

$$1) \gamma = \frac{F}{A}$$

$$dW = \gamma dA = -p dV$$

$$= -p \cdot - \frac{\gamma 2}{r}$$

$$dV = 4\pi r^2 dr$$

$$dA = 8\pi r dr$$

$$2) dV = dW + dQ = \gamma dA + T dS$$

$$c) F = U - TS$$

$$dF = dU - T dS - S dT = \gamma dA + T dS - T dS - S dT = \gamma dA - S dT$$

$$d) \left( \frac{\partial F}{\partial A} \right)_T = \gamma \quad \left( \frac{\partial F}{\partial T} \right)_A = -S$$

$$\Rightarrow \left( \frac{\partial S}{\partial A} \right)_T = - \left( \frac{\partial \gamma}{\partial T} \right)_A \quad \left( \frac{\partial \gamma}{\partial T} \right)_A < 0, \text{ weil Teilchen sich mehr bewegen}$$

$$\Rightarrow \frac{\partial S}{\partial A} > 0$$

$$e) \gamma = \left( \frac{\partial F}{\partial A} \right)_T = \left( \frac{\partial U}{\partial A} \right)_T - \left( \frac{T \partial S}{\partial A} \right)_T = \left( \frac{\partial U}{\partial A} \right)_T + T \left( \frac{\partial \gamma}{\partial T} \right)_A$$

$$\Rightarrow \left( \frac{\partial U}{\partial A} \right)_T = \gamma - T \left( \frac{\partial \gamma}{\partial T} \right)_A$$

$$f) \left( \frac{\partial U}{\partial A} \right)_T = \left( \frac{dQ}{dA} \right)_T + \left( \frac{dW}{dA} \right)_T = -T \left( \frac{\partial \gamma}{\partial T} \right)_A + \gamma$$

$$\Rightarrow \left( \frac{dQ}{dA} \right)_T = -T \left( \frac{\partial \gamma}{\partial T} \right)_A$$

$$\Delta Q = -T \left( \frac{\partial \gamma}{\partial T} \right)_A \Delta A \quad \Delta Q > 0$$

$$\Delta S = \frac{\Delta Q}{T} > 0, \text{ größere Oberfläche} \Rightarrow \text{mehr Mikrozustände} \Rightarrow \text{mehr Entropie}$$



$$2) dU = TdS + Bdm \quad \chi \approx \frac{\mu_0 m}{\rho V} \propto \frac{1}{T} \Rightarrow m \propto \frac{1}{T}$$

$$F = U - TS - mB \quad C_B = T \left( \frac{\partial S}{\partial T} \right)_B$$

$$dF = -SdT - m dB$$

$$\left( \frac{\partial F}{\partial T} \right)_B = -S \quad \left( \frac{\partial F}{\partial B} \right)_T = -m$$

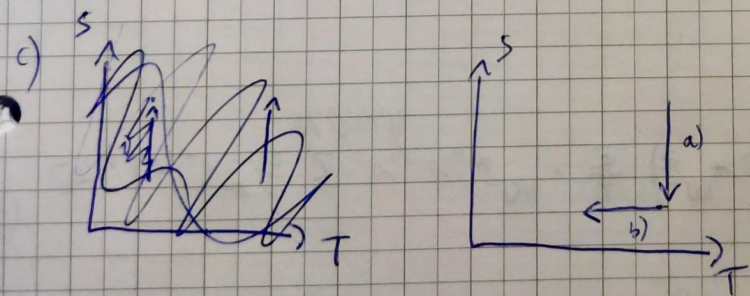
$$\left( \frac{\partial S}{\partial B} \right)_T = \left( \frac{\partial m}{\partial T} \right)_B \Rightarrow \Delta S = \Delta B \left( \frac{\partial m}{\partial T} \right)_B$$

$$\Delta Q = \Delta S T = \Delta B T \left( \frac{\partial m}{\partial T} \right)_B = \Delta B T \left( -\frac{1}{T^2} \right)$$

$$\stackrel{m \propto \frac{1}{T}}{\Rightarrow} \Delta B T \left( -\frac{1}{T^2} \right) = -\frac{\Delta B}{T} < 0 \quad (\Delta B > 0, T > 0)$$

