Einheiten der Kern- & Teilchenphysik & Raumwinkel

Blatt 1

(a)
$$\Delta \phi = 1 \text{ V}$$
; $q_e = e$

$$\Delta E = q_e \Delta \phi = 1.602 \times 10^{-19} \text{ CJ/C} = \underline{1.602 \times 10^{-19} \text{ J} = 1 \text{ eV}}$$

(b)

(i)
$$m_{\rm e}=0.510999~{
m MeV/c^2}=0.510999\frac{e}{c^2}~{
m J/C}=\underline{9.11\times 10^{-31}~{
m kg}}$$

(ii)
$$m_{\rm p}=938.272~{
m MeV/c^2}=938.272\frac{e}{c^2}~{
m J/C}=\underline{1.67\times 10^{-27}~{
m kg}}$$

(iii)
$$m_{\rm d}=1875.613~{\rm MeV/c^2}=1875.613\frac{e}{c^2}~{\rm J/C}=\underline{3.34\times10^{-27}~{\rm kg}}$$

(c) Der Raumwinkel beschreibt das Verhältnis zwischen der Teilfläche einer Kugel zum Kugelradius r zum Quadrat.

$$d\Omega = \frac{dA}{r^2} = \sin(\theta) d\theta d\varphi$$

(d)

$$A = 2\pi r^2$$

$$\Omega = \frac{A}{r^2} = \underline{2\pi \text{ sr}}$$

$$\Omega = 2\pi \left(\frac{180}{\pi}\right)^2 = \frac{64800}{\pi} \deg^2 = 20626.5 \deg^2$$

(e)
$$\theta = 1^{\circ}$$

$$A = 2\pi r^2 \left(1 - \cos(\theta) + \frac{1}{2} \sin^2(\theta) \right)$$

$$\Omega = \frac{A}{r^2} = 2\pi \left(1 - \cos(\theta) + \frac{1}{2}\sin^2(\theta)\right) = \underline{5.11284 \text{ sr}}$$

Relativistische Formel der Energie

(a)
$$p = \gamma m_0 v$$
; $\gamma = \frac{1}{\sqrt{1-\beta^2}}$; $\beta = \frac{v}{c}$; $F = \frac{\mathrm{d}p}{\mathrm{d}t}$

$$E_{kin} = \int_{0}^{r} F \, \mathrm{d}\tilde{r} = \int_{0}^{r} \frac{\mathrm{d}p}{\mathrm{d}t} \, \mathrm{d}\tilde{r} = \int_{0}^{r} \frac{\mathrm{d}}{\mathrm{d}t} \left(\frac{m_{0}v}{\sqrt{1 - \beta^{2}}} \right) \mathrm{d}\tilde{r}$$

(b)
$$dp = m_0 \frac{d(\gamma v)}{dt} dt = m_0 \gamma (1 + \gamma^2 \beta^2) dv$$

$$E_{\rm kin} = \int_{0}^{r} \frac{\mathrm{d}p}{\mathrm{d}t} \, \mathrm{d}\tilde{r} = \int_{0}^{p} \frac{\mathrm{d}r}{\mathrm{d}t} \, \mathrm{d}\tilde{p} = m_0 \int_{0}^{v} \gamma \tilde{v} \left(1 + \gamma^2 \beta^2\right) \mathrm{d}\tilde{v} = \underline{\underline{m_0 c^2 (\gamma - 1)}}$$

(c)
$$\gamma \approx 1 + \frac{1}{2}\beta^2$$

$$E_{\text{kin}} = m_0 c^2 (\gamma - 1) \approx m_0 c^2 \left(1 + \frac{1}{2} \beta^2 - 1 \right) = \frac{m_0 v^2}{2}$$

Streuung an harter Kugel

(a)
$$2\alpha + \theta = \pi$$
; $R = R_1 + R_2$

$$\frac{b}{R} = \sin(\alpha) = \sin(\frac{\pi}{2} - \frac{\theta}{2}) = \cos(\frac{\theta}{2})$$

$$b = R\cos\left(\frac{\theta}{2}\right)$$

$$db = -\frac{R}{2}\sin(\frac{\theta}{2})d\theta$$

(b)

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \left| \frac{b \, \mathrm{d}b}{\sin(\theta) \, \mathrm{d}\theta} \right| = \left| \frac{\left(R \cos\left(\frac{\theta}{2}\right)\right) \left(-\frac{R}{2} \sin\left(\frac{\theta}{2}\right) \, \mathrm{d}\theta \right)}{\sin(\theta) \, \mathrm{d}\theta} \right| = \left| \frac{R^2 \cos\left(\frac{\theta}{2}\right) \sin\left(\frac{\theta}{2}\right)}{2 \sin(\theta)} \right| = \underline{\frac{R^2}{4}}$$

Da der differentielle Wirkungsquerschnitt nicht vom Winkel θ abhängt, treten alle Streuwinkel gleich häufig auf.

(c)

$$\sigma_{\text{tot}} = \int_{0}^{\sigma} d\tilde{\sigma} = \int_{0}^{\Omega} \frac{R^2}{4} d\tilde{\Omega} = \frac{R^2}{4} \int_{0}^{\theta} \sin(\tilde{\theta}) d\tilde{\theta} \int_{0}^{2\pi} d\phi = \frac{R^2\pi}{2} (\cos(\theta) - 1)$$

Der totale Wirkungsquerschnitt hängt vom Winkel θ ab und nimmt Werte zwischen $-R^2\pi$ und $R^2\pi$ an.

(d)
$$\frac{\pi}{2} < \theta < \frac{3\pi}{2}$$

$$\sigma_{\text{rück}} = \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} -\frac{R^2\pi}{2} \left(\cos(\theta) - 1\right) d\theta = -\frac{R^2\pi}{2} \left[\sin(\theta) - \theta\right]_{\frac{\pi}{2}}^{\frac{3\pi}{2}} = \underline{R^2\pi(1 + \frac{\pi}{2})}$$