

1. Influenz in einer Metallplatte

$$k = \frac{1}{4\pi\epsilon_0}; \quad r^2 = x^2 + y^2; \quad \sigma = \epsilon_0 E$$

$$E_1(x, y, z) = k \frac{Q}{x^2 + y^2 + (z+R)^2}$$

$$E_2(x, y, z) = E_1(x, y, z - 2R) = k \frac{Q}{x^2 + y^2 + (z-R)^2}$$

$$E_{ges}(x, y, z) = E_1 + E_2 = kQ \left(\frac{1}{x^2 + y^2 + (z+R)^2} + \frac{1}{x^2 + y^2 + (z-R)^2} \right)$$

$$\begin{aligned} \sigma(r) &= \epsilon_0 E_{ges}(x, y, z) = \epsilon_0 kQ \left(\frac{1}{x^2 + y^2 + (z+R)^2} + \frac{1}{x^2 + y^2 + (z-R)^2} \right) \\ &= \underline{\underline{\frac{Q}{2\pi} \frac{1}{r^2 + R^2}}} \end{aligned}$$

2. Feld einer Hohlkugel

$$k = \frac{1}{4\pi\epsilon_0}$$

a. $\rho = \frac{Q}{V}$

$$\begin{aligned} V &= \int_V dV = \int_0^\pi \int_0^{2\pi} \int_{R_i}^{R_a} r^2 \sin(\theta) \, dr d\varphi d\theta \\ &= \frac{4}{3} \pi (R_a^3 - R_i^3) \\ \rho &= \underline{\underline{\frac{3Q}{4\pi} \frac{1}{R_a^3 - R_i^3}}} \end{aligned}$$

For $r \geq R_a$:

$$\begin{aligned} q_{in} &= Q \\ \frac{q_{in}}{\epsilon_0} &= \oint \vec{E}(\vec{r}) \, d\vec{A} = E(r) \oint dA = E(r) 4\pi r^2 \\ E(r) &= kQ \frac{1}{r^2} \end{aligned}$$

For $R_i < r < R_a$:

$$q_{in} = \rho V_{in} = Q \frac{r^3 - R_i^3}{R_a^3 - R_i^3}$$

$$\frac{q_{in}}{\epsilon_0} = \oint \vec{E}(\vec{r}) \cdot d\vec{A} = E(r) \oint dA = E(r) 4\pi r^2$$

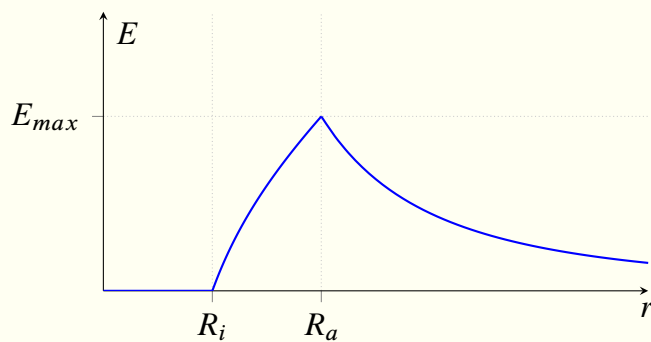
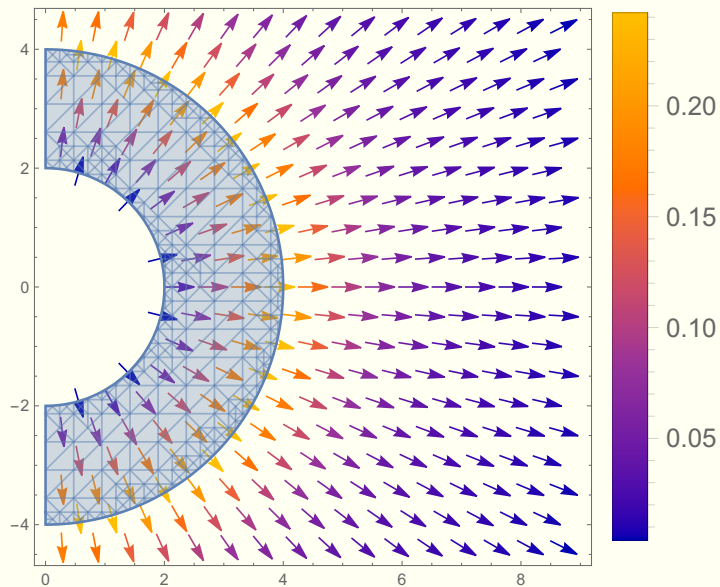
$$E(r) = kQ \frac{r^3 - R_i^3}{R_a^3 - R_i^3} \frac{1}{r^2}$$