$$b) \left( \frac{\partial T}{\partial B} \right)_S = -\frac{T}{C_B} \left( \frac{\partial m}{\partial T} \right)_B \propto +\frac{1}{C_B T}$$

$$\Rightarrow \Delta T = \frac{\Delta B}{C_B T} \quad \Delta B < 0 \Rightarrow \Delta T < 0 \quad (C_B, T > 0)$$



d) a) Mehr Spin // zu B als im Gleichgewicht ohne B  $\Rightarrow$  mehr Ordnung / weniger Mikrozustände

b) Demagnetisierung erhöht # Mikrozustände, senkt aber auch Temperatur, wodurch die Besetzung geändert wird



$$3) H = -J \sum_i^N \sigma_{i1} \sigma_{i2}$$

$$E_M = -2J \quad E_{\uparrow\downarrow} = 2J$$

$$Z = \sum_{\sigma_1, \sigma_2} e^{-\beta H}$$

$$\sigma_1 \sigma_2 = 1$$

$$E_M / E_{\uparrow\downarrow} = -2J$$

$$\sigma_1 \sigma_2 = -1$$

$$E_{\uparrow\downarrow} / E_M = 2J$$

$$Z = 2 \sum_{\sigma_1, \sigma_2 = \pm 1} e^{-\beta(-J \sum_{j=1}^N \sigma_{j1} \sigma_{j2})}$$

$$= 2 \prod_{j=1}^N \sum_{\sigma_{j1}, \sigma_{j2} = \pm 1} e^{+ \beta J \sigma_{j1} \sigma_{j2}} = 2 \prod_{j=1}^N (e^{+ \beta J} + e^{- \beta J}) = 2 \prod_{j=1}^N 2 \cosh(\beta J) = (4 \cosh(\beta J))^N$$

$$a) \langle \sigma_{i1} \sigma_{i2} \rangle = \frac{1}{Z} \sum_{\sigma_1, \sigma_2} \sigma_{i1} \sigma_{i2} e^{-\beta H} = \frac{2}{Z} \left( \frac{-\beta J}{e^{+ \beta J} + e^{- \beta J}} \right) = \frac{4 \sinh(\beta J)}{4 \cosh(\beta J)} = \tanh(\beta J)$$

c)

$$\langle M^2 \rangle = m^2 N^2 \langle \sigma^2 \rangle = m^2 N^2 \lim_{N \rightarrow \infty} \langle \sigma_{i1} \sigma_{i2} \rangle = m^2 N^2 \tanh(\beta J)$$

$$T \rightarrow 0 : \langle M^2 \rangle \rightarrow m^2 N^2$$

$$d) S_J = -k_B \sum_i p_i \ln p_i = -k_B \sum_i e^{-\beta E_i} \ln e^{-\beta E_i} = -k_B \sum_i \left( -\frac{\beta E_i}{2} \right) e^{-\frac{\beta E_i}{2}} \ln(2) =$$

$$= k_B \ln(2) + k_B \sum_i e^{-\frac{\beta E_i}{2}} \frac{\beta}{2} E_i =$$

$$= k_B \ln(2) + k_B \frac{4 \beta J}{2} (e^{-\beta J} - e^{\beta J}) =$$

$$= k_B \ln(2) + k_B T \left( \frac{\partial \ln(2)}{\partial T} \right)$$

$$\frac{\partial \ln(2)}{\partial T} = \frac{1}{2} \frac{4 \beta J}{T} \sinh(\beta J)$$

$$e) S = k_B \ln[(4 \cosh(\beta J))^N] + k_B T \frac{\partial \ln[(4 \cosh(\beta J))^N]}{\partial T} =$$

$$= N k_B \ln(4 \cosh(\beta J)) + \frac{k_B T N}{4 \cosh(\beta J)} (-4 \sinh(\beta J) \frac{\beta J}{T}) =$$

$$= N k_B \ln(4 \cosh(\beta J)) - k_B \beta J \tanh(\beta J)$$

$$T \rightarrow 0 : S \rightarrow N k_B \ln 2$$

