Magnetisches Moment von Kernen

a) $\boldsymbol{j} = \boldsymbol{l} + \boldsymbol{s}$

$$\boldsymbol{j}^2 = (\boldsymbol{l} + \boldsymbol{s})^2 = \boldsymbol{l}^2 + \boldsymbol{s}^2 + 2\boldsymbol{l}\cdot\boldsymbol{s} = \boldsymbol{l}^2 + \boldsymbol{s}^2 + 2\boldsymbol{l}\cdot(\boldsymbol{j} - \boldsymbol{l}) = \boldsymbol{s}^2 - \boldsymbol{l}^2 + 2\boldsymbol{j}\cdot\boldsymbol{l}$$

$$\Rightarrow \boldsymbol{j}\cdot\boldsymbol{l} = \frac{1}{2}\left(\boldsymbol{j}^2 + \boldsymbol{l}^2 - \boldsymbol{s}^2\right)$$

b)

$$\langle jm_j | g_l \boldsymbol{j}^2 | jm_j\rangle = j(j+1)$$

$$\langle jm_j | g_l \boldsymbol{j}\cdot\boldsymbol{l} | jm_j\rangle = \langle jm_j | g_l \frac{1}{2}\left(\boldsymbol{j}^2 + \boldsymbol{l}^2 - \boldsymbol{s}^2\right) | jm_j\rangle =$$
$$= \frac{g_l}{2}\Big[j(j+1) + l(l+1) - s(s+1)\Big]$$

$$\langle jm_j | g_s \boldsymbol{s}\cdot\boldsymbol{j} | jm_j\rangle = \langle jm_j | g_l \frac{1}{2}\left(\boldsymbol{j}^2 + \boldsymbol{s}^2 - \boldsymbol{l}^2\right) | jm_j\rangle =$$
$$= \frac{g_s}{2}\Big[j(j+1) + s(s+1) - l(l+1)\Big]$$

$$\langle jm_j | g_l \boldsymbol{j}\cdot\boldsymbol{l} + g_s \boldsymbol{s}\cdot\boldsymbol{j} | jm_j\rangle =$$
$$= \langle jm_j | g_l \frac{1}{2}\left(\boldsymbol{j}^2 + \boldsymbol{l}^2 - \boldsymbol{s}^2\right) | jm_j\rangle + \langle jm_j | g_s \frac{1}{2}\left(\boldsymbol{j}^2 + \boldsymbol{l}^2 - \boldsymbol{s}^2\right) | jm_j$$

$$\frac{\langle jm_j | g_l \boldsymbol{j}\cdot\boldsymbol{l} + g_s \boldsymbol{s}\cdot\boldsymbol{j} | jm_j\rangle}{\langle jm_j | g_l \boldsymbol{j}^2 | jm_j\rangle} =$$

c)

a)

b)

c)

a)

b)

c)