1) $p: \mathbb{R} \to \mathbb{R}$

$x \mapsto 8x^2+4x+2$

$$\int_a^b p(x)\,dx = \left[\frac{8x^3}{3}+2x^2+2x\right]_a^b = \left(\frac{8b^3}{3}+2b^2+2b\right)-\left(\frac{8a^3}{3}+2a^2+2a\right) =$$

$$= \frac{8}{3}(b^3-a^3)+2(b^2-a^2)+2(b-a)$$

$$\int_a^b p(x)\,dx = \lim_{n\to\infty}(R_n) \qquad R_n = \frac{b-a}{n}\sum_{k=0}^{n-1} p\left(a+(2k+1)\frac{b-a}{2n}\right)$$

$$\Delta := a+(2k+1)\frac{b-a}{2n}$$

$$\Delta^2 = a^2+2ka\frac{b-a}{n}+a\frac{b-a}{n}+\frac{(b-a)^2}{4n^2}+\frac{(b-a)^2}{n^2}\cdot k+\frac{(b-a)^2}{n^2}\cdot k^2$$

$$R_n = \frac{b-a}{n}\cdot\underbrace{\sum_{k=0}^{n-1} p(\Delta)}_{=}$$

$$= \boxed{\sum_{k=0}^{n-1} 8\Delta^2} + \boxed{\sum_{k=0}^{n-1} 4\Delta} + \boxed{\sum_{k=0}^{n-1} 2} = 2n$$

$\frac{n(n-1)}{2}$ =

$$\sum_{k=0}^{n-1} k = \frac{n(n+1)}{2} - n = \frac{n(n-1)}{2}$$

$$\sum_{k=0}^{n-1} k^2 = \frac{n(n-1)(2n-1)}{6}$$

$$\sum_{k=0}^{n-1} 4\left(a+(2k+1)\frac{b-a}{2n}\right) = \sum_{k=0}^{n-1} 4a + \sum_{k=0}^{n-1}\frac{2(b-a)}{n} + 4\frac{(b-a)}{n}\sum_{k=0}^{n-1} k$$

$$= 4an + 2(b-a) + 4\frac{(b-a)}{\cancel{n}}\cdot\frac{\cancel{n}(n-1)}{\cancel{2}} =$$

$$= 4an + \cancel{2b} - \cancel{2a} + 2bn - \cancel{2an} - \cancel{2b} + \cancel{2a} =$$

$$= 2n(b+a) \quad \left(=\sum_{k=0}^{n-1} 4\Delta\right)$$

$$\sum_{k=0}^{n-1} 2 = 2n$$

$$\sum_{k=0}^{n-1} 8(\Delta^2) = 8\left(\sum_{k=0}^{n-1} a^2 + \sum_{k=0}^{n-1} a\frac{b-a}{n} + 2a\frac{b-a}{n}\sum_{k=0}^{n-1} k + \sum_{k=0}^{n-1}\frac{(b-a)^2}{4n^2} + \frac{(b-a)^2}{n^2}\cdot\sum_{k=0}^{n-1} k + \frac{(b-a)^2}{n^2}\sum_{k=0}^{n-1} k^2\right) =$$

$$= \cancel{8}\left(\cancel{a^2 n} + \cancel{a(b-a)} + \cancel{2a\frac{b-a}{n}}\frac{n(n-1)}{2} + \frac{(b-a)^2}{4n^2} + \frac{(b-a)^2}{n^2}\frac{n(n-1)}{2} + \frac{(b-a)^2}{n^2}\frac{n(n-1)(2n-1)}{6}\right) =$$

$$= \cancel{8a(b-a)} \frac{2(b-a)^2}{n^2} + \frac{4(b-a)^2(n-1)}{n} + 4(b-a)^2\frac{(n-1)(2n-1)}{3n} \quad \left(=\sum_{k=0}^{n-1} 8\Delta^2\right)$$

$$\Rightarrow R_n = \frac{b-a}{n}\left(\frac{2(b-a)^2}{n} + \frac{4(b-a)^2(n-1)}{n} + 4(b-a)^2\frac{(n-1)(2n-1)}{3n} + 2n(b+a) + 2n\right)$$

$$\lim_{n\to\infty}(R_n) = \lim_{n\to\infty}\Big[\underbrace{\frac{2(b-a)^3}{n^2}}_{0} + \underbrace{\frac{4(b-a)^3(n-1)}{n^2}}_{0} + 4(b-a)^3\,\frac{2n^2+3n+1}{3n^2} + 2(b^2-a^2) + 2(b-a)\Big] =$$

$$= (b-a)^3 \cdot \underbrace{\lim_{n\to\infty}\Big[\frac{8n^2+12n+1}{3n^2}\Big]}_{\frac{8}{3}} + 2(b^2-a^2) + 2(b-a) =$$

$$= \frac{8}{3}(b-a)^3 + 2(b^2-a^2) + 2(b-a)$$

2) $\int_{-1}^{1} \sqrt{1-x^2}\,dx =$ Für $x \in [-1;1)$: $\sqrt{1-x^2} \neq 0$

$x = \sin(\theta)$ $\theta = \sin^{-1}(x)$

$dx = \cos(\theta)\,d\theta$

$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{1-\sin^2(\theta)}\cos(\theta)\,d\theta =$

