Quasi-Stellar Diameters and Intensity Fluctuations
Author(s): James Terrell
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Reports

Quasi-Stellar Diameters and Intensity Fluctuations

Abstract. *It is shown that there are relativistic limits on the fluctuations in brightness which may be observed for a large spherical source, and also for more general sources, so that quasi-stellar objects are probably no more than light-days in size. There is thus a possibility that they may be close to our galaxy.*

The relatively rapid fluctuation (*1–4*) of photographic magnitude has been one of the most interesting and puzzling features of quasi-stellar objects. These fluctuations indicate that the objects cannot be larger than a few light-years, or perhaps even light-days, in diameter. These size limitations are based on the finite speed of light, and have been discussed in several papers (*1, 2, 5, 6*).

Hoffmann has recently suggested (*6*) that rapid fluctuations and a large size may still be consistent, provided that a central source in a roughly spherical object triggers simultaneous brightening of the entire surface. This interesting suggestion is, unfortunately, not consistent with the special theory of relativity. A very large spherical surface brightening simultaneously at all points would not be seen as such by an observer, because of the finite speed of light and the different distances to various points of the surface. The brightening would be seen to begin at the nearest part of the surface and to spread out from there to the edge in a time given by R/c, where R is the radius of the object and c the velocity of light. If the surface were oscillating in brightness with period T small compared to R/c, a concentric set of bright rings would be seen, expanding continually from the center.

For an object sufficiently far away that no surface details could be seen, there could thus be a large relativistic reduction in the visible fluctuations of magnitude. It is possible to calculate the exact relation between the visible brightness fluctuation and the true fluctuation of Hoffmann's spherical surface. An integration over the surface, using Lambert's cosine intensity law, yields the fraction $F(\alpha)$ of the sinusoidal fluctuation which is visible at a distance:

$$F(\alpha) = (2/\alpha^2)(2 + \alpha^2 - 2\cos\alpha - 2\alpha\sin\alpha)^{\frac{1}{2}} \tag{1}$$

in which α is the ratio of circumference to period, in appropriate units:

$$\alpha \equiv 2\pi R/cT \tag{2}$$

This relativistic effect vanishes, so that $F(\alpha)$ approaches unity, for small sources or long periods:

$$F(\alpha) = 1 - \alpha^2/36 + \cdots, \ \alpha << 1 \tag{3}$$

For large values of α, however, the visible fraction of the fluctuation is inversely proportional to α:

$$F(\alpha) \cong 2/\alpha = cT/\pi R, \ \alpha >> 1 \tag{4}$$

Since the intensity at the source cannot become negative, the oscillating component at the source must have a smaller amplitude than the average brightness. Therefore, at a distance, the ratio of maximum to minimum visible intensity must be less than

$$[1 + F(\alpha)]/[1 - F(\alpha)].$$

This immediately establishes a maximum value α_m of α which can be associated with a given oscillation in brightness. Letting Δm be the change in stellar magnitude from maximum to minimum brightness, we have

$$\Delta m = 2.5 \log_{10} \{[1 + F(\alpha_m)]/[1 - F(\alpha_m)]\} \tag{5}$$

This may be approximated by

$$\alpha_m/\pi = D_m/cT \cong 10(\log_{10} e)/\pi\Delta m = 1.382/\Delta m, \ \alpha >> 1 \tag{6}$$

giving the maximum diameter D_m which could produce sinusoidal fluctuations Δm in visual magnitude of period T.

This asymptotic relation approximates the exact relation, Eq. 5, in a very satisfactory way, giving an error in D_m of the order of ± 10 percent or less for $\Delta m \leqq 0.7$, which is the range of interest for quasi-stellar sources. These relations mean, to give an example, that for a spherical source with $\alpha = 2\pi$, or $D = 2cT$ (for example, 1-year period and 2-light-year diameter), the apparent fluctuations would be reduced to 31.8 percent of the true fluctuations. The maximum and minimum visible brightness could not differ by as much as a factor of 1.93, or 0.71 magnitudes ($\Delta m = 0.69$ from Eq. 6).

