

# 1 Results

## 1.1 Initial PDFs

The Initial PDFs used here and by SAL have the form:

$$\begin{aligned}
\frac{1}{\alpha} x \bar{u}^\gamma(x, Q_0) &= A_Q^{\text{had}} x^{B_Q^{\text{had}}} (1-x)^{C_Q^{\text{had}}} + A_Q^{\text{PL}} \frac{4}{9} x \frac{x^2 + (1-x)^2}{1 - B_Q^{\text{PL}} \ln(1-x)} \\
\frac{1}{\alpha} x \bar{d}^\gamma(x, Q_0) &= A_Q^{\text{had}} x^{B_Q^{\text{had}}} (1-x)^{C_Q^{\text{had}}} + A_Q^{\text{PL}} \frac{1}{9} x \frac{x^2 + (1-x)^2}{1 - B_Q^{\text{PL}} \ln(1-x)} \\
\frac{1}{\alpha} x \bar{s}^\gamma(x, Q_0) &= K_S A_Q^{\text{had}} x^{B_Q^{\text{had}}} (1-x)^{C_Q^{\text{had}}} + A_Q^{\text{PL}} \frac{1}{9} x \frac{x^2 + (1-x)^2}{1 - B_Q^{\text{PL}} \ln(1-x)} \\
\frac{1}{\alpha} x g^\gamma(x, Q_0) &= A_G^{\text{had}} x^{B_G^{\text{had}}} (1-x)^{C_G^{\text{had}}}
\end{aligned} \tag{1.1}$$

Some of the values are set to constant values ( $K_S = 0.3$ ,  $C_Q^{\text{had}} = 1.0$ ,  $C_G^{\text{had}} = 3.0$ ) here and by SAL.

Since SAL doesn't fix any other parameters, therefore they have 6 free parameters.

We however fix another parameter ( $A_G^{\text{had}}$ ) using the FG Momentum Sum Rule. Therefore our basic Initial PDFs (SAL5) have 5 free parameters. We also use Initial PDFs, which are similar to SAL5, but lack the pointlike contribution (SAL3).

## 1.2 Settings

For the APFEL++ and MINUIT-program, the used quark-masses are

$$m_c = 1.3 \text{ GeV}, \quad m_b = 4.5 \text{ GeV}, \quad m_t = 174 \text{ GeV}$$

The input scale is chosen as

$$Q_0 = m_c = 1.3 \text{ GeV}$$

APFEL++ can calculate the strong coupling constant, given a reference energy and the value for  $\alpha_s$  at this reference energy. In this case, the mass of the Z-Boson is used as the reference energy and is given by

$$Q_{\text{ref}} = m_Z = 91.188 \text{ GeV}$$

The  $\alpha_s(Q_{\text{ref}}) = \alpha_{s,\text{ref}}$  at this energy are

$$\alpha_{s,\text{ref}}^{\text{LO}} = 0.1179973 \text{ GeV}, \quad \alpha_{s,\text{ref}}^{\text{HO}} = 0.1179973 \text{ GeV}$$

These values are the same as used by nCTEQ15 [1]. For lack of a better  $\alpha_{s,\text{ref}}$  value for LO, the HO value is also used for LO.

### 1.2.1 Parameter Settings - Minuit

All parameters are initialized with the value 0.5.

The initial errors for the parameters are set to 0.1.

The upper bounds for all  $B^{\text{had}}$ -parameters are set to 1.0, for all other parameters the upper bound is set to 40.0 (just some random number, high enough that it doesn't affect the minimization).

The lower bounds for all  $A$ -parameters are set to 0.0, because the PDFs should be positive. For all  $B^{\text{had}}$ -parameters the lower bound is set to -1.0, because the gamma function in the FG Momentum Sum Rule only takes values bigger than zero. To avoid, that the exponent and exponential integral in the FG Momentum Sum Rule, get too big, the lower bound for  $B_Q^{\text{PL}}$  are set to 0.1.

### 1.2.2 Experimental Data

The experimental data is taken from the following experiments:

ALEPH1, ALEPH2, AMY, DELPHI, JADE, L3, OPAL1, PLUTO, TASSO, TOPAZ

This means, that 134 experimental data points are used for minimizing the  $\chi^2$ -function.

## 1.3 Calculation of the Data

The data will be compared at  $Q = \sqrt{2} \text{ GeV}$ , the energy where SAL calculated their parameters for the Initial PDFs.

