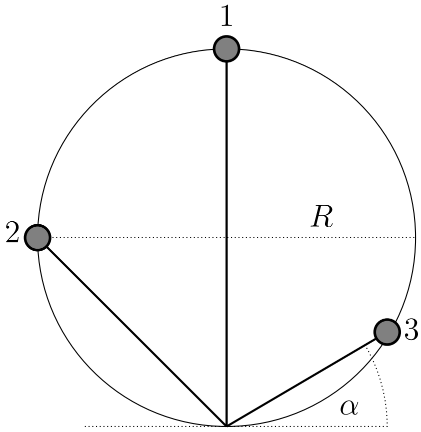
37. Ordnungsaufgabe – Kistenziehen

F_i ... Kraft, mit der das Seil $i = \{\text{A, B, C, D, E, F}\}$ gespannt ist

$$\underline{\underline{F_D > (F_E,\ F_B) > (F_A,\ F_C,\ F_F)}}$$

40. Welche Perle ist schneller?



a)

$$r_1(t) = 2R - \frac{g}{2}t^2$$

$$r_1(t) = 0 \quad \Rightarrow \quad t_1 = 2\sqrt{\frac{R}{g}}$$

$$r_2(t) = \sqrt{2}R - \frac{g}{2\sqrt{2}}t^2$$

$$r_2(t) = 0 \quad \Rightarrow \quad t_2 = 2\sqrt{\frac{R}{g}}$$

$$\Rightarrow \underline{\underline{t_1 = t_2}}$$

b)

$$r_3(t,\alpha) = 2R\sin(\alpha) - \frac{g\sin(\alpha)}{2}t^2$$

$$r_3(t,\alpha) = 0 \quad \Rightarrow \quad \underline{\underline{t_3(\alpha) = \sqrt{\frac{4R\sin(\alpha)}{g\sin(\alpha)}} = 2\sqrt{\frac{R}{g}}}}$$

Die Zeit, die die Perle braucht, um am Ziel anzukommen, hängt nicht vom Winkel α ab

50. Kiste auf schiefer Ebene

a) $F_G = mg; \quad g = 9.81\ \text{m/s}^2$

$$\vec{F}_Z(m,\theta) = \underline{\underline{9.81m\sin(\theta)\begin{pmatrix}\cos(\theta)\\ \sin(\theta)\end{pmatrix}}}$$

$$\vec{F}_N(m,\theta) = \underline{\underline{9.81m\cos(\theta)\begin{pmatrix}\sin(\theta)\\ \cos(\theta)\end{pmatrix}}}$$

b) $g = 9.81\ \text{m/s}^2; \quad m = 5\ \text{kg}; \quad \theta = 60°$

$$F_Z = 9.81\ \text{m/s}^2 * 5\ \text{kg} * \sin(60) = \underline{\underline{42\ \text{N}}}$$
$$F_N = 9.81\ \text{m/s}^2 * 5\ \text{kg} * \cos(60) = \underline{\underline{25\ \text{N}}}$$

c) $f_H = 0.5; \quad f_G = 0.3$

$$F_H = f_H * F_N = \tfrac{gm\cos(\theta)}{2}$$
$$F_\parallel = mg\sin(\theta)$$
$$F_c = 20\ \text{N}$$

