1) $f: \mathbb{R} \to \mathbb{R}$

$$x \mapsto \tan^{-1}(x) - \frac{\pi}{2} + \frac{\sqrt{|x|}}{1+\sqrt{|x|}}$$

$$\lim_{x\to\infty}(f(x)) = \underbrace{\lim_{x\to\infty}(\tan^{-1}(x))}_{=\frac{\pi}{2}} - \frac{\pi}{2} + \lim_{x\to\infty}\left(\frac{\sqrt{|x|}}{1+\sqrt{|x|}}\right) =$$

$$= -\frac{\pi}{2} + \frac{\pi}{2} + \lim_{x\to\infty}\left(\frac{\sqrt{|x|}+1-1}{1+\sqrt{|x|}}\right) = \lim_{x\to\infty}\left(1 - \frac{1}{1+\sqrt{|x|}}\right) =$$

$$= 1 - \underbrace{\lim_{x\to\infty}\left(\frac{1}{1+\sqrt{|x|}}\right)}_{=0} = \underline{\underline{1}}$$

$$\lim_{x\to-\infty}(f(x)) = \underbrace{\lim_{x\to-\infty}(\tan^{-1}(x))}_{=-\frac{\pi}{2}} - \frac{\pi}{2} + \lim_{x\to-\infty}\left(\frac{\sqrt{|x|}}{1+\sqrt{|x|}}\right) =$$

$$= -\frac{\pi}{2} - \frac{\pi}{2} + \lim_{x\to-\infty}\left(\frac{\sqrt{|x|}+1-1}{\sqrt{|x|}+1}\right) = -\pi + 1 - \underbrace{\lim_{x\to-\infty}\left(\frac{1}{\sqrt{|x|}+1}\right)}_{=0}$$

$$= \underline{\underline{1-\pi}}$$

f stetig $\wedge$ geht von $-2{,}14$ bis 1 $\Rightarrow$ min 1 Nullstelle existiert

2) $f : \mathbb{R} \to \mathbb{R}$, $k+1$ diff

$$f(x) = \sum_{k=0}^{k} \frac{1}{k!} f^{(k)}(a)(x-a)^k + \int_0^1 \frac{(1-z)^{k+1}}{k!} f^{(k+1)}(zx+(1-z)a)(x-a)^{k+1}\, dz$$

$$f(x) - f(a) = \int_0^1 f'(zx+(1-z)a)(x-a)\, dz$$

D	I
$+\ f'(zx+(1-z)a)(x-a)$	1
$-\ f''(zx+(1-z)a)(x-a)^2$	$z + c_1$
$+\ f'''(zx+(1-z)a)(x-a)^3$	$\frac{z^2}{2} - z + c_2$
$-\ f''''(zx+(1-z)a)(x-a)^4$	$\frac{z^3}{6} - \frac{z^2}{2} + \frac{z}{2} + c_3$

$$\Rightarrow \left[f'(zx+(1-z)a)(x-a)(z+c) \right]_0^1 = \left(f'(x)(x-a)(c+1) \right) - \left(+ f'(a)(x-a)\, c \right)$$

$c_1 := -1 \Rightarrow \; = f'(a)(x-a)$

$$\Rightarrow -\left[f''(zx+(1-z)a)(x-a)^2 \left(\frac{z^2}{2} - z + c_2\right) \right]_0^1 = \left(f''(x)(x-a)^2\left(-\frac{1}{2} + c_2\right) + \left(f''(a)(x-a)^2 \cdot c_2 \right) \right)$$

$c_2 := \frac{1}{2} \Rightarrow \; = f''(a)(x-a)^2 \cdot \frac{1}{2}$

$$\Rightarrow \left[f'''(zx+(1-z)a)(x-a)^3 \left(\frac{z^3}{6} - \frac{z^2}{2} + \frac{z}{2} + c_3 \right) \right]_0^1 = f'''(a)(x-a)^3 \cdot \frac{1}{6}$$

$$\Rightarrow f(x) - f(a) = f'(a)(x-a) + f''(a)(x-a)^2 \cdot \frac{1}{2} + f'''(a)(x-a)^3 \cdot \frac{1}{6} + \int_0^1 f^{IV}(zx+(1-z)a)(x-a)^4 \frac{(1-z)^3}{3!}\, dz$$

