Hochschule München University of Applied Sciences

Optional Assignment: Coin toss experiment

Objectives

- You will conduct several coin toss experiments
- You will get to know the difference between probabilities and amplitudes
- You will understand the principle of amplitudes of Quantum Computers
- You will practically use Qiskit to get access to a real quantum simulator

Requirements

- Notebook or Desktop Computer with access to the internet
- Access to IBM Quantum Lab (https://lab.quantum-computing.ibm.com/)

Solution Steps

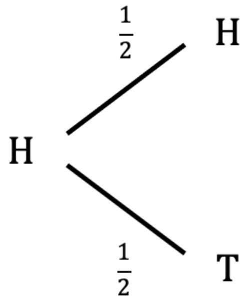
Another fundamental property of Quantum Computers is that they do not follow the principles of classical probabilities, hence Quantum theory is sometimes also described as probability theory with negative numbers [1].

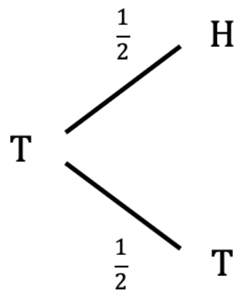
To understand why Quantum theory is called probability theory with negative numbers, it is essential to compare probability trees of Quantum Computers with Classical Computers. To do so let's play a coin toss game.

Please open the following link to play the coin toss experiment and follow the instructions: or copy and paste.
https://github.com/alexanderfechtel/Quantum-Computing/blob/main/Assignment%203_Coin_Toss_Experiments.ipynb

In order to better understand the outcome of the double coin toss experiment, we need to understand how classical probability trees differ from quantum probabilities, called amplitudes.

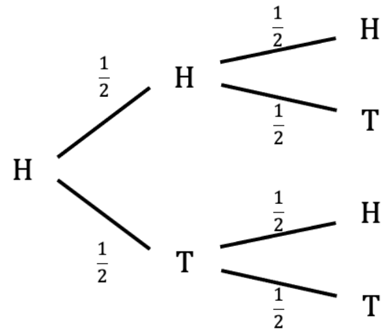
Let's start with a classical probability of tossing a fair coin once. Whenever we toss a fair coin, there is a 50% probability to get "Heads" and a 50% chance to get "Tails" as a result. As we always achieve this result independent from whether the initial coin state was "Heads" or "Tails" the probability trees of the single coin toss experiment with the Classical Computer can be depicted as followed:



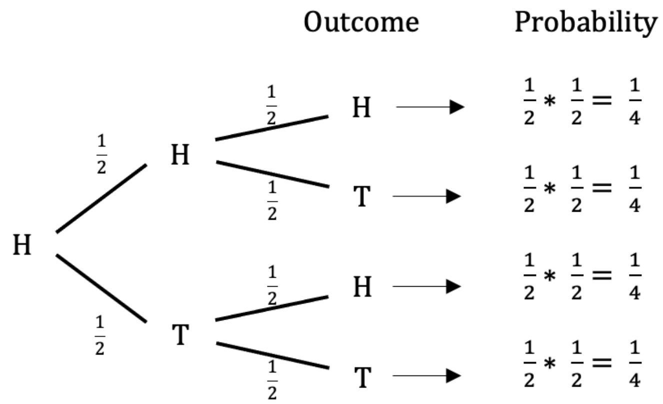


If you have not achieved this result in the single coin toss experiment with a Classical Computer, you can try it again by setting the number of repetitions of the coin toss experiment between 500 to 1,000.

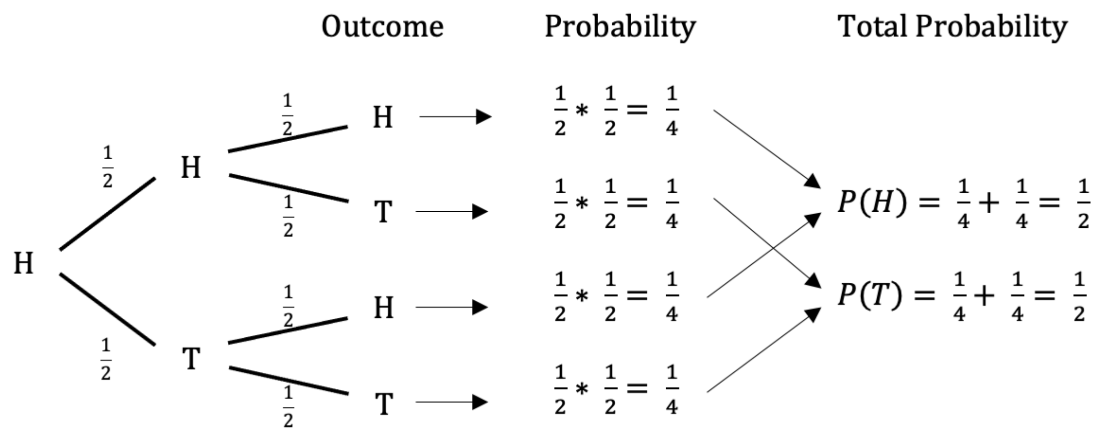
In the next step let's think about how the probability tree looks like when we toss the coin two times in a row.