### 1.3.1 Our SAL-PDFs

We calculated data for our two SAL-type Initial PDFs: SAL3 and SAL5. To calculate the data, we used APFEL++ and MINUIT to calculate the parameters for the Initial PDFs at the input scale  $Q_0 = 1.3 \text{ GeV}$ . With the calculated parameters (see table 1) we were able to use APFEL++ again to evolve the Initial PDFs to  $Q = \sqrt{2} \text{ GeV}$ .

### 1.3.2 GRV

The GRV data at  $Q = \sqrt{2} \text{ GeV}$  is calculated in two ways. It is calculated by:

1. using the functions in `grvphoton.f` at  $\sqrt{2} \text{ GeV}$ .
2. using the functions in `grvphoton.f` at  $1.3 \text{ GeV}$  and evolving the results to  $\sqrt{2} \text{ GeV}$  using APFEL++.

Since GRV has set their charm quark threshold to  $m_c = 1.5 \text{ GeV}$ , the first method produces no data for the charm quark at  $\sqrt{2} \text{ GeV}$ .

### 1.3.3 SAL

The parameters for the Initial PDFs by SAL [2, Table 1, ZEUS-TR] are already given at  $\sqrt{2}$  GeV. Therefore no further calculation is needed.

The parameters can be seen in table 1.

## 1.4 Calculated Initial PDFs - without errors

Running the minimization results in the parameters for the Initial PDFs shown in table 1.

**Table 1:** Parameters of the Initial PDFs and Chi-Squares, resulting from the minimization using Minuit and from SAL. The italic numbers are fixed parameters.

	SAL3		SAL5		SAL
	LO	HO	LO	HO	
$A_G^{\text{had}}$	0.204848	0.200964	0.554251	0.277174	0.027
$B_G^{\text{had}}$	-0.353503	-0.197582	-0.187398	-0.286863	-0.57
$C_G^{\text{had}}$	<i>3.0</i>	<i>3.0</i>	<i>3.0</i>	<i>3.0</i>	<i>3.0</i>
$A_Q^{\text{had}}$	0.423403	0.46428	0.495995	0.563353	0.065
$B_Q^{\text{had}}$	0.244205	0.272071	0.65595	0.599801	-0.16
$C_Q^{\text{had}}$	<i>1.0</i>	<i>1.0</i>	<i>1.0</i>	<i>1.0</i>	<i>1.0</i>
$A_Q^{\text{PL}}$	<i>0.0</i>	<i>0.0</i>	0.352323	0.302575	5.45
$B_Q^{\text{PL}}$	<i>0.0</i>	<i>0.0</i>	0.580187	1.02583	1.9
$K_S$	<i>0.3</i>	<i>0.3</i>	<i>0.3</i>	<i>0.3</i>	<i>0.3</i>
$\chi^2$	120.11	110.439	101.002	99.9632	
$\chi^2/\text{d.o.f.}$	0.896343	0.824173	0.753746	0.745994	1.63

The resulting comparison of the Initial PDFs can be seen in fig. 1 for LO and in fig. 2 for HO.

### 1.4.1 Notes on Initial PDFs without errors

I think, the Evolution itself is as good as it might get. When comparing to Vadim, the difference is very very small. Furthermore, the differences seen between Vadim and me, as explained to me by Valerio, are on QCDNUMs part.

I also think, that the code for the Initial PDFs might be also as good as it gets. The only thing, that might be up for change, are the parameters and parameter bounds.

## 1.5 Calculated Initial PDFs - with Errors

As an example I calculated the Initial PDFs with Errors for SAL5 HO (see fig. 3). From that, I can also calculate the Structure Functions with Errors. They can be seen by running the `plottingStructureFunctionsErrorPDFs.py` program in the `Bestandsaufnahme` from 2022.19.15.

