Atomvibrationen in einem Metall

a) $\rho = \frac{3}{4\pi R^3}; \quad V = \frac{4}{3}\pi r^3; \quad q = e\rho V; \quad k = \frac{1}{4\pi\epsilon_0}$

$$E = \frac{kq}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{er}{R^3}$$
$$F = qE = kr$$
$$\omega = \sqrt{\frac{k}{mM}} = \underline{\underline{\sqrt{\frac{e^2}{4\pi\epsilon_0 MR^3}}}}$$

b)

$$4R^2 = 3a^2 \qquad \Rightarrow \qquad R = \underline{\underline{\frac{\sqrt{3}a}{2}}}$$
$$\omega = \sqrt{\frac{e^2}{4\pi\epsilon_0 MR^3}} = \underline{\underline{5.11 \times 10^{10}\ \text{Hz}}}$$

c) $N = 9; \quad V = a^3$

$$v = \frac{\Omega_\text{D}}{\sqrt[3]{\frac{6\pi^2 N}{V}}} = \underline{\underline{1715}}\ \text{m/s}$$

Zwei-atomige Kette

a) $\omega^2 = G\left(\frac{1}{m} + \frac{1}{M}\right) \pm G\sqrt{\left(\frac{1}{m} + \frac{1}{M}\right)^2 - \frac{4\sin^2(ka)}{Mm}}; \quad ka \ll 1; \quad \mu = \frac{1}{m} + \frac{1}{M}$

$$\omega^2 = G\left(\frac{1}{m} + \frac{1}{M}\right) \pm G\sqrt{\left(\frac{1}{m} + \frac{1}{M}\right)^2 - \frac{4\sin^2(ka)}{Mm}} \approx \frac{G}{\mu} \pm G\sqrt{\frac{1}{\mu^2} - \frac{4(ka)^2}{Mm}} =$$
$$= \frac{G}{\mu} \pm \frac{G}{\mu}\sqrt{1 - \frac{4(ka)^2\mu^2}{Mm}} \approx \frac{G}{\mu} \pm \frac{G}{\mu}\left(1 - \frac{2(ka)^2\mu^2}{Mm}\right)$$

$$\omega_- = \sqrt{\frac{2G(ka)^2\mu}{Mm}} = ka\sqrt{\frac{2G\mu}{Mm}}$$

$$v = \frac{\omega_-}{k} = \underline{\underline{a\sqrt{\frac{2G\mu}{Mm}}}}$$

b)

c)

d)

Eigenschaften eines Natriumkristalls

$$m = 22.9897 \text{ u}; \quad \rho = 0.968 \text{ g/cm}^3$$

a) $V\rho = M$

$$a = \sqrt[3]{\frac{9m}{\rho}} = \underline{\underline{7.08 \times 10^{-10} \text{ m}}}$$

b) $I \propto |F_{\text{hkl}}|^2$

$$|F_{111}|^2 = f^2 \left(\left| \mathrm{e}^0 + \mathrm{e}^{3i\pi} \right|^2 \right) = 0$$

$$|F_{110}|^2 = f^2 \left(\left| \mathrm{e}^0 + \mathrm{e}^{2i\pi} \right|^2 \right) = 2f^2$$

The Intensity for a (111) Lattice is lower than for a (110) lattice.

c) $T_\text{D} = 150 \text{ K}$

$$\Omega_\text{D} = \frac{T_\text{D} k_\text{B}}{\hbar} = \underline{\underline{1.96 \times 10^{13} \text{ Hz}}}$$