Although Eq. 6 was derived on the assumption of a spherical surface with sinusoidally oscillating brightness (*7*), it can be put on a more general basis. Consider, for example, an astronomical source composed of a number n of adjacent spherical oscillating sources, all alike but with completely random relative phases. The fluctuation of the total light intensity I from the average, $\bar{I}$, may be described in terms of the relative standard deviation, $\sigma(I)/\bar{I}$. It may readily be shown that the combination of n sources has a fluctuation which is reduced from that of a single source by the factor $n^{1/2}$, the square root of the ratio of total source areas (*2*). Similar considerations have been discussed previously (*2, 5*). Thus, in this case also, the visible fluctuation Δm is inversely proportional to an effective diameter, so that Eq. 6 should still hold for n independent sources.

Still more generally, the source could merely be assumed to be composed of a number of independently flashing surface areas. This still leads to Eq. 6 if the independent surface areas are taken to have area c^2T^2/π. This exact choice is by no means forced on us, but is at least a reasonable choice for the maximum area for which the oscillating brightness may remain in phase, considering the finite speed of light signals between portions of the area.

Thus we may use Eq. 6 for quasi-stellar objects with some confidence that it will give a physically reasonable maximum diameter. The precision is greatest, of course, for sinusoidal light fluctuations of a spherical source, but the result is as accurate, in less simple cases, as the meanings of "period" and "effective diameter" permit. The fluctuations observed (*1–4*) for the quasi-stellar objects 3C48, 3C196, and 3C273 are of the order of 0.3 magnitudes, or a factor of 1.32 in brightness. A period of 2 years would mean, according to Eq. 6, a source diameter less than 9.2 light-years (8.6 light-years, from Eq. 5). If the night-to-night variations and bright flashes observed for 3C273 and 3C48, amounting to as much as 0.7 magnitude with a period of days (*1–4*), are real and general phenomena, the objects cannot be more than a few light-days in size, as Smith and Hoffleit (*1*) have stated. In the case of a multiple-source disc produced by gravitational contraction, as suggested by Hoyle and Fowler (*5, 8*), the effective diameter would have to be considered to be essentially that of the one source emitting most of the light at a given time, rather than that of the entire assemblage.

It should be kept in mind that Eq. 6 yields an upper limit for the source diameter, not an average, and that the light source would perhaps be considerably smaller than this size. Thus the quasi-stellar objects—at least the optically visible parts—are probably less than a few light-days in size and could be of the order of the solar system in size, or less (*9*). These objects are then many orders of magnitude less than galactic size, simply on the basis of the light fluctuations.

Since the quasi-stellar objects are not of galactic size, it is not absolutely necessary to suppose that they are at the distances of the order of 1000 Mparsec which follow from the application of Hubble's law. The large red shifts observed for these objects (*10–13*) may be due to the same cause as that of the most distant galaxies, or they may be gravitational in origin, or they may simply be due to relativistic velocities of nearby objects. The first explanation requires optical power outputs perhaps 100 times larger than those of the largest and brightest galaxies (*1–3, 5*) and is so far the favorite explanation, although the energy source is not clear. The gravitational explanation is usually ruled out (*5, 10, 12*) primarily on spectroscopic grounds.

The third possibility, that the objects might be much closer to our galaxy, with the red shift due only to the relativistic Doppler shift, has not been much discussed. A minimum distance is determined by the lack of observed proper motion. W. H. Jefferys (*14*) has established the proper motion of 3C273 as unobservably small (0.001 ± 0.0025 second of arc per year), and W. J. Luyten (*15*) has obtained a similar result. If it is assumed, for example, that 3C273 originated near the center of our galaxy, about 8 kparsec (26,000 light-years) from the sun, a proper motion of 0.002 second per year and a recessional velocity of 0.146 c (*10*) would correspond to 190 kparsec observed distance, and to an origin about 5 million years ago. This distance is a few galactic diameters away, further than the clouds of Magellan, but not so far as the Andromeda nebula which is about 500 kparsec distant.

