Mini-project in TSRT04:
Cell Phone Coverage

2022-03-10

1 Problem Formulation

According to the study "Swedes and Internet 2019" (Stiftelsen för Internetinfrastruktur), 99% of all Swedes above 12 years old has a cellular phone (mobile phone) and 92% has a smartphone. The more we use our cell phones, the higher our expectation become at being continuously connected to Internet. It begins to be necessary to have a smartphone and cellular coverage to take care of daily tasks, such as paying for parking.

Cellular telephony is based on having so-called base stations deployed on roof tops in cities and high masts in the country side. Each cell phone communicates wirelessly with the closest base station. This means that radio waves (i.e., light with a wavelength that our eyes cannot see) are sent back and forth between the phone and the base station. The physics of wave propagation tells us that the energy is spread more and more the further the radio waves propagate—it's like blowing up a balloon that first is small and thick, and then becomes large and thin. In other words, the signals get weaker and weaker the further it is between the phone and the base station. When the signals become too weak, it is no longer possible to communicate; this is known as not having cellular coverage.

In this mini-project, we will study how the cellular coverage works. The goal is to write a function that, given a certain area and positions of multiple base stations, can illustrate how the cellular coverage will be at different places in the area. You will have to draw three-dimensional graphs over the data rates that are achieved at different places and compute histograms of how the coverage for a random user position behaves. A typical simulation scenario is illustrated in Figure 1.

1.1 Presentation

This is one of the mini-projects in the course TSRT04. To pass the course you need to solve one of the mini-projects according to the guidelines and present it to a teacher at one of the lab exercises that offers examination. The examination will be in English so please write your code and comments in English (a good exercise since English coding is common at companies). The problem should be solved in groups of two students, or individually. Since it is an examination assignment it is not allowed to share or show MATLAB code or notes to other students. It is, however, permitted to discuss the project orally with other groups; for example, to share some good advice!

- Write a MATLAB function `cellcoverage` that computes the data rates over a grid of user positions, for given positions of the base stations. You should make three-dimensional graphs that show the data rates in the area and illustrate the variations in data rates for a random user location. The function `cellcoverage` should call other functions, that you've written yourself, to solve certain subproblems.
- The solution should be demonstrated and the code should be showed to the teachers at Lab 3 or Lab 5. We will check if you have followed the guidelines, if you have a reasonable amount of comments in the code, if the plots are easy to understand, and if you can describe what the different parts of the code are doing. There is a style guide available on the course

homepage, which elaborates on how a good solution should look like. Please make sure to read it and follow the recommendations!

- When the teachers have told you that you have passed the project, the code should be uploaded to the study room under Submissions. The examiner will then send the code to Urkund for control of plagiarism. The code should be saved in a text document called coursecode_year_studentid1_studentid2.txt (e.g., TSRT04_2022_helan11_halvan22.txt). This document should contain all functions and scripts that were examined, including a short example of how to call the function to solve the project.