$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2(\theta)\,d\theta = \cos(\theta)\sin(\theta) + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2(\theta)\,d\theta =$

	D	I
+	cos	cos
−	−sin	sin

$= \cos(\theta)\sin(\theta) + \int 1-\cos^2(\theta)\,d\theta =$

$= \Big[\frac{\theta}{2} + \frac{1}{2}\cos(\theta)\sin(\theta)\Big]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} =$

$= \cancel{\left(\frac{\pi}{4}\right) - \left(-\frac{\pi}{4}\right) = \frac{\pi}{2}}$

$= \left(\frac{\pi}{4}+0\right) - \left(-\frac{\pi}{4}+0\right) = \frac{\pi}{2}$

$x = \cos(\theta)$ $\theta = \cos^{-1}(x)$

$dx = -\sin(\theta)\,d\theta$

	D	I
+	sin	sin
−	cos	−cos

$\int_{-1}^{1}\sqrt{1-x^2}\,dx = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} -\sin^2(\theta)\,d\theta = \cancel{-\sin(\theta)\cos(\theta)}$

$= -\sin(\theta)\cos(\theta) + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2(\theta)\,d\theta = \Big[\frac{\theta}{2} - \frac{1}{2}\sin(\theta)\cos(\theta)\Big]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} =$

$= \left(\frac{\pi}{4}+0\right) - \left(-\frac{\pi}{4}+0\right) = \frac{\pi}{2}$

Das Resultat ist plausibel

$$\int_{m}^{m+r} \sqrt{r^2-(x-m)^2}\, dx =$$

$\cancel{(x-m) = a\sin(\theta)}$

$x - m = r \sin(\theta) \qquad \theta = \sin^{-1}\left(\frac{x-m}{r}\right)$

$dx = r\cos(\theta)\, d\theta$

$$\cancel{= \int_0^{\frac{\pi}{2}} \sqrt{r^2 - r^2\sin^2(\theta)}\, d\theta = \int_0^{\frac{\pi}{2}} \sqrt{r^2 \cos^2(\theta)}\, d\theta = 2}$$

$$= \cancel{\int_0^{\frac{\pi}{2}} r} \int_0^{\frac{\pi}{2}} \sqrt{r^2(1-\sin^2(\theta))}\, r\cos(\theta)\, d\theta = \int_0^{\frac{\pi}{2}} r^2 \cos^2(\theta)\, d\theta =$$

$$= r^2 \int_0^{\frac{\pi}{2}} \cos^2(\theta)\, d\theta = r^2\left[\frac{\theta}{2} + \frac{1}{2}\cos(\theta)\sin(\theta)\right]_0^{\frac{\pi}{2}} =$$

$$= r^2\left[\frac{\pi}{4} + 0\right] - [0+0] = \underline{\underline{r^2 \frac{\pi}{4}}}$$

$r = 2$

$$\Rightarrow r^2 \frac{\pi}{4} = \underline{\underline{\pi}}$$

3) $\int (x+\frac{1}{2})^2 \sin(3x)\,dx = -\frac{\cos(3x)}{3}(x+\frac{1}{2})^2 + 2(x+\frac{1}{2})\frac{\sin(3x)}{\cancel{3}\,9} + \frac{2\cos(3x)}{27} + c$

D	I
$+\ (x+\frac{1}{2})^2$	$\sin(3x)$
$-\ 2(x+\frac{1}{2})$	$-\frac{\cos(3x)}{3}$
$+\ 2$	$-\frac{\sin(3x)}{9}$
$-\ 0$	$\frac{\cos(3x)}{27}$

für $x = \frac{\pi}{3}$: $(x+\frac{1}{2})^2 \sin(3x) = 0$

$$= \int_0^{\frac{\pi}{2}} (x+\frac{1}{2})^2 \sin(3x)\,dx = \left|\int_0^{\frac{\pi}{3}} (x+\frac{1}{2})^2 \sin(3x)\,dx\right| + \left|\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} (x+\frac{1}{2})^2 \sin(3x)\,dx\right| =$$

$$= \left[-\frac{\cos(3x)}{3}\left(\left(x+\frac{1}{2}\right)^2 - \frac{2}{9}\right) + \frac{2\sin(3x)}{9}\left(x+\frac{1}{2}\right)\right]_0^{\frac{\pi}{3}} =$$

