1. Zyklotron

(a)

$$
\begin{aligned}
F &= qvB = q\omega rB = ma = m\frac{v^2}{r} = m\omega^2 r \\
\omega &= \underline{\underline{\frac{qB}{m}}} \\
f &= \underline{\underline{\frac{qB}{2\pi m}}}
\end{aligned}
$$

(b)

$$
\begin{aligned}
v &= \frac{qBr}{m} \\
K &= \frac{mv^2}{2} = \underline{\underline{\frac{(qBr)^2}{2m}}}
\end{aligned}
$$

(c)

$$
\begin{aligned}
r &= \frac{mv}{qB} \\
v_n &= v_{n-1} + \sqrt{\frac{8W_0}{m}} = n\sqrt{\frac{8W_0}{m}} \\
r_n &= r_{n-1} + \frac{\sqrt{8W_0 m}}{qB} = \underline{\underline{n\frac{\sqrt{8m}}{qB}\sqrt{W_0}}}
\end{aligned}
$$

(d) The Cyclotron is used to accelerate charged Particles to up to 50 MeV. At these energies relativistic effects come into play and disrupt the cyclotron motion. To reach higher energies a similar machine with an adjustable B-Field (the Synchrotron) can be used.

2. Geschwindigkeitsfilter

$$
\vec{E} = \begin{pmatrix} 0 \\ 0 \\ E \end{pmatrix}; \quad \vec{B} = \begin{pmatrix} 0 \\ B \\ 0 \end{pmatrix}
$$

(a)

$$\vec{a} = \frac{q}{m}(\vec{E} + \vec{v} \times \vec{B}) = \frac{q}{m} \begin{pmatrix} -v_z B \\ 0 \\ E + v_x B \end{pmatrix}$$

$$\frac{\mathrm{d}v_x}{\mathrm{d}t} = -\tfrac{qB}{m} v_z(t)$$

$$\frac{\mathrm{d}v_z}{\mathrm{d}t} = \tfrac{qE}{m} + \tfrac{qB}{m} v_x(t)$$

(b) $v_x(0) = v_0; \quad v_z(0) = 0$

$$v_x(t) = -\tfrac{E}{B} + \left(\tfrac{E}{B} + v_0\right)\cos\left(\tfrac{qB}{m}t\right)$$

$$v_z(t) = \left(\tfrac{E}{B} + v_0\right)\sin\left(\tfrac{qB}{m}t\right)$$

$$\vec{v}(t) = \underline{\underline{\begin{pmatrix} -\tfrac{E}{B} + \left(\tfrac{E}{B} + v_0\right)\cos\left(\tfrac{qB}{m}t\right) \\ 0 \\ \left(\tfrac{E}{B} + v_0\right)\sin\left(\tfrac{qB}{m}t\right) \end{pmatrix}}}$$

(c) $\vec{r}(0) = \vec{0}$

$$\vec{r}(t) = \int \vec{v}(t)\ \mathrm{d}t = \underline{\underline{\begin{pmatrix} -\tfrac{E}{B}t + \left(\tfrac{E}{B} + v_0\right)\tfrac{m}{qB}\sin\left(\tfrac{qB}{m}t\right) \\ 0 \\ -\left(\tfrac{E}{B} + v_0\right)\tfrac{m}{qB}\cos\left(\tfrac{qB}{m}t\right) \end{pmatrix}}}$$

(d)

$$v_z(t) = \left(\tfrac{E}{B} + v_0\right)\sin\left(\tfrac{qB}{m}t\right) = 0 \quad \Rightarrow \quad \tfrac{E}{B} = -v_0$$

$$v_x(t) = -\tfrac{E}{B} + \left(\tfrac{E}{B} + v_0\right)\cos\left(\tfrac{qB}{m}t\right) = -(-v_0) + (-v_0 + v_0)\cos\left(\tfrac{qB}{m}t\right) = v_0$$

$$\Rightarrow \underline{\underline{\tfrac{E}{B} = -v_0; \quad m, q \text{ choose freely}}}$$

3. Der magnetische Spiegel

(a)

$$\vec{v} = \vec{v}_\perp + \vec{v}_\parallel = \begin{pmatrix} v_x \\ v_y \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ v_z \end{pmatrix}$$

$$F_L = q v_\perp B = m \frac{v_\perp^2}{r} = F_Z$$

$$r_L = \underline{\underline{\frac{m v_\perp}{qB}}}$$

(b) $T = \frac{2\pi m}{qB}; \quad r = \frac{m v_\perp}{qB}$

$$E = E_z + E_\perp = \frac{m {v_\parallel}^2}{2} + \frac{m v_\perp}{2}$$

$$\mu = I\vec{A} = \frac{q}{T} r^2 \pi = \frac{q}{\frac{2\pi m}{qB}} \left(\frac{m v_\perp}{qB}\right)^2 \pi \vec{n}_a = \frac{q^2 B}{2\pi m} \frac{m^2 {v_\perp}^2}{q^2 B^2} \pi = \frac{m {v_\perp}^2}{2} \frac{1}{B} =$$

$$= \underline{\underline{\frac{E_\perp}{B}}}$$

(c) $E = E_\perp + E_\parallel = \text{const.}; \quad \mu = \frac{E_\perp}{B} = \text{const.}$

$$\frac{m {v_\parallel}^2}{2} = E - \frac{m {v_\perp}^2}{2} = E - \mu B$$

$$v_z(B) = \underline{\underline{\sqrt{\frac{2E}{m} - \frac{2\mu B}{m}}}}$$

(d) When $v_z = 0$ the particle has come to a halt and will reverse its z-direction (due to the non-homogeneity of the B-Field). Earths Magnetic Field acts like a magnetic bottle, trapping particles emitted by the sun in the van Allen belts.