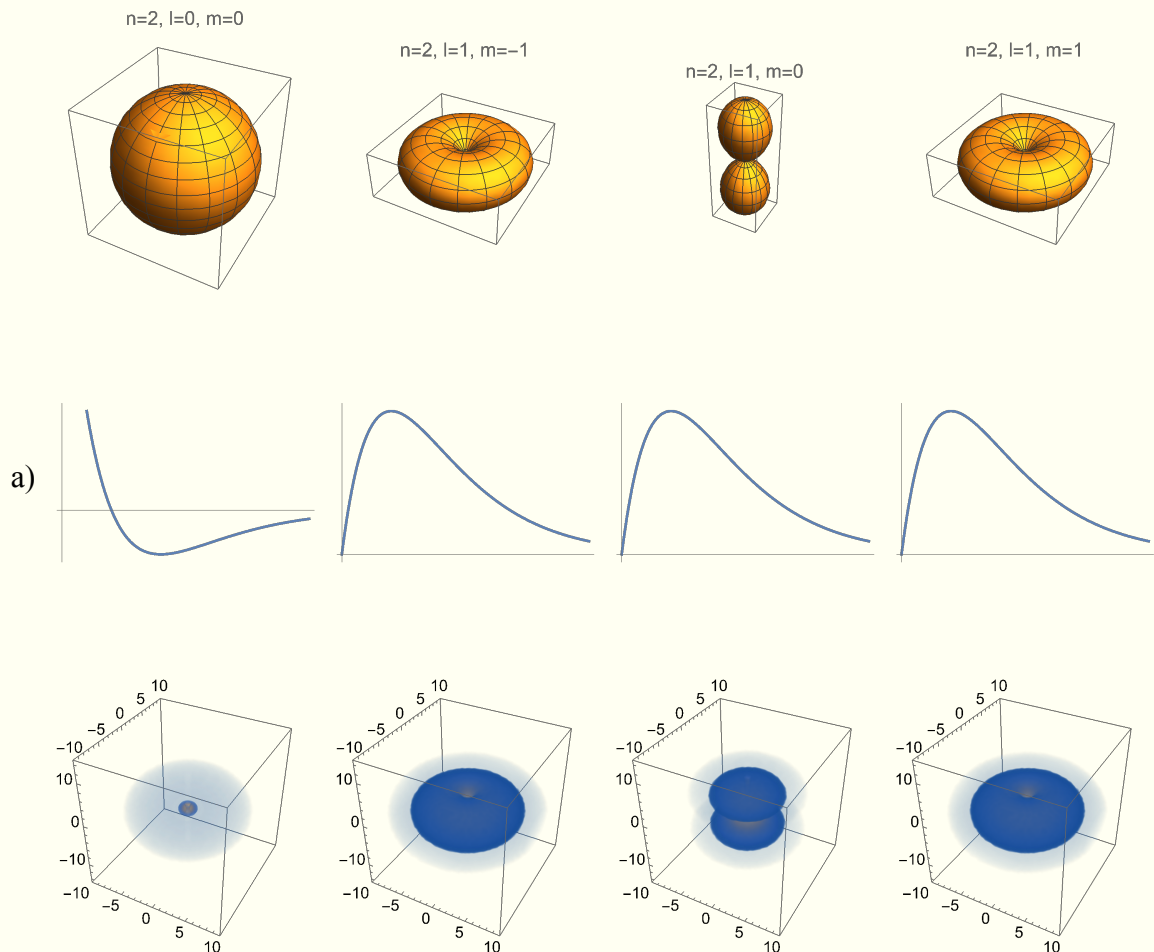


Wasserstoffatom: Aufenthaltswahrscheinlichkeiten



On top you can see the spherical and radial probability of an electron in a hydrogen Atom for different Quantum numbers. Below is a density plot of the square modulus of the whole wave function (as product of the upper 2 plots). If you sum all 4 states up you can kind of recognize a radial symmetry, as the $l = 0, m = 0$ state is already symmetric and the other three apparently add up to a sphere.

b)

$$\begin{aligned}\langle \hat{r} \rangle &= \int_V r |\psi_{2,1,1}|^2 dV = \int_0^{2\pi} \int_0^\pi \int_0^\infty \frac{1}{24a_0^5} e^{-\frac{r}{a_0}} r^2 \frac{3}{8\pi} \sin(\theta)^2 r r^2 \sin(\theta) dr d\theta d\phi = \\ &= \frac{1}{32a_0^5} \int_0^\infty r^5 e^{-\frac{r}{a_0}} dr \int_0^\pi \sin(\theta)^3 d\theta = \frac{1}{32a_0^5} 120a_0^6 \frac{4}{3} = \underline{\underline{5a_0}}\end{aligned}$$

$$\langle \hat{r}^2 \rangle = \int_V r^2 |\psi_{2,1,1}|^2 dV = \frac{1}{32a_0^5} \int_0^\infty r^6 e^{-\frac{r}{a_0}} dr \int_0^\pi \sin(\theta)^3 d\theta = \underline{\underline{30a_0^2}}$$

$$\langle \Delta \hat{r}^2 \rangle = \langle \hat{r}^2 \rangle - \langle \hat{r} \rangle^2 = 30a_0^2 - 25a_0^2 = \underline{\underline{5a_0^2}}$$

c) $d = 1.75 \text{ fm}$

$$\begin{aligned}P &= \int_V |\psi_{2,0,0}|^2 dV = \int_0^{2\pi} \int_0^\pi \int_0^d \frac{1}{4a_0^3} e^{-\frac{r}{a_0}} \left(1 - \frac{r}{a_0} + \frac{r^2}{4a_0^2}\right) \frac{1}{4\pi} r^2 \sin(\theta) dr d\theta d\phi = \\ &= \frac{1}{2a_0^3} \int_0^d e^{-\frac{r}{a_0}} \left(1 - \frac{r}{a_0} + \frac{r^2}{4a_0^2}\right) r^2 dr = \underline{\underline{6.11 \times 10^{-15}}}\end{aligned}$$

Drehimpulsquantenzahlen

a) $\hat{L}_z = -i\hbar \frac{\partial}{\partial \phi}; \quad \psi_{n,l,m} = a_m R_{n,l} \cos(\theta) e^{im\phi} P_l^m$

$$\langle \hat{L}_z \rangle = \int_V \psi^* \hat{L}_z \psi dV = -i\hbar \int_V \psi^* \frac{\partial \psi}{\partial \phi} dV = \hbar m \int_V \psi^* \psi dV = \underline{\underline{\hbar m}}$$

b) $\hat{\mathbf{L}}^2 Y_l^m(\theta, \phi) = l(l+1)\hbar^2 Y_l^m(\theta, \phi)$

$$\langle \hat{\mathbf{L}}^2 \rangle = \int_V \psi^* \hat{\mathbf{L}}^2 \psi dV = \int_V \psi^* R_{n,l} \hat{\mathbf{L}}^2 Y_l^m dV = l(l+1)\hbar^2 \int_V \psi^* \psi dV = \underline{\underline{l(l+1)\hbar^2}}$$

c)

$$\begin{aligned} [\hat{\mathbf{L}}^2, \hat{L}_z] \psi_{n,l,m} &= \hat{\mathbf{L}}^2(\hat{L}_z \psi_{n,l,m}) - \hat{L}_z(\hat{\mathbf{L}}^2 \psi_{n,l,m}) = \\ &= \hat{\mathbf{L}}^2 \hbar m \psi_{n,l,m} - \hat{L}_z l(l+1) \hbar^2 \psi_{n,l,m} \end{aligned}$$

Zeeman–Effekt

a)

b)

c)

d)