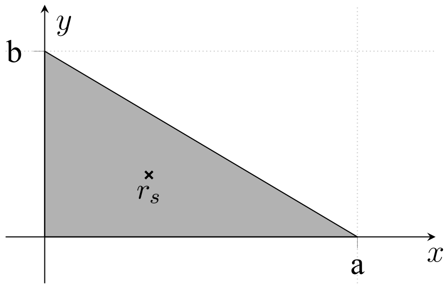
75. Schwerpunkt

a)

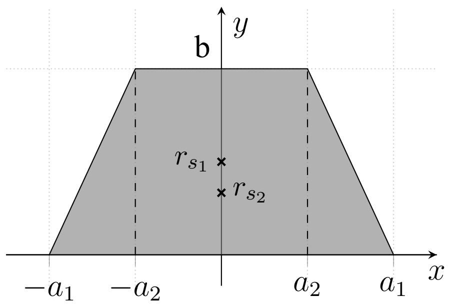


$$\vec{r}_s = \begin{pmatrix} x_s \\ y_s \end{pmatrix}$$
$$dA = l(y) * dy$$
$$l(y) = a(1 - \tfrac{y}{b})$$
$$y_s = \tfrac{1}{A} \int_0^b l(y) * y\, dy = \tfrac{2}{ab} \int_0^b a(1 - \tfrac{y}{b}) y\, dy$$
$$y_s = \tfrac{1}{3}\, b$$

Für x_s wird das Koordinatensystem gedreht $\Rightarrow$ Formeln werden wiederverwendet

$$x_s = \tfrac{1}{3}\, a$$
$$\underline{\underline{\vec{r}_s = \tfrac{1}{3} \begin{pmatrix} a \\ b \end{pmatrix}}}$$

b)



$$x_s = 0 \quad \text{(Symmetrie)}$$
$$l(y) = \tfrac{a_2 - a_1}{b}\, y + a_1$$
$$y_s = \tfrac{1}{A} \int_0^b l(y) * y\, dy = \tfrac{1}{A} \int_0^b \tfrac{a_2 - a_1}{b}\, y + a_1 * y\, dy$$
$$y_s = \tfrac{1}{3}\, b\, \tfrac{a_1 + 2a_2}{a_1 + a_2}$$
$$\underline{\underline{\vec{r}_s = \begin{pmatrix} 0 \\ \tfrac{1}{3}\, b\, \tfrac{a_1 + 2a_2}{a_1 + a_2} \end{pmatrix}}}$$

Für $a_1 = a_2$

$$\vec{r}_{s_1} = \begin{pmatrix} 0 \\ \tfrac{1}{3}\, b\, \tfrac{a_1 + 2a_1}{a_1 + a_1} \end{pmatrix}$$
$$\underline{\underline{\vec{r}_{s_1} = \begin{pmatrix} 0 \\ \tfrac{1}{2}\, b \end{pmatrix}}}$$

Für $a_2 = 0$

$$\vec{r}_{s_2} = \begin{pmatrix} 0 \\ \tfrac{1}{3}\, b\, \tfrac{a_1 + 2*0}{a_1 + 0} \end{pmatrix}$$
$$\underline{\underline{\vec{r}_{s_2} = \begin{pmatrix} 0 \\ \tfrac{1}{3}\, b \end{pmatrix}}}$$

76. Anheben eines Seils

a)

$$\underline{\underline{F(y) = \tfrac{mgy}{l}}}$$

b)

$$W = \int_0^l F(y)\,dy = \tfrac{mg}{l} \int_0^l y\,dy$$
$$W = \underline{\underline{\tfrac{mgl}{2}}}$$

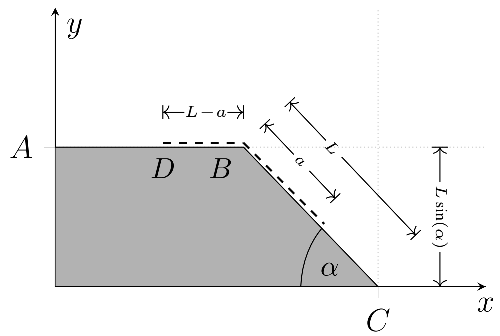
c) $m = \rho r^2 \pi l$

$$x_s = 0 \qquad \text{(Mittelpunkt vom Seil)}$$
$$y_s = \tfrac{1}{m} \int_0^l \rho r^2 \pi * y\,dy = \tfrac{\rho r^2 \pi}{m} \int_0^l y\,dy$$
$$y_s = \tfrac{\rho r^2 \pi}{m} * \tfrac{l^2}{2} = \tfrac{l}{2}$$
$$\vec{r}_s = \underline{\underline{\begin{pmatrix} 0 \\ \frac{l}{2} \end{pmatrix}}}$$

$$E_{pot} = g \int_0^l \rho r^2 \pi * y\,dy = g * \rho r^2 \pi \int_0^l y\,dy$$
$$E_{pot} = \tfrac{\rho r^2 \pi g l^2}{2} = \underline{\underline{\tfrac{mgl}{2}}}$$

d) W stellt die Energieumwandlung/übertragung dar. Im Fall des Seils, welches zu Beginn 0 und am ende E_{pot} Energie besitzt, wird mittels Arbeit $E_{pot} - 0 = E_{pot}$ zugeführt. Demnach gilt $W = E_{pot}$

77. Kette auf schiefer Ebene



$$t_0 = \text{ start ; } \quad t_1 = \text{ wenn } a = L$$

$$E_{pot}(t_0) = g\left(m\,\tfrac{L-a}{L}\right) L\sin(\alpha) + g\left(m\,\tfrac{a}{L}\right)\left(L - \tfrac{a}{2}\right)\sin(\alpha)$$
$$= gm\sin(\alpha)\left(L - \tfrac{a^2}{2L}\right)$$
$$E_{ges}(t_0) = E_{pot}(t_0)$$

$$E_{pot}(t_1) = mg\,\tfrac{L\sin(\alpha)}{2}$$
$$E_{kin}(t_1) = E_{ges}(t_0) - E_{pot}(t_1)$$

$$\tfrac{mv^2}{2} = gm\sin(\alpha)\left(L - \tfrac{a^2}{2L}\right) - mg\,\tfrac{L\sin(\alpha)}{2}$$
$$v = \underline{\underline{\sqrt{g\sin(\alpha)\left(L - \tfrac{a^2}{L}\right)}}}$$