Einheiten der Kern- & Teilchenphysik & Raumwinkel

(a) $\Delta\phi = 1\ \text{V}; \quad q_\text{e} = e$

$$\Delta E = q_\text{e}\Delta\phi = 1.602 \times 10^{-19}\ \text{CJ/C} = \underline{\underline{1.602 \times 10^{-19}\ \text{J} = 1\ \text{eV}}}$$

(b)

(i) $m_\text{e} = 0.510999\ \text{MeV/c}^2 = 0.510999\frac{e}{c^2}\ \text{J/C} = \underline{\underline{9.11 \times 10^{-31}\ \text{kg}}}$

(ii) $m_\text{p} = 938.272\ \text{MeV/c}^2 = 938.272\frac{e}{c^2}\ \text{J/C} = \underline{\underline{1.67 \times 10^{-27}\ \text{kg}}}$

(iii) $m_\text{d} = 1875.613\ \text{MeV/c}^2 = 1875.613\frac{e}{c^2}\ \text{J/C} = \underline{\underline{3.34 \times 10^{-27}\ \text{kg}}}$

(c) Der Raumwinkel beschreibt das Verhältnis zwischen der Teilfläche einer Kugel zum Kugelradius r zum Quadrat.

$$\mathrm{d}\Omega = \frac{\mathrm{d}A}{r^2} = \underline{\underline{\sin(\theta)\,\mathrm{d}\theta\,\mathrm{d}\varphi}}$$

(d)

$$A = 2\pi r^2$$
$$\Omega = \frac{A}{r^2} = \underline{\underline{2\pi\ \text{sr}}}$$
$$\Omega = 2\pi\left(\frac{180}{\pi}\right)^2 = \underline{\underline{\frac{64800}{\pi}\ \text{deg}^2 = 20626.5\ \text{deg}^2}}$$

(e) $\theta = 1°$

$$A = 2\pi r^2\left(1 - \cos(\theta) + \frac{1}{2}\sin^2(\theta)\right)$$
$$\Omega = \frac{A}{r^2} = 2\pi\left(1 - \cos(\theta) + \frac{1}{2}\sin^2(\theta)\right) = \underline{\underline{5.11284\ \text{sr}}}$$

Relativistische Formel der Energie

(a) $p = \gamma m_0 v; \quad \gamma = \frac{1}{\sqrt{1-\beta^2}}; \quad \beta = \frac{v}{c}; \quad F = \frac{\mathrm{d}p}{\mathrm{d}t}$

$$E_{\text{kin}} = \int_0^r F\,\mathrm{d}r = \int_0^r \frac{\mathrm{d}p}{\mathrm{d}t}\,\mathrm{d}r = \int_0^r \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{m_0 v}{\sqrt{1-\beta^2}}\right)\mathrm{d}r$$

(b) $\mathrm{d}p = m_0 \frac{\mathrm{d}(\gamma v)}{\mathrm{d}t}\,\mathrm{d}t = m_0 \gamma (1 + \gamma^2\beta^2)\,\mathrm{d}v$

$$E_{\text{kin}} = \int_0^r \frac{\mathrm{d}p}{\mathrm{d}t}\,\mathrm{d}\tilde{r} = \int_0^p \frac{\mathrm{d}r}{\mathrm{d}t}\,\mathrm{d}\tilde{p} = m_0 \int_0^v \gamma\tilde{v}\left(1 + \gamma^2\beta^2\right)\mathrm{d}\tilde{v} = \underline{\underline{m_0 c^2(\gamma - 1)}}$$

(c) $\gamma \approx 1 + \frac{1}{2}\beta^2$

$$E_{\text{kin}} = m_0 c^2(\gamma - 1) \approx m_0 c^2\left(1 + \frac{1}{2}\beta^2 - 1\right) = \underline{\underline{\frac{m_0 v^2}{2}}}$$

Streuung an harter Kugel

(a) $2\alpha + \theta = \pi; \quad R = R_1 + R_2$

$$\frac{b}{R} = \sin(\alpha) = \sin\left(\frac{\pi}{2} - \frac{\theta}{2}\right) = \cos\left(\frac{\theta}{2}\right)$$

$$b = R\cos\left(\frac{\theta}{2}\right)$$

$$\mathrm{d}b = -\frac{R}{2}\sin\left(\frac{\theta}{2}\right)\mathrm{d}\theta$$

(b)

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \left|\frac{b\,\mathrm{d}b}{\sin(\theta)\,\mathrm{d}\theta}\right| = \left|\frac{\left(R\cos\left(\frac{\theta}{2}\right)\right)\left(-\frac{R}{2}\sin\left(\frac{\theta}{2}\right)\mathrm{d}\theta\right)}{\sin(\theta)\,\mathrm{d}\theta}\right| = \left|\frac{R^2\cos\left(\frac{\theta}{2}\right)\sin\left(\frac{\theta}{2}\right)}{2\sin(\theta)}\right| = \underline{\underline{\frac{R^2}{4}}}$$

Da der differentielle Wirkungsquerschnitt nicht vom Winkel θ abhängt, treten alle Streuwinkel gleich häufig auf.

(c)

$$\sigma_{\text{tot}} = \int_0^{\sigma} \mathrm{d}\tilde{\sigma} = \int_0^{\Omega} \frac{R^2}{4} \,\mathrm{d}\tilde{\Omega} = \frac{R^2}{4} \int_0^{\theta} \sin\left(\tilde{\theta}\right) \mathrm{d}\tilde{\theta} \int_0^{2\pi} \mathrm{d}\phi = \underline{\underline{-\frac{R^2\pi}{2} \left(\cos(\theta) - 1\right)}}$$

Der totale Wirkungsquerschnitt hängt vom Winkel θ ab und nimmt Werte zwischen $-R^2\pi$ und $R^2\pi$ an.

(d) $\frac{\pi}{2} < \theta < \frac{3\pi}{2}$

$$\sigma_{\text{rück}} = \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} -\frac{R^2\pi}{2} \left(\cos(\theta) - 1\right) \mathrm{d}\theta = -\frac{R^2\pi}{2} \left[\sin(\theta) - \theta \right]_{\frac{\pi}{2}}^{\frac{3\pi}{2}} = \underline{\underline{R^2\pi(1 + \tfrac{\pi}{2})}}$$