### 1.5.1 Notes on Initial PDFs with errors

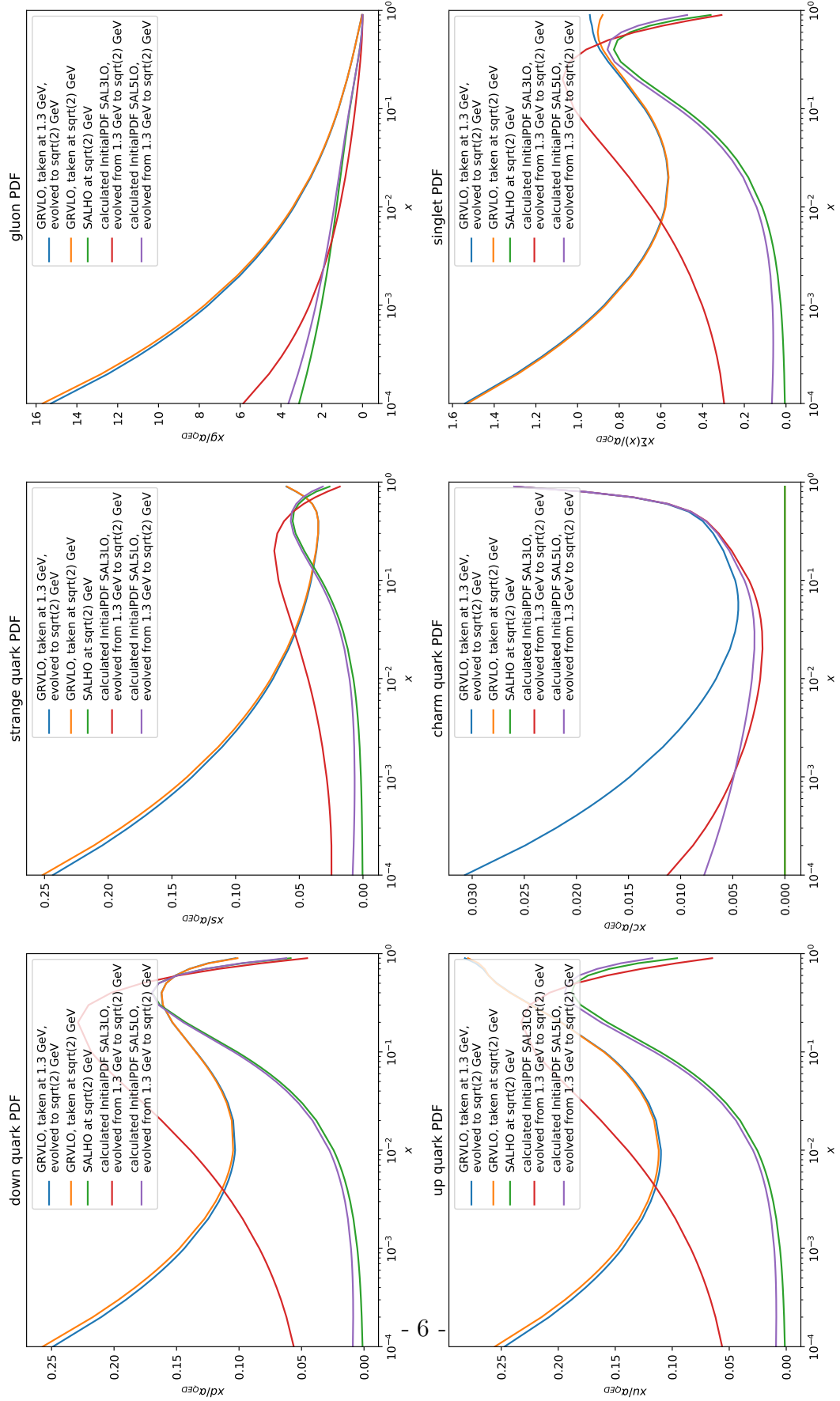
There are still some things to do with the calculation of the ErrorPDFs:

- momentum sum rule for the errors is violated, because the  $A_G^{\text{had}}$  (co-eff. calculated using momentum sum rule) is the same as for the InitialPDFs
  - I asked Karol if nCTEQ ErrorPDFs violates momentum sum rule and he doesn't really remember, but they probably don't
- we can change our choice for `deltaChi2` - I just took the "basic" value of 1
  - there is a reference in the nCTEQ15 paper on how to do it

