

Radiokarbonmethode

$$t_{1/2} \approx 5730 \text{ yr}; \quad \lambda = \frac{\ln(2)}{t_{1/2}}; \quad M = 15.7 \text{ g}; \quad m_C \approx 12 \text{ u}; \quad n = \frac{M}{m_C}$$

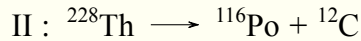
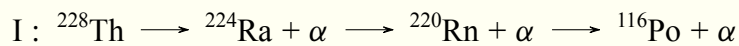
$$N_0 = \frac{n}{8 \cdot 10^{11}}$$

$$\frac{1}{3} \text{ Bq} = \lambda N_0 e^{-\lambda t}$$

$$t = -\frac{\ln\left(\frac{1 \text{ Bq}}{3\lambda N_0}\right)}{\lambda} = \underline{\underline{9.85 \cdot 10^{11} \text{ s}}} \approx 20070 \text{ yr}$$

Theorie des Alpha-Zerfalls und Alternative

a)



b)

$$\Delta M_{\text{I}} = M_{{}^{228}\text{Th}} - 3M_{\alpha} - M_{{}^{116}\text{Po}} = \underline{\underline{17.71 \text{ MeV}/c^2}}$$

$$\Delta M_{\text{II}} = M_{{}^{228}\text{Th}} - M_{{}^{12}\text{C}} - M_{{}^{116}\text{Po}} = \underline{\underline{24.99 \text{ MeV}/c^2}}$$

Der ${}^{12}\text{C}$ -Zerfall sollte häufiger vorkommen, da in Summe mehr Energie freigesetzt wird und das Endprodukt somit einen energetisch niedrigeren Zustand einnimmt.

$$\text{c) } k = \frac{1}{4\pi\epsilon_0}; \quad R_0 = 1.3 \text{ fm}$$

$$V_{\text{I}} = k \frac{e^2 Z_{\alpha} Z_{\text{Ra}}}{R_0 \sqrt[3]{A_{\alpha} + A_{\text{Ra}}}} = \underline{\underline{25.45 \text{ MeV}}}$$

$$V_{\text{II}} = k \frac{e^2 Z_{\text{C}} Z_{\text{Po}}}{R_0 \sqrt[3]{A_{\text{C}} + A_{\text{Po}}}} = \underline{\underline{115.60 \text{ MeV}}}$$

$$d) \quad G = \frac{2}{\hbar} \int \sqrt{2m(V_i(r) - E_i)} \, dr; \quad r_i = k \frac{e^2 Z_i Z_j}{E_i} = \frac{K_i}{E_i}; \quad V_i(r) = k \frac{e^2 Z_i Z_j}{r} = \frac{K_i}{r}$$

$$\begin{aligned} G_i &= \frac{2}{\hbar} \int_R^{r_i} \sqrt{2m(V_i(r) - E_i)} \, dr = \frac{\sqrt{8m}}{\hbar} \int_R^{r_i} \sqrt{\frac{K_i}{r} - E_i} \, dr \\ &= \frac{\sqrt{8mE_i}}{\hbar} \int_R^{r_i} \sqrt{\frac{r_i}{r} - 1} \, dr = \frac{\sqrt{8mE_i}}{\hbar} \left[r \sqrt{\frac{r_i}{r} - 1} - r_i \arctan\left(\sqrt{\frac{r_i}{r} - 1}\right) \right]_R^{r_i} \end{aligned}$$

$$r_\alpha = 4 \cdot 10^{-14} \, \text{m}; \quad E_\alpha = 5.9 \, \text{MeV}; \quad r_C = 4.01 \cdot 10^{-14} \, \text{m}; \quad E_C = 24.99 \, \text{MeV}$$

$$G_\alpha = \underline{\underline{64.52}}$$

$$G_C = \underline{\underline{205.26}}$$

$$e) \quad \lambda = \lambda_0 e^{-G}$$

$$\frac{m_i v_i^2}{2} = E_i + V_0 \quad \Rightarrow \quad v_i = \sqrt{\frac{2(E_i + V_0)}{m_i}}$$

$$\lambda_0(i) = \frac{v_i}{2R} = \sqrt{\frac{E_i + V_0}{2m_i R^2}}$$

$$\lambda_0(\alpha) = \underline{\underline{2.36 \cdot 10^{21} \, 1/s}}$$

$$\lambda_0(^{12}\text{C}) = \underline{\underline{1.86 \cdot 10^{21} \, 1/s}}$$

f)

$$t_{1/2}(\alpha) = \frac{\ln(2)}{\lambda} = \frac{\ln(2)}{\lambda_0} e^G = \underline{\underline{1.47 \cdot 10^6 \, \text{s}}}$$

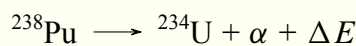
$$t_{1/2}(^{12}\text{C}) = \underline{\underline{5.16 \cdot 10^{67} \, \text{s}}}$$

Der ^{12}C -Zerfall ist energetisch zwar günstiger, wird aber so gut wie nicht beobachtet, da die Halbwertszeit sehr sehr groß ist.

Energieversorgung des Perseverance Rovers

- a) Der MMRTG startet mit einer Leistung von 110 W und hat eine operational lifetime von 14 Jahren.

b)



$$\Delta E = (M(^{238}\text{Pu}) - M(^{234}\text{U}) - M(\alpha))c^2 = \underline{\underline{5.59 \text{ MeV}}}$$

c)

$$P = 0.06A\Delta E = 0.06N_0\lambda\Delta E \stackrel{!}{=} 110 \text{ MeV}$$

$$N_0 = \frac{110 \text{ MeV}}{0.06\lambda\Delta E} = \underline{\underline{8.17 \cdot 10^{24}}}$$

d)

$$N_0 M_{\text{PuO}_2} = N_0 (M_{\text{Pu}} + 2M_{\text{O}}) = \underline{\underline{3.66 \text{ kg}}}$$

Website: 4.8 kg

- e) $t_1 = 14 \text{ d}; \quad t_2 = 14 \text{ yr}$

$$P_1 = N_0 e^{-\lambda t_1} 0.06\lambda\Delta E = \underline{\underline{109.52 \text{ W}}}$$

$$P_2 = N_0 e^{-\lambda t_2} 0.06\lambda\Delta E = \underline{\underline{98.48 \text{ W}}}$$

- f) $P_1 = 0.9 \text{ kWh}; \quad P_2 = 110 \text{ W}; \quad A_1 = 1.3 \text{ m}^2; \quad t = 1 \text{ Marstag} = 88642.66 \text{ s}$

$$\frac{P_1}{t} = 36.55 \text{ W}$$

$$A_2 = \frac{P_2 t}{P_1} A_1 = \underline{\underline{3.91 \text{ m}^2}}$$