Wiensches Verschiebungsgesetz

$$\rho(\nu)\,\mathrm{d}\nu = \frac{8\pi h\nu^3}{c^3}\frac{\mathrm{d}\nu}{\mathrm{e}^{\frac{h\nu}{k_B T}}-1}$$

a) $\nu = \frac{c}{\lambda}; \quad \mathrm{d}\nu = -\frac{c}{\lambda^2}\,\mathrm{d}\lambda$

$$\rho(\nu)\,\mathrm{d}\nu = \frac{8\pi h\nu^3}{c^3}\frac{\mathrm{d}\nu}{\mathrm{e}^{\frac{h\nu}{k_B T}}-1}$$

$$\rho(\lambda)\,\mathrm{d}\lambda = \underline{\underline{\frac{8\pi ch}{\lambda^5}\frac{\mathrm{d}\lambda}{\mathrm{e}^{\frac{hc}{\lambda k_B T}}-1}}}$$

b)

$$\frac{\partial\rho(\lambda)}{\partial\lambda} = \frac{8\pi c^2h^2\,\mathrm{e}^{\frac{hc}{\lambda k_B T}}}{k_B\lambda^7 T\left(\mathrm{e}^{\frac{hc}{\lambda k_B T}}-1\right)^2} - \frac{40\pi ch}{\lambda^6\left(\mathrm{e}^{\frac{hc}{\lambda k_B T}}-1\right)} = 0$$

$$\frac{hc}{\lambda k_B T}\frac{\mathrm{e}^{\frac{hc}{\lambda k_B T}}}{\mathrm{e}^{\frac{hc}{\lambda k_B T}}-1} - 5 = 0$$

Using the Lambert W function and numeric approximation one get that

$$\lambda_{\max} \approx \underline{\underline{\frac{2.88\ \mu\mathrm{K}}{T}}}$$

Photoeffekt

$$W = 2.9\ \mathrm{eV}$$

a) $E > \underline{\underline{W = 2.9\ \mathrm{eV}}}$

b) $E = hf; \quad \lambda = \frac{c}{f}$

$$\lambda = \frac{ch}{E} = \underline{\underline{4.28\times10^{-7}\ \mathrm{m}}}$$

c) $\lambda = 400\ \mathrm{nm}; \quad I_0 = 1\ \mathrm{mA}$

$$U = -\frac{h\nu - W}{e} = \frac{\lambda W - hc}{e\lambda}$$
$$P_0 = UI_0 = \underline{\underline{-1.996 \times 10^{-4}\ \mathrm{W}}}$$

d)

$$UI_1 = \frac{P_0}{2} = \frac{UI_0}{2}$$
$$I_1 = \frac{I_0}{2} = \underline{\underline{I_1 = 0.5\ \mathrm{mA}}}$$

e)

$$UI_2 = \frac{\lambda W - 2hc}{e\lambda} I_2 = P_0 = \frac{\lambda W - hc}{e\lambda} I_0$$
$$I_2 =$$

f) $\lambda > 450\ \mathrm{nm}$

$$U_3 = \underline{\underline{-0.14\ \mathrm{V}}}$$

For $\lambda > 428\ \mathrm{nm}$ the Voltage U drops below 0 and therefore no no electrons are freed from their atoms. The photons don't carry enough energy at longer wavelengths for the electrons to overcome the attractive force of the atom core.

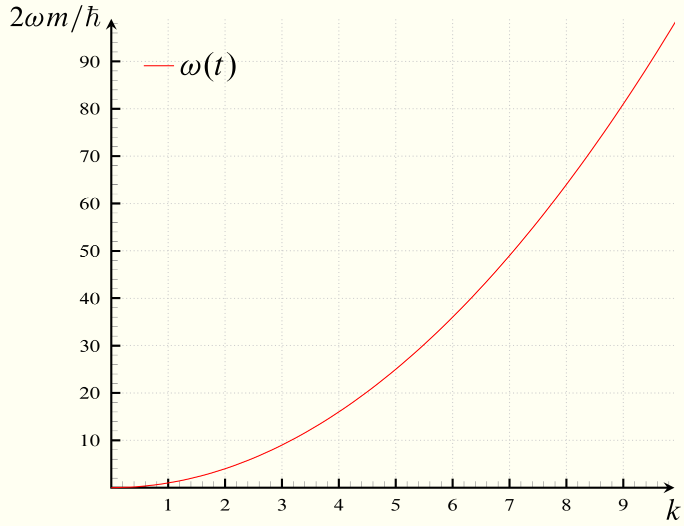
Zerfließen eines Gauß-Pakets

$$\psi(x,t) = \frac{\sqrt{a}}{(2\pi)^{3/4}} \int_{-\infty}^{\infty} \exp\left(-\frac{a^2}{4}(k-k_0)^2 \right) \exp\Big(i\,(kx - \omega(k)t) \Big)\,\mathrm{d}k$$

a) $\omega(k) = \frac{\hbar k^2}{2m}$

$$v_\mathrm{g} = \frac{\partial \omega}{\partial k} = \underline{\underline{\frac{k\hbar}{m}}}$$

