Materiewellen und Wellenfunktionen

$$\Psi(x) = \begin{cases} Ax(a-x) & \text{für } 0 \leq x \leq a \\ 0 & \text{für } x < 0 \text{ und } x > a \end{cases}$$

a)

$$1 = \int_{-\infty}^{\infty} \underbrace{|\Psi(x)|^2}_{[\mathrm{m}^{-\frac{1}{2}}]} \,\mathrm{d}x = \int_{0}^{a} A^2 x^2 (a-x)^2 \,\mathrm{d}x = \frac{a^5 A^2}{30}$$

$$A = \underline{\underline{\sqrt{\frac{30}{a^5}}}}$$

b) $k = \frac{2\pi}{\lambda} = \frac{m v_\mathrm{T}}{\hbar} = \frac{p}{\hbar}; \quad \omega = \frac{2\pi v_\mathrm{T}}{\lambda} = \frac{m v_\mathrm{T}^2}{2\hbar} = \frac{E}{\hbar} = \frac{p^2}{2m\hbar} = \frac{k^2\hbar}{2m}$

$$v_\mathrm{g} = \frac{\mathrm{d}\omega}{\mathrm{d}k} = \underline{\underline{\frac{k\hbar}{m} = \frac{p}{m}}}$$

c) $a_1 = 1\ \mathrm{m}; \quad a_2 = 0.5 \times 10^{-10}\ \mathrm{m}$

$$E_\mathrm{ges} = \int_{-\infty}^{r} k\,\frac{q^2}{s^2}\,\mathrm{d}s = k\,\frac{q^2}{r}$$

$$\lambda_\mathrm{db} = \frac{h}{m v_\mathrm{T}} = \frac{h}{\sqrt{2 m E_\mathrm{ges}}}$$

$$\lambda_1 = \underline{\underline{3.23 \times 10^{-5}\ \mathrm{m}}}$$
$$\lambda_2 = \underline{\underline{2.29 \times 10^{-10}\ \mathrm{m}}}$$

Doppelspaltversuch mit Elektronen

a)

$$
\begin{aligned}
E_1 &= E_0 \cos(kr - \omega t) \\
E_2 &= E_0 \cos(k(r + d\sin(\alpha)) - \omega t) \\
E &= E_1 + E_2 = E_0 \cos(kr - \omega t) + E_0 \cos(k(r + d\sin(\alpha)) - \omega t) = \\
&= \underbrace{2E_0 \cos\big(\tfrac{1}{2}kd\sin(\alpha)\big)}_{:= A} \cos\big(kr - \omega t + \tfrac{1}{2}kd\sin(\alpha)\big) \\
I &\propto A^2 \\
I(\alpha) &= \underline{\underline{I_0 \cos^2(\tfrac{d\pi}{\lambda}\sin(\alpha))}}
\end{aligned}
$$

b) $E = 10$ eV; $\quad d = 3$ nm

$$
\begin{aligned}
I(\alpha) &= 0 \\
\frac{d\pi}{\lambda}\sin(\alpha) &= \frac{\pi}{2} \\
\alpha &= \arcsin\Big(\frac{\lambda}{2d}\Big) = \arcsin\left(\frac{h}{2d\sqrt{2Em_e}}\right) = \underline{\underline{3.71°}}
\end{aligned}
$$

c) The probability of finding a single electron at the first minima is zero because destructive interference is happening. If you close one slit, no interference happens and you have a gaussian distribution centered on the remaining slit.

Neutronen im Interferometer

a) $\phi_{\mathrm{AC}} - \phi_{\mathrm{BD}} = 0$ due to symmetry

$$\frac{p_{\mathrm{f}}^2}{2m} = \frac{p_{\mathrm{i}}^2}{2m} - mgd$$

$$p_{\mathrm{f}} = \sqrt{p_{\mathrm{i}}^2 - 2gdm^2} = \sqrt{\frac{h^2}{\lambda_0^2} - 2gdm^2}$$

$$\lambda_{\mathrm{f}} = \frac{h}{p_{\mathrm{f}}} = \frac{h}{\sqrt{\frac{h^2}{\lambda_0^2} - 2gdm^2}} = \frac{1}{\sqrt{\frac{1}{\lambda_0^2} - \frac{2gdm^2}{h^2}}}$$

$$\phi_{\mathrm{AB}} - \phi_{\mathrm{CD}} = k_{\mathrm{i}}L - k_{\mathrm{f}}L = 2\pi L\left(\frac{1}{\lambda_0} - \frac{1}{\lambda_{\mathrm{f}}}\right) = \underline{\underline{2\pi L\left(\frac{1}{\lambda_0} - \sqrt{\frac{1}{\lambda_0^2} - \frac{2gdm^2}{h^2}}\right)}}$$

b) $\lambda_0 = 0.2$ m; $L = 45$ mm; $d = 24$ mm; $\Delta\phi = 81°$

$$\psi_1 = \psi_2\,\mathrm{e}^{\frac{2\Delta\phi+\pi}{2}i}$$

$$\psi_2 = \frac{\psi_0}{\sqrt{2}}$$

$$\psi_1' = \frac{1}{\sqrt{2}}(\psi_1 + i\,\psi_2) = \frac{\psi_2}{\sqrt{2}}(\mathrm{e}^{\frac{2\Delta\phi+\pi}{2}i} + i) = \frac{\psi_2}{\sqrt{2}}\,\mathrm{e}^{\frac{\pi}{2}i}(\mathrm{e}^{\Delta\phi i} + 1)$$

$$\psi_2' = \frac{1}{\sqrt{2}}(\psi_2 + i\,\psi_1) = \frac{\psi_2}{\sqrt{2}}(i\,\mathrm{e}^{\frac{2\Delta\phi+\pi}{2}i} + 1) = \frac{\psi_2}{\sqrt{2}}(\mathrm{e}^{(\Delta\phi+\pi)i} + 1)$$

$$I_0 \propto |\psi_0|^2 = 5000\ 1/\mathrm{s}$$

$$I_1 \propto |\psi_1'|^2 = \left|\frac{\psi_2}{\sqrt{2}}\,\mathrm{e}^{\frac{\pi}{2}i}(\mathrm{e}^{\Delta\phi i} + 1)\right|^2 = \frac{\psi_0^2}{2}|1 + \cos(\Delta\phi)| = \underline{\underline{2891\ 1/\mathrm{s}}}$$

$$I_2 \propto \left|\frac{\psi_2}{\sqrt{2}}(\mathrm{e}^{(\Delta\phi+\pi)i} + 1)\right|^2 = \frac{\psi_0^2}{2}|1 - \cos(\Delta\phi)| = \underline{\underline{2109\ 1/\mathrm{s}}}$$