

132. Freier Fall einer rotierenden Hantel

$$\begin{array}{lll}
 m_1 = 1 \text{ kg}; & \vec{v}_1 = (4, 0, 0) \text{ m/s}; & \vec{r}_1 = (0, 0, 2.5) \text{ m} \\
 m_2 = 0.5 \text{ kg}; & \vec{v}_2 = (1, 0, 0) \text{ m/s}; & \vec{r}_2 = (0, 0, 2.2) \text{ m}
 \end{array}$$

$$\vec{r}_{sp} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2} = \begin{pmatrix} 0 \\ 0 \\ 2.4 \text{ m} \end{pmatrix}$$

a) Im bewegten Bezugssystem, das sich mit 3 m/s mitbewegt reine Rotation

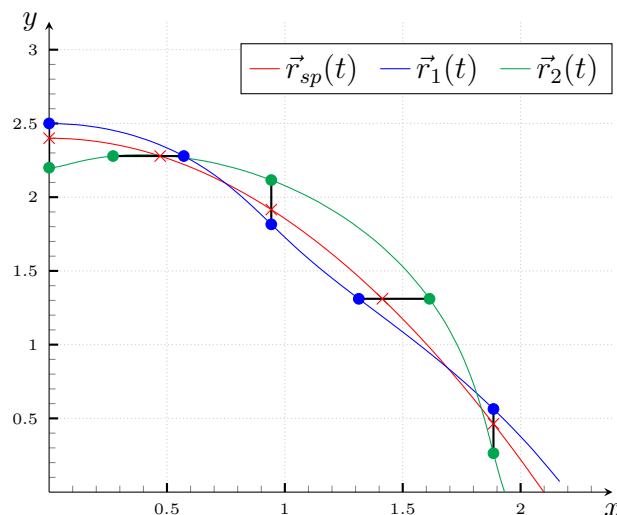
$$\Rightarrow \vec{r}_{sp}(t) = \begin{pmatrix} 0 \\ 0 \\ 2.4 \text{ m} \end{pmatrix} + \begin{pmatrix} 3 \text{ m/s} \\ 0 \\ 0 \end{pmatrix} t + \frac{1}{2} \begin{pmatrix} 0 \\ 0 \\ -9.81 \text{ m/s}^2 \end{pmatrix} t^2$$

b) Im bewegten Bezugssystem: $\vec{v}_1 = (1, 0, 0) \text{ m/s}$; $\vec{v}_2 = (-2, 0, 0) \text{ m/s}$

$$\omega = \frac{|\vec{v}_1|}{r_1} = \underline{\underline{10 \text{ s}^{-1}}}$$

$$T = \frac{2\pi}{\omega} = \frac{\pi}{5} = \underline{\underline{0.63 \text{ s}}}$$

c)



d) Im Schwerpunktsystem:

$$\vec{r}_{1,sp}(t) = 0.1 \begin{pmatrix} \sin(\omega t) \\ 0 \\ \cos(\omega t) \end{pmatrix} \text{ m}$$

$$\vec{r}_{2,sp}(t) = -0.2 \begin{pmatrix} \sin(\omega t) \\ 0 \\ \cos(\omega t) \end{pmatrix} \text{ m}$$

Im Laborsystem:

$$\begin{aligned}\vec{r}_1(t) &= \vec{r}_{sp}(t) + \vec{r}_{1,sp}(t) \\ &= \begin{pmatrix} 0.1 \sin(\omega t) \text{ m} \\ 0 \\ 0.1 \cos(\omega t) \text{ m} + 2.4 \text{ m} \end{pmatrix} + \begin{pmatrix} 3 \text{ m/s} \\ 0 \\ 0 \end{pmatrix} t + \frac{1}{2} \begin{pmatrix} 0 \\ 0 \\ -9.81 \text{ m/s}^2 \end{pmatrix} t^2\end{aligned}$$

$$\begin{aligned}\vec{r}_2(t) &= \vec{r}_{sp}(t) + \vec{r}_{2,sp}(t) \\ &= \begin{pmatrix} -0.2 \sin(\omega t) \text{ m} \\ 0 \\ -0.2 \cos(\omega t) \text{ m} + 2.4 \text{ m} \end{pmatrix} + \begin{pmatrix} 3 \text{ m/s} \\ 0 \\ 0 \end{pmatrix} t + \frac{1}{2} \begin{pmatrix} 0 \\ 0 \\ -9.81 \text{ m/s}^2 \end{pmatrix} t^2\end{aligned}$$