$$= \left[\frac{1}{3}\left(\left(\frac{2\pi+3}{6}\right)^2 - \frac{2}{9}\right) + 0\right] - \left[-\frac{1}{3}\left(\frac{1}{4} - \frac{2}{9}\right) + 0\right] = \frac{1}{3}\left(\left(\frac{2\pi+3}{6}\right)^2 + \frac{1}{4} - \frac{4}{9}\right)$$

$$\left(= \int_0^{\frac{\pi}{3}} (x+\frac{1}{2})^2 \sin(3x)\,dx\right)$$

$$\left[-\frac{\cos(3x)}{3}\left(\left(x+\frac{1}{2}\right)^2 - \frac{2}{9}\right) + 2\,\frac{\sin(3x)}{9}\left(x+\frac{1}{2}\right)\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}} =$$

$$= \left[0 + \frac{2}{9}\left(\frac{\pi+1}{2}\right)\right] - \left[-\frac{1}{3}\left(\left(\frac{2\pi+3}{6}\right)^2 - \frac{2}{9}\right) + 0\right] = \frac{\pi+1}{9} + \frac{1}{3}\left(\left(\frac{2\pi+3}{6}\right)^2 - \frac{2}{9}\right)$$

$$\left(= \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} (x+\frac{1}{2})^2 \sin(3x)\,dx\right)$$

$$\Rightarrow \int_0^{\frac{\pi}{2}} (x+\frac{1}{2})^2 \sin(3x)\,dx = \frac{1}{3}\left(\left(\frac{2\pi+3}{6}\right)^2 + \frac{1}{4} - \frac{4}{9}\right) + \frac{\pi+1}{9} + \frac{1}{3}\left(\left(\frac{2\pi+3}{6}\right)^2 - \frac{2}{9}\right) =$$

$$= \frac{\pi+1}{9} + \frac{1}{3}\left(2\left(\frac{4\pi^2+12\pi+9}{36}\right) - \frac{15}{36}\right) = \cancel{\frac{4\pi+4}{36}} + \frac{12\pi+12}{108} + \frac{8\pi^2+24\pi+4}{108} =$$

$$= \frac{8\pi^2+36\pi+16}{108} \approx 1{,}93$$

4) $f(x) = \sqrt{r^2 - (x - m)^2}$

$r = 2,\ m = 3$

$T(x) = f(3) + f'(3)(x-3) =$

$= 2 + 0(x-3) = \underline{\underline{2}}$

$f(3) = \sqrt{r^2 - (3-m)^2} = \sqrt{4 - 0} = \underline{\underline{2}}$

$f'(x) = \dfrac{1}{2\sqrt{r^2 - (x-m)^2}} \cdot (2x - 2m) = \dfrac{x - m}{\sqrt{r^2 - (x-m)^2}}$

$f'(3) = \dfrac{3 - m}{\sqrt{r^2 - (3-m)^2}} = 0$

$P(x) = f(3) + f'(3)(x-3) + f''(3) \cdot \dfrac{(x-3)^2}{2} =$

$= 2 + 0(x-3) + \left(-\dfrac{1}{2} \cdot \dfrac{(x-3)^2}{2}\right) =$

$= \underline{\underline{-\dfrac{1}{4}x^2 + \dfrac{3}{2}x - \dfrac{1}{4}}}$

$f''(x) = \dfrac{-(r^2 - (x-m)^2)^{\frac{1}{2}} - (x-m)^2 (r^2 - (x-m)^2)^{-\frac{1}{2}}}{(r^2 - (x-m)^2)} =$

$= -\dfrac{1}{\sqrt{r^2 - (x-m)^2}} - \dfrac{(x-m)^2}{(r^2 - (x-m)^2)^{\frac{3}{2}}}$

$f''(3) = -\dfrac{1}{\sqrt{r^2 - (3-m)^2}} - \dfrac{(3-m)^2}{(r^2 - (3-m)^2)^{\frac{3}{2}}} = \underline{\underline{-\dfrac{1}{2}}}$

=

$\int \sqrt{1 - x^2}\, dx = \int \cos^2(\theta)\, d\theta =$

$x = \sin(\theta)$

$dx = \cos(\theta)\, d\theta$

$\cos(\theta) = \sqrt{1 - \sin^2\theta}$

SUDOKU

KNIFFLIG

1	9	7	4	6	8	5	2	3
5	8	2	1	7	3	9	4	6
4	3	6	5	9	2	8	1	7
2	4	1	6	3	5	7	8	9
7	5	3	2	8	9	1	6	4
8	6	9	7	1	4	3	5	2
9	2	8	3	5	6	4	7	1
3	7	4	8	2	1	6	9	5
6	1	5	9	4	7	2	3	8

Sudoku: Füllen Sie das Diagramm so aus, dass in jeder Zeile, in jeder Spalte und in jedem der neun 3x3-Quadrate jede Ziffer von 1 bis 9 genau ein Mal vorkommt.
Weitere Sudokus aller Schwierigkeitsgrade finden Sie unter diepresse.com/sudoku.