1) $S = k_B \ln\left(\frac{N!}{n!(N-n)!}\right) \approx \cancel{N} k_B \left(N\ln N - N - n\ln n + n - (N-n)\ln N-n + N-n\right) =$

$= k_B\left(N \ln \frac{N}{N-n} + n \ln \frac{N-n}{\cancel{N-n}}\right)$

$p_1 = \frac{n}{N},\ p_2 = 1-p_1$

$S_g = -k_B \sum_i p_i \ln p_i = -k_B\left(p_1 \ln p_1 + p_2 \ln p_2\right) = -k_B\left(\frac{n}{N}\ln\left(\frac{n}{N}\right) + \frac{N-n}{N}\ln\left(\frac{N-n}{N}\right)\right) =$

$= -k_B\left(\frac{n}{N}\ln\left(\frac{n}{N-n}\right) + \ln\left(\frac{N-n}{N}\right)\right) = \cancel{\frac{k_B}{N}\left(n \ln\frac{n}{N-n} + N \ln\right.}$

$= \frac{k_B}{N}\left(n \ln \frac{N-n}{n} + N\ln\left(\frac{N}{N-n}\right)\right) = \frac{S}{N}$

b)

$\Omega(E) = \sum_{a_1=0}^{1} \cdots \sum_{a_N=0}^{1} \delta(E - \mu B \sum_i^N a_i) = \sum_{a_1=0}^{1} \cdots \sum_{a_N=0}^{1} \int \frac{dk}{2\pi} e^{ik(E - \mu B \sum_i^N a_i)} =$

$= \int \frac{dk}{2\pi} e^{ikE} \sum_{a_1}^{N} \cdots \sum_{a_N}^{N} e^{-ik\mu B \sum_i^N a_i} = \int \frac{dk}{2\pi} e^{ikE} \sum_{a_1} \cdots \sum_{a_N} \prod^N e^{-ik\mu B a_i} =$

$= \int \frac{dk}{2\pi} e^{ikE}\left(1 + e^{-ik\mu B}\right)^N = \int \frac{dk}{2\pi} e^{N\left(ik\frac{E}{N} + \ln\left(1 + e^{-ik\mu B}\right)\right)}$

c) $I = \lim_{N\to\infty} \int e^{N\varepsilon(x)} dx = \lim_{N\to\infty} e^{N\varepsilon(x_0)} \sqrt{\frac{2\pi}{N|\varepsilon''(x_0)|}}$

$\varepsilon(k) = ik\frac{E}{N} + \ln(1 + e^{-ik\mu B})$

$\varepsilon'(k) = i\frac{E}{N} + \frac{1}{1+e^{-ik\mu B}} e^{-ik\mu B}(-iB\mu)$

$\varepsilon''(k) = (-iB\mu)^2 \frac{e^{-ik\mu B}}{1+e^{-ik\mu B}} + (-iB\mu)^2 \frac{(e^{-ik\mu B})^2}{(1+e^{-ik\mu B})^2} =$

$= -(B\mu)^2 \frac{1}{4\cos^2\left(\frac{Bk\mu}{2}\right)} = -\left(\frac{B\mu}{2\cos\left(\frac{Bk\mu}{2}\right)}\right)^2$

$\to \Omega(E) = \int \frac{dk}{2\pi} e^{N\left(ik\frac{E}{N} + \ln(1 + e^{-ik\mu B})\right)}$

$\approx \frac{e^{N\left(ik_0\frac{E}{N} + \ln(1 + e^{-ik_0\mu B})\right)}}{2\pi} \sqrt{\frac{2\pi}{N\left(\frac{B\mu}{2\cos\left(\frac{k_0 B\mu}{2}\right)}\right)^2}}$

$= e^{N\left(ik_0\frac{E}{N} + \ln(1 + e^{-ik_0\mu B})\right)} \frac{2\cos\left(k_0\frac{B\mu}{2}\right)}{B\mu} \sqrt{\frac{1}{2\pi N}}$

$k_0 : \varepsilon'(k) \overset{!}{=} 0 \Rightarrow k_0 = \frac{i\left(2i\pi m + \ln\left(-\frac{E}{E - B m\mu}\right)\right)}{B\mu}$

$\Omega(E)\big|_{k=k_0} = \frac{1}{BN\mu} \sqrt{\frac{N}{2\pi E(E - B\mu N)}} \left(BN\mu \ln\left(\frac{BN\mu}{BN\mu - E}\right) - E\ln\left(\frac{E}{BN\mu - E}\right)\right)$ (Mathematica)

