2) $f: \mathbb{R}^2 \to \mathbb{R} : \begin{pmatrix} x \\ y \end{pmatrix} \mapsto e^{-(x^{\cancel{2}}+y)^2}$

$$T_0 = f\begin{pmatrix} 0 \\ 0 \end{pmatrix} + \underbrace{\frac{\partial f}{\partial x}\begin{pmatrix} 0 \\ 0 \end{pmatrix}}_{=0}(x-0) + \underbrace{\frac{\partial f}{\partial y}\begin{pmatrix} 0 \\ 0 \end{pmatrix}}_{=0}(y-0)$$

$$= 1 + 0 + 0$$

$$= 1$$

$$P_{0,3} = T_0 + \frac{1}{2}\left(\frac{\partial^2 f}{\partial x^2}\begin{pmatrix} 0 \\ 0 \end{pmatrix}(x-0)^2 + \frac{\partial^2 f}{\partial y^2}\begin{pmatrix} 0 \\ 0 \end{pmatrix}(y-0)^2 + 2\frac{\partial^2 f}{\partial y \partial x}\begin{pmatrix} 0 \\ 0 \end{pmatrix}(x-0)(y-0)\right) + \frac{1}{6}\left(\frac{\partial^3 f}{\partial x^3}\begin{pmatrix} 0 \\ 0 \end{pmatrix}(x-0)^3 + \frac{\partial^3 f}{\partial^3 y}\begin{pmatrix} 0 \\ 0 \end{pmatrix}(y-0)^3 + 3\frac{\partial f}{\partial x \partial x \partial y}\begin{pmatrix} 0 \\ 0 \end{pmatrix}(x-0)^2(y-0) + 3\frac{\partial f}{\partial x \partial y \partial x}\begin{pmatrix} 0 \\ 0 \end{pmatrix}(x-0)(y-0)^2\right)$$

$$= 1 - x^2 - y^2 - 2xy = -(x+y)^2 + 1$$

$$\frac{\partial f}{\partial x} = -2(x+y)\, e^{-(x+y)^2}$$

$$\frac{\partial f}{\partial y} = -2(x+y)\, e^{-(x+y)^2}$$

$$\frac{\partial^2 f}{\partial x^2} = e^{-(x+y)^2}(-2 + 4x^2 + 8xy + 4y^2) = \frac{\partial^2 f}{\partial y^2}$$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial x}\,\frac{\partial f}{\partial y} = \frac{\partial^2 f}{\partial x^2}$$

Alle 3 Abl gleich.

$$\frac{\partial^3 f}{\partial x^3} = 4\, e^{-(x+y)^2}(x+y)(3 - 2(x+y)^2)$$

3) $f: \mathbb{R}^2 \to \mathbb{R}^2 : \begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} x^2y^2 - 8xy + 4x + 8y - 4 \\ 3x^2y^2 - xy^2 \end{pmatrix}$

$M = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$$\frac{\partial f}{\partial x} = \begin{pmatrix} 2xy^2 - 8y + 4 \\ 6xy^2 - y^2 \end{pmatrix}$$

$$\frac{\partial f}{\partial y} = \begin{pmatrix} 2x^2y - 8x + 8 \\ 6x^2y - 2xy \end{pmatrix}$$

$$\frac{\partial^2 f}{\partial x^2} = \begin{pmatrix} 2y^2 \\ 6y^2 \end{pmatrix}$$

$$\frac{\partial^2 f}{\partial y} = \begin{pmatrix} 2x^2 \\ 6x^2 \end{pmatrix}$$

$$\frac{\partial^2 f}{\partial x \partial y} = \begin{pmatrix} 4xy - 8 \\ 12xy - 2y \end{pmatrix}$$

$$\frac{\partial^3 f}{\partial x^3} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \frac{\partial^3 f}{\partial y^3}$$

$$\frac{\partial^3 f}{\partial x^2 \partial y} = \begin{pmatrix} 4y \\ 12y \end{pmatrix} \qquad \frac{\partial^3 f}{\partial x \partial y^2} = \begin{pmatrix} 4x \\ 12x - 2 \end{pmatrix}$$

$$P_{M,1} = \begin{pmatrix} 0 \\ 10 \end{pmatrix} + \begin{pmatrix} 0 \\ 11 \end{pmatrix}(x-2) + \begin{pmatrix} 0 \\ 20 \end{pmatrix}(y-1)$$

$$= \begin{pmatrix} 0 \\ 11x + 20y - 32 \end{pmatrix}$$

$$P_{M,3} = P_{M,1} + \frac{1}{2}\left[\begin{pmatrix} 2 \\ 6 \end{pmatrix}(x-2)^2 + \begin{pmatrix} 8 \\ 24 \end{pmatrix}(y-1)^2 + 2\begin{pmatrix} 0 \\ 22 \end{pmatrix}(x-2)(y-1)\right] + \frac{1}{6}\left[3\begin{pmatrix} 4 \\ 12 \end{pmatrix}(x-2)^2(y-1) + 3\begin{pmatrix} 8 \\ 22 \end{pmatrix}(x-2)(y-1)^2\right] =$$

$$= \begin{pmatrix} 0 \\ 11x + 20y - 32 \end{pmatrix} + \begin{pmatrix} x^2 + 4y^2 - 4x - 8y + 8 \\ 3x^2 + 12y^2 + 22xy - 34x - 68y + 68 \end{pmatrix} + \begin{pmatrix} -2x^2 - 8y^2 + 2x^2y + 4xy^2 - 16xy + 12x + 24y - 16 \\ -6x^2 - 22y^2 + 6x^2y + 11xy^2 - 46xy + 35x + 68y - 46 \end{pmatrix} =$$

$$= \begin{pmatrix} -x^2 - 4y^2 + 2x^2y + 4xy^2 - 16xy + 8x + 23y - 8 \\ -3x^2 - 10y^2 + 6x^2y + 11xy^2 - 46xy + 12x + 20y - 10 \end{pmatrix}$$

4) $f: \{(x,y) \in \mathbb{R}^2 : x+y \neq 1\} \to \mathbb{R}, \binom{x}{y} \mapsto \frac{1}{1-x-y}$

$\frac{\partial f}{\partial x} = \frac{1}{(1-x-y)^2} = \frac{\partial f}{\partial y}$

$\frac{\partial^2 f}{\partial y^2} = \frac{1}{(1-x-y)^3} = \frac{\partial^2 f}{\partial x^2} = \frac{\partial^2 f}{\partial x \partial y}$

$P_0 = 1 + x + y + x^2 + y^2 + 2xy + y^3 + x^3 + 3x^2y + 3xy^2 + \dots$

$= \underline{\underline{\sum_{k=0}^{n} (x+y)^k}}$

Geometrische Reihe: $\sum_{k=0}^{n} q^k$ Konvergiert für $|q| < 1$

$\Rightarrow \sum_{k=0}^{n} (x+y)^k$ konvergiert für $|x+y| < 1$