Intuitive Explanation of the Black-Scholes Model (Part 2)

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Introduction to Volatility

Volatility reflects the uncertainty in the stock's price and the higher it is, the greater the potential for options to fluctuate in price, this means that there is a higher chance of finishing in the money. Thus, higher volatility will increase options prices.

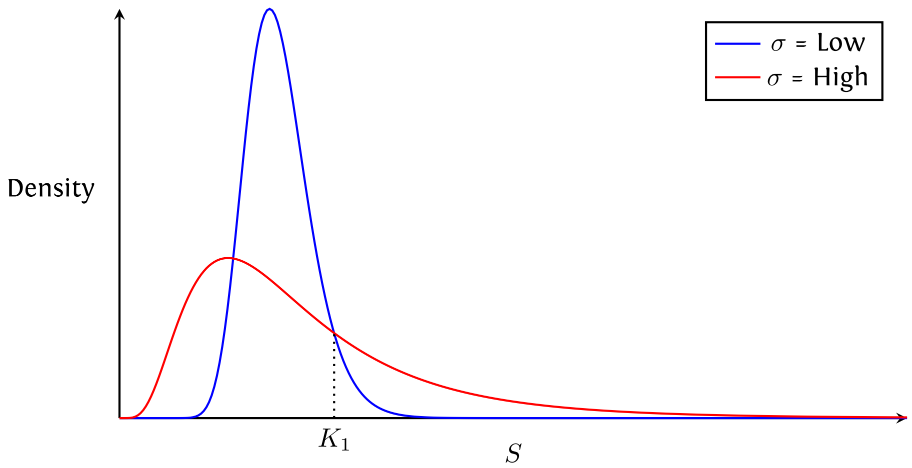

Figure 1: Lognormal distributions with low and high volatility

In Figure (1), we have a visual representation that the same option with a strike price above K_1 and a current stock price below K_1 will be priced higher when its volatility is higher since it has a greater change of expiring above K_1, represented by a larger area under the curve.

Intuition of the Black-Scholes Model (continued)

To calculate these effects, like the Greeks and the call price, the Black-Scholes equation must be solved. In Article No.1, I explained that the the fair price of a European-style call option is given by the following:

$$C = S\Phi\left(d_1\right) - Ee^{-r(T-t)}\Phi\left(d_2\right) \tag{1}$$

Where

$$d_1 = \frac{\ln\left(\frac{S}{E}\right) + \left(r + \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}}, \qquad d_2 = d_1 - \sigma\sqrt{T-t} \tag{2}$$

However, I simply quoted the solution, let us further analyse and see what each of the terms in equation (1) really mean. The term $S\Phi\left(d_1\right)$ denotes that if I held an option until expiration, what would be the average value of the stock above the strike price? i.e. it denotes the amount you will earn for owning this option on average (which we can denote by Profit_{Avg}). The term $e^{-r(T-t)}\Phi(d_2)$ denotes what is the likelihood of paying the exercise price at expiration? i.e. likelihood of finishing in the money. Multiplying by the strike price E we can see how much it will cost on average to own this option (whih we can denote by Cost_{Avg}). Thus:

$$C = \text{Profit}_{Avg} - \text{Cost}_{Avg} \tag{3}$$

These terms represent the current expected value of the option by subtracting the average cost from the average profit we will earn from the option, so we can work out its fair value.

No Explanation is Served Better than with an Example

I personally think that whilst theory can be exciting and valuable, a practical example is often what makes an idea truly stick and allow yourself to understand the logic.

Imagine a lognormal distribution of stock prices at expiration, with possible prices labelled along the x-axis against the density on the y-axis. The lognormal distribution works for stock prices because stocks cannot fall below 0 and historically, they tend to follow this skewed, right-tailed behaviour.

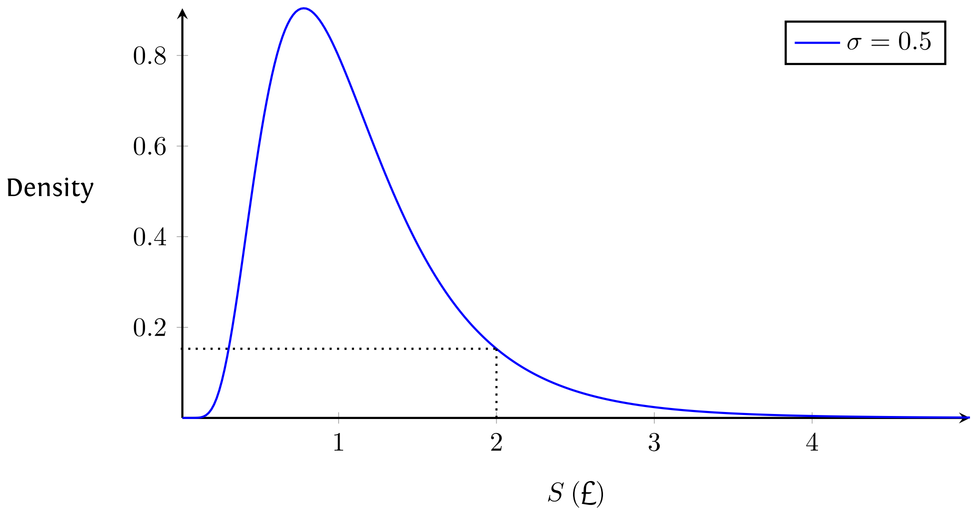

Figure 2: An Example Lognormal Distribution

Let us evaluate a call option with a strike price E of £2. To find Profit$_{Avg}$, this is where we want to find the average stock price only when finishing in the money. Mathematically, we want to find the answers to:

$$\mathbb{E}\left[S \cdot \mathbf{1}_{S>2}\right] = \int_2^\infty x f(x)dx, \qquad \mathbb{P}\left(S > 2\right) = \int_2^\infty f(x)dx \tag{4}$$

