87. Ein sehr elastischer Ball

a) $h = 0.5 \text{ m}; \quad m_1 \ll m_2$

$$z(t) = 0.5 - \frac{g}{2}t^2 \quad \Rightarrow \quad t_1 = 0.32 \text{ s}$$

 $v(t_1) = 3.13 \text{ m/s}$

Nach dem Stoß:

$$v_1 = \underline{\underline{9.4 \text{ m/s}}}$$

$$v_2 = 3.13 \text{ m/s}$$

b)

$$z_1(t) = 9.4t - \frac{g}{2}t^2$$

$$v_1(t) = 9.4 - gt$$

$$0 = 9.4 - gt \implies t_1 = 0.96 \text{ s}$$

$$z_1(t_1) = 4.5 \text{ m}$$

96. Zwei Himmelskörper

$$\vec{v}_1 = \begin{pmatrix} v_1 \\ 0 \end{pmatrix}$$

$$\vec{v}_2 = \begin{pmatrix} 0 \\ v_2 \end{pmatrix}$$

$$\vec{v}_E = \frac{1}{m_1 + m_2} \, \begin{pmatrix} m_1 v_1 \\ m_2 v_2 \end{pmatrix}$$

$$|v_1| = v_1$$

$$|v_2| = v_2$$

$$|v_E| = \frac{\sqrt{(m_1 v_1)^2 + (m_2 v_2)^2}}{m_1 + m_2}$$

a)
$$\alpha = \arctan\left(\frac{m_2 v_2}{m_1 v_1}\right)$$

b) $K_1 := E_{kin}$ vor dem Stoß; $K_2 := E_{kin}$ nach dem Stoß

$$K_1 = K_2 + \Delta E$$

$$\frac{m_1 v_1^2}{2} + \frac{m_2 v_2^2}{2} = \frac{1}{2} (m_1 + m_2) \left(\frac{\sqrt{(m_1 v_1)^2 + (m_2 v_2)^2}}{m_1 + m_2} \right)^2 + \Delta E$$

$$\Delta E = \frac{(m_1 v_1)^2 + (m_2 v_2)^2 + m_1 m_2 v_1^2 + m_1 m_2 v_2^2 - (m_1 v_1)^2 - (m_2 v_2)^2}{2(m_1 + m_2)}$$

$$\Delta E = \frac{(m_1 v_1)^2 + (m_2 v_2)^2 + m_1 m_2 v_1^2 + m_1 m_2 v_2^2 - (m_1 v_1)^2 - (m_2 v_2)^2}{2(m_1 + m_2)}$$

$$\Delta E = \frac{m_1 m_2 (v_1^2 + v_2^2)}{2(m_1 + m_2)}$$

c) Für
$$m_1 = m_2 = m$$

$$\Delta E = \frac{1}{4}m\left(v_1^2 + v_2^2\right)$$

Für
$$m_1 \gg m_2$$

$$\Delta E = \frac{1}{2} m_2 \left(v_1^2 + v_2^2 \right)$$

102. Trägheitsmoment eines Hohlzylinders

a)
$$V = (r_1^2 - r_0^2)\pi z$$

$$I = \int r^2 dm = \rho \int_V r^2 dV = \rho \int_0^z \int_0^{2\pi} \int_{r_0}^{r_1} r^3 dr \, d\varphi \, dz = \frac{r_1^4 - r_0^4}{2} \pi z \rho =$$
$$= \frac{1}{2} M(r_1^2 + r_0^2)$$

113. Getränkedosen auf schiefer Ebene

$$r = 0.03 \text{ m}; \quad \alpha = 25^{\circ}; \quad l = 1.5 \text{ m}; \quad h = l \sin(\alpha)$$

a)
$$m = 0.06 \text{ kg}$$

$$mgh = \frac{I\omega^2 + mv_s^2}{2}$$

$$mgh = \frac{I\omega^2 + mv_s^2}{2}$$
$$mgh = \frac{mv_s^2 + mv_s^2}{2}$$

$$v_s = \sqrt{gh}$$

$$= \underline{2.49 \text{ m/s}}$$

b)
$$m_w = 0.33 \text{ kg}$$
; $M = m + m_w = 0.39 \text{ kg}$

$$Mgh = \frac{I\omega^2 + Mv_s^2}{2}$$

$$Mgh = \frac{mv_s^2 + Mv_s^2}{2}$$

$$Mgh = \frac{mv_s^2 + Mv_s^2}{2}$$

$$v_s = \sqrt{\frac{2ghM}{2m + m_w}}$$
$$= 3.28 \text{ m/s}$$

c)

$$Mgh = \frac{I_w\omega^2 + I\omega^2 + Mv_s^2}{2}$$

$$Mgh = \frac{I_w\omega^2 + I\omega^2 + Mv_s^2}{2}$$

$$Mgh = \frac{0.5m_wv_s^2 + mv_s^2 + Mv_s^2}{2}$$

$$v_s = \sqrt{\frac{4Mgh}{3M+m}}$$

$$v_s = \sqrt{\frac{4Mgh}{3M+m}}$$

$$= 2.81 \text{ m/s}$$