Einheiten der Kern- & Teilchenphysik & Raumwinkel

(a) $\Delta\phi = 1\ \mathrm{V}; \quad q_\mathrm{e} = e$

$$\Delta E = q_\mathrm{e}\Delta\phi = 1.602 \times 10^{-19}\ \mathrm{CJ/C} = \underline{\underline{1.602 \times 10^{-19}\ \mathrm{J} = 1\ \mathrm{eV}}}$$

(b)

(i) $m_\mathrm{e} = 0.510999\ \mathrm{MeV/c^2} = 0.510999\frac{e}{c^2}\ \mathrm{J/C} = \underline{\underline{9.11 \times 10^{-31}\ \mathrm{kg}}}$

(ii) $m_\mathrm{p} = 938.272\ \mathrm{MeV/c^2} = 938.272\frac{e}{c^2}\ \mathrm{J/C} = \underline{\underline{1.67 \times 10^{-27}\ \mathrm{kg}}}$

(iii) $m_\mathrm{d} = 1875.613\ \mathrm{MeV/c^2} = 1875.613\frac{e}{c^2}\ \mathrm{J/C} = \underline{\underline{3.34 \times 10^{-27}\ \mathrm{kg}}}$

(c) Der Raumwinkel beschreibt das Verhältnis zwischen der Teilfläche einer Kugel zum Kugelradius r zum Quadrat.

$$\mathrm{d}\Omega = \frac{\mathrm{d}A}{r^2} = \underline{\underline{\sin(\theta)\,\mathrm{d}\theta\,\mathrm{d}\varphi}}$$

(d)

$$A = 2\pi r^2$$
$$\Omega = \frac{A}{r^2} = \underline{\underline{2\pi\ \mathrm{sr}}}$$
$$\Omega = 2\pi\left(\frac{180}{\pi}\right)^2 = \underline{\underline{\frac{64800}{\pi}\ \mathrm{deg}^2 = 20626.5\ \mathrm{deg}^2}}$$

(e) $\theta = 1°$

$$A = 2\pi r^2\left(1 - \cos(\theta) + \frac{1}{2}\sin^2(\theta)\right)$$
$$\Omega = \frac{A}{r^2} = 2\pi\left(1 - \cos(\theta) + \frac{1}{2}\sin^2(\theta)\right) = \underline{\underline{5.11284\ \mathrm{sr}}}$$

Relativistische Formel der Energie

(a) $p = \gamma m_0 v; \quad \gamma = \frac{1}{\sqrt{1-\beta^2}}; \quad \beta = \frac{v}{c}; \quad F = \frac{\mathrm{d}p}{\mathrm{d}t}$

$$E_{\text{kin}} = \int_0^r F \,\mathrm{d}\tilde{r} = \int_0^r \frac{\mathrm{d}p}{\mathrm{d}t} \,\mathrm{d}\tilde{r} = \int_0^r \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{m_0 v}{\sqrt{1-\beta^2}}\right) \mathrm{d}\tilde{r}$$

(b) $\mathrm{d}p = m_0 \frac{\mathrm{d}(\gamma v)}{\mathrm{d}t} \,\mathrm{d}t = m_0 \gamma (1 + \gamma^2 \beta^2) \,\mathrm{d}v$

$$E_{\text{kin}} = \int_0^r \frac{\mathrm{d}p}{\mathrm{d}t} \,\mathrm{d}\tilde{r} = \int_0^p \frac{\mathrm{d}r}{\mathrm{d}t} \,\mathrm{d}\tilde{p} = m_0 \int_0^v \gamma \tilde{v} \left(1 + \gamma^2 \beta^2\right) \mathrm{d}\tilde{v} = \underline{\underline{m_0 c^2 (\gamma - 1)}}$$

(c) $\gamma \approx 1 + \frac{1}{2}\beta^2$

$$E_{\text{kin}} = m_0 c^2 (\gamma - 1) \approx m_0 c^2 \left(1 + \tfrac{1}{2}\beta^2 - 1\right) = \underline{\underline{\frac{m_0 v^2}{2}}}$$

Streuung an harter Kugel

(a) $2\alpha + \theta = \pi; \quad R = R_1 + R_2$

$$\begin{aligned} \frac{b}{R} &= \sin(\alpha) = \sin\left(\tfrac{\pi}{2} - \tfrac{\theta}{2}\right) = \cos\left(\tfrac{\theta}{2}\right) \\ b &= R\cos\left(\tfrac{\theta}{2}\right) \\ \mathrm{d}b &= -\tfrac{R}{2}\sin\left(\tfrac{\theta}{2}\right) \mathrm{d}\theta \end{aligned}$$

(b)

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \left|\frac{b\,\mathrm{d}b}{\sin(\theta)\,\mathrm{d}\theta}\right| = \left|\frac{\left(R\cos\left(\frac{\theta}{2}\right)\right)\left(-\frac{R}{2}\sin\left(\frac{\theta}{2}\right)\mathrm{d}\theta\right)}{\sin(\theta)\,\mathrm{d}\theta}\right| = \left|\frac{R^2\cos\left(\frac{\theta}{2}\right)\sin\left(\frac{\theta}{2}\right)}{2\sin(\theta)}\right| = \underline{\underline{\frac{R^2}{4}}}$$

Da der differentielle Wirkungsquerschnitt nicht vom Winkel θ abhängt, treten alle Streuwinkel gleich häufig auf.

(c)

$$\sigma_{\text{tot}} = \int_0^{\sigma} \mathrm{d}\tilde{\sigma} = \int_0^{\Omega} \frac{R^2}{4}\,\mathrm{d}\tilde{\Omega} = \frac{R^2}{4}\int_0^{\theta} \sin\left(\tilde{\theta}\right)\mathrm{d}\tilde{\theta}\int_0^{2\pi}\mathrm{d}\phi = \underline{\underline{-\frac{R^2\pi}{2}\left(\cos(\theta)-1\right)}}$$

Der totale Wirkungsquerschnitt hängt vom Winkel θ ab und nimmt Werte zwischen $-R^2\pi$ und $R^2\pi$ an.

(d) $\frac{\pi}{2} < \theta < \frac{3\pi}{2}$

$$\sigma_{\text{rück}} = \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} -\frac{R^2\pi}{2}\left(\cos(\theta)-1\right)\mathrm{d}\theta = -\frac{R^2\pi}{2}\Big[\sin(\theta)-\theta\Big]_{\frac{\pi}{2}}^{\frac{3\pi}{2}} = \underline{\underline{R^2\pi(1+\tfrac{\pi}{2})}}$$