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② $T(n) = 1$ if $n = 1$

$T(n) = 2T(n-1) + n$ for $n > 1$

$T(1) = 1$ $\quad$ $T(3) = 11$

$T(2) = 4$ $\quad$ $T(4 = 26$

$$
\begin{aligned}
T(n) &= 2(T(n-1)) + n \\
&= 2(2T(n-2) + (n-1)) + n \\
&= 4T(n-2) + 3n - 2 \\
&= 8T(n-3) + 7n - 10 \\
&= 16T(n-4) + 15n - 34 \\
&= 32T(n-5) + 31n - 98 \\
&= 2^{k+1}(T(n-(k+1))) + (2^{k+1} - 1)n - \sum_{i=0}^{k} i \cdot 2^i \\
&= 2^{k+1}(T(n-k-1) + n - k + 1) - n - 2
\end{aligned}
$$

recursion ends when $n-k-1 = 1$ $\quad$, $k = n-2$

$$
\begin{aligned}
&= 2^{n-1}(1 + n - (n-2) + 1) - n - 2 \\
&= 2^{n-1}(4) - n - 2 \\
&= 2^{n+1} - n - 2
\end{aligned}
$$

③ Ordercards (n)

```
ArrayList<Integer> a ← new ArrayList<Integer>()
for i=n down to 1 (inclusive) do
    a.shift  (moves last element in arraylist to index 0)
    a.add(0, i)  (adds i to the beginning of the arraylist)
print a
```

④ prove $\log(n!) \in O(n\log n)$

$$\log(n!) = \sum_{i=1}^{n} \log(i) = \log(1) + \log(2) + \ldots + \log(n)$$

$$n\log(n) = \sum_{i=1}^{n} \log(n) = \log(n) + \log(n) + \ldots + \log(n)$$

$$\sum_{i=1}^{n} \log(i) \leq \sum_{i=1}^{n} \log(n) \qquad \Rightarrow \log(n!) \leq c(n\log(n)) \checkmark$$

find c:

$\log(n!)$ is at most $\frac{n}{2}$ (the mid value) + $\frac{n}{2}$ (the lowest value)

$$\Rightarrow \log(n!) \geq \frac{n}{2}\log\left(\frac{n}{2}\right) + \frac{n}{2}\log(1)$$

$$\geq \frac{n}{2}\log\left(\frac{n}{2}\right)$$

$$\geq \frac{n}{2}(\log(n) - \log(2))$$

$$\geq \frac{n}{2}(\log(n) - \log(\sqrt{n})) \quad \text{if } n \geq 4 \text{ and assuming base 2}$$

$$\geq \frac{n}{2}\left(\log(n) - \frac{1}{2}\log(n)\right)$$

$$\geq \frac{n}{2}\left(\frac{1}{2}\log(n)\right)$$

$$\geq \frac{1}{4} n\log n$$

statement is true of $n \geq 4$ $(N=4)$ and $c=4$