How such highly relativistic objects could be produced by an explosion or explosions in our galaxy is not known, and this is probably the principal objection to the idea. Considerable kinetic energy alone would be required, amounting to 1.1 percent of the rest mass for 3C273, and to 9.6 percent for 3C147, the fastest such object yet observed, with $v/c = 0.410$, and $\Delta\lambda/\lambda = 0.545$ (*13*). The optical energy requirements are reduced by a factor of 10^7 at this distance, however, amounting to about 10^5 times the output of our sun. A possible source of high velocities, if the objects are assumed to be local, would be a gravitational collapse (*5, 8*) in our galaxy.

Whatever their distance, it is clear that the quasi-stellar objects are not of galactic size, but are, at least optically, probably less than light-days in size (*16*).

JAMES TERRELL

University of California, Los Alamos Scientific Laboratory, Los Alamos, New Mexico

References and Notes

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5. J. L. Greenstein, *Sci. Am.* **209**, No. 6, 54 (1963); H. Y. Chiu, *Phys. Today* **17**, No. 5, 21 (1964).
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7. It is interesting to note that a somewhat different case, that of a luminous disc viewed perpendicularly, with waves of brightness spreading out from the center, has precisely the same solution, Eq. 5. In either case an assumed recession of the source would not change the result, but the period T should be as measured in the rest frame of the source.
8. F. Hoyle, W. A. Fowler, G. R. Burbidge, E. M. Burbidge, *Astrophys. J.* **139**, 909 (1964).
9. The solar system is about ½ light-day across. These considerations would not rule out an extremely small, dense, region in the center of a galaxy or cluster as a quasi-stellar object, as suggested by S. M. Ulam and W. E. Walden [*Nature* **201**, 1202 (1964)].
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14. W. H. Jefferys, III, *Astron. J.* **69**, 255 (1964).
15. W. J. Luyten, *Minnesota University Astron. Observatory Publ.* **3**, No. 13 (1963).
16. A letter by A. T. Young has just been published [*Science* **145**, 72 (1964)], giving some similar considerations on the size of quasi-stellar objects. Young's results are based on square-wave modulation of a light source. If a large spherical source of radius R is "turned off" for a time interval t, it will undergo a maximum reduction in apparent brightness amounting to $(2ct/R) - (ct/R)^2$. This is, as in Young's example, 19 percent decrease for a 10-light-year-radius source turned off for 1 year. If such a fluctuation were seen, the correct maximum radius would be obtained from Eq. 4 if the period were taken to be $T = \pi t$. This is the period corresponding to the amplitude of the visible fluctuation and its maximum rate of change, for a sinusoidal oscillation. Thus the results of this paper are as applicable to monotonic and single-pulse fluctuations as to sinusoidal oscillations, if the "period" is suitably estimated. Young's formulation and the one given here both give a maximum rate of change of luminosity equal to $2c/R$ logarithmically, or to 1.0857 $(2c/R)$ in units of stellar magnitude.

17. Supported by the AEC. Some of the matters considered have been clarified in interesting discussions with G. R. Burbidge, A. N. Cox, and S. M. Ulam, but they should not be held responsible for the opinions expressed here.

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Pulse Radiolysis of Potassium Bromide Solutions

Abstract. *After application of a 2-microsecond pulse of an electron beam to aqueous, aerated, acid solutions of KBr, transient spectra of* Br_2^- *and* Br_3^- *were observed. The kinetic analysis of the reactions as well as the measured values of* $G(H_2O_2)$, $G(Br_2)$, *and* $G(Br_3^-)$ *revealed a reaction mechanism differing from the one which is accepted for radiolysis at low intensity.*

In recent pulse radiolysis experiments we have studied the transient species and the yields of hydrogen peroxide in aqueous, aerated $10^{-2}M$ KBr in sulfuric acid at pH 2. Pulses of 2-μsec duration of 4 Mev electrons giving up to 5000 rad/pulse were used. The technique has been described elsewhere (*1*). Hydrogen peroxide was determined by the iodide reagent (*2*) after removal of bromine from the solution by bubbling with inert gas.

As shown in Fig. 1, a transient species appears immediately after the pulse, showing a strong absorption with a maximum at 3600 Å. This transient disappears in about 100 μsec and a new, strongly absorbing transient appears, with a maximum absorption at 2700 Å.