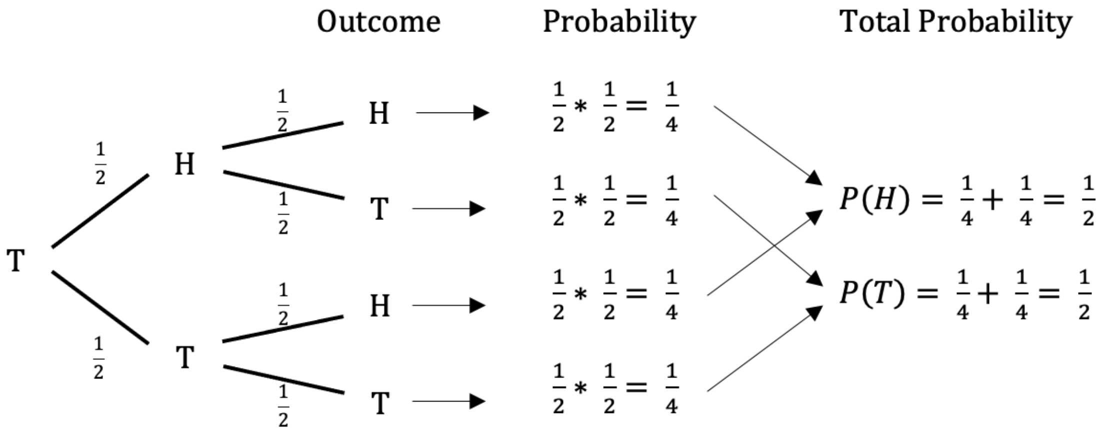
As you can see flipping a coin two times in a row leads to four different paths. To calculate the probability for each of those outcomes we need to multiply the individual probabilities of each step in the path. Due to the fairness of the coin the probability of each path is 25%.



As always two paths lead to the same outcome we can calculate the total probability of one outcome by adding up the probabilities of the two paths leading to the same outcome. By doing so the probability of the coin showing "Heads" after it was tossed twice is 50%. Receiving "Tails" after the second toss has the same probability of 50%.

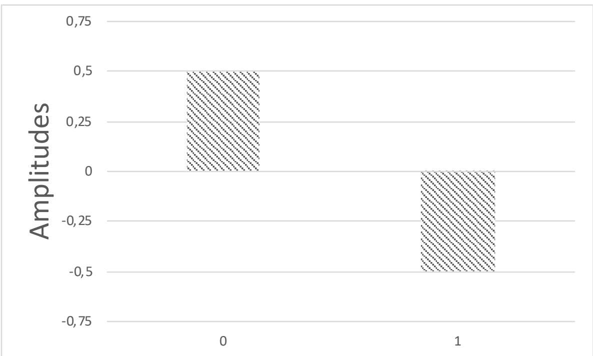


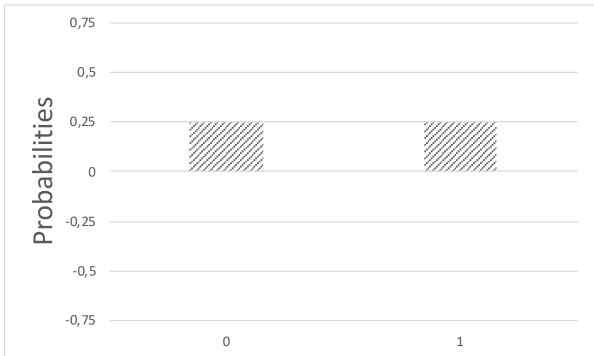
In the aforementioned example “Heads” was chosen to be the initial coin state of the double coin toss experiment. However, we would receive the same results when “Tails” was the initial coin state as the probability of having “Tails” or “Heads” is still evenly distributed and thus 50%.