$k := 3$

$$\Rightarrow f(x) - f(a) = f^{(k-2)}(a)(x-a) + f^{(k-1)}(a)(x-a)^{k-1} \cdot \frac{1}{(k-1)!} + f^{k}(a)(x-a)^k \frac{1}{k!} + \int_0^1 f^{k+1}(zx+(1-z)a)(x-a)^{k+1} \frac{(1-z)^{k+1}}{(k)!}\, dz$$

$$f(x) - f(a) = \sum_{k=1}^{k} f^{(k)}(a)(x-a)^k \cdot \frac{1}{k!} + \int_0^1 \frac{(1-z)^{k+1}}{k!} f^{(k+1)}(zx+(1-z)a)(x-a)^{k+1}\, dz$$

$$f(x) = \underbrace{f(a) + \sum_{k=1}^{k} f^{(k)}(a)(x-a)^k \frac{1}{k!}}_{= \sum_{k=0}^{k} f^{(k)}(a)(x-a)^k} + \int_0^1 \frac{(1-z)^{k+1}}{k!} f^{(k+1)}(zx+(1-z)a)(x-a)^{k+1}\, dz$$

$$\Rightarrow f(x) = \sum_{k=0}^{k} f^{(k)}(a)(x-a)^k \cdot \frac{1}{k!} + \int_0^1 \frac{(1-z)^{k+1}}{k!} f^{(k+1)}(xz+(1-z)a)(x-a)^{k+1}\, dz$$

3) $f: \mathbb{R}\setminus\{0\} \to \mathbb{R}$

$x \mapsto e^{-\frac{1}{x^2}}$

$$\lim_{x\to 0} \left(e^{-x^{-2}}\right) \Big|_{x\to 0,\, y\to\infty}^{y = x^{-2}} \Rightarrow \lim_{y\to\infty} \left(e^{-y}\right) = \underline{\underline{0}}$$

$$\lim_{x\to 0^+} \left(e^{-x^{-2}}\right) = 0 = \lim_{x\to 0^-} \left(e^{-x^{-2}}\right)$$

$\Rightarrow$ f ist auf $\mathbb{R}$ stetig fortsetzbar

$$f'(x) = \frac{2e^{-x^{-2}}}{x^3}$$

L'hôpital

$$\lim_{x\to 0} \left(\frac{2e^{-x^{-2}}}{x^3}\right) = 2 \lim_{x\to 0} \left(\frac{x^{-3}}{e^{x^{-2}}}\right) \stackrel{\downarrow}{=} 2 \lim_{x\to 0} \left(\frac{3}{2}\, \underbrace{\frac{x^{-1}}{e^{x^{-2}}}}_{=0}\right) =$$

$$= 3 \cancel{\lim_{x\to 0}} \cdot 0 = \underline{\underline{0}}$$

$$\lim_{x\to 0^+} \left(\frac{2e^{-x^{-2}}}{x^3}\right) = 0 = \lim_{x\to 0^-} \left(\frac{2e^{-x^{-2}}}{x^3}\right)$$

$\Rightarrow$ f' ist auf $\mathbb{R}$ stetig fortsetzbar

$$f''(x) = \frac{2e^{-x^{-2}}}{x^4} \left(2 - \frac{3}{x^2}\right)$$

$\lim_{x\to 0} (f''(x)) = 0$ (mittels L'hôpital)

$\lim_{x\to 0^+} (f''(x)) = 0 = \lim_{x\to 0^-} (f''(x)) \Rightarrow f''(x)$ stetig fortsetzbar

Gleiche Vorgehensweise lässt sich auf alle weitere Abl. von f anwenden.

4) $2\int_0^\infty x^{-3} e^{-\frac{1}{x^2}} dx = \int_0^\infty 2x^{-3} e^{-x^{-2}} dx =$ $\left| \begin{array}{l} u = -x^{-2} \\ du = -2x^{-3} dx \end{array} \right.$

$= \int_{-\infty}^{0} e^u du = \left[e^{-x^{-2}} \right]_{-\infty}^{0} = e^{2\cdot 0} - \underbrace{\lim_{a \to -\infty} \left(e^{2a} \right)}_{=0} = 1 - 0 = \underline{1}$

$\int_0^\infty \frac{e^{-3x}}{e^{-2x} + 3e^{-x} + 2} dx = -\int_1^0 \frac{u^2}{u^2+3u+2} du = -\int_1^0 \frac{u^2}{(u+2)(u+1)} du =$ $\left| \begin{array}{l} u = e^{-x} \\ du = -e^{-x} dx \end{array} \right.$

$= -\int_1^0 1 + \frac{1}{u+1} - \frac{4}{u+2} du = -\int_1^0 du - \int_1^0 \frac{1}{u+1} du + \int_1^0 \frac{4}{u+2} du =$

$= -[u]_1^0 - [\ln(u+1)]_1^0 + [4\ln(u+2)]_1^0 =$

$= -(0-1) - (\ln(1) - \ln(2)) + (4\ln(2) - 4\ln(3)) =$

$= 1 + 5\ln(2) - 4\ln(3) = \underline{\underline{1 + \ln\left(\frac{2^5}{3^4}\right) \approx 0.0729}}$

$\cancel{\frac{A}{u+2} + \frac{B}{u+1}}$

$\cancel{Au + A + Bu + 2B = u^2}$

$u^2 : u^2 + 3u + 2 = \underline{1 + \frac{-3u-2}{u^2+3u+2}}$

$\frac{A}{u+2} + \frac{B}{u+1} = \frac{-3u-2}{(u+2)(u+1)}$

$Au + Bu = -3u$
$A + 2B = -2$
$\Rightarrow B = 1$, $\underline{A = -4}$