$\frac{E}{\mu B} = n$

$= \frac{n}{NE} \sqrt{\frac{N}{2\pi E^2\left(1 - \frac{N}{n}\right)}} \left(\frac{NE}{n} \ln\left(\frac{\frac{EN}{n}}{\frac{EN}{n} - E}\right) - E\ln\left(\frac{E}{\frac{EN}{n} - E}\right)\right) =$

$= \frac{n}{N} \sqrt{\frac{N}{2\pi E^2\left(1 - \frac{N}{n}\right)}} \left(\frac{N}{n}\ln\left(\frac{N}{N-n}\right) - \ln\left(\frac{n}{N-n}\right)\right)$

$k_B = $

$= \frac{\sqrt{\frac{N}{2\pi E^2\left(1 - \frac{N}{n}\right)}}}{N} \left(n \ln\left(\frac{N-n}{n}\right) + N\ln\left(\frac{N}{N-n}\right)\right) = \frac{S_g}{k_B}$

2) $f(\vec{x}_0+\vec{h}) - f(\vec{x}_0) \approx f(\vec{x}_0) + \nabla f(\vec{x}_0)\vec{h} - f(\vec{x}_0) = \sum_i \frac{\partial f(\vec{x}_0)}{\partial x_i}\vec{e}_{x_i} \, h_i \vec{e}_{x_i} = \underline{\underline{\sum_i \frac{\partial f(\vec{x}_0)}{\partial x_i} h_i}}$

a) $df = f(\vec{x}_0 + d\vec{x}) - f(\vec{x}_0) = \sum_i \frac{\partial f(\vec{x}_0)}{\partial x_i} dx_i \Rightarrow \underline{\underline{\frac{df}{dt} = \sum_i \frac{\partial f(x_0)}{\partial x_i} \frac{dx_i}{dt}}}.$

c) $\cancel{I_\gamma = \int_\gamma f(x,y)\,dx + g(x,y)\,dy = \int_\gamma \vec{\nabla}\vec{F} = \int_\gamma}$

$I_\gamma = \int_\gamma f(x,y)\,dx + g(x,y)\,dy \overset{\wedge}{=} \int_\gamma \frac{\partial F}{\partial x} dx + \frac{\partial F}{\partial y} dy \overset{\wedge}{=} \int_\gamma dF = F(\gamma_b) - F(\gamma_a)$

$\vec{\nabla} F = \binom{f}{g}$ b)

Assumption

Für $\gamma_a = \gamma_b \Rightarrow I_\gamma = 0$

d) $pV = Nk_BT \quad E = \frac{3}{2} Nk_BT$

$dS = \frac{dE}{T} + \frac{p\,dV}{T} = \frac{3}{2}\frac{Nk_B\,dT}{T} + \frac{Nk_B\,dV}{V}$

$\Delta S = S_a - S_b = \int_{V_1}^{2V_1} \frac{Nk_B\,dV}{V} + \int_{T_1}^{T_2} \frac{3}{2} \frac{Nk_B\,dT}{T} = Nk_B\left(\ln(2) + \frac{3}{2}\ln\left(\frac{35}{30}\right)\right) \approx 7{,}69 \text{ J/K}$

3a)

```
In[1]:= V1 := ⊟ 10 dm³ ✓

V2 := ⊟ 50 dm³ ✓

T1 := ⊟ 300 K ⋯ ✓

k := ⊟ k ✓

n := ⊟ N_A ✓ * ⊟ 1 mol ✓

e[T_] := 3 / 2 n * k * T
ΔW =
 UnitConvert[Integrate[-2 / 3 e[T1] / V, {V, V1, V2}], "J"] // N

Out[7]= -4014.48 J
```

b)