Dispersion happens, as the group velocity still depends upon k



b) $b = k - k_0$; $\quad \alpha = \sqrt{\frac{a^2}{4} + \frac{i\hbar t}{2m}}$; $\quad \beta = i\,x - \frac{i\hbar k_0 t}{m}$; $\quad \phi = -\frac{k_0^2\hbar}{2m}t - \frac{1}{2}\arctan\left(\frac{2\hbar t}{ma^2}\right)$

$$\psi(x,t) = \frac{\sqrt{a}}{(2\pi)^{3/4}} \int\limits_{-\infty}^{\infty} \exp\left(-\frac{a^2}{4}(k-k_0)^2\right) \exp\left(i\,(kx - \omega(k)t)\right) \mathrm{d}k =$$

$$\overset{\text{u-sub}}{=} \frac{\sqrt{a}}{(2\pi)^{3/4}} \int\limits_{-\infty}^{\infty} \exp\left(-\frac{a^2}{4}b^2\right) \exp\left(i\,(k(b+k_0) - \frac{\hbar(b+k_0)^2}{2m}t)\right) \mathrm{d}b =$$

$$= \frac{\sqrt{a}}{(2\pi)^{3/4}} \exp\left(i\,k_0 x\right) \exp\left(-i\,t\,\frac{k_0^2\hbar}{2m}\right) \int\limits_{-\infty}^{\infty} \exp\left(\beta b\right) \exp\left(-\alpha^2 b^2\right) \mathrm{d}b =$$

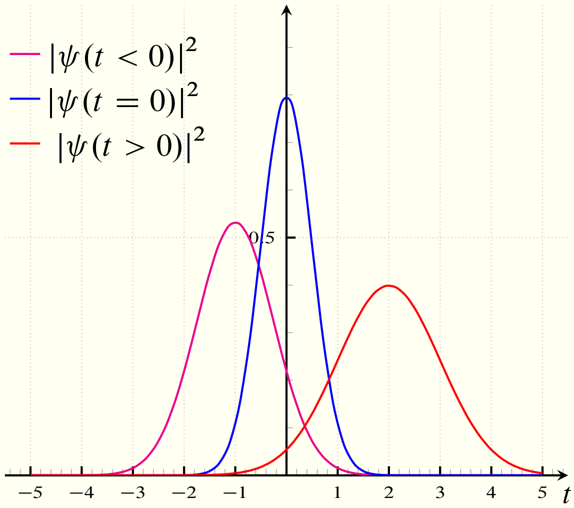
$$= \frac{\sqrt{a}}{(2\pi)^{3/4}} \exp\left(i\,k_0 x\right) \exp\left(i\,\phi\right) \frac{\exp\left(\frac{\beta^2}{4\alpha^2}\right)\sqrt{\pi}}{\alpha} =$$

$$= \underline{\underline{\left(\frac{2a^2}{\pi}\right)^{1/4} \frac{\exp\left(i\,\phi\right)}{\left(a^4 + \frac{4\hbar^2 t^2}{m^2}\right)^{1/4}} \exp\left(i\,k_0 x\right) \exp\left(\frac{\left(x - \frac{\hbar k_0}{m}t\right)^2}{a^2 + \frac{2i\hbar t}{m}}\right)}}$$

$$|\psi(x,t)|^2 = \sqrt{\frac{2a^2}{\pi\left(a^4 + \frac{4\hbar^2 t^2}{m^2}\right)}} \exp\left(2i\,k_0 x\right) \exp\left(2i\,\phi\right) \exp\left(\frac{\left(x - \frac{\hbar k_0}{m}t\right)^2}{a^2 + \frac{2i\hbar t}{m}}\right) =$$

$$= \underline{\underline{\sqrt{\frac{1}{2\sigma^2(t)}} \exp\left(-\frac{\left(x - \frac{\hbar k_0}{m}t\right)^2}{2\sigma^2(t)}\right)}}$$

c)



$$-\frac{1}{2} = -\frac{\left(2\delta x(t) - \frac{\hbar k_0}{m} t\right)^2}{2\sigma^2(t)} \quad \Leftrightarrow \quad 2\delta x(t) = \sqrt{\sigma^2(t)} - \frac{\hbar k_0}{m} t$$

$$\delta x(t) = \underline{\underline{\frac{\sqrt{\sigma^2(t)} - \frac{\hbar k_0}{m} t}{2}}}$$