1. Magnetfeld eines asymmetrischen Leiters

a) $r = \sqrt{x^2 + y^2}$

$$\begin{aligned}
I_+ &= I(1 + \tfrac{a^2}{R^2}) \\
I_- &= -I\,\tfrac{a^2}{R^2} \\
B_+ &= \tfrac{\mu_0 I_+}{2r\pi} = \tfrac{\mu_0 I_+}{2\pi\sqrt{x^2+y^2}} \\
B_- &= \tfrac{\mu_0 I_-}{2r\pi} = \tfrac{\mu_0 I_-}{2\pi\sqrt{(x-b)^2+y^2}} \\
B &= B_+ + B_- = \tfrac{\mu_0 I_+}{2\pi\sqrt{x^2+y^2}} + \tfrac{\mu_0 I_-}{2\pi\sqrt{(x-b)^2+y^2}} = \\
&= \tfrac{\mu_0 I}{2\pi}\left[\tfrac{1}{\sqrt{x^2+y^2}} + \tfrac{a^2}{R^2}\left(\tfrac{1}{\sqrt{x^2+y^2}} - \tfrac{1}{\sqrt{(x-b)^2+y^2}}\right)\right] \\
B(2R,0) &= \underline{\underline{\tfrac{\mu_0 I}{4R\pi}\left[1 + \tfrac{a^2}{R}\left(\tfrac{1}{R} - \tfrac{1}{R-b}\right)\right]}}
\end{aligned}$$

b)

$$B(0,2R) = \underline{\underline{\tfrac{\mu_0 I}{2R\pi}\left[\tfrac{1}{2} + \tfrac{a^2}{R}\left(\tfrac{1}{2R} - \tfrac{1}{\sqrt{b^2+4R^2}}\right)\right]}}$$

2. Induktion

$$\vec{B} = B_x \hat{x}$$

a)

$$\Phi(t) = \int \vec{B}\ \mathrm{d}\vec{A} = B\cos(\omega t)\int 1\ \mathrm{d}\vec{A} = r^2\pi B\cos(\omega t)$$

$$U(t) = -\frac{\mathrm{d}\Phi}{\mathrm{d}t} = \underline{\underline{r^2\pi\omega B\sin(\omega t)}}$$

b)

c)

$$\begin{aligned} I &= \tfrac{U}{R} \\ \frac{\mathrm{d}Q}{\mathrm{d}t} &= -\tfrac{1}{R}\frac{\mathrm{d}\Phi}{\mathrm{d}t} \\ \Delta Q &= -\tfrac{r^2\pi B}{R}\int_0^{\pi/2\omega}\cos(\omega t)\,\mathrm{d}t = \underline{\underline{\tfrac{r^2\pi B}{R}}} \end{aligned}$$

3. Indunktionsspannung - Stab

$$I := I_{\text{Bat}}$$

a)

$$\begin{aligned} \Phi &= Blx \\ U &= -Blv \\ I_{\text{Ind}} &= -\tfrac{Blv}{R} \\ \vec{F}_{\text{Ind}} &= I_{\text{Ind}}\vec{l}\times\vec{B} = -\tfrac{B^2l^2v}{R}\hat{x} \\ \vec{F}_L &= I\vec{l}\times\vec{B} = IlB\hat{x} \\ \vec{F}_{\text{ges}}(v) &= \vec{F}_{\text{Ind}} + \vec{F}_L = \underline{\underline{lB(I - \tfrac{Blv}{R})\hat{x}}} \end{aligned}$$

b)

$$\begin{aligned} m\frac{\mathrm{d}v}{\mathrm{d}t} &= lB\left(I - \tfrac{Bl}{R}v(t)\right) \\ v(t) &= \tfrac{IR}{Bl}\left(1 - \mathrm{e}^{-\frac{B^2l^2}{mR}t}\right) \\ \lim_{t\to\infty} v(t) &= \underline{\underline{\tfrac{IR}{Bl}}} \end{aligned}$$

c)

$$I_{\text{ges}} = I + I_{\text{Ind}} = I - \tfrac{Blv}{R} = I - \tfrac{BlRI}{BlR} = \underline{\underline{0}}$$