In the next section we find out how the quantum coin behaves.

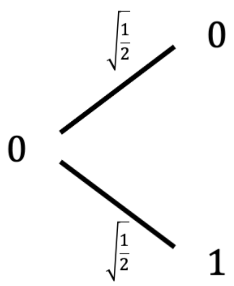
As mentioned in the beginning of this tutorial quantum theory is also called probability theory with negative numbers. As the following two graphics depict, amplitudes can be either negative or positive. However, as probabilities can’t be negative, we need to transfer the amplitudes to probabilities. As the graphs show an amplitude of 0.5 and -0.5 can be transferred to a probability of 0.25, indicating that we can calculate the probability by squaring the amplitude.



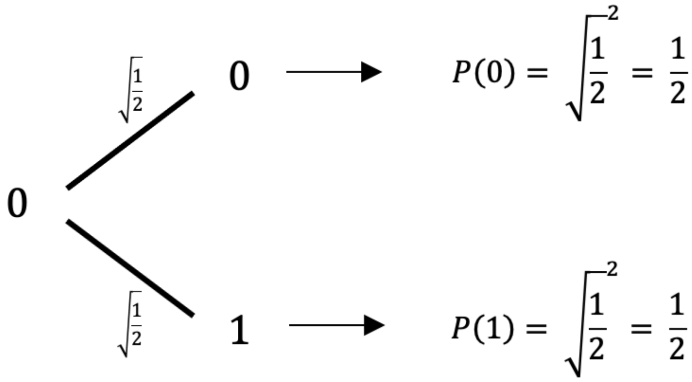


Our quantum coin is called qubit and when we measure it, it shows either 0 for “Heads” or 1 for “Tails”.

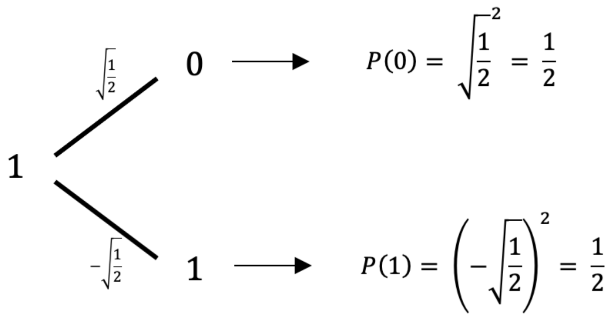
For the single coin toss experiment with a Quantum Computer, we get the following amplitude tree, when “Heads” was chosen to be the initial coin state.



As probabilities always must add up to 100% the amplitudes are squared to calculate the probabilities. Following this logic, when we choose “Heads” as the initial coin state we have a 50/50 chance to receive “Heads” or “Tails” as the outcome of the single coin toss game.

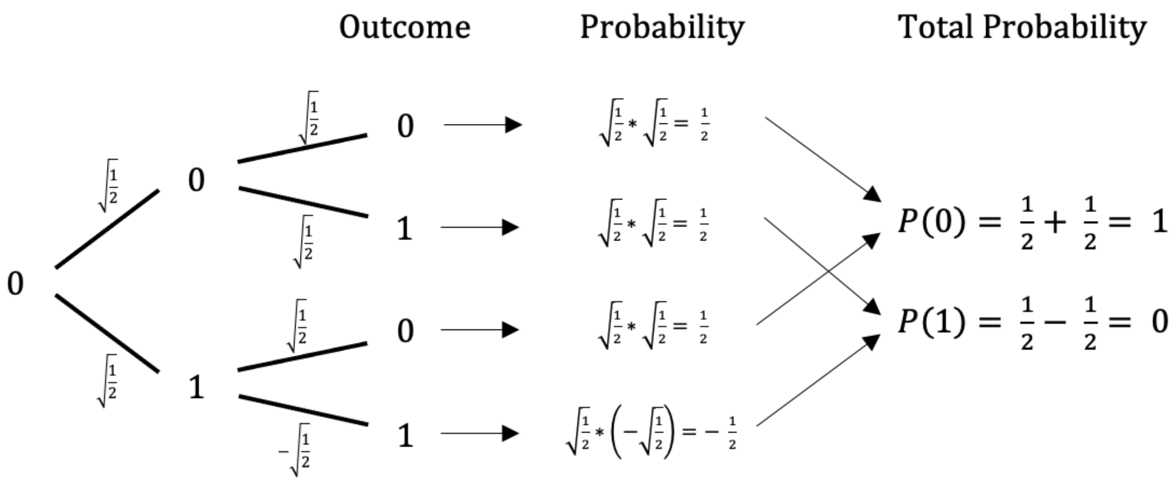


When we decide to choose “Tails” as the initial coin state of the single coin toss game we get the following amplitude tree. Although the tree looks different it leads to the same outcome.