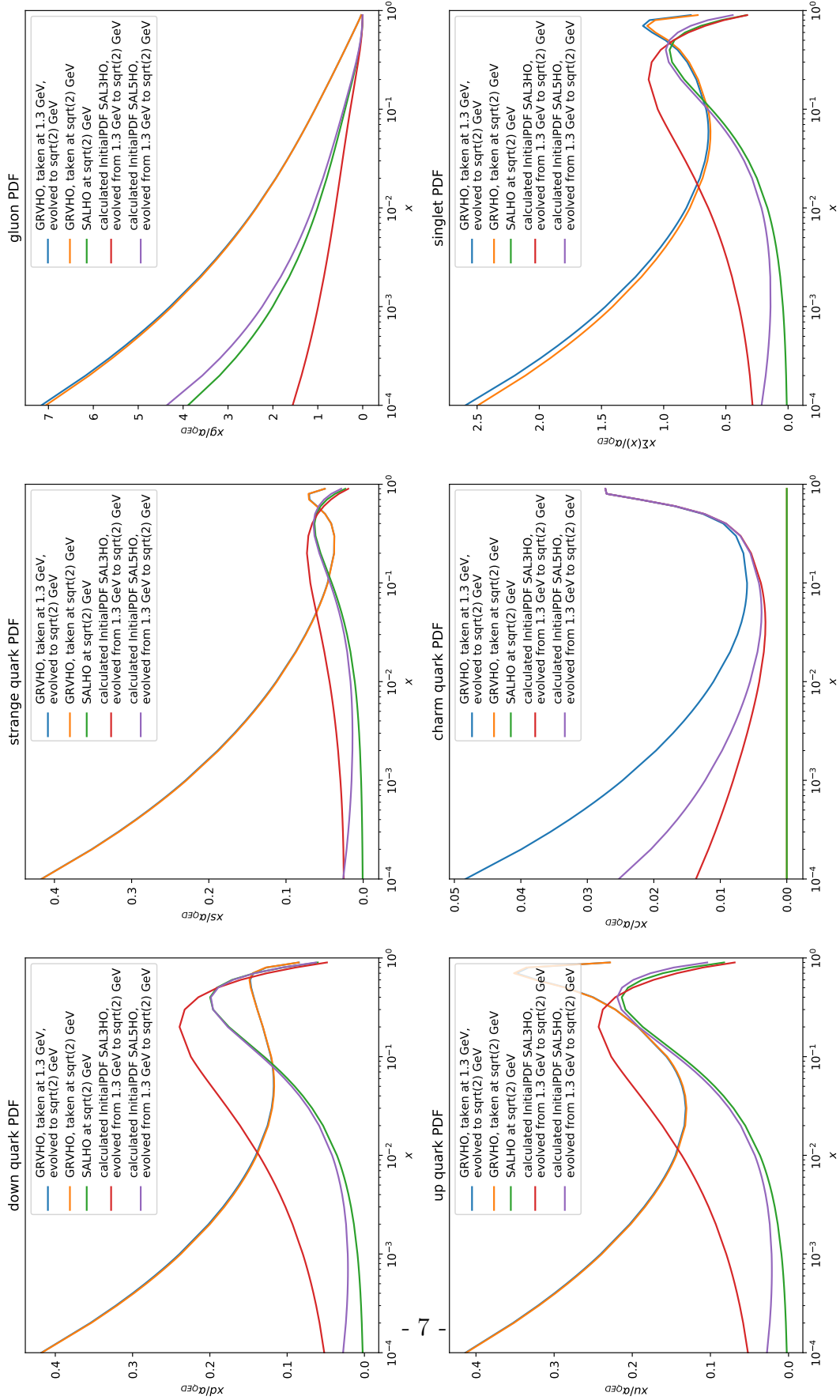
## 2 General Things To Discuss

- I find it hard to structure the results etc and save them in a good way
- tying into that: I think it would be nice, to finally get the Evolution to our satisfaction and decide on things like the parameters etc
  - makes it a whole lot easier for me to work and I can tidy up my programs etc
  - however, for some comparisons (for example comparison with GRV) we need specific parameters (and also different ways of calculating as) to match their results as best as possible
    - \* should we compare our results to them using our standard parametrization or using their parameters etc? And also: should this comparison be part of the final thesis?
- hard for me to motivate, if there are no “short-term goals” or “direct” responsibilities
  - weekly meeting with Vadim to discuss the goals for next week?

