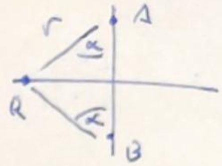
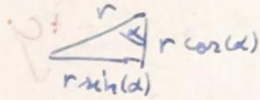
$\vec{x} = \begin{pmatrix} x \\ 0 \end{pmatrix} \Rightarrow |\vec{x}| = x$



$|r| = \sqrt{1+x^2}$

$\alpha = \sin^{-1}\left(\frac{x}{r}\right)$



1) $F_G = m\, MG \cdot \frac{1}{r^2}$

$\vec{F}_A = m_A MG \cdot \frac{1}{1+x^2} \begin{pmatrix} \sin(\sin^{-1}(\frac{x}{r})) \\ \cos(\sin^{-1}(\frac{x}{r})) \end{pmatrix}$

$\vec{F}_B = m_B MG \frac{1}{1+x^2} \begin{pmatrix} \frac{x}{r} \\ \cos(\sin^{-1}(\frac{x}{r})) \end{pmatrix}$

$\vec{F}(x) = \frac{m_{A,B}\, MG \cdot 2x}{\sqrt{1+x^2}^{\,3}} \begin{pmatrix} -1 \\ 0 \end{pmatrix}$

? Annahme aus Angabe $m_A = m_B$?

? Annahme aus Angabe $mMG = 1$?

$\vec{F}(x) = \frac{2x}{(1+x^2)^{\frac{3}{2}}} \begin{pmatrix} -1 \\ 0 \end{pmatrix}$ ✓

b) $\vec{F}'(x) = \frac{6x^2}{(1+x^2)^{\frac{5}{2}}} + \frac{2}{(1+x^2)^{\frac{3}{2}}} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ ✓

$\vec{F}''(x) = \frac{30x^3}{(1+x^2)^{\frac{7}{2}}} - \frac{18x}{(1+x^2)^{\frac{5}{2}}} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ ✓

c) $F'(x) = 0$

$x = \pm \frac{1}{\sqrt{2}}$ Bei $x = \frac{1}{\sqrt{2}}$ $F''(x) \lessgtr 0 \Rightarrow$ ~~maximum~~ minimum ✓

Bei $x = -\frac{1}{\sqrt{2}}$ $F''(x) > 0 \Rightarrow$ maximum ✓

d) $F''(x) = 0$

$x = \pm\sqrt{\frac{3}{2}}$ ✓

$x = 0$ ✓

e) $\lim_{x \to \infty} F(x) = 0$ ✓, F_G wird mit zunehmender Entfernung ab ~~...~~ $\left(\frac{1}{r^2}\right)$

f) $\ddot{x} = F(x)$ $\frac{dx^2}{d^2t} = -\frac{2x}{(1+x^2)^{\frac{3}{2}}}$ Für $|x| \ll 1 \Rightarrow -2x$

~~$\frac{dF^2}{d^2x} = -2x$~~

g) $\underbrace{\frac{dx^2}{d^2t}}_{a(t)} = -\underbrace{2}_{\omega^2}\underbrace{x}_{x(t)}$ $\underline{\underline{\omega = \sqrt{2}}}$ ✓

$a(t) = -\omega^2 x(t)$

e) $W = \int F(x)\,dx$

$u = 1 + x^2$

$du = 2x$

$= \int -\frac{2x}{(1+x^2)^{3/2}}\,dx =$

$= -\int \frac{1}{u^{3/2}}\,du =$

$= -\left(-2u^{-\frac{1}{2}}\right)$

$= \frac{2}{\sqrt{1+x^2}} + C$

f) $\Delta W = \int_0^\infty F(x)\,dx$

$= \lim_{x \to \infty} \left[\frac{2}{\sqrt{1+x^2}} - 2\right] =$

$= -2 \,\frac{kg\,m^2}{s^2}$

ΔW ist die Änderung der Pot. Gravitationsenergie über den Weg $r \in [0;\infty]$.

2) $\frac{dx}{dt} - kx(x_m - x) = 0$

$$\frac{dx}{dt} = k \cdot x\left(1 - \frac{x}{x_m}\right)$$

$$k = \frac{dx}{dt} \frac{1}{x\left(1 - \frac{x}{x_m}\right)}$$

$$k = \left(\frac{1}{x} - \frac{-\frac{1}{x_m}}{1 - \frac{x}{x_m}}\right) \frac{dx}{dt}$$

$$k - \frac{1}{x}\frac{dx}{dt} - \frac{-\frac{1}{x_m}}{1 - \frac{x}{x_m}} \frac{dx}{dt}$$

$$kt = \ln(x) - \ln\left(1 - \frac{x}{x_m}\right) + c$$

$$kt + c = \ln\left(\frac{x}{1 - \frac{x}{x_m}}\right)$$

$$\frac{x}{1 - \frac{x}{x_m}} = C e^{kt}$$

$$x(t) = \frac{1}{c\, e^{-kt} + \frac{1}{x_m}}$$

$$\frac{A}{x} + \frac{B}{1 - \frac{x}{x_m}} = \frac{1}{x\left(1 - \frac{x}{x_m}\right)}$$

$$A = 1$$

$$-\frac{Ax}{x_m} + Bx = 0 \Rightarrow B = \frac{1}{x_m}$$

b) $x_m = 130.000$, $x(0) = 100$, $x(20) = 1000$

$\cancel{x(t) = 100 \cdot 13.000}$

$$100 = \frac{1}{c + \frac{1}{x_m}} \Rightarrow c = \frac{1}{100} - \frac{1}{130000} = \underline{0.00992}$$

$$x(t) = \frac{100 \cdot 130000}{(130000 - 100)\, e^{-kt} + 100}$$

$$1000 = \frac{1.3 \cdot 10^7}{129900\, e^{-20k} + 100}$$

$$-20k = \ln\left(\frac{1.3 \cdot 10^4 - 100}{129900}\right)$$

$$k = -\ln\left(\frac{129}{1299}\right) \cdot \frac{1}{20}$$

$$\underline{k = 0{,}1155}$$

$$\Rightarrow x(t) = \frac{1{,}3 \cdot 10^7}{129900\, e^{-0{,}1155t} + 100}$$

c)

$\cancel{x(t) = 120\,000}$ $\frac{1{,}3 \cdot 10^7}{129900\, e^{-0{,}1155t} + 100} = 120000 \Rightarrow t = \underline{83{,}6 \text{ tage}}$