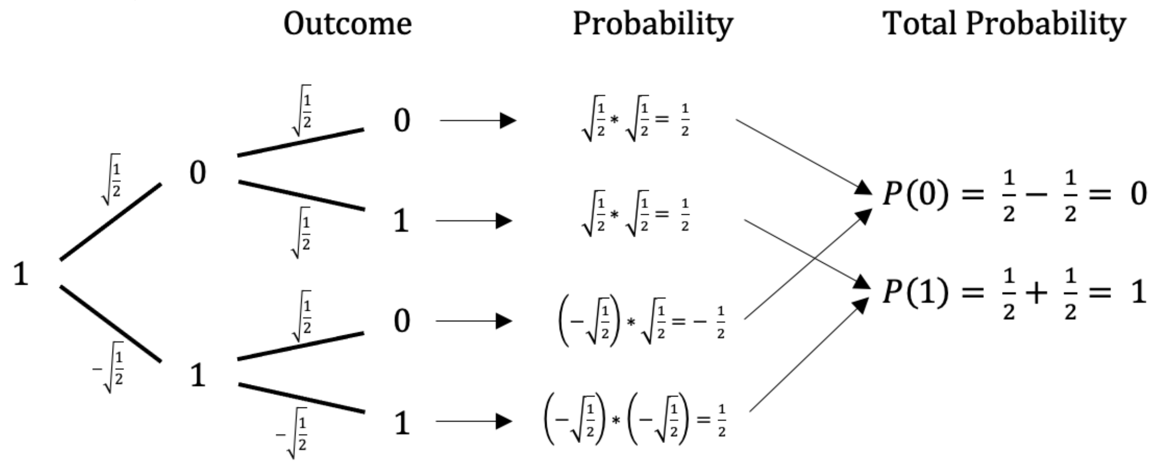


Therefore, we can conclude that when the coin is only tossed once before tracking the result, we get the same probabilities for “Heads” and “Tails”, no matter if the single coin toss game is performed with a Classical or a Quantum Computer.

Let’s figure out what happens when we toss the quantum coin twice in a row and record the result after the second toss. Similar to the normal double coin toss experiment we first calculate the probability of each path and then add up the probabilities of the paths that lead to the same outcome. This amplitude tree and its’ transition to probabilities shows why the quantum double coin toss experiment always leads to “Heads” as an outcome, when “Heads” was chosen to be the initial coin state.



When we start with “Tails” as initial coin state the amplitude tree looks as follows and leads to the following probabilities.



Based on the amplitude trees we can conclude that the double coin toss experiment with a Quantum Computer leads to a certain outcome of “Heads” when “Heads” was the initial coin state and “Tails” whenever “Tails” was the initial state the coin showed.

In general, it has to be noted that quantum particles can be described with wave functions and that the amplitude indicates the distance between the center and the crest of this wave. Due to this characteristic amplitudes can be negative and positive [2].

How can we benefit from this feature of quantum computing?
Combining operations like in the double coin toss experiment and making use of amplitudes enables us to build more efficient algorithms on Quantum Computers. These algorithms can make wrong answers cancel out quickly and give us a high probability of measuring the right answer for mathematical problems of high complexity like the route optimization example of the previous assignment [1].

Useful Resources for Own Research

Where the quantum advantage comes from:
https://towardsdatascience.com/where-the-quantum-advantage-comes-from-5accd926eb7a

Where quantum probability comes from:
https://www.quantamagazine.org/where-quantum-probability-comes-from-20190909/

TED talk about the power of Quantum Computers:
https://www.youtube.com/watch?v=QuR969uMICM

Retrospective

Please answer the following questions:

1. What is the main difference between the double coin toss experiment with a Classical Computer and a Quantum Computer?
2. How can amplitudes be transferred to classical measurement probabilities?
3. Why are “negative probabilities” possible in quantum mechanics?
4. What is the advantage of amplitudes?

Sources

[1] https://qiskit.org/textbook/what-is-quantum.html
[2] https://towardsdatascience.com/quantum-amplitudes-and-probabilities-b49a6969b0b9