1. Der Dipol

$$k = \frac{1}{4\pi\epsilon_0}; \quad x_1 = -\frac{d}{2}; \quad x_2 = \frac{d}{2}$$

(a)

$$V_{ges} = V_+ + V_- = kq\left(\frac{1}{r_+} - \frac{1}{r_-}\right)$$

(b) $d \ll r; \quad \vec{p} := q\vec{d}; \quad d\cos(\theta) = \vec{d}$

$$V = kq\left(\frac{1}{r_+} - \frac{1}{r_-}\right) = kq\left(\frac{r_- - r_+}{r_+ r_-}\right) \approx kq\frac{d\cos(\theta)}{r^2}$$
$$= k\frac{p\cos(\theta)}{r^2} = \underline{\underline{k\frac{\vec{p}\cdot\vec{r}}{r^3}}}$$

$$\boxed{\begin{aligned} r_- - r_+ &\approx d\cos(\theta) \\ r_+ r_- \pm l &\approx r^2 \end{aligned}}$$

(c) $|\vec{M}| = |\vec{p}\times\vec{E}| = pE\sin(\theta); \quad E_{pot} = \int_\theta^{\pi/2} |\vec{M}|\, \mathrm{d}\tilde{\theta}$

$$E_{pot} = \int_\theta^{\pi/2} |\vec{M}|\, \mathrm{d}\tilde{\theta} = pE\int_\theta^{\pi/2} \sin\left(\tilde{\theta}\right) \mathrm{d}\tilde{\theta} = pE\cos(\theta)$$
$$= \underline{\underline{\vec{p}\cdot\vec{E}}}$$

2. Kondensator mit Dielektrikum

$$k = \frac{1}{4\pi\epsilon_0}; \quad C = \frac{\epsilon A}{d}; \quad \epsilon_{rel} = \frac{\epsilon}{\epsilon_0}$$

(a)

$$C_1 = \frac{\epsilon_0 a(a-h)}{d}$$
$$C_2 = \frac{\epsilon_r a h}{d}$$
$$C_{ges} = C_1 + C_2 = \underline{\underline{\frac{\epsilon_0 a}{d}\left(\epsilon_{rel} h + a - h\right)}}$$

(b)

$$E = \frac{CU^2}{2} = \underline{\underline{\frac{U^2\epsilon_0 a(\epsilon_{rel}h+a-h)}{2d}}}$$

(c)

$$E' = \frac{C'U^2}{2} = \frac{U^2\epsilon_0 a(\epsilon_{rel}(h+\Delta h)+a-(h+\Delta h))}{2d}$$

$$\Delta E = \underline{\underline{\frac{U^2\epsilon_0 a}{2d}(\epsilon_{rel}\Delta h - \Delta h)}}$$

$$\Delta E_{pot} = \frac{mg\Delta h}{2} = \underline{\underline{\frac{\rho a^2 dg\Delta h}{2}}}$$

(d)

$$\Delta q = 2\frac{\Delta E}{U} = \underline{\underline{\frac{U\epsilon_0 a}{d}(\epsilon_{rel}\Delta h - \Delta h)}}$$

(e)

(f) $a = 0.1$ m; $d = 5.0 * 10^{-3}$ m; $h = 0.05$ m; $U = 7$ kV
$\epsilon_r = 80$; $\rho = 1.0 * 10^3$ kg/m^3

3. Kapazität eines Koaxialkabels

$$k = \frac{1}{4\pi\epsilon_0}; \quad C = \frac{\epsilon A}{d}; \quad \epsilon_{rel} = \frac{\epsilon}{\epsilon_0}$$

(a) For $r \leq d \leq R$:

$$\frac{q_{in}}{\epsilon_0} = \oint \vec{E}(\vec{d})\,\mathrm{d}\vec{A} = E(d)\oint \mathrm{d}A = E(d)2\pi dl$$

$$E(d) = kq\frac{2}{dl}$$

For $R < d$:

$$q_{in} = 0$$
$$E(d) = 0$$

$$\boxed{E(d) = \begin{cases} kQ\,\frac{2}{lr} & \text{for } r \leq d \leq R, \\ 0 & \text{for } R < d. \end{cases}}$$

(b) For $r \leq d \leq R$:

$$E(d) = kq\,\frac{2}{dl}$$

$$V(d) = \int_d^\infty \vec{E}(r)\,\mathrm{d}\vec{r} = \int_d^R kq\,\frac{2}{rl}\,\mathrm{d}r + \underbrace{\int_R^\infty 0\,\mathrm{d}r}_{=0}$$

$$V(d) = \frac{2kq}{l}\ln\left(\frac{R}{d}\right)$$

$$\boxed{V(d) = \begin{cases} \frac{2kq}{l}\ln\left(\frac{R}{d}\right) & \text{for } r \leq d \leq R, \\ 0 & \text{for } R < d. \end{cases}}$$

(c) $r = 7.0 * 10^{-4}$ m; $R = 2.5 * 10^{-3}$ m; $C = 500$ pF

$$C = \frac{q}{V(r)} = \frac{l}{2k\ln\left(\frac{R}{r}\right)}$$

$$l = \frac{C}{2k\ln\left(\frac{R}{r}\right)}$$

$$= \underline{\underline{11.44 \text{ m}}}$$

(d) $I = 0.1$ A; $U = 10$ V

$$\Delta q = 2\,\frac{\Delta E}{U} = \underline{\underline{\frac{U\epsilon_0 a}{d}\left(\epsilon_{rel}\Delta h - \Delta h\right)}}$$