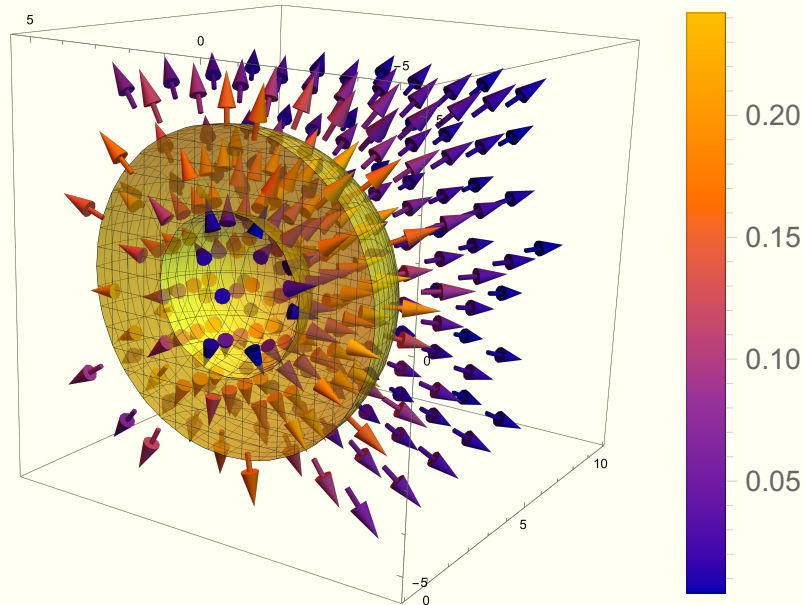
For $r \leq R_i$:

$$q_{in} = 0$$

$$E(r) = 0$$

$$E(r) = \begin{cases} 0 & \text{for } 0 < r \leq R_i, \\ \frac{kQ}{R_a^3 - R_i^3} \left(r - \frac{R_i^3}{r^2} \right) & \text{for } R_i < r < R_a, \\ kQ \frac{1}{r^2} & \text{for } R_a \leq r. \end{cases}$$

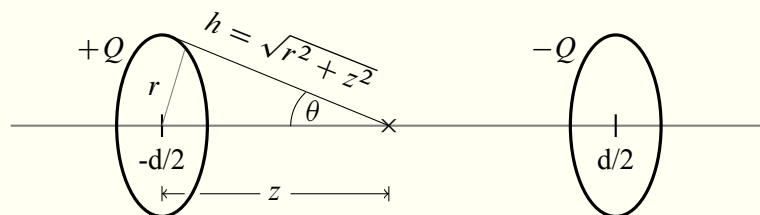




3. Zwei ringförmige Ladungsträger

$$k = \frac{1}{4\pi\epsilon_0}$$

a. $h = \sqrt{r^2 + z^2}; \quad \cos(\theta) = \frac{z}{h}; \quad dQ = \lambda dr; \quad dr = r d\varphi; \quad Q = 2r\pi\lambda$



Due to the symmetric nature of this problem we can neglect vertical components of forces

$$\begin{aligned}
 dE_1(z) &= k \frac{dQ}{h^2} \cos(\theta) = k \frac{\lambda dr}{r^2+z^2} \frac{z}{\sqrt{r^2+z^2}} = k \frac{\lambda rz}{(r^2+z^2)^{3/2}} d\varphi \\
 E_1(z) &= \int dE_1 = k \frac{\lambda rz}{(r^2+z^2)^{3/2}} \int_0^{2\pi} d\varphi = k \frac{\lambda 2\pi rz}{(r^2+z^2)^{3/2}} = k \frac{Qz}{(r^2+z^2)^{3/2}} \\
 E_2(z) &= -E_1(z-d) = -k \frac{Q(z-d)}{(r^2+(z-d)^2)^{3/2}} \\
 E_{ges}(z) &= E_1 + E_2 = k \frac{Qz}{(r^2+z^2)^{3/2}} - k \frac{Q(z-d)}{(r^2+(z-d)^2)^{3/2}} \\
 &= kQ \left(\frac{z+\frac{d}{2}}{\left(r^2+\left(z+\frac{d}{2}\right)^2\right)^{3/2}} + \frac{\frac{d}{2}-z}{\left(r^2+\left(z-\frac{d}{2}\right)^2\right)^{3/2}} \right)
 \end{aligned}$$

$$\begin{aligned}
 \text{b. } \int_{-d/2}^{d/2} qE(z) dz &= kQq \left(\int_{-d/2}^{d/2} \frac{z+\frac{d}{2}}{\left(r^2+\left(z+\frac{d}{2}\right)^2\right)^{3/2}} dz + \int_{-d/2}^{d/2} \frac{\frac{d}{2}-z}{\left(r^2+\left(z-\frac{d}{2}\right)^2\right)^{3/2}} dz \right) \\
 &= \frac{kQq}{2} \left(\int_{r^2}^{r^2+d^2} \frac{1}{u^{3/2}} du - \int_{r^2-d^2}^{r^2} \frac{1}{v^{3/2}} dv \right) \\
 &= \underline{\underline{2kQq \left(\frac{1}{r} - \frac{1}{\sqrt{r^2+d^2}} \right)}}
 \end{aligned}$$

$$\begin{aligned}
 u &= r^2 + \left(z + \frac{d}{2}\right)^2 \\
 du &= 2 \left(z + \frac{d}{2}\right) dz \\
 v &= r^2 + \left(z - \frac{d}{2}\right)^2 \\
 dv &= 2 \left(z - \frac{d}{2}\right) dz
 \end{aligned}$$

$qE(z)$ represents a force, which in turn gives the amount of work done by the field when integrated over a distance

$$\text{c. } r = 0.1 \text{ m}; \quad d = 0.5 \text{ m}; \quad Q = 10^{-6} \text{ C}; \quad q = 1.6 * 10^{19} \text{ C}; \quad m = 200.592u$$

$$W = 2kQq \left(\frac{1}{r} - \frac{1}{\sqrt{r^2+d^2}} \right) = \underline{\underline{2.3 * 10^{24} \text{ J}}}$$

$$W = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2W}{m}} = \underline{\underline{3.73 * 10^{24} \text{ m/s}}}$$