Wellenfunktion und Aufenthaltswahrscheinlichkeit eines Teilchens

$$\Psi(x) = \begin{cases} 2 & \text{für } 0 \leq x \leq 1 \\ 1 & \text{für } 1 \leq x \leq 2 \\ -2 & \text{für } 2 \leq x \leq 3 \end{cases}$$

a) The probability of finding the particle is the lowest in sector II

b) the unit of $\psi(x)$ is $\sqrt{\mathrm{m}}$ and the unit of x is meter m

c) The normalization condition states that

$$\int_{-\infty}^{\infty} |A\psi(x)|^2 \, \mathrm{d}x = 1$$

In our case

$$\int_{-\infty}^{\infty} |A\psi(x)|^2 \, \mathrm{d}x = A^2 \left[\int_0^1 1 \, \mathrm{d}x + \int_1^2 4 \, \mathrm{d}x + \int_2^3 4 \, \mathrm{d}x \right] = 9A^2 \overset{!}{=} 1$$

$\Rightarrow A = \frac{1}{3}$

d)

$$P = \int_2^3 \frac{4}{9} \, \mathrm{d}x = \underline{\underline{\frac{4}{9}}}$$

e)

$$P = \int_2^3 \left| \frac{2}{3} \, \mathrm{e}^{\frac{\pi}{6} i} \right|^2 \mathrm{d}x = \int_2^3 \frac{4}{9} \, \mathrm{d}x = \underline{\underline{\frac{4}{9}}}$$

Teilchen im asymmetrischen Potentialtopf

$$V(x) = \begin{cases} \infty & \text{für } x \leq 0 \\ -V_0 & \text{für } 0 \leq x \leq a \\ 0 & \text{für } a \leq x \end{cases}$$

a) $-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} + E_{\text{pot}}\psi(x) = E\psi(x)$

$\psi_1(x) = 0$

b)

For II: $\psi_{\text{II}}(x) = A\sin(kx) + B\cos(kx)$

For III: $\psi_{\text{III}}(x) = C\,\mathrm{e}^{-\kappa x} + D\,\mathrm{e}^{\kappa x}$

Boundary conditions:

$$\psi_{\text{II}}(x=0) = 0$$
$$\psi_{\text{II}}(x=a) = \psi_{\text{III}}(x=a)$$
$$\frac{\mathrm{d}}{\mathrm{d}x}\psi_{\text{II}}(x=0) = 0$$
$$\frac{\mathrm{d}}{\mathrm{d}x}\psi_{\text{II}}(x=a) = \frac{\mathrm{d}}{\mathrm{d}x}\psi_{\text{III}}(x=a)$$

c)

$$k = \pm\frac{\sqrt{2Em}}{\hbar}$$
$$\kappa = \pm\frac{\sqrt{2m(V_0-E)}}{\hbar}$$

d)

e)

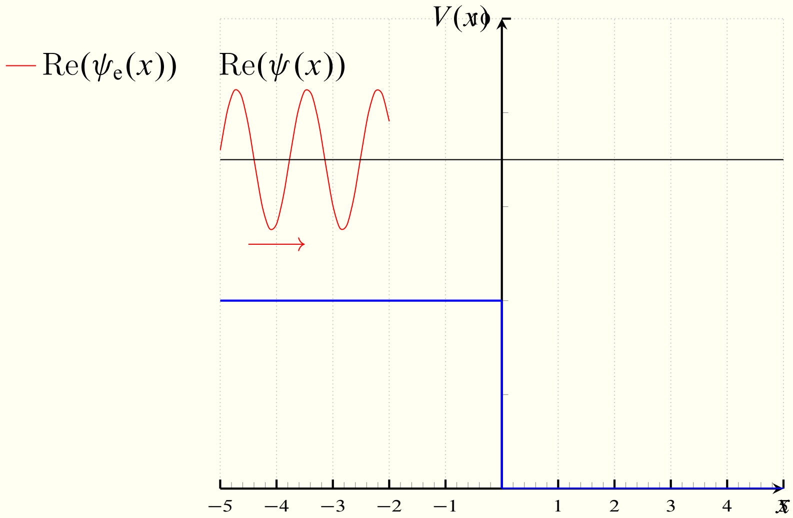
f)

g)

Teilchen an einem Potentialabfall

$$V(x) = \begin{cases} V_0 & \text{für } x \leq 0 \\ 0 & \text{für } x > 0 \end{cases}$$

a)



b)

$$\psi_t(x=0) = \psi_e(x=0) + \psi_r(x=0)$$
$$\frac{d}{dx}\psi_t(x=0) = \frac{d}{dx}\left(\psi_e(x=0) + \psi_r(x=0)\right)$$

c)

d)

e)

f)