193. Gekoppelte physikalische Pendel

$$m = 5 \text{ kg}$$
; $k = 2 \text{ N/m}$; $a = 0.1 \text{ m}$; $h = 0.4 \text{ m}$; $l = 0.1 \text{ m}$

a)
$$\omega = \sqrt{\frac{mgl}{I}}$$

$$I = \frac{1}{12}m(a^2 + h^2) + ml^2 = 0.12 \text{ kg m}^2$$

$$\omega_a = \sqrt{\frac{mgl}{I}} = \underline{6.37 \text{ rad/s}}$$

b)
$$F = -m\omega^2 x$$
; $\Delta x = x_1 - x_2$
 $m\ddot{x}_1 = -m\omega_a^2 x_1 - k(x_1 - x_2)$
 $m\ddot{x}_2 = -m\omega_a^2 x_2 - k(x_2 - x_1)$
 $\Rightarrow m(\ddot{x}_1 - \ddot{x}_2) = -m\omega_a^2 (x_1 - x_2) - 2k(x_1 - x_2) = -(m\omega_a^2 + 2k)(x_1 - x_2)$
 $\Delta \ddot{x} = -(\omega_a^2 + \frac{2k}{m})\Delta x$
 $= \omega_b$
 $\omega_b = \sqrt{\omega_a^2 + \frac{2k}{m}} = \underline{6.43 \text{ rad/s}}$

c)
$$\delta \omega = \omega_b - \omega_a$$

$$0 = \cos(\frac{1}{2}\delta\omega t)$$

$$t = \frac{2\arccos(0)}{\delta\omega} = \underline{50.29 \text{ s}}$$

196. Schallgeschwindigkeit und Elastizität

$$\rho = 4500 \text{ kg/m}^3$$
; $v_{\parallel} = 5050 \text{ m/s}$; $v_{\perp} = 3100 \text{ m/s}$

a)
$$v_{\parallel} = \sqrt{\frac{E}{\rho}};$$
 $v_{\perp} = \sqrt{\frac{G}{\rho}}$
 $E = v_{\parallel}^2 \rho = \underline{1.15 * 10^{11} \text{ Pa}}$
 $G = v_{\perp}^2 \rho = \underline{4.32 * 10^{10} \text{ Pa}}$

b)
$$\mu = \frac{E}{2G} - 1$$

$$\mu = \frac{v_{\parallel}^2}{2v_{\parallel}^2} - 1 = \underline{0.33}$$

c) $\mu = \underbrace{\frac{v_{\parallel}^2}{2v_{\perp}^2} - 1}_{2v_{\perp}} \quad \text{oder} \quad \frac{v_{\parallel}}{v_{\perp}} = \underbrace{\sqrt{2\mu + 2}}_{v_{\perp}}$

199. Kompensation der Wärmeausdehnung

$$a=h=0.1 \text{ m}; \quad \Delta T=1 \text{ K}; \quad E=2.1*10^{11} \text{ N/m}^2$$

 $\mu=0.29; \quad \rho=7870 \text{ kg/m}^3; \quad \alpha=1.2*10^{-5} \text{ K}^{-1}$

a)
$$\frac{\Delta h}{h} = \alpha \Delta T$$
; $\frac{F}{A} = E \frac{\Delta h}{h}$
 $\frac{mg}{h^2} = E \alpha \Delta T$
 $m = \frac{E \alpha \Delta T h^2}{g} = \underline{2568.8 \text{ kg}}$

b)
$$\Delta a_1 = \mu \Delta h$$
; $\Delta a_2 = a\alpha \Delta T$
$$\Delta a_{\text{ges}} = \Delta a_1 + \Delta a_2 = \underline{1.5 * 10^{-6} \text{ m}}$$