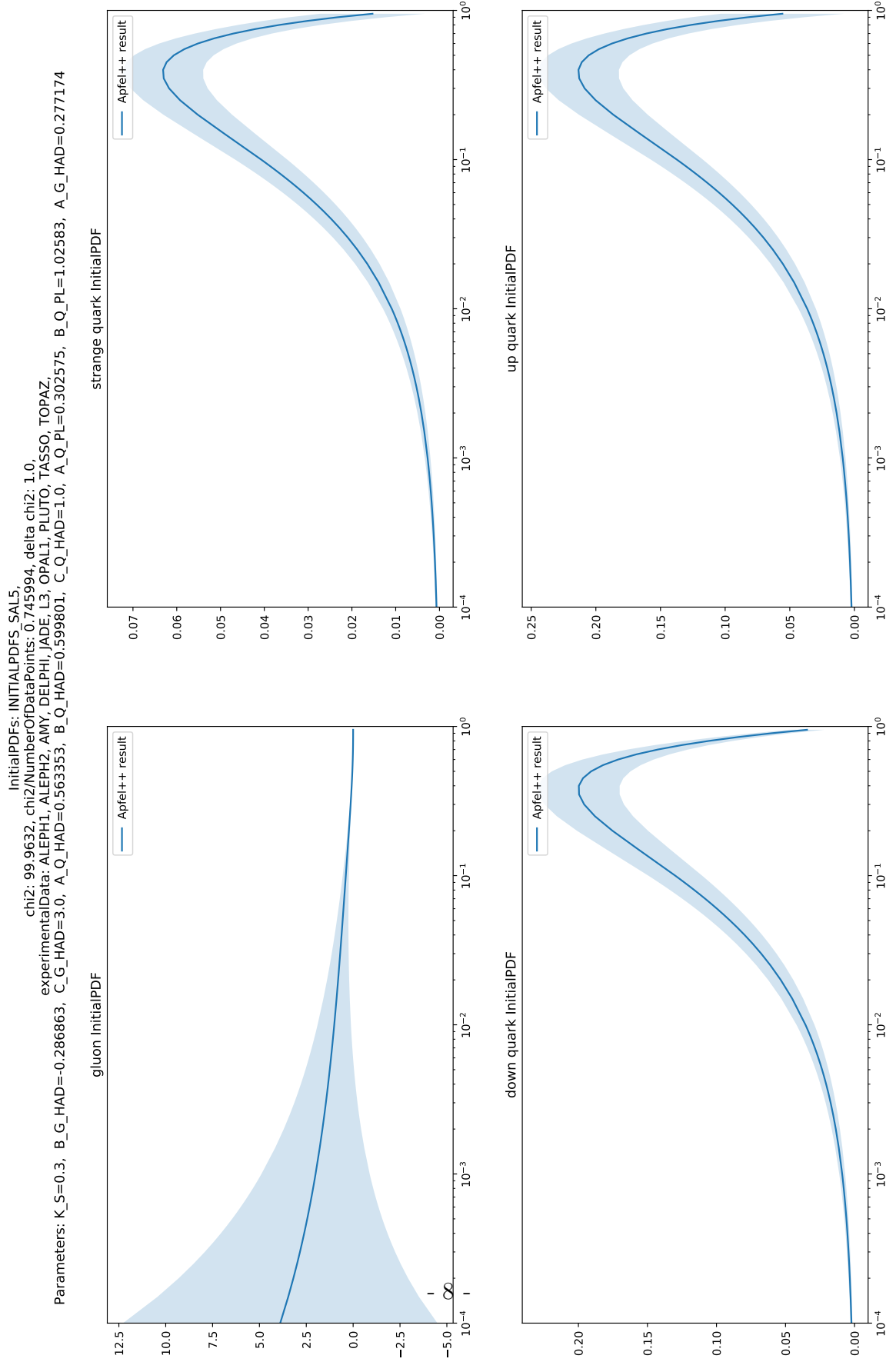
## 2 General Things To Discuss



**Figure 1:** Comparison of all Initial PDFs at LO at  $\sqrt{2}$  GeV



**Figure 2:** Comparison of all Initial PDFs at HO at  $\sqrt{2}$  GeV



**Figure 3:** Initial PDFs with ErrorPDFs at HO at  $Q_0 = 1.3\text{GeV}$



## References

- [1] K. Kovarik et al. “nCTEQ15: Global analysis of nuclear parton distributions with uncertainties in the CTEQ framework”. In: *Physical Review D* 93.8 (Apr. 2016). DOI: 10.1103/physrevd.93.085037. URL: <https://doi.org/10.1103/PhysRevD.93.085037>.
- [2] W. Slominski, H. Abramowicz, and A. Levy. “NLO photon parton parametrization using ee and ep data\*”. In: *The European Physical Journal C* 45.3 (Jan. 2006). DOI: 10.1140/epjc/s2005-02458-7. URL: <https://doi.org/10.1140/epjc/s2005-02458-7>.