

Elastische und tief-inelastische Elektron-Nukleonstreuung

$$\mathcal{P}_1^\mu = (E_1, \mathbf{p}_1); \quad \mathcal{P}_2^\mu = (m_p, \mathbf{0}); \quad \mathcal{P}_3^\mu = (E_3, \mathbf{p}_3); \quad \mathcal{P}_4^\mu = (E_4, \mathbf{p}_4)$$

$$\text{a) } \mathcal{P}_1^\mu + \mathcal{P}_2^\mu = \mathcal{P}_3^\mu + \mathcal{P}_4^\mu \iff \mathcal{P}_1^\mu - \mathcal{P}_3^\mu = \mathcal{P}_4^\mu - \mathcal{P}_2^\mu$$

$$(\mathcal{P}_1^\mu - \mathcal{P}_3^\mu)^2 = (\mathcal{P}_4^\mu - \mathcal{P}_2^\mu)^2$$

$$(E_1 - E_3)^2 - (\mathbf{p}_1 - \mathbf{p}_3)^2 = (E_4 - m_p)^2 - \mathbf{p}_4^2$$

$$E_1^2 - 2E_1E_3 + E_3^2 - E_1^2 + 2\mathbf{p}_1\mathbf{p}_3 - E_3^2 = E_4^2 - 2E_4m_p + m_p^2 - \mathbf{p}_4^2$$

$$-2E_1E_3 + 2p_1p_3\cos(\theta) = -2E_4m_p + 2m_p^2$$

$$E_1E_3(\cos(\theta) - 1) = m_p^2 - (E_1 + m_p - E_3)m_p$$

$$\Rightarrow E_3 = \frac{E_1m_p}{E_1 + m_p - E_1\cos(\theta)} = \underline{\underline{374 \text{ MeV}}}$$

b)

$$q^2 = (\mathcal{P}_1^\mu - \mathcal{P}_3^\mu)^2 = 2E_1E_3(\cos(\theta) - 1)$$

$$Q = \sqrt{-q^2} = \sqrt{-2E_1E_3(\cos(\theta) - 1)} = \underline{\underline{542 \text{ MeV}}}$$

$$\text{c) } E_4 = E_1 - E_3 + m_p; \quad \mathbf{p}_4 = \sqrt{(E_1 - E_3\cos(\theta))^2 + E_3^2\sin(\theta)^2}$$

$$\beta = \frac{p}{E} = \underline{\underline{0.52}}$$

d)

$$x = \frac{Q^2}{2m_p(E_1 - E_3)} = \underline{\underline{1}}$$

e)

$$x = \frac{Q^2}{2m_p(E_1 - E_3)} = \underline{\underline{0.40}}$$

f)

$$W^2 = p_4^2 = (E_1 - E_3 \cos(\theta))^2 + E_3^2 \sin(\theta)^2 = \underline{\underline{0.318 \text{ GeV}^2}}$$

Zentraler relativistischer Stoß und Bethe-Bloch-Formel

$$\mathcal{P}_1^\mu = (E_1, \mathbf{p}_1), \quad \mathcal{P}_2^\mu = (m_e, \mathbf{0})$$

$$\text{a) } \tilde{\beta} = \frac{p}{E} = \frac{p_1}{E_1 + m_e} = \frac{\gamma \beta M}{\gamma M + m_e}$$

$$\mathcal{P}_{2i}^{v'} = \Lambda_{\mu}^{v'} \mathcal{P}_{2i}^{\mu} = (\tilde{\gamma} m_e, -\tilde{\beta} \tilde{\gamma} m_e)$$

$$\mathcal{P}_{2f}^{v'} = (\tilde{\gamma} m_e, \tilde{\beta} \tilde{\gamma} m_e)$$

$$\mathcal{P}_{2f}^{\mu} = \Lambda_{\nu}^{\mu} \mathcal{P}_{2f}^{v'} = (\tilde{\gamma}^2 m_e + \tilde{\beta}^2 \tilde{\gamma}^2 m_e, \tilde{\beta} \tilde{\gamma}^2 m_e + \tilde{\beta} \tilde{\gamma}^2 m_e)$$

