193. Gekoppelte physikalische Pendel

$$m = 5 \text{ kg}; \quad k = 2 \text{ N/m}; \quad a = 0.1 \text{ m}; \quad h = 0.4 \text{ m}; \quad l = 0.1 \text{ m}$$

a) $\omega = \sqrt{\frac{mgl}{I}}$

$$I = \tfrac{1}{12}m(a^2 + h^2) + ml^2 = 0.12 \text{ kg m}^2$$
$$\omega_a = \sqrt{\tfrac{mgl}{I}} = \underline{\underline{6.37 \text{ rad/s}}}$$

b) $F = -m\omega^2 x; \quad \Delta x = x_1 - x_2$

$$m\ddot{x}_1 = -m{\omega_a}^2 x_1 - k(x_1 - x_2)$$
$$m\ddot{x}_2 = -m{\omega_a}^2 x_2 - k(x_2 - x_1)$$
$$\Rightarrow \; m(\ddot{x}_1 - \ddot{x}_2) = -m{\omega_a}^2(x_1 - x_2) - 2k(x_1 - x_2) = -(m{\omega_a}^2 + 2k)(x_1 - x_2)$$
$$\Delta\ddot{x} = -\underbrace{({\omega_a}^2 + \tfrac{2k}{m})}_{=\,\omega_b}\Delta x$$
$$\omega_b = \sqrt{\omega_a^2 + \tfrac{2k}{m}} = \underline{\underline{6.43 \text{ rad/s}}}$$

c) $\delta\omega = \omega_b - \omega_a$

$$0 = \cos\left(\tfrac{1}{2}\delta\omega t\right)$$
$$t = \tfrac{2\arccos(0)}{\delta\omega} = \underline{\underline{50.29 \text{ s}}}$$

196. Schallgeschwindigkeit und Elastizität

$$\rho = 4500 \text{ kg/m}^3; \quad v_\parallel = 5050 \text{ m/s}; \quad v_\perp = 3100 \text{ m/s}$$

a) $v_\parallel = \sqrt{\frac{E}{\rho}}; \quad v_\perp = \sqrt{\frac{G}{\rho}}$

$$E = {v_\parallel}^2 \rho = \underline{\underline{1.15 * 10^{11} \text{ Pa}}}$$
$$G = {v_\perp}^2 \rho = \underline{\underline{4.32 * 10^{10} \text{ Pa}}}$$

b) $\mu = \frac{E}{2G} - 1$

$$\mu = \frac{{v_\parallel}^2}{2{v_\perp}^2} - 1 = \underline{\underline{0.33}}$$

c)

$$\mu = \underline{\underline{\frac{{v_\parallel}^2}{2{v_\perp}^2} - 1}} \qquad \text{oder} \qquad \frac{v_\parallel}{v_\perp} = \underline{\underline{\sqrt{2\mu + 2}}}$$

199. Kompensation der Wärmeausdehnung

$$a = h = 0.1 \text{ m}; \quad \Delta T = 1 \text{ K}; \quad E = 2.1 * 10^{11} \text{ N/m}^2$$
$$\mu = 0.29; \quad \rho = 7870 \text{ kg/m}^3; \quad \alpha = 1.2 * 10^{-5} \text{ K}^{-1}$$

a) $\frac{\Delta h}{h} = \alpha \Delta T; \quad \frac{F}{A} = E \frac{\Delta h}{h}$

$$\frac{mg}{h^2} = E \alpha \Delta T$$
$$m = \frac{E \alpha \Delta T h^2}{g} = \underline{\underline{2568.8 \text{ kg}}}$$

b) $\Delta a_1 = \mu \Delta h; \quad \Delta a_2 = a \alpha \Delta T$

$$\Delta a_{\text{ges}} = \Delta a_1 + \Delta a_2 = \underline{\underline{1.5 * 10^{-6} \text{ m}}}$$