## Elastische und tief-inelastische Elektron-Nukleonstreuung

$$\mathcal{P}_1^\mu = (E_1, \boldsymbol{p_1}); \quad \mathcal{P}_2^\mu = (m_\mathrm{p}, \boldsymbol{0}); \quad \mathcal{P}_3^\mu = (E_3, \boldsymbol{p_3}); \quad \mathcal{P}_4^\mu = (E_4, \boldsymbol{p_4})$$

a)
$$\mathcal{P}_1^\mu + \mathcal{P}_2^\mu = \mathcal{P}_3^\mu + \mathcal{P}_4^\mu \iff \mathcal{P}_1^\mu - \mathcal{P}_3^\mu = \mathcal{P}_4^\mu - \mathcal{P}_2^\mu$$
$$(\mathcal{P}_1^\mu - \mathcal{P}_3^\mu)^2 = (\mathcal{P}_4^\mu - \mathcal{P}_2^\mu)^2$$
$$(E_1 - E_3)^2 - (\boldsymbol{p_1} - \boldsymbol{p_3})^2 = (E_4 - m_\mathrm{p})^2 - \boldsymbol{p_4}^2$$
$$E_1^2 - 2E_1E_3 + E_3^2 - E_1^2 + 2\boldsymbol{p_1}\boldsymbol{p_3} - E_3^2 = E_4^2 - 2E_4m_\mathrm{p} + m_\mathrm{p}^2 - \boldsymbol{p_4}^2$$
$$-2E_1E_3 + 2p_1p_3\cos(\theta) = -2E_4m_\mathrm{p} + 2m_\mathrm{p}^2$$
$$E_1E_3(\cos(\theta) - 1) = m_\mathrm{p}^2 - (E_1 + m_\mathrm{p} - E_3)m_\mathrm{p}$$

$$\Rightarrow E_3 = \frac{E_1 m_\mathrm{p}}{E_1 + m_p - E_1\cos(\theta)} = \underline{\underline{374 \text{ MeV}}}$$

b)

$$q^2 = (\mathcal{P}_1^\mu - \mathcal{P}_3^\mu)^2 = 2E_1E_3(\cos(\theta) - 1)$$
$$Q = \sqrt{-q^2} = \sqrt{-2E_1E_3(\cos(\theta) - 1)} = \underline{\underline{542 \text{ MeV}}}$$

c)
$$E_4 = E_1 - E_3 + m_p; \quad p_4 = \sqrt{(E_1 - E_3\cos(\theta))^2 + E_3^2\sin(\theta)^2}$$
$$\beta = \frac{p_4}{E_4} = \underline{\underline{0.52}}$$

d)

$$x = \frac{Q^2}{2m_\mathrm{p}(E_1 - E_3)} = \underline{\underline{1}}$$

e) $E_3 = 134.2\ \text{MeV}$

$$x = \frac{Q^2}{2m_\text{p}(E_1 - E_3)} = \underline{\underline{0.40}}$$

f)

$$W^2 = p_4^2 = (E_1 - E_3\cos(\theta))^2 + E_3^2\sin(\theta)^2 = \underline{\underline{0.318\ \text{GeV}^2}}$$

## Zentraler relativistischer Stoß und Bethe-Bloch-Formel

$\mathcal{P}_i^\mu \dots$ LS, vor dem Stoß, $\mathcal{P}_f^\mu \dots$ LS, nach dem Stoß
$\mathcal{P}_i^{\nu'} \dots$ CMS, vor dem Stoß, $\mathcal{P}_f^{\nu'} \dots$ CMS, nach dem Stoß

a) $\mathcal{P}_i^\mu = (m_\text{e}, \mathbf{0})$; $\tilde\beta = \frac{p}{E} = \frac{p_1}{E_1 + m_\text{e}} = \frac{\gamma\beta M}{\gamma M + m_\text{e}}$

$$\mathcal{P}_i^{\nu'} = \Lambda^{\nu'}_{\mu}\mathcal{P}_i^\mu = (\tilde\gamma m_\text{e}, -\tilde\beta\tilde\gamma m_\text{e})$$

$$\mathcal{P}_f^{\nu'} = (\tilde\gamma m_\text{e}, \tilde\beta\tilde\gamma m_\text{e})$$

$$\mathcal{P}_f^{\mu} = \Lambda^{\mu}_{\nu'}\mathcal{P}_f^{\nu'} = (\tilde\gamma^2 m_\text{e} + \tilde\beta^2\tilde\gamma^2 m_\text{e}, \tilde\beta\tilde\gamma^2 m_\text{e} + \tilde\beta\tilde\gamma^2 m_\text{e})$$

