Expressed as $\dot{x} = Ax + Bu, \quad y = Cx + Du.$
- System matrices:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{g}{L} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\frac{mgr}{I} & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{I} \end{bmatrix}.$$

Controller Design

Objective: Minimize the cost function to balance performance and control effort:

$$J = \int_0^\infty \left(x^T Q x + u^T R u \right) dt$$

Design Parameters:

- `Q = diag([200, 10, 10, 10])`: Penalizes state deviations.
  - Prioritizes ball position (x) and beam angle (θ).
- `R = 1`: Penalizes excessive control effort.

Result: The feedback gain K is computed in MATLAB using:

```
K = lqr(A, B, Q, R);
```

Control Law: The control law uses K to stabilize the ball-and-beam system and minimize deviations in state variables. The control input is calculated as

$$u = -Kx.$$

MATLAB Integration

Purpose: Use MATLAB for controller design and performance evaluation.

MATLAB Script Features:

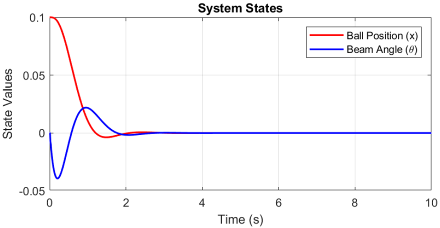
- Defines system parameters and state-space matrices.
- Controller Design.
- Performance Metrics:
  - **IAE:** Measures overall error magnitude for control precision.
  - **ISE:** Emphasizes large errors for stability assessment.
  - **ITAE:** Penalizes sustained errors for improved settling time.
  - Overshoot and settling time: Assess transient response behavior.

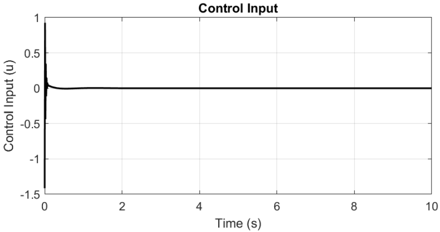
Output: Plots system states (ball position, beam angle) and control effort over time.

Simulation Results

State Trajectories and Control Input:

- Ball position (x) and beam angle (θ) stabilize within 5 seconds.
- Control input (u) is smooth and free of oscillations, ensuring stability.





Performance Metrics:

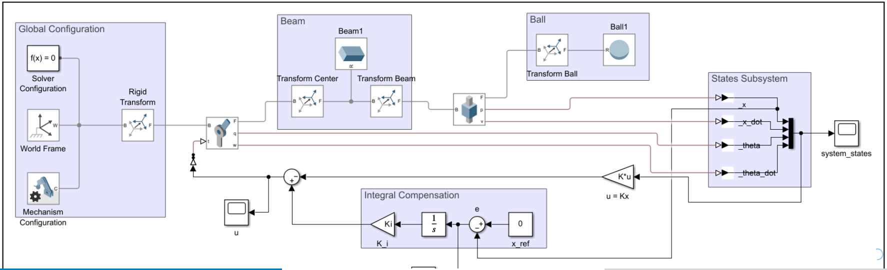
- Integral Absolute Error (IAE): 0.06657.
- Integral Square Error (ISE): 0.00494.
- Integral Time-weighted Absolute Error (ITAE): 0.02813.

Simulink System Diagram

Purpose: Simulate real-world dynamics of the ball-and-beam system using MATLAB's Simscape library.

Key Features:

- Models system dynamics with realistic physics (ball and beam motion).
- Includes key components:
  - System Dynamics: Ball and beam subsystems.
  - Integral Compensation: Corrects steady-state errors.
  - LQR Controller: Stabilizes the system.



3D Model Simulation Results

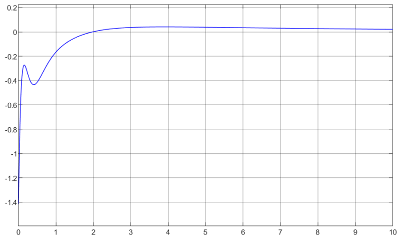


Figure: Control input - simulation scope.

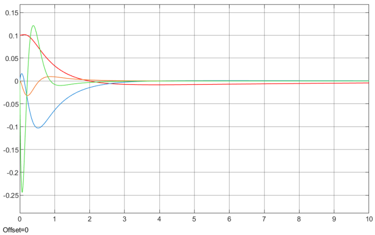


Figure: State trajectories - simulation scope.

- Red: x (Ball position)
- Blue: $\dot{x}$ (Ball velocity)
- Orange: θ (Beam angle)
- Green: $\dot{\theta}$ (Beam angular velocity)

Insights:

- The system stabilizes within 5 seconds with minimal oscillations.
- The control input indicates energy efficiency.

Challenges and Future Work

Challenges:

- Sensitivity to parameter changes: ball mass (m) and beam length (L).
- Limitations of linearization for large beam angles (θ).
- Initial controller design using pole placement was ineffective:
  - Resulted in excessive oscillations, reducing stability.
  - Required high control input, impractical for physical systems.
  - LQR was chosen for its ability to balance control effort and system stability.

Potential Improvements:

- Model external disturbances (e.g., friction) to test system robustness.
- Integrate filtering for noise reduction and state estimation.
- Develop dynamic reference tracking for moving target positions.
- Investigate nonlinear control strategies for large-angle behaviors.

Conclusion

Key Achievements:

- Successfully modeled the ball-and-beam system dynamics using Newtonian and Lagrangian mechanics.
- Designed an LQR controller to stabilize the system, achieving:
  - Minimal overshoot and short settling time.
  - Smooth and stable control input (u).
- Validated the system through MATLAB analysis and Simulink simulations.

Outcomes:

- Visualized results through 2D plots and 3D animations, providing insights into system performance.
- Identified limitations (e.g., sensitivity to parameter variations, large-angle behaviors) for future exploration.

Thank You

Questions or Comments?

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