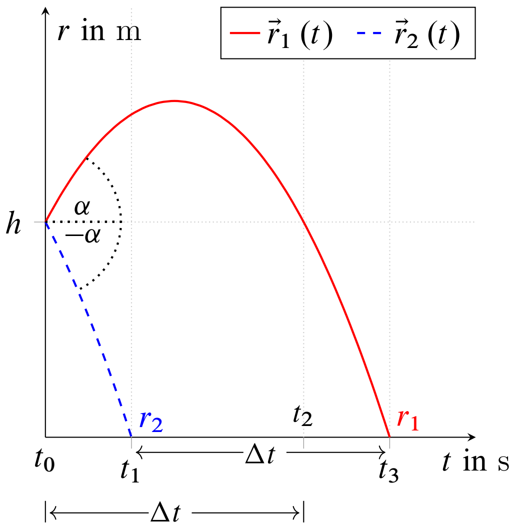
20. Wurf von einem Balkon



a)

$$\vec{r}_1(t) = \underline{\underline{\begin{pmatrix} v_0\cos(\alpha)t \\ h + v_0\sin(\alpha)t - \frac{g}{2}t^2 \end{pmatrix}}}$$

$$\vec{r}_2(t) = \begin{pmatrix} v_0\cos(-\alpha)t \\ h + v_0\sin(-\alpha)t - \frac{g}{2}t^2 \end{pmatrix}$$

$$r_x(t) = v_0\cos(\pm\alpha)t$$

$$r_z(t) = h + v_0\sin(\pm\alpha)t - \frac{g}{2}t^2$$

$$\vec{v}(t) = \underline{\underline{\begin{pmatrix} v_0\cos(\alpha) \\ v_0\sin(\alpha) - gt \end{pmatrix}}}$$

$$v_x(t) = v_0\cos(\alpha)$$

$$v_z(t) = v_0\sin(\alpha) - gt$$

b)

$$0 = r_z(t)$$

$$0 = h + v_0\sin\alpha t - \frac{g}{2}t^2$$

$$t_{i,ii} = \frac{-v_0\sin\alpha \pm \sqrt{v_0^2\sin^2\alpha + 2gh}}{-g}$$

$$t_3 = \underline{\underline{\frac{-v_0\sin\alpha - \sqrt{v_0^2\sin^2\alpha + 2gh}}{-g}}}$$

c)

$$|v_1| = \sqrt{v_x(t_3)^2 + v_z(t_3)^2} = \sqrt{(v_0\cos\alpha)^2 + (v_0\sin\alpha - gt_3)^2} =$$

$$= \sqrt{(v_0\cos\alpha)^2 + \left(v_0\sin\alpha - g\,\frac{v_0\sin\alpha + \sqrt{v_0^2\sin^2\alpha + 2gh}}{g}\right)^2} =$$

$$= \sqrt{v_0^2\cos^2\alpha + v_0^2\sin^2\alpha + 2gh} =$$

$$|v_1| = \underline{\underline{\sqrt{v_0^2 + 2gh}}}$$

d)

$$
\begin{aligned}
r_z(t) &= h \\
h &= h + v_0 \sin \alpha t - \tfrac{g}{2} t^2 \\
t_{i,ii} &= \tfrac{-v_0 \sin\alpha \pm \sqrt{v_0^2 \sin^2\alpha}}{-g} \\
t_i &= t_0 = \tfrac{-v_0 \sin\alpha + \sqrt{v_0^2 \sin^2\alpha}}{-g} = 0 \\
t_{ii} &= t_3 = \tfrac{-v_0 \sin\alpha - \sqrt{v_0^2 \sin^2\alpha}}{-g} = \tfrac{2v_0 \sin\alpha}{g}
\end{aligned}
$$

$$
\begin{aligned}
\Delta t &= t_3 - t_1 = t_2 - t_0 \\
\Delta t &= \tfrac{2v_0 \sin\alpha}{g} - 0 \\
\Delta t &= \underline{\underline{\tfrac{2v_0 \sin\alpha}{g}}}
\end{aligned}
$$

e) Δt ist die Zeit, die die Kugel mit der steileren Trajektorie braucht, um wieder auf der Starthöhe h zu sein, weil ab dem Zeitpunkt t_2 die Bewegungen der zwei Kugeln ident sind. Hierbei ist es egal auf welcher Höhe man startet, da es nicht um den Betrag der Höhe geht, sonder nur darum, wieder auf der anfänglichen Höhe zu sein.