$$T = \tilde{\gamma}^2 m_e + \tilde{\beta}^2 \tilde{\gamma}^2 m_e - m_e = m_e \frac{1 + \tilde{\beta}^2}{1 - \tilde{\beta}^2} - m_e = 2m_e \frac{\beta^2}{1 - \beta^2}$$

$$\begin{aligned} &= 2m_e \frac{\frac{\gamma^2 \beta^2 M^2}{(\gamma M + m_e)^2}}{\frac{(\gamma M + m_e)^2 - \gamma^2 \beta^2 M^2}{(\gamma M + m_e)^2}} = 2m_e \frac{\gamma^2 \beta^2 M^2}{(\gamma M + m_e)^2 - \gamma^2 \beta^2 M^2} \\ &= \frac{2m_e \gamma^2 \beta^2 M^2}{2\gamma M m_e + m_e^2 + \gamma^2 M^2 (1 - \beta^2)} = \underline{\underline{\frac{2m_e \gamma^2 \beta^2 M^2}{2\gamma M m_e + m_e^2 + M^2}}} \end{aligned}$$

$$\text{b) } \mathcal{P}_{\text{ges}}^{\mu} = (E_{\text{ges}}, \mathbf{p}_{\text{ges}})$$

$$\mathcal{P}_{\text{ges}}^{v'} = \Lambda_{\nu}^{\mu} \mathcal{P}_{\text{ges}}^{v'} = (E', \gamma E_{\text{ges}} - \gamma \beta \mathbf{p}_{\text{ges}}) \stackrel{!}{=} (E', \mathbf{0})$$

$$\mathbf{0} = \gamma E_{\text{ges}} - \gamma \beta \mathbf{p}_{\text{ges}} \quad \Longleftrightarrow \quad \beta = \frac{\mathbf{p}_{\text{ges}}}{E_{\text{ges}}}$$

Das Mesonen-Nonett

a) $\psi_1 = N_1 (|u\bar{u}\rangle + |d\bar{d}\rangle + |s\bar{s}\rangle)$

$$\langle\psi_1|\psi_1\rangle = N_1^2 (\langle u\bar{u}|u\bar{u}\rangle + \langle d\bar{d}|d\bar{d}\rangle + \langle s\bar{s}|s\bar{s}\rangle) = 3N_1^2 \stackrel{!}{=} 1$$

$$N_1 = \frac{1}{\sqrt{3}}$$

b) $\psi_2 = N_2 (|u\bar{u}\rangle \pm |d\bar{d}\rangle); \quad N_2 = \frac{1}{\sqrt{2}}$

$$\langle\psi_1|\psi_2\rangle = \frac{1}{\sqrt{6}} (\langle u\bar{u}|u\bar{u}\rangle \pm \langle d\bar{d}|d\bar{d}\rangle) = \frac{1}{\sqrt{6}} (1 \pm 1) \stackrel{!}{=} 0$$

$$\psi_2 = \frac{1}{\sqrt{2}} (|u\bar{u}\rangle - |d\bar{d}\rangle)$$

c) $\psi_3 = N_3 (|u\bar{u}\rangle + |d\bar{d}\rangle - 2|s\bar{s}\rangle); \quad N_3 = \frac{1}{\sqrt{6}}$

$$\langle\psi_3|\psi_3\rangle = 1$$

$$\langle\psi_3|\psi_2\rangle = 0$$

$$\langle\psi_3|\psi_1\rangle = 0$$

d)

$$\eta = |\psi_1\rangle = \frac{1}{\sqrt{3}} (|u\bar{u}\rangle + |d\bar{d}\rangle + |s\bar{s}\rangle)$$

$$\pi^0 = |\psi_2\rangle = \frac{1}{\sqrt{2}} (|u\bar{u}\rangle \pm |d\bar{d}\rangle)$$

$$\eta' = |\psi_3\rangle = \frac{1}{\sqrt{6}} (|u\bar{u}\rangle + |d\bar{d}\rangle - 2|s\bar{s}\rangle)$$