1. Magnetfeld eines Koaxialkabels

For $r \leq R_1$:

$$I_{in} = I\,\frac{r^2}{R_1^2}$$
$$I_{in}\mu_0 = \oint \vec{B}(\vec{r})\,\mathrm{d}\vec{s} = B(r)\oint \mathrm{d}s = B(r)2r\pi$$
$$B(r) = \frac{I\mu_0}{2\pi R_1^2}r$$

For $R_1 \leq r \leq R_2$:

$$I_{in} = I$$
$$I_{in}\mu_0 = \oint \vec{B}(\vec{r})\,\mathrm{d}\vec{s} = B(r)\oint \mathrm{d}s = B(r)2r\pi$$
$$B(r) = \frac{I\mu_0}{2\pi}\frac{1}{r}$$

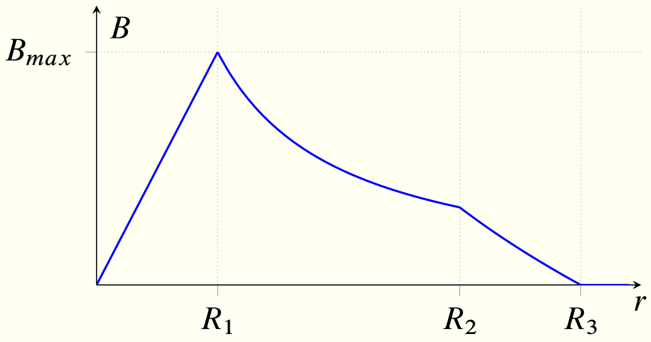
For $R_2 \leq r \leq R_3$:

$$I_{in} = I\left(1 - \frac{r^2-R_2^2}{R_3^2-R_2^2}\right)$$
$$I_{in}\mu_0 = \oint \vec{B}(\vec{r})\,\mathrm{d}\vec{s} = B(r)\oint \mathrm{d}s = B(r)2r\pi$$
$$B(r) = \frac{I\mu_0}{2\pi}\left(\frac{1}{r} - \frac{r^2-R_2^2}{r(R_3^2-R_2^2)}\right)$$

For $R_3 \leq r$:

$$I_{in} = 0$$
$$B(r) = 0$$

$$\boxed{B(r) = \begin{cases} \frac{I\mu_0}{2\pi R_1^2}r & \text{for } 0 < r \leq R_1, \\ \frac{I\mu_0}{2\pi}\frac{1}{r} & \text{for } R_1 \leq r \leq R_2, \\ \frac{I\mu_0}{2\pi}\left(\frac{1}{r} - \frac{r^2-R_2^2}{r(R_3^2-R_2^2)}\right) & \text{for } R_2 \leq r \leq R_3, \\ 0 & \text{for } R_3 \leq r. \end{cases}}$$



2. Anwendung des Gesetzes von Biot-Savart – „Haarnadel“

$$R = \sqrt{x^2 + y^2}; \quad r = \sqrt{x^2 + y^2 + z^2} = \sqrt{R^2 + z^2}$$

$$\mathrm{d}B_1 = \frac{\mu_0 I}{4\pi} \frac{|\mathrm{d}\vec{l} \times \vec{r}|}{r^3} = \frac{\mu_0 I}{4\pi} \frac{\mathrm{d}l}{r^2} = \frac{\mu_0 I}{4\pi} \frac{\mathrm{d}\varphi R}{r^2}$$

$$B_1 = \frac{\mu_0 I}{4\pi} \frac{R}{r^2} \int_0^\pi 1 \,\mathrm{d}\varphi = \frac{\mu_0 I}{4} \frac{R}{r^2}$$

$$B_{x,1} = B_1 \sin(\theta) = \frac{\mu_0 I}{4} \frac{R}{r^2} \sin(\theta) = \frac{\mu_0 I}{4} \frac{R^2}{r^3}$$

$$B_{z,1} = B_1 \cos(\theta) = \frac{\mu_0 I}{4} \frac{R}{r^2} \cos(\theta) = \frac{\mu_0 I}{4} \frac{Rz}{r^3}$$