21. Golf

a) $l = 220\ \text{m}, \quad v_0 = 50\ \text{m/s}, \quad g = 9.81\ \text{m/s}^2$

$$
\vec{r}(t) = \begin{pmatrix} v_0 \cos \alpha t \\ v_0 \sin \alpha t - \frac{g}{2} t^2 \end{pmatrix} \quad \Rightarrow \quad \begin{aligned} x(t) &= v_0 \cos \alpha t \\ z(t) &= v_0 \sin \alpha t - \tfrac{g}{2} t^2 \end{aligned}
$$

$$
\begin{aligned}
z(t) &= 0 \\
t_1 &= 0, \quad t_2 = \tfrac{2v_0}{g} \sin\alpha \\
x(t_2) &= l = \tfrac{v_0^2}{g} \sin 2\alpha \\
\alpha &= \frac{\arcsin\left(\frac{200g}{v_0^2}\right)}{2} = 29.84° \\
\alpha_{1,2} &= 45° \pm (45° - \alpha) \\
\alpha_1 &= \underline{\underline{29.84°}}, \quad \alpha_2 = \underline{\underline{60.16°}}
\end{aligned}
$$

b)

$$
\begin{aligned}
l_{max} &= \tfrac{v_0^2}{g} \quad \text{für } \alpha = 45° \\
l_{max} &= \tfrac{50^2\ \text{m}}{9.81\ \text{m/s}} = \underline{\underline{255\ \text{m}}}
\end{aligned}
$$

29. Beschleunigte Kreisbewegung

a) $\vec{r}(t) = R \begin{pmatrix} \cos \gamma t^2 \\ \sin \gamma t^2 \end{pmatrix}$

$$\vec{v}(t) = \underline{\underline{2R\gamma t \begin{pmatrix} -\sin \gamma t^2 \\ \cos \gamma t^2 \end{pmatrix}}}$$

$$|v(t)| = \sqrt{(2R\gamma t \sin \gamma t^2)^2 + (2R\gamma t \cos \gamma t^2)^2} =$$
$$|v(t)| = \underline{\underline{2R\gamma t}}$$

$$s(t) = R\varphi(t) = \underline{\underline{R\gamma t^2}}$$

b)

$$\varphi(t) = \gamma t^2$$
$$\omega(t) = \underline{\underline{2\gamma t}}$$
$$\alpha(t) = \underline{\underline{2\gamma}}$$

c)

$$\vec{a}(t) = \underline{\underline{\overbrace{2R\gamma \begin{pmatrix} -\sin \gamma t^2 \\ \cos \gamma t^2 \end{pmatrix}}^{\text{Tangentialbeschleunigung}} + \overbrace{4R\gamma^2 t^2 \begin{pmatrix} -\cos \gamma t^2 \\ -\sin \gamma t^2 \end{pmatrix}}^{\text{Zentripetalbeschleunigung}}}}$$

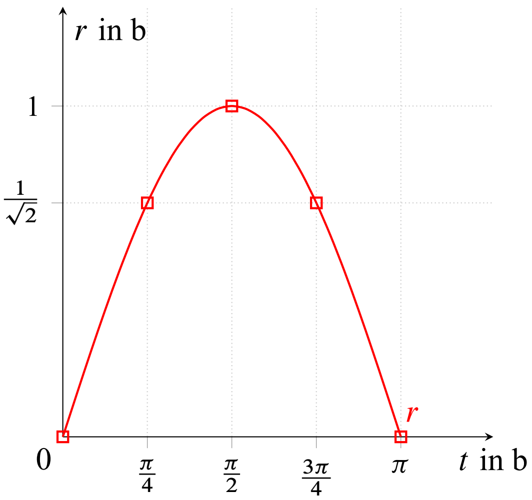
$$|a(t)| = \sqrt{(-2R\gamma \sin \gamma t^2 - 4R\gamma^2 t^2 \cos \gamma t^2)^2 + (2R\gamma \cos \gamma t^2 - 4R\gamma^2 t^2 \sin \gamma t^2)^2} =$$
$$|a(t)| = \sqrt{\underbrace{4R^2\gamma^2}_{\text{Tangentialbeschl.}} + \underbrace{16R^2\gamma^4 t^4}_{\text{Zentripetalbeschl.}}}$$

