132. Freier Fall einer rotierenden Hantel

$m_1 = 1$ kg; $\vec{v}_1 = (4, 0, 0)$ m/s; $\vec{r}_1 = (0, 0, 2.5)$ m
$m_2 = 0.5$ kg; $\vec{v}_2 = (1, 0, 0)$ m/s; $\vec{r}_2 = (0, 0, 2.2)$ m

$$\vec{r}_{sp} = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2} = \begin{pmatrix} 0 \\ 0 \\ 2.4 \text{ m} \end{pmatrix}$$

a) Im bewegten Bezugssystem, das sich mit 3 m/s mitbewegt reine Rotation

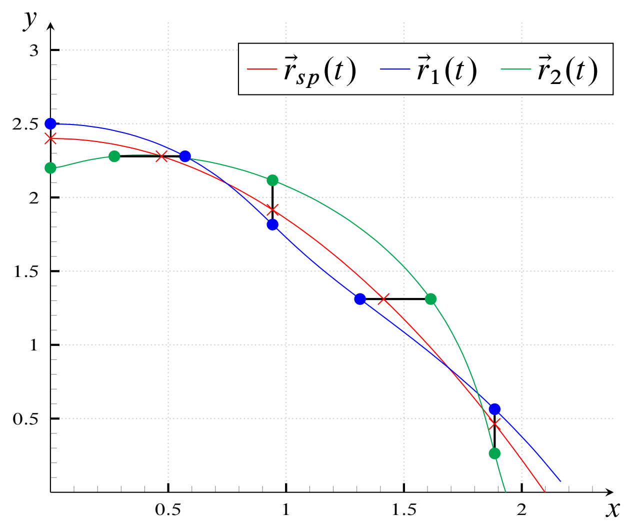
$$\Rightarrow \vec{r}_{sp}(t) = \underline{\underline{\begin{pmatrix} 0 \\ 0 \\ 2.4 \text{ m} \end{pmatrix} + \begin{pmatrix} 3 \text{ m/s} \\ 0 \\ 0 \end{pmatrix} t + \frac{1}{2} \begin{pmatrix} 0 \\ 0 \\ -9.81 \text{ m/s}^2 \end{pmatrix} t^2}}$$

b) Im bewegten Bezugssystem: $\vec{v}_1 = (1, 0, 0)$ m/s; $\vec{v}_1 = (-2, 0, 0)$ m/s

$\omega = \frac{|v_1|}{r_1} = \underline{\underline{10 \text{ s}^{-1}}}$

$T = \frac{2\pi}{\omega} = \frac{\pi}{5} = \underline{\underline{0.63 \text{ s}}}$

c)



d) Im Schwerpunktsystem:

$$\vec{r}_{1,sp}(t) = 0.1 \begin{pmatrix} \sin(\omega t) \\ 0 \\ \cos(\omega t) \end{pmatrix} \text{ m}$$

$$\vec{r}_{2,sp}(t) = -0.2 \begin{pmatrix} \sin(\omega t) \\ 0 \\ \cos(\omega t) \end{pmatrix} \text{ m}$$

Im Laborsystem:

$$\vec{r}_1(t) = \vec{r}_{sp}(t) + \vec{r}_{1,sp}(t)$$

$$= \underline{\underline{\begin{pmatrix} 0.1\sin(\omega t)\ \mathrm{m} \\ 0 \\ 0.1\cos(\omega t)\ \mathrm{m} + 2.4\ \mathrm{m} \end{pmatrix} + \begin{pmatrix} 3\ \mathrm{m/s} \\ 0 \\ 0 \end{pmatrix} t + \frac{1}{2} \begin{pmatrix} 0 \\ 0 \\ -9.81\ \mathrm{m/s^2} \end{pmatrix} t^2}}$$

$$\vec{r}_2(t) = \vec{r}_{sp}(t) + \vec{r}_{2,sp}(t)$$

$$= \underline{\underline{\begin{pmatrix} -0.2\sin(\omega t)\ \mathrm{m} \\ 0 \\ -0.2\cos(\omega t)\ \mathrm{m} + 2.4\ \mathrm{m} \end{pmatrix} + \begin{pmatrix} 3\ \mathrm{m/s} \\ 0 \\ 0 \end{pmatrix} t + \frac{1}{2} \begin{pmatrix} 0 \\ 0 \\ -9.81\ \mathrm{m/s^2} \end{pmatrix} t^2}}$$

140. International Space Station

$$R_E = 6.37 * 10^6\ \mathrm{m}; \quad m = 4.55 * 10^5\ \mathrm{kg}; \quad h = 3.3 * 10^5\ \mathrm{m}$$

a) $h = 0$ m

$$F_G = mg \frac{1}{\left(1 + \frac{h}{R_E}\right)^2}$$
$$= 4.55 * 10^5\ \mathrm{kg} * 9.81\ \mathrm{m/s^2}$$
$$F_G = \underline{\underline{4.46 * 10^6\ \mathrm{kg\,m/s^2}}}$$

b) $h = 3.3 * 10^5$ m

$$F_G = mg \frac{1}{\left(1 + \frac{h}{R_E}\right)^2}$$
$$= 4.55 * 10^5\ \mathrm{kg} * 9.81\ \mathrm{m/s^2} \frac{1}{\left(1 + \frac{3*10^5\ \mathrm{m}}{6.37*10^6\ \mathrm{m}}\right)}$$
$$F_G = \underline{\underline{4.03 * 10^6\ \mathrm{kg\,m/s^2}}}$$

$$g(h) = \frac{g}{\left(1 + \frac{h}{R_E}\right)^2}$$
$$g(3.3 * 10^5) = \underline{\underline{8.87\ \mathrm{m/s^2}}}$$

$$v = r\omega; \quad a = \frac{v^2}{r} = r\omega^2; \quad r = R_E + h$$

c)

$$\omega = \sqrt{\frac{g(h)}{r}} = \underline{\underline{1.2 * 10^{-3}\ \mathrm{s}^{-1}}}$$

d)

$$T = \frac{2\pi}{\omega} = \underline{\underline{5461.58\ \mathrm{s}}}$$

e)

$$v = r\omega = \underline{\underline{7707.91\ \mathrm{m/s}}}$$

152. Komprimierter Metallblock

$$a = h = 0.2\ \mathrm{m} \qquad \Delta h = 1.3 * 10^{-6}\ \mathrm{m} \qquad m = 500\ \mathrm{kg}$$

$$V = 8 * 10^{-3}\ \mathrm{m}^3 \qquad \Delta V = 3 * 10^{-7}\ \mathrm{m}^3 \qquad p = 2 * 10^6\ \mathrm{kg/m\,s^2}$$

a) $p = K\ \frac{\Delta V}{V}; \quad \frac{F}{A} = E\ \frac{\Delta h}{a}$

$$K = p\ \frac{V}{\Delta V} = \underline{\underline{\frac{16}{3} * 10^{10}\ \mathrm{kg/m\,s^2}}}$$

$$E = \frac{mga}{a^2 \Delta h} = \underline{\underline{1.89 * 10^{10}\ \mathrm{kg/m\,s^2}}}$$

b) $\Delta a = \mu \Delta h; \quad K = \frac{E}{3(1-2\mu)}$

$$K = \frac{E}{3(1-2\mu)}$$
$$\frac{3K}{E} = \frac{1}{1-2\mu}$$
$$\mu = \frac{1}{2} - \frac{E}{6K} = \underline{\underline{0.44}}$$

$$\Delta a = \mu \Delta h = \underline{\underline{5.73 * 10^{-7}\ \mathrm{m}}}$$