1. Potentialverlauf einer geladenen Hohlkugel

$$k = \frac{1}{4\pi\epsilon_0}, V = \int_{V_i}^{\infty} \vec{E}\ \mathrm{d}\vec{s}$$

1. $$\boxed{E(r) = \begin{cases} 0 & \text{for } 0 < r \leq R_i, \\ \frac{kQ}{R_a^3 - R_i^3}\left(r - \frac{R_i^3}{r^2}\right) & \text{for } R_i < r < R_a, \\ kQ\,\frac{1}{r^2} & \text{for } R_a \leq r. \end{cases}}$$

<u>For $r \geq R_a$:</u>

$$E(r) = kQ\,\frac{1}{r^2}$$

$$V(R_a) = \int_{R_a}^{\infty} \vec{E}(r)\ \mathrm{d}\vec{r} = kQ \int_{R_a}^{\infty} \frac{1}{r^2}\ \mathrm{d}r = kQ\,\frac{1}{R_a}$$

$$V(r) = kQ\,\frac{1}{r}$$

<u>For $R_i < r < R_a$:</u>

$$E(r) = \frac{kQ}{R_a^3 - R_i^3}\left(r - \frac{R_i^3}{r^2}\right)$$

$$V(R_i) = \int_{R_i}^{\infty} \vec{E}(r)\ \mathrm{d}\vec{r} = kQ\left(\int_{R_i}^{R_a} \frac{r^3 - R_i^3}{r^2(R_a^3 - R_i^3)}\ \mathrm{d}r + \underbrace{\int_{R_a}^{\infty} \frac{1}{r^2}\ \mathrm{d}r}_{= V(R_a)}\right) = kQ\left(\frac{1}{R_i} - \frac{1}{R_a} + \frac{1}{R_a}\right)$$

$$V(r) = kQ\,\frac{1}{r^2}$$

<u>For $r \leq R_i$:</u>

$$E(r) = 0$$

$$V(r) = \int_{0}^{\infty} \vec{E}(s)\ \mathrm{d}\vec{s} = kQ \int_{R_a}^{\infty} \frac{1}{s^2}\ \mathrm{d}s = kQ\,\frac{1}{r^2}$$

$$V(r) = kQ\,\frac{1}{r^2}$$

$$\boxed{V(r) = \begin{cases} 0 & \text{for } 0 < r \leq R_i, \\ \frac{kQ}{R_a^3 - R_i^3}\left(r - \frac{R_i^3}{r^2}\right) & \text{for } R_i < r < R_a, \\ kQ\,\frac{1}{r^2} & \text{for } R_a \leq r. \end{cases}}$$

2. $F = -m\omega^2 x; \quad \Delta x = x_1 - x_2$

$$m\ddot{x}_1 = -m{\omega_a}^2 x_1 - k(x_1 - x_2)$$
$$m\ddot{x}_2 = -m{\omega_a}^2 x_2 - k(x_2 - x_1)$$
$$\Rightarrow \; m(\ddot{x}_1 - \ddot{x}_2) = -m{\omega_a}^2(x_1 - x_2) - 2k(x_1 - x_2) = -(m{\omega_a}^2 + 2k)(x_1 - x_2)$$
$$\Delta\ddot{x} = -\underbrace{({\omega_a}^2 + \tfrac{2k}{m})}_{= \omega_b}\Delta x$$
$$\omega_b = \sqrt{\omega_a^2 + \tfrac{2k}{m}} = \underline{\underline{6.43 \text{ rad/s}}}$$

3. $\delta\omega = \omega_b - \omega_a$

$$0 = \cos\left(\tfrac{1}{2}\delta\omega t\right)$$
$$t = \frac{2\arccos(0)}{\delta\omega} = \underline{\underline{50.29 \text{ s}}}$$