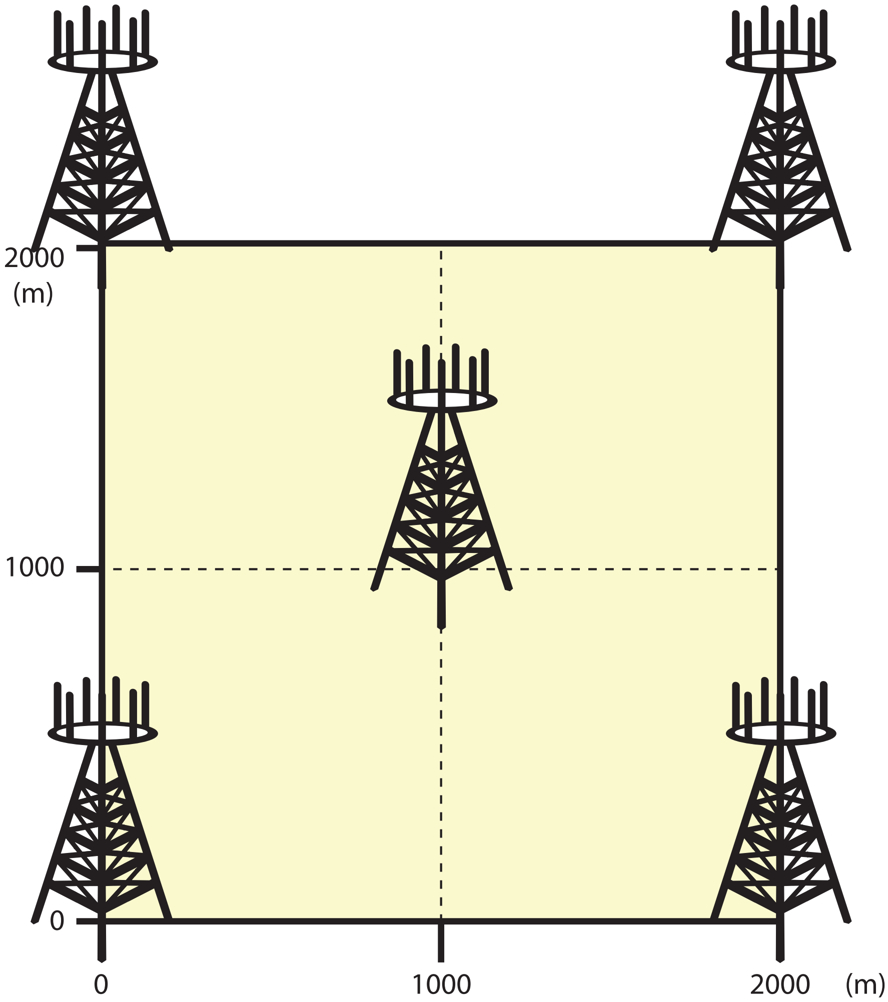

Figure 1: Example of deployment of five base stations.

2 Suggested Solution Outline

Begin by reading through the whole document so that you understand the problem specification and our suggested way to divide the problem into smaller subproblems.

The steps below provide one way to split the main programming problem into subproblems that can be solved one at a time. We have chosen the subproblems to make it possible to verify each part before you continue with the next part. Make sure to take these opportunities! Although the solution outline suggests that you write small functions that solve subproblem, this doesn't mean that it is good or efficient to use (call) all these small functions to solve the original problem.

Some functions take care of such small pieces that it is better to copy the code into a new larger function, than to call the small function.

Note that this solution outline is intended for students with no or little previous programming experience. An experienced programmer would probably, based on previous experience, divide the problem into different and/or fewer parts. If you consider yourself an experienced programmer and think that you have a better approach to solve the problem, you are allowed to solve it in your own way. However, if you need help, the teaching assistant has more experience of the outlined solution. If you don't follow the solution outline you must at least make sure that the solution gives the same functionality.

2.1 Theory: Cellular Coverage

In order to compute the cellular coverage at different places, we first need to define what coverage actually means. We will do this in terms of the *data rate*, which is the number of information bits (zeros and ones) that can be transferred per second. If the data rate is larger than zero, then we have coverage. For simplicity, we assume a two-dimensional world; that is, we do not consider that the base stations and cell phones can have different elevation. We can then describe a position with two coordinates, (x, y), just as on a conventional map.

If a cell phone is located at (x_0, y_0) and a base station is deployed at the position (x_1, y_1), then we can compute the distance by the classic distance formula

$$d = \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2}, \tag{1}$$

which follows from Pythagoras' theorem. If we let x_0, x_1, y_0, y_1 be measured in meter, then the distance d will also be in meter.

Suppose that the cell phone or base station send signals with power P (in Watt). The signals will decay with the distance d. A simple model is that the power decays as $P/(1+d)^\kappa$. In other words, the power is P at $d = 0$ (no distance) and decays as P/d^κ when the distance d is much larger than a meter. The exponent κ is equal to 2 if there are *no* disturbing objects (e.g., buildings and asphalt) in the area, while measurements of κ in urban areas give values in the interval 3-6.

The distant-dependent loss of signal power is a problem since the received signal power should be compared to the noise power at the receiver. Let σ^2 denote the noise power (in Watt). The signal-to-noise ratio is defined as

$$\frac{P}{(1+d)^\kappa \sigma^2} \tag{2}$$

in our model. This expression should preferably be much larger than 1, since then the received signal power $P/(1+d)^\kappa$ is much larger than the noise power σ^2.

