

## 1. Zyklotron

(a)

$$F = qvB = q\omega rB = ma = m \frac{v^2}{r} = m\omega^2 r$$

$$\omega = \frac{qB}{m}$$

$$f = \frac{qB}{2\pi m}$$

(b)

$$v = \frac{qBr}{m}$$

$$K = \frac{mv^2}{2} = \frac{(qBr)^2}{2m}$$

(c)

$$r = \frac{mv}{qB}$$

$$v_n = v_{n-1} + \sqrt{\frac{8W_0}{m}} = n \sqrt{\frac{8W_0}{m}}$$

$$r_n = r_{n-1} + \frac{\sqrt{8W_0 m}}{qB} = n \frac{\sqrt{8m}}{qB} \sqrt{W_0}$$

(d) The Cyclotron is used to accelerate charged Particles to up to 50 MeV. At these energies relativistic effects come into play and disrupt the cyclotron motion. To reach higher energies a similar machine with an adjustable B-Field (the Synchrotron) can be used.

## 2. Geschwindigkeitsfilter

$$\vec{E} = \begin{pmatrix} 0 \\ 0 \\ E \end{pmatrix}; \quad \vec{B} = \begin{pmatrix} 0 \\ B \\ 0 \end{pmatrix}$$

(a)

$$\vec{a} = \frac{q}{m}(\vec{E} + \vec{v} \times \vec{B}) = \frac{q}{m} \begin{pmatrix} -v_z B \\ 0 \\ E + v_x B \end{pmatrix}$$

$$\frac{dv_x}{dt} = -\frac{qB}{m} v_z(t)$$

$$\frac{dv_z}{dt} = \frac{qE}{m} + \frac{qB}{m} v_x(t)$$

(b)  $v_x(0) = v_0; \quad v_z(0) = 0$ 

$$v_x(t) = -\frac{E}{B} + \left(\frac{E}{B} + v_0\right) \cos\left(\frac{qB}{m} t\right)$$

$$v_z(t) = \left(\frac{E}{B} + v_0\right) \sin\left(\frac{qB}{m} t\right)$$

$$\vec{v}(t) = \underline{\underline{\begin{pmatrix} -\frac{E}{B} + \left(\frac{E}{B} + v_0\right) \cos\left(\frac{qB}{m} t\right) \\ 0 \\ \left(\frac{E}{B} + v_0\right) \sin\left(\frac{qB}{m} t\right) \end{pmatrix}}}$$

(c)  $\vec{r}(0) = \vec{0}$ 

$$\vec{r}(t) = \int \vec{v}(t) dt = \underline{\underline{\begin{pmatrix} -\frac{E}{B} t + \left(\frac{E}{B} + v_0\right) \frac{m}{qB} \sin\left(\frac{qB}{m} t\right) \\ 0 \\ -\left(\frac{E}{B} + v_0\right) \frac{m}{qB} \cos\left(\frac{qB}{m} t\right) \end{pmatrix}}}$$

(d)

$$v_z(t) = \left(\frac{E}{B} + v_0\right) \sin\left(\frac{qB}{m} t\right) = 0 \quad \Rightarrow \quad \frac{E}{B} = -v_0$$

$$v_x(t) = -\frac{E}{B} + \left(\frac{E}{B} + v_0\right) \cos\left(\frac{qB}{m} t\right) = -(-v_0) + (-v_0 + v_0) \cos\left(\frac{qB}{m} t\right) = v_0$$

$$\Rightarrow \underline{\underline{\frac{E}{B} = -v_0; \quad m, q \text{ choose freely}}}$$

### 3. Der magnetische Spiegel

(a)

$$\vec{v} = \vec{v}_{\perp} + \vec{v}_{\parallel} = \begin{pmatrix} v_x \\ v_y \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ v_z \end{pmatrix}$$

$$F_L = qv_{\perp}B = m \frac{v_{\perp}^2}{r} = F_Z$$

$$r_L = \underline{\underline{\frac{mv_{\perp}}{qB}}}$$

$$(b) \quad T = \frac{2\pi m}{qB}; \quad r = \frac{mv_{\perp}}{qB}$$

$$E = E_z + E_{\perp} = \frac{mv_{\parallel}^2}{2} + \frac{mv_{\perp}^2}{2}$$

$$\begin{aligned} \mu = I \vec{A} &= \frac{q}{T} r^2 \pi = \frac{q}{2\pi m} \left( \frac{mv_{\perp}}{qB} \right)^2 \pi n_a = \frac{q^2 B}{2\pi m} \frac{m^2 v_{\perp}^2}{q^2 B^2} \pi = \frac{mv_{\perp}^2}{2} \frac{1}{B} = \\ &= \underline{\underline{\frac{E_{\perp}}{B}}} \end{aligned}$$

$$(c) \quad E = E_{\perp} + E_{\parallel} = \text{const.}; \quad \mu = \frac{E_{\perp}}{B} = \text{const.}$$

$$\frac{mv_{\parallel}^2}{2} = E - \frac{mv_{\perp}^2}{2} = E - \mu B$$

$$v_z(B) = \underline{\underline{\sqrt{\frac{2E}{m} - \frac{2\mu B}{m}}}}$$

(d) When  $v_z = 0$  the particle has come to a halt and will reverse its z-direction (due to the non-homogeneity of the B-Field). Earth's Magnetic Field acts like a magnetic bottle, trapping particles emitted by the sun in the van Allen belts.