47)

$$A = \begin{pmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{pmatrix}$$

$$p_A = \det(t \cdot I - A) = \det\begin{pmatrix} t-1 & 3 & -3 \\ -3 & t+5 & -3 \\ -6 & 6 & t-4 \end{pmatrix} =$$

$$= (t-1)(t+5)(t-4) + 3(-3)(-6) + (-3)(-3)(6) - (-3)(-6)(t+5) - 6(-3)(t-1) - (t-4)(-3)3 =$$

$$= (t^2+4t-5)(t-4) \cancel{+54} \cancel{+54} \cancel{-18t} \cancel{-90} \cancel{+18t} \cancel{-18} + 9t - 36 =$$

$$= t^3 \cancel{-4t^2} - 5t - 16t + 20 + 9t - 36 =$$

$$0 = t^3 - 12t - 16 \qquad t_1 = 4 \quad \text{erraten}$$

$$t^3 - 12t - 16 : (t-4) = t^2 + 4t + 4$$

$$\begin{array}{l} t^3 - 4t^2 \\ \hline 4t^2 - 12t - 16 \\ 4t^2 - 16t \\ \hline 4t - 16 \end{array}$$

$$t^2 + 4t + 4 = (t+2)^2$$

$$\Rightarrow t_1 = 4, \; t_2 = -2$$

Eigenwerte: $-2, 4$

Für $\lambda = 4$

$$(A - \lambda I) \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$\left(\begin{array}{ccc|c} -3 & -3 & 3 & 0 \\ 3 & -9 & 3 & 0 \\ 6 & -6 & 0 & 0 \end{array}\right) \begin{array}{l} I = I \cdot (-\frac{1}{3}) \\ \rightsquigarrow \\ II = II - 3I \\ III = III - 6I \end{array} \left(\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & -12 & 6 & 0 \\ 0 & -12 & 6 & 0 \end{array}\right) \begin{array}{l} III = III + II \\ \rightsquigarrow \\ II = II \cdot -\frac{1}{12} \end{array} \left(\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$x_3 = x_3$

$x_2 = \frac{1}{2} x_3 \quad \Rightarrow (A - \lambda I) \cdot \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} = 0$

$x_1 = \frac{1}{2} x_3$

Eigenwert -2:

$$\left(A - \lambda I\right) \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$\left(\begin{array}{ccc|c} 3 & -3 & 3 & 0 \\ 3 & -3 & 3 & 0 \\ 6 & -6 & 6 & 0 \end{array}\right) \underset{\mathrm{II} = \mathrm{II} - 3\mathrm{I}}{\overset{\mathrm{I} = \frac{1}{3}\mathrm{I}}{\rightsquigarrow}} \left(\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 6 & -6 & 6 & 0 \end{array}\right) \overset{\mathrm{III} = \mathrm{III} - 6\mathrm{I}}{\rightsquigarrow} \left(\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$x_3 = x_3 \quad x_2 = x_2 \quad x_1 = x_2 - x_3$

Allg Lösung: $\left(x_2 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}\right)$

oder für $x_2 = 1$, $x_3 = 0 \Rightarrow (A - \lambda I) \cdot \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = 0$

für $x_2 = 0$, $x_3 = 1 \Rightarrow (A - \lambda I) \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} = 0$

(48) Sei $A \in GL_n(K)$, $B \in Mat_n(K)$, $\lambda \in K$

z.z. $\det(I_n \lambda - AB) = \det(\lambda I_n - BA)$

$\det(I_n\lambda - AB) = \det(\lambda A A^{-1} - AB) = \det(A(\lambda A^{-1} - B)) =$

$= \det(A) \cdot \det(\lambda A^{-1} - B) = \det(\lambda A^{-1} - B) \cdot \det(A) = \det((\lambda A^{-1} - B) \cdot A) =$

$= \det(\lambda A^{-1} \cdot A - BA) = \underline{\underline{\det(\lambda I_n - BA)}}$

(48)

i) $z_1 = i$ $\quad z_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $\quad z_1 = (1, \frac{\pi}{2})$

ii) $z_2 = 1 - i$ $\quad z_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ $\quad z_2 = (\sqrt{2}, \frac{7\pi}{4})$

iii) $z_3 = z_1 \cdot \overline{z_2} = i(1+i) = -1+i$ $\quad z_3 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ $\quad z_3 = (\sqrt{2}, \frac{3\pi}{4})$

iv) $z_4 = (2{,}5, 4\pi) = (2{,}5, 0)$ $\quad z_4 = \begin{pmatrix} 2{,}5 \\ 0 \end{pmatrix}$ $\quad z_4 = 2{,}5$

v) $z_5 = |z_2| \cdot \overline{z_4} = \cancel{(1+i)(2{,}5) = 2{,}5 + 2{,}5i}$

$= \sqrt{2} \cdot 2{,}5 = \frac{5\sqrt{2}}{2}$ $\quad z_5 = \begin{pmatrix} \frac{5\sqrt{2}}{2} \\ 0 \end{pmatrix}$ $\quad z_5 = (5\frac{\sqrt{2}}{2}, 0)$

b) Löse in $\mathbb{C}$

$z^8 + 4z^4 + 4 = 0$ $\qquad$ sub: $u = z^4$

$\Rightarrow u^2 + 4u + 4 = 0$

$(u+2)^2 = 0 \Rightarrow u = -2$

$z = (u)^{\frac{1}{4}} = (-2)^{\frac{1}{4}}$

$\underline{z = 0{,}841\ldots + 0{,}841\ldots i}$