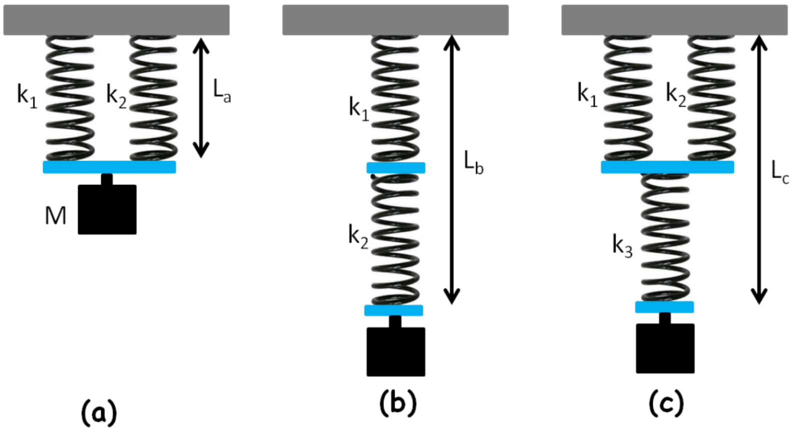
$$\begin{aligned}
F_\parallel &= F_c + F_H \\
mg\sin(\theta_c) &= 20 + \tfrac{gm\cos(\theta_c)}{2} && |\ ^2 \\
(mg\sin(\theta_c))^2 &= (20 + \tfrac{gm\cos(\theta_c)}{2})^2 && |\ -(20 + \tfrac{gm\cos(\theta_c)}{2})^2 \\
0 &= m^2g^2\sin^2(\theta_c) - 400 - 20mg\cos(\theta_c) - \tfrac{m^2g^2\cos^2(\theta_c)}{4} \\
0 &= m^2g^2 - 400 - 20mg\cos(\theta_c) - \tfrac{m^2g^2\cos^2(\theta_c)}{4} - \tfrac{4m^2g^2\cos^2(\theta_c)}{4} \\
0 &= -\tfrac{5m^2g^2\cos^2(\theta_c)}{4} - 20mg\cos(\theta_c) + m^2g^2 - 400 && |\ *(-\tfrac{4}{5m^2g^2}) \\
0 &= \cos^2(\theta_c) + \tfrac{80}{5mg}\cos(\theta_c) - \tfrac{4}{5} + \tfrac{1600}{5m^2g^2} \\
\cos(\theta_c) &= -\tfrac{80}{10mg} \pm \sqrt{(\tfrac{80}{10mg})^2 - (-\tfrac{4}{5} + \tfrac{1600}{5m^2g^2})} \\
\cos(\theta_c) &= -0.163... \pm 0.832... \\
\cos(\theta_c) &= 0.669... \\
\theta_c &= \underline{\underline{47.95°}}
\end{aligned}$$

d)

$$F = F_\parallel - f_G * F_N$$
$$ma = mg\sin(\theta_c) - \tfrac{3mg\cos(\theta_c)}{10}$$
$$a = \tfrac{g\ (10\sin(\theta_c) - 3\cos(\theta_c))}{10} = 5.31\ \text{m/s}^2$$

$$r(t) = \tfrac{a}{2}t^2$$
$$1.3\ \text{m} = \tfrac{5.31\ \text{m/s}^2}{2}t^2$$
$$t = \sqrt{\tfrac{2.6\ \text{m}}{5.31\ \text{m/s}^2}} = \underline{\underline{0.7\ \text{s}}}$$

52. Gekoppelte Federn



$$F = -k * x = Mg; \quad g = 9.81 \text{ m/s}^2; \quad L_{a,b,c} = L_0 + x_{a,b,c}$$

a)

$$k = \underline{\underline{k_1 + k_2}}$$
$$L_a = L_0 + \frac{F}{-k}$$
$$L_a = \underline{\underline{L_0 + Mg\left(\frac{1}{k1+k2}\right)}}$$

b)

$$k = \underline{\underline{\frac{k_1 k_2}{k_1+k_2}}}$$
$$L_b = 2L_0 + \frac{F}{-k}$$
$$L_b = \underline{\underline{2L_0 + Mg\left(\frac{k_1+k_2}{k_1 k_2}\right)}}$$

c)

$$k = \underline{\underline{\frac{k_3(k_1+k_2)}{k_1+k_2+k_3}}}$$
$$L_c = 2L_0 + \frac{F}{-k}$$
$$L_c = \underline{\underline{2L_0 + Mg\left(\frac{k_1+k_2+k_3}{k_3(k_1+k_2)}\right)}}$$