Based on the signal-to-noise ratio, one can compute the data rate as

$$R = \begin{cases} 0, & \frac{P}{(1+d)^\kappa \sigma^2} < 0.3 \\ B \log_2\left(1 + \frac{P}{(1+d)^\kappa \sigma^2}\right), & 0.3 \leq \frac{P}{(1+d)^\kappa \sigma^2} \leq 63 \\ B \log_2\left(1 + 63\right), & \frac{P}{(1+d)^\kappa \sigma^2} > 63 \end{cases} \quad \text{[bit/s]} \tag{3}$$

where B is the bandwidth (in Hertz) and the logarithm of "one plus the signal-to-noise ratio" gives the number of information bits than can be transferred per second per Hertz of bandwidth. We do not expect you to understand where this formula comes from (you can learn this in later courses), but only to apply it within this project. It can be seen from (3) that the data rate is zero if the signal-to-noise ratio is lower than 0.3. This number originates from a 4G standard for mobil telephony and says that the received signal power cannot be too small. This is our definition of not having coverage: the data rate is zero. The middle formula in (3) gives larger values as the signal-to-noise ratio increases, but when it grows beyond 63 the rate will not continue to grow any more. This is a type of hardware limitation that originates from real systems.

To get practically reasonable results, you can use the parameter values $P = 1$ Watt, $B = 10$ MHz, exponent $\kappa = 4$, and noise power $\sigma^2 = 10^{-11,2}$ Watt.

2.2 Compute Distance between a Base Station and a Phone

Begin by writing a function that computes the distance between a cell phone and a base station. It can, for example, have four arguments: two parameters for the phone's position (x_0 and y_0) and two parameters for the base station's position (x_1 and y_1). Compute the distance by using the formula in (1) and let the function return this distance.

Test your function on some simple positions. If the phone has position $(x_0, y_0) = (50, 15)$ and the base station has position $(x_1, y_1) = (10, -15)$, then the distance should be 50 meter.

2.3 Compute Distance between a Base Station and Multiple Phones

Modify your function so that it can compute the distance between a base station and multiple phones at different positions. Use the same number of arguments as before, but let x_0 and y_0 be *matrices* where each combination of elements (with the same index) describe the position of a phone.

Test your function on some simple positions of the phones.

2.4 Create a Coverage Area and Visualize the Distances

You can now test your function in the case where the base station is deployed in the origin ($x_1 = y_1 = 0$) and the phone can be anywhere in the area $0 \leq x_0 \leq 1000$ meter and $0 \leq y_0 \leq 1000$ meter. The function `meshgrid` can be used to create matrices that represent different phone positions. Compute the distance to the base station from each position, by using the function from the previous part.

Plot the result in a three-dimensional graph where two of the dimensions represent positions and the third is the distance from the base station. Put descriptive names on the axes and verify that the scales and numbers are correct. The distance should be zero for a phone in the origin, while the distance should be approximately 1414 meter for a phone in the position $x_0 = y_0 = 1000$.

Hint: If the distance between each phone position is too short, the graph will be "heavy" for MATLAB to draw and it will be hard to rotate it. Test some different distances between the phones (t.ex. 1 meter, 10 meter, and 50 meter) and see what happens!

2.5 Compute Signal-to-Noise Ratios and Visualize

Write a function that takes a matrix with distances as argument (e.g., the one created in the previous part) and also the signal power P, noise power σ^2, and exponent κ. The function should compute the signal-to-noise ratio in (2) and return a matrix that contains this ratio for each distance in the matrix.

Plot the result in a three-dimensional graph where two of the dimensions represent positions and the third is the signal-to-noise ratio. Put descriptive names on the axes and verify that the scales and numbers are correct by putting in some different values in the formula in (2). Hint: The graph will have extremely large values close to the base station. Is there anything we can do about it?

2.6 Compute Data Rates and Visualize the Coverage

Next step is to write a function that takes a matrix with different signal-to-noise ratios as argument (e.g., the one created in the previous part) and returns the data rates for each values, according to the formula in (3).

Hint: When you have a double inequality like $0.3 \leq x \leq 63$, you cannot implement it by writing `0.3 <= x <= 63`. This is because MATLAB first computes `0.3 <= x` and then checks if the results (1 for true and 0 for false) is smaller than 63. You need to find another way to implement it!

Plot the result in a three-dimensional graph where two of the dimensions represent positions and the third is the data rates. Put descriptive names on the axes and verify that the scales and numbers are reasonable (e.g., that the maximal and minimal number are in accordance to (3)). It might be good to use the unit Mbit/s on the data rate, instead of bit/s, and thereby divide all rates by 10^6.