$$\mathrm{d}B_2 = \frac{\mu_0 I}{4\pi} \frac{|\mathrm{d}\vec{l} \times \vec{r}|}{r^3} = \frac{\mu_0 I}{4\pi} \frac{\mathrm{d}l \sin(\theta)}{r^2} = \frac{\mu_0 I}{4\pi} \frac{\mathrm{d}l \sqrt{y^2+z^2}}{r^3}$$

$$B_2 = \frac{\mu_0 I}{2\pi} \int_{-\infty}^{0} \frac{\sqrt{y^2+z^2}}{r^3} \,\mathrm{d}x = \frac{\mu_0 I}{2\pi} \frac{1}{\sqrt{y^2+z^2}}$$

$$B_{z,2} = B_2 \cos(\theta) = \frac{\mu_0 I}{2\pi} \frac{1}{\sqrt{y^2+z^2}} \cos(\theta) = \frac{\mu_0 I}{2\pi} \frac{z}{\sqrt{y^2+z^2}\sqrt{x^2+y^2+z^2}}$$

$$\underline{\underline{\vec{B}(z) = \frac{\mu_0 I}{4} \begin{pmatrix} R^2 / \left(R^2 + z^2\right)^{3/2} \\ 0 \\ \frac{Rz}{\left(R^2+z^2\right)^{3/2}} + \frac{2z}{\pi \sqrt{y^2+z^2}\sqrt{R^2+z^2}} \end{pmatrix}}}$$

3. Drehmoment auf rechteckige Leiterschleife

a) $\varphi = \arctan(\frac{d}{2x})$; $\quad r = \sqrt{\frac{4x^2+d^2}{4}}$

$$\vec{B}(r) = \frac{\mu_0 I_1}{2r\pi}\begin{pmatrix}-\sin(\varphi)\\ \cos(\varphi)\\ 0\end{pmatrix}$$

$$\vec{F}_1 = I_2\vec{h}\times\vec{B} = \frac{\mu_0 I_1 I_2 h}{2\pi}\begin{pmatrix}-\cos(\varphi)\\ -\sin(\varphi)\\ 0\end{pmatrix}$$

$$\vec{F}_2 = I_2\vec{h}\times\vec{B} = \frac{\mu_0 I_1 I_2 h}{2r\pi}\begin{pmatrix}\cos(\varphi)\\ \sin(\varphi)\\ 0\end{pmatrix}$$

$$\vec{M}_1 = \vec{r}_1\times\vec{F}_1 = \begin{pmatrix}0\\ d/2\\ 0\end{pmatrix}\times\frac{\mu_0 I_1 I_2 h}{2r\pi}\begin{pmatrix}-\cos(\varphi)\\ -\sin(\varphi)\\ 0\end{pmatrix} = \frac{\mu_0 I_1 I_2 hd}{4r\pi}\begin{pmatrix}0\\ 0\\ \cos(\varphi)\end{pmatrix}$$

$$\vec{M}_2 = \vec{r}_2\times\vec{F}_2 = \begin{pmatrix}0\\ -d/2\\ 0\end{pmatrix}\times\frac{\mu_0 I_1 I_2 h}{2r\pi}\begin{pmatrix}\cos(\varphi)\\ \sin(\varphi)\\ 0\end{pmatrix} = \frac{\mu_0 I_1 I_2 hd}{4r\pi}\begin{pmatrix}0\\ 0\\ \cos(\varphi)\end{pmatrix}$$

$$\vec{M} = \vec{M}_1 + \vec{M}_2 = \underline{\underline{\frac{\mu_0 I_1 I_2 hd}{2r\pi}\begin{pmatrix}0\\ 0\\ \cos(\varphi)\end{pmatrix}}}$$

b) $\mu_0 = 4\pi * 10^{-7}$ H/m; $\quad I_1 = 16$ A; $\quad I_2 = 1$ A; $\quad d = h = x = 0.1$ m

$$\vec{M} = \frac{\mu_0 I_1 I_2 hd}{2r\pi}\cos(\varphi)\hat{z} = \underline{\underline{2.56 * 10^{-7}\ \mathrm{Nm}}}$$

Bolts on Bike : $5 - 30$ Nm
Basic Electromotor : $0.3 - 0.5$ Nm
Car wheels : $110 - 120$ Nm