1)

$$M = \{1,2,3,4,5,6,7,8,9\}$$
$$G = \{(1,3),(2,1),(3,3),(4,7),(5,1),(6,9),(7,2),(8,1),(9,1)\}$$
$$N = \{1,2,8,9\} \supset M$$

$$Bild\ G = \{3,1,3,7,1,9,2,1,1\} = \{1,2,3,7,9\}$$
$$Bild\ N = \{3,1,1,1\} = \{1,3\}$$
$$Urbild\ N = \{1,2,3,5,8,9\}$$

2)

$A = \{a,b,c\} \quad B = \{1,2\}$

i) $A \to A$

$$G_1 = \{(a,a),(b,a),(c,a)\} \quad G_2 = \{(a,a),(b,a),(c,b)\} \quad G_3 = \{(a,a),(b,a),(c,c)\}$$
$$G_4 = \{(a,a),(b,b),(c,a)\} \quad G_5 = \{(a,a),(b,b),(c,b)\} \quad G_6 = \{(a,a),(b,b),(c,c)\}$$
$$G_7 = \{(a,a),(b,c),(c,a)\} \quad G_8 = \{(a,a),(b,c),(c,b)\} \quad G_9 = \{(a,a),(b,c),(c,c)\}$$
$$G_{10} = \{(a,b),(b,a),(c,a)\} \quad G_{11} = \{(a,b),(b,a),(c,b)\} \quad G_{12} = \{(a,b),(b,a),(c,c)\}$$
$$G_{13} = \{(a,b),(b,b),(c,a)\} \quad G_{14} = \{(a,b),(b,b),(c,b)\} \quad G_{15} = \{(a,b),(b,b),(c,c)\}$$
$$G_{16} = \{(a,b),(b,c),(c,a)\} \quad G_{17} = \{(a,b),(b,c),(c,b)\} \quad G_{18} = \{(a,b),(b,c),(c,c)\}$$
$$G_{19} = \{(a,c),(b,a),(c,a)\} \quad G_{20} = \{(a,c),(b,a),(c,b)\} \quad G_{21} = \{(a,c),(b,a),(c,c)\}$$
$$G_{22} = \{(a,c),(b,b),(c,a)\} \quad G_{23} = \{(a,c),(b,b),(c,b)\} \quad G_{24} = \{(a,c),(b,b),(c,c)\}$$
$$G_{25} = \{(a,c),(b,c),(c,a)\} \quad G_{26} = \{(a,c),(b,c),(c,b)\} \quad G_{27} = \{(a,c),(b,c),(c,c)\}$$

ii) $A \to B$

$$G_1 = \{(a,1),(b,1),(c,1)\} \qquad G_2 = \{(a,1),(b,1),(c,2)\}$$
$$G_3 = \{(a,1),(b,2),(c,1)\} \qquad G_4 = \{(a,1),(b,2),(c,2)\}$$
$$G_5 = \{(a,2),(b,1),(c,1)\} \qquad G_6 = \{(a,2),(b,1),(c,2)\}$$
$$G_7 = \{(a,2),(b,2),(c,1)\} \qquad G_8 = \{(a,2),(b,2),(c,2)\}$$

iii) $B \to A$

$$G_1 = \{(1,a),(2,a)\} \quad G_2 = \{(1,a),(2,b)\} \quad G_3 = \{(1,a),(2,c)\}$$
$$G_4 = \{(1,b),(2,a)\} \quad G_5 = \{(1,b),(2,b)\} \quad G_6 = \{(1,b),(2,c)\}$$
$$G_7 = \{(1,c),(2,a)\} \quad G_8 = \{(1,c),(2,b)\} \quad G_9 = \{(1,c),(2,c)\}$$

iv) $B \to B$

$$G_1 = \{(1,1),(2,1)\} \qquad G_2 = \{(1,1),(2,2)\}$$
$$G_3 = \{(1,2),(2,1)\} \qquad G_4 = \{(1,2),(2,2)\}$$

3)

$A = \{a,b,c\} \quad B = \{1,2\}$

i) $A \to A$, wobei die Abbildung G injektiv ist

$$G_1 = \{(a,a),(b,b),(c,c)\} \quad G_2 = \{(a,b),(b,a),(c,c)\} \quad G_3 = \{(a,c),(b,a),(c,b)\}$$
$$G_4 = \{(a,a),(b,c),(c,b)\} \quad G_5 = \{(a,b),(b,c),(c,a)\} \quad G_6 = \{(a,c),(b,b),(c,a)\}$$

ii) $A \to B$, wobei die Abbildung G injektiv ist

Es existieren keine injektive Abbildungen, da A mehr Elemente als B enthält und somit doppelte Zuordnungen nicht vermieden werden können.

iii) $B \to A$, wobei die Abbildung G injektiv ist

$$G_1 = \{(1,a),(2,b)\} \qquad G_2 = \{(1,a),(2,c)\} \qquad G_3 = \{(1,b),(2,a)\}$$
$$G_4 = \{(1,b),(2,c)\} \qquad G_5 = \{(1,c),(2,a)\} \qquad G_6 = \{(1,c),(2,b)\}$$

iv) $B \to B$, wobei die Abbildung G injektiv ist

$$G_1 = \{(1,1),(2,2)\} \qquad G_2 = \{(1,2),(2,1)\}$$

4)

$f : A \to B; \quad g : B \to A; \quad f \circ g; \quad g \circ f; \quad f \circ g \neq g \circ f$

$f(x) = x + 1; \quad g(x) = \frac{1}{x}$

$(f \circ g)(x) = f(g(x)) = \frac{1}{x} + 1$
$(g \circ f)(x) = g(f(x)) = \frac{1}{x+1}$

Für x = 1
$(f \circ g)(1) = \frac{1}{1} + 1$
$(g \circ f)(1) = \frac{1}{1+1}$

$2 \neq \frac{1}{2} \Rightarrow (f \circ g)(x) \neq (g \circ f)(x)$