1. Influenz in einer Metallplatte

$$k = \frac{1}{4\pi\epsilon_0}; \quad r^2 = x^2 + y^2; \quad \sigma = \epsilon_0 E$$

$$\begin{aligned}
E_1(x,y,z) &= k\,\frac{Q}{x^2+y^2+(z+R)^2} \\
E_2(x,y,z) &= E_1(x,y,z-2R) = k\,\frac{Q}{x^2+y^2+(z-R)^2} \\
E_{ges}(x,y,z) &= E_1 + E_2 = kQ\left(\frac{1}{x^2+y^2+(z+R)^2} + \frac{1}{x^2+y^2+(z-R)^2}\right)
\end{aligned}$$

$$\begin{aligned}
\sigma(r) &= \epsilon_0 E_{ges}(x,y,z) = \epsilon_0 kQ\left(\frac{1}{x^2+y^2+(z+R)^2} + \frac{1}{x^2+y^2+(z-R)^2}\right) \\
&= \underline{\underline{\frac{Q}{2\pi}\,\frac{1}{r^2+R^2}}}
\end{aligned}$$

2. Feld einer Hohlkugel

$$k = \frac{1}{4\pi\epsilon_0}$$

a. $\rho = \frac{Q}{V}$

$$\begin{aligned}
V &= \int_V dV = \int_0^{\pi}\int_0^{2\pi}\int_{R_i}^{R_a} r^2\sin(\theta)\,\mathrm{d}r\mathrm{d}\varphi\mathrm{d}\theta \\
&= \frac{4}{3}\pi(R_a^3 - R_i^3) \\
\rho &= \underline{\underline{\frac{3Q}{4\pi}\,\frac{1}{R_a^3-R_i^3}}}
\end{aligned}$$

For $r \geq R_a$:

$$\begin{aligned}
q_{in} &= Q \\
\frac{q_{in}}{\epsilon_0} &= \oint \vec{E}(\vec{r})\,\mathrm{d}\vec{A} = E(r)\oint \mathrm{d}A = E(r)4\pi r^2 \\
E(r) &= kQ\,\frac{1}{r^2}
\end{aligned}$$

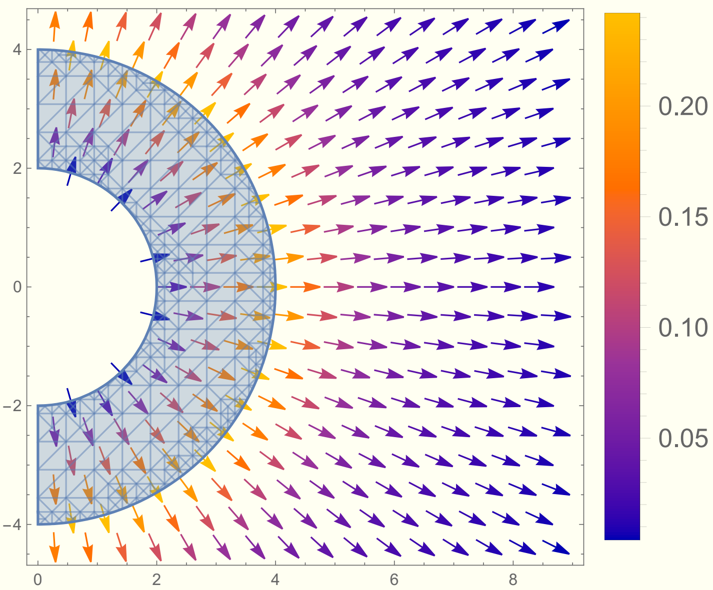
For $R_i < r < R_a$:

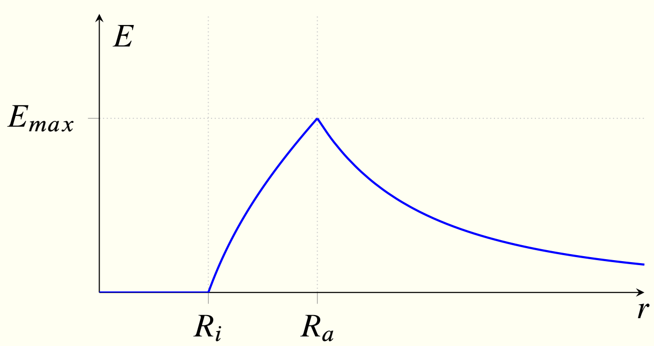
$$q_{in} = \rho V_{in} = Q\,\frac{r^3 - R_i^3}{R_a^3 - R_i^3}$$
$$\frac{q_{in}}{\epsilon_0} = \oint \vec{E}(\vec{r})\,\mathrm{d}\vec{A} = E(r)\oint \mathrm{d}A = E(r)4\pi r^2$$
$$E(r) = kQ\,\frac{r^3 - R_i^3}{R_a^3 - R_i^3}\,\frac{1}{r^2}$$

