177. Stabpendel mit Kugeln

$$L=0.3 \ {
m m}; \hspace{1cm} r_1=0.02 \ {
m m}; \hspace{1cm} r_2=0.03 \ {
m m} \ m_0=0.1 \ {
m kg}; \hspace{1cm} m_1=0.1 \ {
m kg}; \hspace{1cm} m_2=0.3 \ {
m kg}$$

a)
$$\begin{split} I_{ges} &= I_{Stab} + I_{Kugel_1} + I_{Kugel_2} \\ I_{Stab} &= \frac{1}{12} m_0 L^2 = 7.5 * 10^{-4} \text{ kg m}^2 \\ I_{Kugel_1} &= \frac{2}{5} m_1 r_1^2 + m_1 (\frac{L}{2} + r_1)^2 = 2.9 * 10^{-3} \text{ kg m}^2 \\ I_{Kugel_2} &= \frac{2}{5} m_2 r_2^2 + m_2 (\frac{L}{2} + r_2)^2 = 9.8 * 10^{-3} \text{ kg m}^2 \\ &\Rightarrow I_{ges} &= \underline{1.3 * 10^{-2} \text{ kg m}^2} \end{split}$$

b)

$$x_{Stabsp} = 0 \text{ m}$$

$$x_{Kugeln_{sp}} = \frac{m_1 \left(-\frac{2r_1 + L}{2}\right) + m_2 \left(\frac{2r_2 + L}{2}\right)}{m_1 + m_2} = 0.0925 \text{ m}$$

$$x_{Stabsp} * m_0 + x_{Kugeln_{sp}} * (m_1 + m_2)$$

$$x_{sp} = \frac{x_{Stabsp} * m_0 + x_{Kugeln_{sp}} * (m_1 + m_2)}{m_0 + m_1 + m_2} = \underline{0.074} \text{ m}$$

c)
$$T = 2\pi \sqrt{\frac{I}{Mgl}}$$

 $T = 2\pi \sqrt{\frac{I}{(m_0 + m_1 + m_2) g\left(\frac{2r_2 + L}{2}\right)}}$
 $= 0.69 \text{ s}$

184. Resonanzverhalten einer Stimmgabel

$$f_0 = 440 \text{ Hz}; \quad \omega_0 = 2\pi f_0; \quad x(\tau) = x_m e^{-\beta \tau/2}; \quad x(5) = \frac{1}{10}x(0)$$

a)

$$\frac{x_m}{10} = x_m e^{-\beta 5/2}$$
 $\beta = -\frac{2}{5} \ln(0.1)$

$$\tau = \frac{1}{\beta} = \underline{1.09 \text{ s}}$$

b)
$$\omega = \sqrt{\omega_0^2 - \frac{\beta^2}{4}}$$

$$\omega = 2764.6 \text{ Hz} \qquad (\omega_0 \approx \omega)$$

$$f = \frac{\omega}{2\pi} = 440 \text{ Hz} \qquad (f_0 \approx f)$$

$$f * \tau = \underline{477.7}$$

c)
$$Q = \frac{\omega}{\beta}$$

$$Q = 3001.63$$

d)
$$\delta f = \frac{f}{Q}$$

$$\delta f = 0.15 \text{ Hz}$$

e)

185. Resonante Anregung eines Federpendels

$$m = 0.1 \text{ kg}; \quad k = 40 \text{ N/m}$$

a)
$$\omega = \sqrt{\frac{k}{m}} = 20 \text{ rad/s}; \quad \omega_0 = \sqrt{\frac{k}{m} - \frac{\beta^2}{4m^2}}$$

$$\begin{array}{l} \frac{x_m}{2} = x_m * e^{-\beta t/2} \\ \beta = -\frac{\ln(0.5)2}{t} = \underline{1.4 * 10^{-2} \text{ kg/s}} \end{array}$$

$$\omega_0 = \sqrt{\frac{k}{m} - \frac{\beta^2}{4m^2}} = \underline{19.99988 \text{ rad/s}}$$

b)

$$Q = \omega_0 \frac{m}{\beta} = \underline{144.269}$$