d)

$$\begin{aligned} 4R^2\gamma^2 &< 16R^2\gamma^4 t^4 && \mid \sqrt{} \\ 2R\gamma &< 4R\gamma^2 t^2 && \mid :2R\gamma \\ 1 &< 2\gamma t^2 && \mid :2\gamma \mid \sqrt{} \\ t &> \underline{\underline{\sqrt{\tfrac{1}{2\gamma}}}} \end{aligned}$$

für $t > \frac{1}{\sqrt{2\gamma}}$ liefert die Zentripetalbeschleunigung den Hauptteil der Beschleunigung.

31. Zweidimensionale Bahnkurve



a) $\vec{r}(t) = b \begin{pmatrix} \omega t \\ \sin \omega t \end{pmatrix}$

$$r(t_0) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$
$$r(t_1) = \begin{pmatrix} b\pi/4 \\ b/\sqrt{2} \end{pmatrix}$$
$$r(t_2) = \begin{pmatrix} b\pi/2 \\ b \end{pmatrix}$$
$$r(t_3) = \begin{pmatrix} 3b\pi/4 \\ b/\sqrt{2} \end{pmatrix}$$
$$r(t_4) = \begin{pmatrix} \pi \\ 0 \end{pmatrix}$$

b)

$$\vec{v}(t) = \underline{\underline{b\omega \begin{pmatrix} 1 \\ \cos \omega t \end{pmatrix}}}$$
$$|v(t)| = \sqrt{(b\omega)^2 + (b\omega \cos \omega t)^2} = \underline{\underline{b\omega \sqrt{1 + \cos^2 \omega t}}}$$

c)

$$\vec{a}(t) = \underline{\underline{b\omega^2 \begin{pmatrix} 0 \\ -\sin \omega t \end{pmatrix}}}$$
$$|a(t)| = \sqrt{(0b\omega^2)^2 + (-b\omega^2 \sin \omega t)^2} = \underline{\underline{b\omega^2 |\sin \omega t|}}$$

d)

$$\left|a_\parallel(t)\right| = \frac{\mathrm{d}\,|v(t)|}{\mathrm{d}t} = \underline{\underline{-\frac{b\omega^2 \sin 2\omega t}{2\sqrt{1+\cos^2 \omega t}}}}$$

e)

$$\left|\vec{a}_{\parallel}\right| = \sqrt{(-\frac{b\omega^2 \sin 2\omega t_1}{2\sqrt{1+\cos^2 \omega t_1}})^2} = \underline{\underline{\frac{b\omega^2}{\sqrt{6}}}}$$

$$|a_{\perp}(t_1)| = \sqrt{|a(t_1)|^2 - \left|a_{\parallel}(t_1)\right|^2} = \sqrt{(b\omega^2 \sin \omega t_1)^2 - (-\frac{b\omega^2 \sin 2\omega t_1}{2\sqrt{1+\cos^2 \omega t_1}})^2} = \sqrt{\frac{b^2\omega^4}{2} - \frac{b^2\omega^4}{6}} =$$

$$|a_{\perp}(t_1)| = \frac{b\omega^2}{\sqrt{3}}$$

$$\rho(t_1) = \frac{|v(t_1)|^2}{|a_{\perp}(t_1)|} = \frac{\left(b\omega \sqrt{1+\cos^2 \omega t_1}\right)^2}{\frac{b\omega^2}{\sqrt{3}}} = \frac{b^2\omega^2 \frac{3}{2}}{\frac{b\omega^2}{\sqrt{3}}} = \frac{3\sqrt{3}b^2\omega^2}{2b\omega^2} =$$

$$\rho(t_1) = \underline{\underline{\frac{\sqrt{27}b}{2}}}$$