140. International Space Station

$$R_E = 6.37 * 10^6 \text{ m}; \quad m = 4.55 * 10^5 \text{ kg}; \quad h = 3.3 * 10^5 \text{ m}$$

a) $h = 0 \text{ m}$

$$\begin{aligned}F_G &= mg \frac{1}{\left(1 + \frac{h}{R_E}\right)^2} \\ &= 4.55 * 10^5 \text{ kg} * 9.81 \text{ m/s}^2 \\ F_G &= \underline{\underline{4.46 * 10^6 \text{ kg m/s}^2}}\end{aligned}$$

b) $h = 3.3 * 10^5 \text{ m}$

$$\begin{aligned}F_G &= mg \frac{1}{\left(1 + \frac{h}{R_E}\right)^2} \\ &= 4.55 * 10^5 \text{ kg} * 9.81 \text{ m/s}^2 \frac{1}{\left(1 + \frac{3 * 10^5 \text{ m}}{6.37 * 10^6 \text{ m}}\right)} \\ F_G &= \underline{\underline{4.03 * 10^6 \text{ kg m/s}^2}} \\ g(h) &= \frac{g}{\left(1 + \frac{h}{R_E}\right)^2} \\ g(3.3 * 10^5) &= \underline{\underline{8.87 \text{ m/s}^2}}\end{aligned}$$

$$v = r\omega; \quad a = \frac{v^2}{r} = r\omega^2; \quad r = R_E + h$$

c)

$$\begin{aligned}\omega &= \sqrt{\frac{g(h)}{r}} \\ &= \underline{\underline{1.2 * 10^{-3} \text{ s}^{-1}}}\end{aligned}$$

d)

$$\begin{aligned}T &= \frac{2\pi}{\omega} \\ &= \underline{\underline{5461.58 \text{ s}}}\end{aligned}$$

e)

$$\begin{aligned}v &= r\omega \\ &= \underline{\underline{7707.91 \text{ m/s}}}\end{aligned}$$

152. Komprimierter Metallblock

$$a = h = 0.2 \text{ m}$$

$$\Delta h = 1.3 * 10^{-6} \text{ m}$$

$$m = 500 \text{ kg}$$

$$V = 8 * 10^{-3} \text{ m}^3$$

$$\Delta V = 3 * 10^{-7} \text{ m}^3$$

$$p = 2 * 10^6 \text{ kg/ms}^2$$

$$\text{a) } p = K \frac{\Delta V}{V}; \quad \frac{F}{A} = E \frac{\Delta h}{a}$$

$$\begin{aligned}K &= p \frac{V}{\Delta V} \\ &= \underline{\underline{\frac{16}{3} * 10^{10} \text{ kg/ms}^2}}\end{aligned}$$

$$\begin{aligned}E &= \frac{mga}{a^2 \Delta h} \\ &= \underline{\underline{1.89 * 10^{10} \text{ kg/ms}^2}}\end{aligned}$$

$$\text{b) } \Delta a = \mu \Delta h; \quad K = \frac{E}{3(1-2\mu)}$$

$$\begin{aligned}K &= \frac{E}{3(1-2\mu)} \\ \frac{3K}{E} &= \frac{1}{1-2\mu} \\ \mu &= \frac{1}{2} - \frac{E}{6K} \\ &= \underline{\underline{0.44}}\end{aligned}$$

$$\begin{aligned}\Delta a &= \mu \Delta h \\ &= \underline{\underline{5.73 * 10^{-7} \text{ m}}}\end{aligned}$$