$$\begin{aligned}
T &= \tilde\gamma^2 m_\text{e} + \tilde\beta^2\tilde\gamma^2 m_\text{e} - m_\text{e} = m_\text{e}\tfrac{1+\tilde\beta^2}{1-\tilde\beta^2} - m_\text{e} = 2m_\text{e}\tfrac{\tilde\beta^2}{1-\tilde\beta^2} \\
&= 2m_\text{e}\frac{\frac{\gamma^2\beta^2M^2}{(\gamma M + m_\text{e})^2}}{\frac{(\gamma M + m_\text{e})^2 - \gamma^2\beta^2M^2}{(\gamma M + m_\text{e})^2}} = 2m_\text{e}\frac{\gamma^2\beta^2M^2}{(\gamma M + m_\text{e})^2 - \gamma^2\beta^2M^2} \\
&= \frac{2m_\text{e}\gamma^2\beta^2M^2}{2\gamma M m_\text{e} + m_\text{e}^2 + \gamma^2M^2(1-\beta^2)} = \underline{\underline{\frac{2m_\text{e}\gamma^2\beta^2M^2}{2\gamma M m_\text{e} + m_\text{e}^2 + M^2}}}
\end{aligned}$$

b) $\mathcal{P}^{\mu}_{\text{ges}} = (E_{\text{ges}}, \boldsymbol{p}_{\text{ges}})$

$$\mathcal{P}^{\nu'}_{\text{ges}} = \Lambda^{\mu}_{\nu'} \mathcal{P}^{\nu'}_{\text{ges}} = (E', \gamma E_{\text{ges}} - \gamma \boldsymbol{\beta} \boldsymbol{p}_{\text{ges}}) \overset{!}{=} (E', \boldsymbol{0})$$

$$\boldsymbol{0} = \gamma E_{\text{ges}} - \gamma \boldsymbol{\beta} \boldsymbol{p}_{\text{ges}} \quad \Longleftrightarrow \quad \boldsymbol{\beta} = \frac{\boldsymbol{p}_{\text{ges}}}{E_{\text{ges}}}$$

## Das Mesonen-Nonett

a) $\psi_1 = N_1 \left( |u\bar{u}\rangle + |d\bar{d}\rangle + |s\bar{s}\rangle \right)$

$$\langle \psi_1 | \psi_1 \rangle = N_1^2 \left( \langle u\bar{u} | u\bar{u} \rangle + \langle d\bar{d} | d\bar{d} \rangle + \langle s\bar{s} | s\bar{s} \rangle \right) = 3N_1^2 \overset{!}{=} 1$$

$$N_1 = \tfrac{1}{\sqrt{3}}$$

b) $\psi_2 = N_2 \left( |u\bar{u}\rangle \pm |d\bar{d}\rangle \right)$; $\quad N_2 = \tfrac{1}{\sqrt{2}}$

$$\langle \psi_1 | \psi_2 \rangle = \tfrac{1}{\sqrt{6}} \left( \langle u\bar{u} | u\bar{u} \rangle \pm \langle d\bar{d} | d\bar{d} \rangle \right) = \tfrac{1}{\sqrt{6}} (1 \pm 1) \overset{!}{=} 0$$

$$\psi_2 = \tfrac{1}{\sqrt{2}} \left( |u\bar{u}\rangle - |d\bar{d}\rangle \right)$$

c) $\psi_3 = N_3 \left( |u\bar{u}\rangle + |d\bar{d}\rangle - 2\,|s\bar{s}\rangle \right)$; $\quad N_3 = \tfrac{1}{\sqrt{6}}$

$$\langle \psi_3 | \psi_3 \rangle = 1$$

$$\langle \psi_3 | \psi_2 \rangle = 0$$

$$\langle \psi_3 | \psi_1 \rangle = 0$$

d)

$$\eta = |\psi_1\rangle = \tfrac{1}{\sqrt{3}} \left( |u\bar{u}\rangle + |d\bar{d}\rangle + |s\bar{s}\rangle \right)$$

$$\pi^0 = |\psi_2\rangle = \tfrac{1}{\sqrt{2}} (|u\bar{u}\rangle \pm |d\bar{d}\rangle)$$

$$\eta' = |\psi_3\rangle = \tfrac{1}{\sqrt{6}} \left( |u\bar{u}\rangle + |d\bar{d}\rangle - 2\,|s\bar{s}\rangle \right)$$