

Materiewellen und Formfaktoren

a)

b)

$$\begin{aligned}
 F(\mathbf{q}^2) &= \int_{\mathbb{R}^3} e^{\frac{i\mathbf{q}\mathbf{x}}{\hbar}} f(\mathbf{x}) d\mathbf{x} = \int_0^{2\pi} \int_0^\pi \int_0^\infty e^{\frac{iqr\cos(\theta)}{\hbar}} f(r)r^2 \sin(\theta) dr d\theta d\varphi \\
 &= 2\pi \int_0^\infty f(r)r^2 \int_0^\pi e^{\frac{iqr\cos(\theta)}{\hbar}} \sin(\theta) d\theta dr = 2\pi \int_0^\infty f(r)r^2 \int_{-1}^1 e^{\frac{iqr u}{\hbar}} du dr \\
 &= 4\pi \int_0^\infty f(r)r^2 \left(e^{\frac{iqr}{\hbar}} - e^{-\frac{iqr}{\hbar}} \right) \frac{\hbar}{2iqr} dr = 4\pi \int_0^\infty \frac{\sin\left(\frac{qr}{\hbar}\right)}{\frac{qr}{\hbar}} f(r)r^2 dr
 \end{aligned}$$

c) $\langle r^2 \rangle = 4\pi \int_0^\infty r^4 f(r) dr; \quad \frac{\sin\left(\frac{qr}{\hbar}\right)}{\frac{qr}{\hbar}} \approx 1 + \frac{r^2 q^2}{6\hbar^2}$

$$F(\mathbf{q}^2) \approx 4\pi \int_0^\infty f(r) \left(r^2 + \frac{r^4 q^2}{6\hbar^2} \right) dr = 4\pi \int_0^\infty f(r)r^2 dr + \langle r^2 \rangle \frac{6\hbar^2}{q^2} = 1 + \langle r^2 \rangle \frac{6\hbar^2}{q^2}$$

$$\Rightarrow \langle r^2 \rangle = \underline{\underline{\frac{F(\mathbf{q}^2) \frac{6\hbar^2}{q^2} - 1}{4\pi}}}$$

d) $f(r) = f_0 e^{-ar}$

$$\int_{\mathbb{R}^3} f(\mathbf{x}) d\mathbf{x} \stackrel{!}{=} 1 \iff \int_0^\infty f_0 e^{-ar} r^2 dr = \frac{1}{4\pi} \iff \underline{\underline{f_0 = \frac{a^3}{8\pi}}}$$

e) $F(q^2) = (1 + \alpha^2)^{-2}$

$$F(q^2) = 4\pi \int_0^\infty \frac{\sin\left(\frac{qr}{\hbar}\right)}{\frac{qr}{\hbar}} f(r) r^2 dr = \frac{\hbar a^3}{2q} \int_0^\infty \sin\left(\frac{qr}{\hbar}\right) e^{-ar} r dr = \frac{a^4 \hbar^4}{(a^2 \hbar^2 + q^2)^2}$$

$$\Rightarrow \alpha(q, a) = \frac{q}{a\hbar}$$

Diese Ladungsverteilung beschreibt Protonen.

f) $f(r) = \begin{cases} \frac{3}{4\pi R_0^3} & \text{für } 0 \leq r \leq R_0 \\ 0 & \text{für } R_0 < r \end{cases}$

$$F(q^2) = 4\pi \int_0^\infty \frac{\sin\left(\frac{qr}{\hbar}\right)}{\frac{qr}{\hbar}} f(r) r^2 dr = 3 \frac{\hbar}{q R_0^3} \int_0^{R_0} \sin\left(\frac{qr}{\hbar}\right) r dr =$$

$$= 3 \frac{\sin\left(\frac{qR_0}{\hbar}\right) - \frac{qR_0}{\hbar} \cos\left(\frac{qR_0}{\hbar}\right)}{\frac{q^3 R_0^3}{\hbar^3}} = 3 \frac{\sin(x) - x \cos(x)}{x^3}, \text{ mit } x(q) = \frac{qR_0}{\hbar}$$

Diese Ladungsverteilung wird in der Natur nicht wiedergefunden, man kann schwerere Kerne aber dadurch approximieren.

g) $x(0) = 0$

$$F(0) = \lim_{x \rightarrow 0} \left(3 \frac{\sin(x) - x \cos(x)}{x^3} \right) = 3 \lim_{x \rightarrow 0} \left(\frac{1}{x^2} \right)$$

Stabilstes Nuklid einer Isobare

a)

b)

c)

Luminosität des LHC

a)

b) Die meisten pp-Wechselwirkungen fanden 2018 statt.