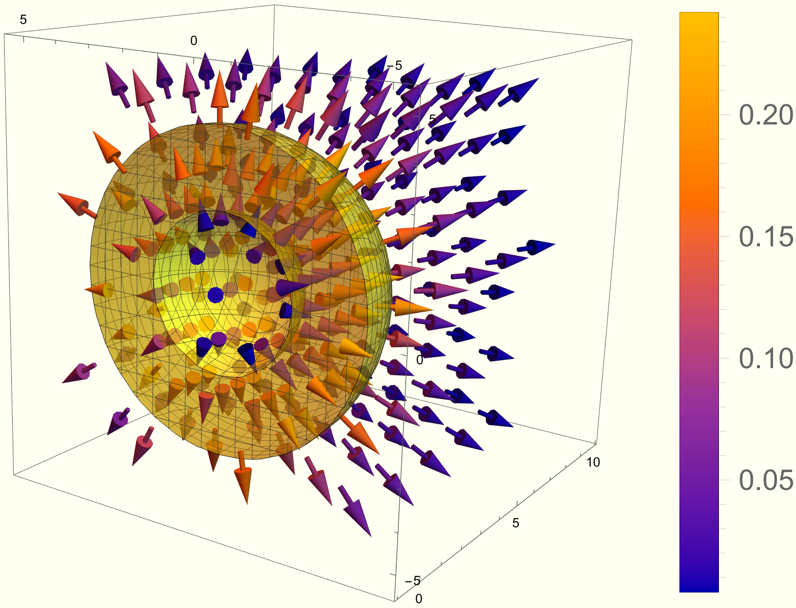
For $r \leq R_i$:

$$q_{in} = 0$$
$$E(r) = 0$$

$$\boxed{E(r) = \begin{cases} 0 & \text{for } 0 < r \leq R_i, \\ \frac{kQ}{R_a^3 - R_i^3}\left(r - \frac{R_i^3}{r^2}\right) & \text{for } R_i < r < R_a, \\ kQ\,\frac{1}{r^2} & \text{for } R_a \leq r. \end{cases}}$$



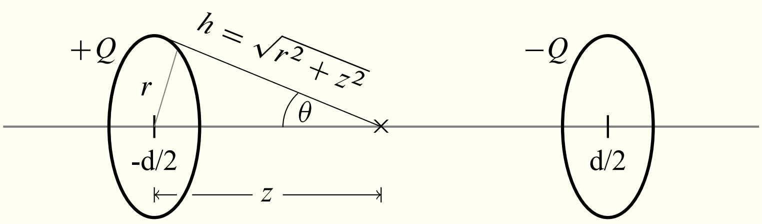




3. Zwei ringförmige Ladungsträger

$$k = \frac{1}{4\pi\epsilon_0}$$

a. $h = \sqrt{r^2 + z^2}$; $\cos(\theta) = \frac{z}{h}$; $\mathrm{d}Q = \lambda \mathrm{d}r$; $\mathrm{d}r = r\mathrm{d}\varphi$; $Q = 2r\pi\lambda$



Due to the symmetric nature of this problem we can neglect vertical components of forces

$$\mathrm{d}E_1(z) = k\,\frac{\mathrm{d}Q}{h^2}\cos(\theta) = k\,\frac{\lambda \mathrm{d}r}{r^2+z^2}\,\frac{z}{\sqrt{r^2+z^2}} = k\,\frac{\lambda r z}{(r^2+z^2)^{3/2}}\,\mathrm{d}\varphi$$

$$E_1(z) = \int \mathrm{d}E_1 = k\,\frac{\lambda r z}{(r^2+z^2)^{3/2}}\int_0^{2\pi}\mathrm{d}\varphi = k\,\frac{\lambda 2\pi r z}{(r^2+z^2)^{3/2}} = k\,\frac{Qz}{(r^2+z^2)^{3/2}}$$

$$E_2(z) = -E_1(z-d) = -k\,\frac{Q(z-d)}{(r^2+(z-d)^2)^{3/2}}$$

$$E_{ges}(z) = E_1 + E_2 = k\,\frac{Qz}{(r^2+z^2)^{3/2}} - k\,\frac{Q(z-d)}{(r^2+(z-d)^2)^{3/2}}$$

$$= kQ\left(\frac{z+\frac{d}{2}}{\left(r^2+\left(z+\frac{d}{2}\right)^2\right)^{3/2}} + \frac{\frac{d}{2}-z}{\left(r^2+\left(z-\frac{d}{2}\right)^2\right)^{3/2}}\right)$$

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b. $$\int_{-d/2}^{d/2} qE(z)\,\mathrm{d}z = kQq\left(\int_{-d/2}^{d/2}\frac{z+\frac{d}{2}}{\left(r^2+\left(z+\frac{d}{2}\right)^2\right)^{3/2}}\,\mathrm{d}z + \int_{-d/2}^{d/2}\frac{\frac{d}{2}-z}{\left(r^2+\left(z-\frac{d}{2}\right)^2\right)^{3/2}}\,\mathrm{d}z\right)$$

$$= \frac{kQq}{2}\left(\int_{r^2}^{r^2+d^2}\frac{1}{u^{3/2}}\,\mathrm{d}u - \int_{r^2-d^2}^{r^2}\frac{1}{v^{3/2}}\,\mathrm{d}v\right)$$

$$= 2kQq\left(\frac{1}{r} - \frac{1}{\sqrt{r^2+d^2}}\right)$$

$$\boxed{\begin{aligned} u &= r^2 + \left(z+\tfrac{d}{2}\right)^2 \\ \mathrm{d}u &= 2\left(z+\tfrac{d}{2}\right)\mathrm{d}z \\ v &= r^2 + \left(z-\tfrac{d}{2}\right)^2 \\ \mathrm{d}v &= 2\left(z-\tfrac{d}{2}\right)\mathrm{d}z \end{aligned}}$$

$qE(z)$ represents a force, which in turn gives the amount of work done by the field when integrated over a distance

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c. $r = 0.1$ m; $d = 0.5$ m; $Q = 10^{-6}$ C; $q = 1.6 * 10^{19}$ C; $m = 200.592u$

$$W = 2kQq\left(\frac{1}{r} - \frac{1}{\sqrt{r^2+d^2}}\right) = 2.3 * 10^{24}\ \mathrm{J}$$

$$W = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2W}{m}} = 3.73 * 10^{24}\ \mathrm{m/s}$$