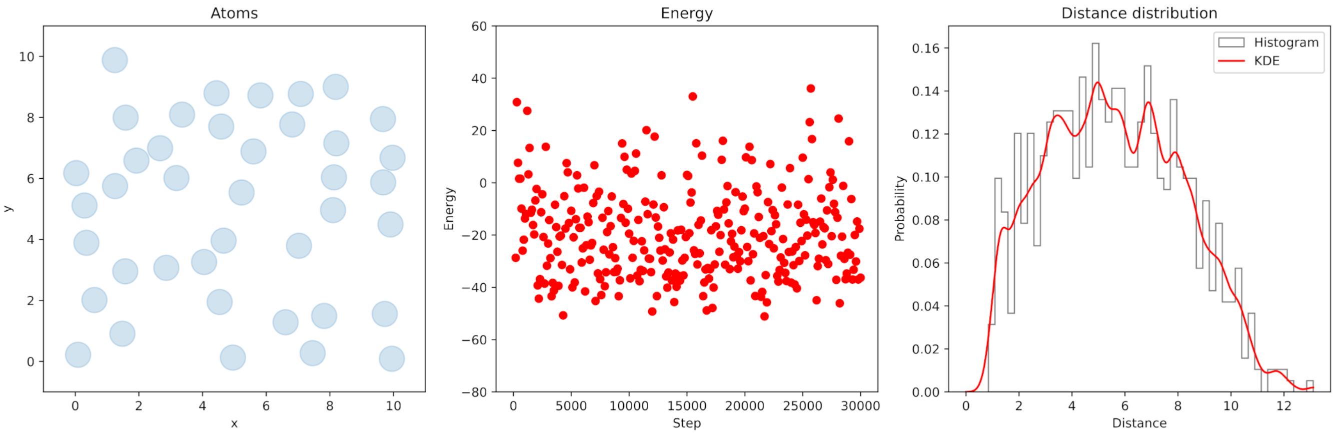
```
In[8]:= γAr := 5 / 3
T2 := T1 (V1 / V2) ^ (γAr - 1)
ΔW = UnitConvert[Integrate[e[T] / T, {T, T1, T2}], "J"] // N

Out[10]= -2461.93 J
```

c)

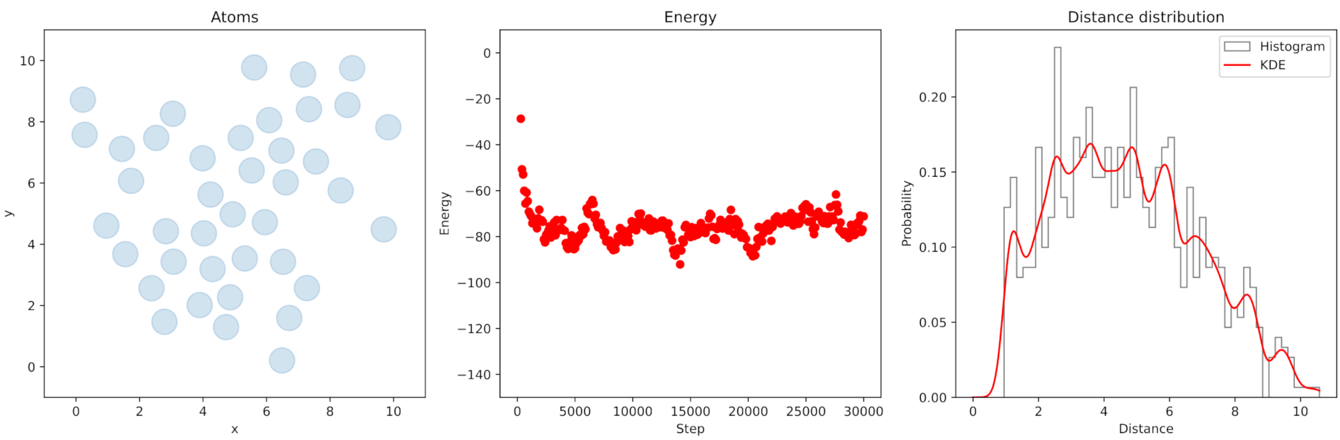
```
In[95]:= a := ⊟ 0.1363 m³J ⋯ ✓

b := ⊟ 0.0000322 m³ ⋯ ✓

eqn := n * k * T == (p + a / V ^ 2) (V - b)
ΔW = UnitConvert[Integrate[-p /. Solve[eqn, p] /. T → T1, {V, V1, V2}], "J"][[1]]

Out[98]= -4010.02 J
```

In a) wird am meisten Arbeit geleistet, weil keine Energieumwandlung in Wärme berücksichtigt wird, während in b) Energieverlust durch Erhitzen des Gases miteinbezogen wird. In Realität lässt sich Argon sehr gut als ideales Gas nähern, weshalb c) dem Wert aus a) ähnelt.

Gas:



Flüssig:



Fest: