1. Elektromagnetische Welle im Vakuum

$$E_x = 0; \quad E_y = 30\cos\left(2\pi * 10^8 t - \frac{2\pi}{3}x\right); \quad E_z = 0$$

(a) $\omega = 2\pi * 10^8 \ 1/\text{s}$

$$f = \frac{\omega}{2\pi} = \underline{\underline{10^8 \ 1/\text{s}}}$$

(b) $k = \frac{2\pi}{3}$

$$\lambda = \frac{2\pi}{k} = \underline{\underline{3 \text{ m}}}$$

(c)

Direction: $\underline{\underline{\hat{x}}}$

(d) $B_0 = \frac{E_0}{c}$

$$\vec{B} = \underline{\underline{10^7 \cos\left(2\pi * 10^8 t - \frac{2\pi}{3}x\right)}}$$

2. Photonen-Ping-Pong

(a)

Linear Polarization	Circular Polarization	Elliptical Polarization
$\vec{E}_1(t) = E_0 \begin{pmatrix} \cos(\omega t) \\ 0 \\ 0 \end{pmatrix}$	$\vec{E}_2(t) = E_0 \begin{pmatrix} \cos(\omega t) \\ \sin(\omega t) \\ 0 \end{pmatrix}$	$\vec{E}_3(t) = E_0 \begin{pmatrix} a\cos(\omega t) \\ b\sin(\omega t) \\ 0 \end{pmatrix}$
$w_{rms} = \frac{\epsilon_0 E_0^2}{2}$	$w_{rms} = \epsilon_0 E_0^2$	$w_{rms} = \frac{\epsilon_0 E_0^2 (a^2+b^2)}{2}$
$I_{rms} = \frac{c\epsilon_0 E_0^2}{2}$	$I_{rms} = c\epsilon_0 E_0^2$	$I_{rms} = \frac{c\epsilon_0 E_0^2 (a^2+b^2)}{2}$

(b) $\vec{F} = \frac{\mathrm{d}\vec{p}}{\mathrm{d}t} = \frac{P}{c} = \epsilon_0 E_{\text{rms}}^2 A$

$$F_1 = \epsilon_0 E_{\text{rms}}^2 A$$
$$F_2 = 2F_1 = 2\epsilon_0 E_{\text{rms}}^2 A$$
$$F_3 = \frac{F_1+F_2}{2} = \frac{3}{2}\epsilon_0 E_{\text{rms}}^2 A$$

(c) pair Wave with largest E_{rms} with F_2

$\Rightarrow$ $\underline{\underline{F_2 + \vec{E}_2}}$ generate the highest mean Force $\overline{F}$

3. Stehende Wellen

$$\vec{E}(\vec{r}, t) = \mathrm{Re}\left(\vec{E}_0\, \mathrm{e}^{i\vec{k}\vec{r} - i\omega t}\right)$$

(a)

$$\vec{E}_1(x\hat{x}, t) = \mathrm{Re}\left(\vec{E}_0\, \mathrm{e}^{ik_x x - i\omega t}\right)$$
$$\vec{E}_2(-x\hat{x}, t) = \mathrm{Re}\left(\vec{E}_0\, \mathrm{e}^{-ik_x x - i\omega t}\right)$$

$$\begin{aligned}\vec{E} = \vec{E}_1 + \vec{E}_2 &= \mathrm{Re}\left(\vec{E}_0\left[\mathrm{e}^{ik_x x - i\omega t} + \mathrm{e}^{-ik_x x - i\omega t}\right]\right) = \\ &= \mathrm{Re}\left(\vec{E}_0\, \mathrm{e}^{-i\omega t}\right) * 2\cos(k_x x)\end{aligned}$$

For $x = (2n+1)\frac{\pi}{2k_x}$, $n \in \mathbb{N}$: $\quad \cos(k_x x) = 0 \quad \Rightarrow \quad \vec{E} = 0$

$\Rightarrow$ $\vec{E}$ describes a standing Wave with $\lambda = \pi$

(b)

$$\vec{E}_1(x\hat{x},t) = \mathrm{Re}\left(\vec{E}_0\,\mathrm{e}^{ik_x x - i\omega t}\right)$$
$$\vec{E}_2(y\hat{y},t) = \mathrm{Re}\left(\vec{E}_0\,\mathrm{e}^{ik_y y - i\omega t}\right)$$

$$\begin{aligned}\vec{E} = \vec{E}_1 + \vec{E}_2 &= \mathrm{Re}\left(\vec{E}_0\left[\mathrm{e}^{ik_x x - i\omega t} + \mathrm{e}^{ik_y y - i\omega t}\right]\right) = \\ &= \mathrm{Re}\left(\vec{E}_0\,\mathrm{e}^{-i\omega t}\underbrace{\left[\mathrm{e}^{ik_x x} + \mathrm{e}^{ik_y y}\right]}_{>0\,\Rightarrow\,\text{no Root}}\right)\end{aligned}$$

$\Rightarrow$ $\vec{E}$ does not describe a standing Wave

(c) $\vec{r}_1 = \cos(\frac{\pi}{10})\hat{x} + \sin(\frac{\pi}{10})\hat{y}$

$$\vec{E}_1(x\hat{x},t) = \mathrm{Re}\left(\vec{E}_0\,\mathrm{e}^{ik_x x - i\omega t}\right)$$
$$\vec{E}_2(\vec{r}_1,t) = \mathrm{Re}\left(\vec{E}_0\,\mathrm{e}^{i\vec{k}\vec{r}_1 - i\omega t}\right)$$

$$\begin{aligned}\vec{E} = \vec{E}_1 + \vec{E}_2 &= \mathrm{Re}\left(\vec{E}_0\left[\mathrm{e}^{ik_x x - i\omega t} + \mathrm{e}^{i\vec{k}\vec{r}_1 - i\omega t}\right]\right) = \\ &= \mathrm{Re}\left(\vec{E}_0\,\mathrm{e}^{-i\omega t}\underbrace{\left[\mathrm{e}^{ik_x x} + \mathrm{e}^{i\vec{k}\vec{r}_1}\right]}_{>0\,\Rightarrow\,\text{no Root}}\right)\end{aligned}$$

$\Rightarrow$ $\vec{E}$ does not describe a standing Wave

(d) $\omega_1 = 2\omega_2$

$$\vec{E}_1(\hat{x},t) = \mathrm{Re}\left(\vec{E}_0\,\mathrm{e}^{i\vec{k}\hat{x} - i\omega_1 t}\right)$$
$$\vec{E}_2(-\hat{x},t) = \mathrm{Re}\left(\vec{E}_0\,\mathrm{e}^{-i\vec{k}\hat{x} - i\omega_2 t}\right)$$

$$\begin{aligned}\vec{E} = \vec{E}_1 + \vec{E}_2 &= \mathrm{Re}\left(\vec{E}_0\left[\mathrm{e}^{ik_x x - i2\omega_2 t} + \mathrm{e}^{-ik_x x - i\omega_2 t}\right]\right) = \\ &= \mathrm{Re}\left(\vec{E}_0\,\mathrm{e}^{-i\omega_2 t}\underbrace{\left[\mathrm{e}^{ik_x x - i\omega_2 t} + \mathrm{e}^{ik_x x}\right]}_{\text{time dependent Root}}\right)\end{aligned}$$

$\Rightarrow$ $\vec{E}$ does not describe a standing Wave