1. Influenz in einer Metallplatte

$$k = \frac{1}{4\pi\epsilon_0}; \quad r^2 = x^2 + y^2; \quad \sigma = \epsilon_0 E$$

$$\begin{aligned}
E_1(x, y, z) &= k \frac{Q}{x^2+y^2+(z+R)^2} \\
E_2(x, y, z) &= E_1(x, y, z - 2R) = k \frac{Q}{x^2+y^2+(z-R)^2} \\
E_{ges}(x, y, z) &= E_1 + E_2 = kQ \left(\frac{1}{x^2+y^2+(z+R)^2} + \frac{1}{x^2+y^2+(z-R)^2} \right)
\end{aligned}$$

$$\begin{aligned}
\sigma(r) &= \epsilon_0 E_{ges}(x, y, z) = \epsilon_0 kQ \left(\frac{1}{x^2+y^2+(z+R)^2} + \frac{1}{x^2+y^2+(z-R)^2} \right) \\
&= \underline{\underline{\frac{Q}{2\pi} \frac{1}{r^2+R^2}}}
\end{aligned}$$

2. Feld einer Hohlkugel

$$k = \frac{1}{4\pi\epsilon_0}$$

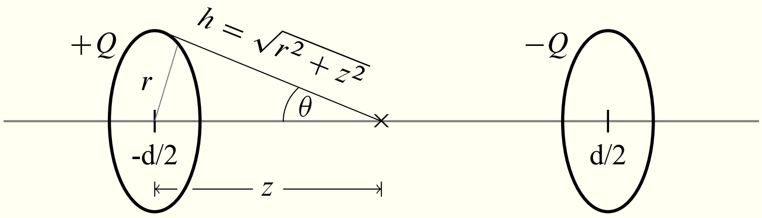
a. $\rho = \frac{Q}{V}$

$$\begin{aligned}
V &= \int_V dV = \int_0^{\pi} \int_0^{2\pi} \int_{R_i}^{R_a} r^2 \sin(\theta)\, \mathrm{d}r \mathrm{d}\varphi \mathrm{d}\theta \\
&= \frac{4}{3}\pi (R_a^3 - R_i^3) \\
\rho &= \underline{\underline{\frac{3Q}{4\pi} \frac{1}{R_a^3 - R_i^3}}}
\end{aligned}$$

3. Zwei ringförmige Ladungsträger

$$k = \frac{1}{4\pi\epsilon_0}$$

a. $h = \sqrt{r^2 + z^2}; \quad \cos(\theta) = \frac{z}{h}; \quad \mathrm{d}Q = \lambda \mathrm{d}r; \quad \mathrm{d}r = r\mathrm{d}\varphi; \quad Q = 2r\pi\lambda$



Due to the symmetric nature of this problem we can neglect vertical components of forces

$$\mathrm{d}E_1(z) = k\frac{\mathrm{d}Q}{h^2}\cos(\theta) = k\frac{\lambda \mathrm{d}r}{r^2+z^2}\frac{z}{\sqrt{r^2+z^2}} = k\frac{\lambda r z}{(r^2+z^2)^{3/2}}\mathrm{d}\varphi$$

$$E_1(z) = \int \mathrm{d}E_1 = k\frac{\lambda r z}{(r^2+z^2)^{3/2}}\int_0^{2\pi}\mathrm{d}\varphi = k\frac{\lambda 2\pi r z}{(r^2+z^2)^{3/2}} = k\frac{Qz}{(r^2+z^2)^{3/2}}$$

$$E_2(z) = -E_1(z-d) = -k\frac{Q(z-d)}{(r^2+(z-d)^2)^{3/2}}$$

$$E_{ges}(z) = E_1 + E_2 = k\frac{Qz}{(r^2+z^2)^{3/2}} - k\frac{Q(z-d)}{(r^2+(z-d)^2)^{3/2}}$$

$$= kQ\left(\frac{z+\frac{d}{2}}{\left(r^2+\left(z+\frac{d}{2}\right)^2\right)^{3/2}} + \frac{\frac{d}{2}-z}{\left(r^2+\left(z-\frac{d}{2}\right)^2\right)^{3/2}}\right)$$

b. $F = -m\omega^2 x; \quad \Delta x = x_1 - x_2$

$$m\ddot{x}_1 = -m\omega_a^2 x_1 - k(x_1 - x_2)$$

$$m\ddot{x}_2 = -m\omega_a^2 x_2 - k(x_2 - x_1)$$

$$\Rightarrow m(\ddot{x}_1 - \ddot{x}_2) = -m\omega_a^2(x_1 - x_2) - 2k(x_1 - x_2) = -(m\omega_a^2 + 2k)(x_1 - x_2)$$

$$\Delta\ddot{x} = -\underbrace{(\omega_a^2 + \tfrac{2k}{m})}_{=\omega_b}\Delta x$$

$$\omega_b = \sqrt{\omega_a^2 + \frac{2k}{m}} = \underline{\underline{6.43\ \mathrm{rad/s}}}$$

c. $\delta\omega = \omega_b - \omega_a$

$0 = \cos\left(\frac{1}{2}\delta\omega t\right)$

$t = \frac{2\arccos(0)}{\delta\omega} = \underline{\underline{50.29\ \mathrm{s}}}$