Where $f(x)$ is the lognormal PDF and $\mathbf{1}_A$ is the indicator function defined as:

$$\mathbf{1}_A(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A \end{cases}$$

If we use the fact that S is a lognormally distributed random variable, therefore the underlying normal distribution is $Y = \ln(S)$. To convert to standard normal ϕ, we can use the formula $\phi = \frac{Y-\mu}{\sigma} = \frac{\ln(S)-\mu}{\sigma}$. Note that in this example, we have used $\mu = 0, \sigma = \frac{1}{2}$. Thus:

$$\mathbb{P}\left(S > 2\right) = \mathbb{P}\left(\ln(S) > \ln(2)\right) = \mathbb{P}\left(\phi > \left(\frac{\ln(2)}{\frac{1}{2}}\right)\right) = 1 - \Phi\left(\frac{\ln(2)}{\frac{1}{2}}\right)$$

Now, when evaluating the first equation in (4), the key to solving the integral is to notice similarities after a substitution between the new integral and that of a standard normal (by completing the square). Note:

$$\begin{aligned}\mathbb{E}\left[S \cdot \mathbf{1}_{S>2}\right] &= \int_2^\infty x f(x)dx \\ &= \int_2^\infty x \frac{1}{\frac{1}{2}x\sqrt{2\pi}} exp\left(-\frac{\left(\ln(x)\right)^2}{2\left(\frac{1}{2}\right)^2}\right) dx\end{aligned}$$

Where we can see the x's will cancel each other.

Now we can use the substitution $y = \ln(x)$, so $x = e^y, dx = e^y dy$. Thus:

$$
\begin{aligned}
\mathbb{E}\left[S \cdot \mathbf{1}_{S>2}\right] &= \int_{\ln(2)}^{\infty} \frac{1}{\frac{1}{2}\sqrt{2\pi}} exp\left(-\frac{y^2}{2\left(\frac{1}{2}\right)^2}\right) e^y dy \\
&= \int_{\ln(2)}^{\infty} \frac{1}{\frac{1}{2}\sqrt{2\pi}} exp\left(y-\frac{y^2}{2\left(\frac{1}{2}\right)^2}\right) dy \\
&= \int_{\ln(2)}^{\infty} \frac{1}{\frac{1}{2}\sqrt{2\pi}} exp\left(-\frac{y^2-2\left(\frac{1}{2}\right)^2 y}{2\left(\frac{1}{2}\right)^2}\right) dy \\
&= \int_{\ln(2)}^{\infty} \frac{1}{\frac{1}{2}\sqrt{2\pi}} exp\left(-\frac{y^2-2\left(\frac{1}{2}\right)^2 y}{2\left(\frac{1}{2}\right)^2}\right) dy \\
&= \int_{\ln(2)}^{\infty} \frac{1}{\frac{1}{2}\sqrt{2\pi}} exp\left(\frac{1}{8}-\frac{\left(y-\frac{1}{4}\right)^2}{2\left(\frac{1}{2}\right)^2}\right) dy \\
&= e^{\frac{1}{8}} \int_{\ln(2)}^{\infty} \frac{1}{\frac{1}{2}\sqrt{2\pi}} exp\left(-\frac{\left(y-\frac{1}{4}\right)^2}{2\left(\frac{1}{2}\right)^2}\right) dy
\end{aligned}
$$

Note, we now have a normal distribution with $\mu = \frac{1}{4}, \sigma = \frac{1}{2}$ multiplied by $e^{\frac{1}{8}}$. Therefore:

$$\mathbb{E}\left[S \cdot \mathbf{1}_{S>2}\right] = e^{\frac{1}{8}}\left(1-\Phi\left(\frac{\ln(2)-\frac{1}{4}}{\frac{1}{2}}\right)\right)$$

And so, taking these results:

$$\text{Profit}_{Avg} = \mathbb{E}\left[S \cdot \mathbf{1}_{S>2}\right] \approx £0.21$$

The next step is to determine the probability that the option finishes in the money, which is simply the area under the curve ($\mathbb{P}(S > 2)$) which we have already calculated. Thus, to find Cost_{Avg}, the expected cost is the likelihood multiplied by the strike price, £2. It represents the average amount we expect to pay for the option at expiration. Thus:

$$\text{Cost}_{Avg} = 2\mathbb{P}\left(S > 2\right) \approx £0.17$$

And so, the fair value of the option would be:

$$C \approx £0.05$$

Conclusion

By stepping through the distribution of future stock prices, the chance of finishing in the money, and the expected payoff, we can see how an option may be priced. Essentially, an option's value comes from understanding how much of the distribution lies above the strike and how large those outcomes tend to be.

In the next article, I will expand at how we move from the standard normal distribution to the lognormal distribution, and how this transformation ties into the Black–Scholes model.

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