

2. Feld einer Hohlkugel

$$m = 5 \text{ kg}; \quad k = 2 \text{ N/m}; \quad a = 0.1 \text{ m}; \quad h = 0.4 \text{ m}; \quad l = 0.1 \text{ m}$$

a) $\rho = \frac{Q}{V}$

$$\begin{aligned} V &= \int_V dV = \int_0^\pi \int_0^{2\pi} \int_{R_i}^{R_a} r^2 \sin(\theta) \, dr d\varphi d\theta \\ &= \frac{4}{3} \pi (R_a^3 - R_i^3) \\ \rho &= \underline{\underline{\frac{3Q}{4\pi} (R_a^3 - R_i^3)}} \end{aligned}$$

b) $F = -m\omega^2 x; \quad \Delta x = x_1 - x_2$

$$\begin{aligned} m\ddot{x}_1 &= -m\omega_a^2 x_1 - k(x_1 - x_2) \\ m\ddot{x}_2 &= -m\omega_a^2 x_2 - k(x_2 - x_1) \\ \Rightarrow m(\ddot{x}_1 - \ddot{x}_2) &= -m\omega_a^2 (x_1 - x_2) - 2k(x_1 - x_2) = -(m\omega_a^2 + 2k)(x_1 - x_2) \\ \Delta\ddot{x} &= -\underbrace{(\omega_a^2 + \frac{2k}{m})}_{= \omega_b^2} \Delta x \\ \omega_b &= \sqrt{\omega_a^2 + \frac{2k}{m}} = \underline{\underline{6.43 \text{ rad/s}}} \end{aligned}$$

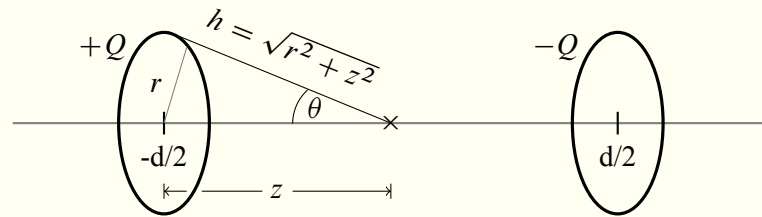
c) $\delta\omega = \omega_b - \omega_a$

$$\begin{aligned} 0 &= \cos\left(\frac{1}{2} \delta\omega t\right) \\ t &= \frac{2 \arccos(0)}{\delta\omega} = \underline{\underline{50.29 \text{ s}}} \end{aligned}$$

3. Zwei ringförmige Ladungsträger

$$k = \frac{1}{4\pi\epsilon_0}$$

a) $h = \sqrt{r^2 + z^2}; \quad \cos(\theta) = \frac{z}{h}; \quad dQ = \lambda dr; \quad dr = r d\varphi; \quad Q = 2r\pi\lambda$



Due to the symmetric nature of this problem we can neglect vertical components of forces

$$dE_1(z) = k \frac{dQ}{h^2} \cos(\theta) = k \frac{\lambda dr}{r^2 + z^2} \frac{z}{\sqrt{r^2 + z^2}} = k \frac{\lambda r z}{(r^2 + z^2)^{3/2}} d\varphi$$

$$E_1(z) = \int dE_1 = k \frac{\lambda r z}{(r^2 + z^2)^{3/2}} \int_0^{2\pi} d\varphi = k \frac{\lambda 2\pi r z}{(r^2 + z^2)^{3/2}} = k \frac{Qz}{(r^2 + z^2)^{3/2}}$$

$$E_2(z) = -E_1(z - d) = -k \frac{Q(z - d)}{(r^2 + (z - d)^2)^{3/2}}$$

$$\begin{aligned} E_{ges}(z) &= E_1 + E_2 = k \frac{Qz}{(r^2 + z^2)^{3/2}} - k \frac{Q(z - d)}{(r^2 + (z - d)^2)^{3/2}} \\ &= kQ \left(\frac{z + \frac{d}{2}}{\left(r^2 + \left(z + \frac{d}{2}\right)^2\right)^{3/2}} + \frac{\frac{d}{2} - z}{\left(r^2 + \left(z - \frac{d}{2}\right)^2\right)^{3/2}} \right) \end{aligned}$$

b) $F = -m\omega^2 x; \quad \Delta x = x_1 - x_2$

$$m\ddot{x}_1 = -m\omega_a^2 x_1 - k(x_1 - x_2)$$

$$m\ddot{x}_2 = -m\omega_a^2 x_2 - k(x_2 - x_1)$$

$$\Rightarrow m(\ddot{x}_1 - \ddot{x}_2) = -m\omega_a^2(x_1 - x_2) - 2k(x_1 - x_2) = -(m\omega_a^2 + 2k)(x_1 - x_2)$$

$$\Delta\ddot{x} = -\underbrace{\left(\omega_a^2 + \frac{2k}{m}\right)}_{=\omega_b^2} \Delta x$$

$$\omega_b = \sqrt{\omega_a^2 + \frac{2k}{m}} = \underline{\underline{6.43 \text{ rad/s}}}$$

$$\text{c) } \delta\omega = \omega_b - \omega_a$$

$$0 = \cos\left(\frac{1}{2}\delta\omega t\right)$$

$$t = \frac{2 \arccos(0)}{\delta\omega} = \underline{\underline{50.29 \text{ s}}}$$