2.7 Deploy Multiple Base Stations

Modify your previous functions to handle more than one base station. You can have the example in Figure 1 in mind, but make sure that the final implementation can handle any number of base stations. You can, for example, let x_1 and y_1 be vectors where each pair of elements represents the position of a base station. Each cell phone connects to the base station that gives the highest signal-to-noise ratio.

You should have a final main function that solves the whole problem by calling other functions that you have written. You can choose the input and output arguments and name the main function as you like, as long as the name describes its purpose for a potential user (it should not be called `problem2.7.m` or similar). One possibility is to name and define the function in the following way:

```
>> datarates = cellcoverage(X0,Y0,x1,y1,bandwidth,P,sigma2,kappa)
```

Hint: Highest signal-to-noise ratio is the same as shortest distance. If you first compute the shortest distance from each phone position to any of the base stations (which will be different base stations for different positions), then you can reuse the rest of the code without having to change anything.

Plot the data rates for the example in Figure 1 and verify that the results is symmetric and that the density of phone locations is high enough to get a smooth graph. Use the same type of three-dimensional graph as before.

2.8 Statistics on Data Rates and Coverage

Finally, you need to compute some statistics of which data rates that a user can expect. Since the user is unaware of the base station locations, we assume that the user can be anywhere in the area (e.g., the yellow area in Figure 1) with equal probability.

Plot the data rates for different phone positions by using a histogram with a reasonable number of bars. Compute the probability of not having coverage (e.g., the percentage of phone position that gives zero data rate) and the mean value of the data rates.

You can think about which data rate that one should use to market the cellular network in Figure 1. What rates can a user be guaranteed? What happens if we decrease/increase the distance between the base stations? What happens if we decrease/increase the signal power P?

2.9 Preparations for the Presentation

To prepare for the project presentation, you need to read through the section "Presentation" in the beginning of this document and also read the Examination and Coding Guide that you find on the course homepage. At these places you will find that your need to have at least two functions in your code, functions and variables need to have descriptive names, there should be a reasonable amount of comments in the code, all figures should be self-explaining, comments should be in English, etc. If there is any requirements in the Examination and Coding Guide that you do *not* fulfill, then you need to take care of this *before* you present the project, otherwise you might have to wait a long to get a second chance to present.

Finally: Don't forget to submit the code to Urkund when you have passed to project assignment. You will find the instructions in the section "Presentation" in the beginning of this document.

3 Voluntary Extra Assignments

This section describes a number of voluntary extra assignments. If you are experienced programmers and feel that the project was too easy, we recommend that you also do some of these extra assignments, to learn MATLAB properly.

Shadow fading: The model in (2) for computing signal-to-noise ratios is simplified as compared to reality. Depending on whether buildings and trees "shadow" the way between the user and the base station or not, the received signal power can vary greatly. This phenomenon is known as shadow fading. A common way to model this is to replace the expression for signal-to-noise ratio in (2) by

$$\frac{P}{z(1+d)^{\kappa}\sigma^2} \tag{4}$$

where z is a random variable with a log-normal distribution. This means that $z = 10^{\zeta}$ where ζ has a normal distribution. Modify your function so that the signal-to-noise ratios are computed according to (4) and assume that ζ is different at different positions. Let ζ have mean value of 0 and let the variance be an argument. A typical value of the variance of ζ is 0.6.

Uplink and downlink: Signals that are sent from a base station to a phone is called the downlink, while signals from a phone to a base station is known as the uplink. The signal power can be very different between the uplink and downlink. A typical battery-powered cell phone has a maximal signal power of 0.2 Watt, while a base station can send out 20 Watt. This major difference can often give different data rates in the uplink and downlink. Compare the data rates; in particular, how it affects the coverage.

A simple way to "extend" the coverage is to concentrate the signal power to a part of the bandwidth B. If we reduce the bandwidth by a factor c, to B/c, then the noise power will be reduced as σ^2/c. Exploit this property to handle cases when there is no coverage in the uplink.

Multiple active users: If there are multiple active cell phones, these have to share the available bandwidth. Throw out a certain number users (the number can be an argument) at random locations in the area of Figure 1 and compute how many phones that connect to each base station. Assume that each base station divides the bandwidth equally over its users (as described in previous problem), while phones that connect to different base station don't affect each other. Plot a histogram that shows how the average data rate per user depends on the number of active users.