193. Gekoppelte physikalische Pendel

$$m = 5\ \text{kg}; \quad k = 2\ \text{N/m}; \quad a = 0.1\ \text{m}; \quad h = 0.4\ \text{m}; \quad l = 0.1\ \text{m}$$

a) $\omega_a =$

$$I = \tfrac{1}{12} m(a^2 + h^2) + ml^2 = 0.12\ \text{kg m}^2$$
$$\omega_a = \sqrt{\tfrac{mgl}{I}} = \underline{\underline{6.37\ \text{rad/s}}}$$

b) $F = -m\omega^2 x; \quad \Delta x = x_1 - x_2$

$$m\ddot{x_1} = -m{\omega_a}^2 x_1 - k(x_1 - x_2)$$
$$m\ddot{x_2} = -m{\omega_a}^2 x_2 - k(x_2 - x_1)$$
$$\Rightarrow m(a_1 - a_2) = -m{\omega_a}^2(x_1 - x_2) - 2k(x_1 - x_2) = -(m{\omega_a}^2 + 2k)(x_1 - x_2)$$
$$\Delta\ddot{x} = -\underbrace{({\omega_a}^2 + \tfrac{2k}{m})}_{=\omega_b}\Delta x$$
$$\omega_b = \sqrt{\omega_a^2 + \tfrac{2k}{m}} = \underline{\underline{6.43\ \text{rad/s}}}$$

c)

196. Schallgeschwindigkeit und Elastizitätseigenschaften

$$\rho = 4500\ \text{kg/m}^3; \quad v_\parallel = 5050\ \text{m/s}; \quad v_\perp = 3100\ \text{m/s}$$

a) $v_\parallel = \sqrt{\tfrac{E}{\rho}}; \quad v_\perp = \sqrt{\tfrac{G}{\rho}}$

$$E = {v_\parallel}^2 \rho = \underline{\underline{1.15 * 10^{11}\ \text{Pa}}}$$
$$G = {v_\perp}^2 \rho = \underline{\underline{4.32 * 10^{10}\ \text{Pa}}}$$

b) $\mu = \tfrac{E}{2G} - 1$

$$\mu = \tfrac{{v_\parallel}^2}{2{v_\perp}^2} - 1 = \underline{\underline{0.33}}$$

c)

$$\mu = \underline{\underline{\tfrac{{v_\parallel}^2}{2{v_\perp}^2} - 1}} \qquad \text{oder} \qquad \underline{\underline{\tfrac{v_\parallel}{v_\perp} = \sqrt{2\mu + 2}}}$$

199. Kompensation der Wärmeausdehnung

$$a = h = 0.1\ \mathrm{m}; \quad \Delta T = 1\ \mathrm{K}; \quad E = 2.1 * 10^{11}\ \mathrm{N/m^2}$$
$$\mu = 0.29; \quad \rho = 7870\ \mathrm{kg/m^3}; \quad \alpha = 1.2 * 10^{-5}\ \mathrm{K^{-1}}$$

a) $\frac{\Delta h}{h} = \alpha \Delta T; \quad \frac{F}{A} = E \frac{\Delta h}{h}$

$\frac{mg}{h^2} = E\alpha\Delta T$

$m = \frac{E\alpha\Delta T h^2}{g} = \underline{\underline{2568.8\ \mathrm{kg}}}$

b) $\Delta a_1 = \mu \Delta h; \quad \Delta a_2 = \alpha \Delta T$

$\Delta a_{\mathrm{ges}} = \Delta a_1 + \Delta a_2 = \underline{\underline{1.5 * 10^{-6}\ \mathrm{m}}}$