Materiewellen und Wellenfunktionen

$$\Psi(x) = \begin{cases} Ax(a-x) & \text{für } 0 \le x \le a \\ 0 & \text{für } x < 0 \text{ und } x > a \end{cases}$$

a)

$$1 = \int_{-\infty}^{\infty} |\Psi(x)|^2 dx = \int_{0}^{a} A^2 x^2 (a - x)^2 dx = \frac{a^5 A^2}{30}$$
$$[m^{-\frac{1}{2}}]$$

$$A = \sqrt{\frac{30}{a^5}}$$

b)
$$k = \frac{2\pi}{\lambda} = \frac{mv_{\text{T}}}{\hbar} = \frac{p}{\hbar}; \quad \omega = \frac{2\pi v_{\text{T}}}{\lambda} = \frac{mv_{\text{T}}^2}{2\hbar} = \frac{E}{\hbar} = \frac{p^2}{2m\hbar} = \frac{k^2\hbar}{2m}$$

$$v_{\text{g}} = \frac{\mathrm{d}\omega}{\mathrm{d}k} = \frac{k\hbar}{m} = \frac{p}{m}$$

c)
$$a_1 = 1 \text{ m}$$
; $a_2 = 0.5 \times 10^{-10} \text{ m}$

$$E_{\text{ges}} = \int_{-\infty}^{r} k \frac{q^2}{s^2} \, \mathrm{d}s = k \frac{q^2}{r}$$

$$\lambda_{\rm db} = \frac{h}{mv_{\rm T}} = \frac{h}{\sqrt{2mE_{\rm ges}}}$$

$$\lambda_1 = \underline{3.23 \times 10^{-5} \text{ m}}$$

$$\lambda_2 = \underline{2.29 \times 10^{-10} \text{ m}}$$

$$\lambda_2 = \underline{2.29 \times 10^{-10} \text{ m}}$$

Doppelspaltversuch mit Elektronen

a)

$$E_{1} = E_{0} \cos(kr - \omega t)$$

$$E_{2} = E_{0} \cos(k(r + d \sin(\alpha)) - \omega t)$$

$$E = E_{1} + E_{2} = E_{0} \cos(kr - \omega t) + E_{0} \cos(k(r + d \sin(\alpha)) - \omega t) =$$

$$= 2E_{0} \cos(\frac{1}{2}kd \sin(\alpha)) \cos(kr - \omega t + \frac{1}{2}kd \sin(\alpha))$$

$$:= A$$

$$I \propto A^{2}$$

$$I \propto A^2$$

$$I(\alpha) = I_0 \cos^2(\frac{d\pi}{\lambda} \sin(\alpha))$$

b)
$$E = 10 \text{ eV}; \quad d = 3 \text{ nm}$$

$$I(\alpha) = 0$$

$$\frac{d\pi}{\lambda} \sin(\alpha) = \frac{\pi}{2}$$

$$\alpha = \arcsin\left(\frac{\lambda}{2d}\right) = \arcsin\left(\frac{h}{2d\sqrt{2Em_a}}\right) = \underline{3.71^{\circ}}$$

c) The probability of finding a single electron at the first minima is zero because destructive interference is happening. If you close one slit, no interference happens and you have a gaussian distribution centered on the remaining slit.

Neutronen im Interferometer

a)
$$\phi_{AC} - \phi_{BD} = 0$$
 due to symmetry

$$\begin{split} \frac{p_{\rm f}^2}{2m} &= \frac{p_{\rm i}^2}{2m} - mgd \\ p_{\rm f} &= \sqrt{p_{\rm i}^2 - 2gdm^2} = \sqrt{\frac{h^2}{\lambda_0^2} - 2gdm^2} \\ \lambda_{\rm f} &= \frac{h}{p_{\rm f}} = \frac{h}{\sqrt{\frac{h^2}{\lambda_0^2} - 2gdm^2}} = \frac{1}{\sqrt{\frac{1}{\lambda_0^2} - \frac{2gdm^2}{h^2}}} \end{split}$$

$$\phi_{\mathrm{AB}} - \phi_{\mathrm{CD}} = k_{\mathrm{i}}L - k_{\mathrm{f}}L = 2\pi L \left(\frac{1}{\lambda_{0}} - \frac{1}{\lambda_{\mathrm{f}}}\right) = 2\pi L \left(\frac{1}{\lambda_{0}} - \sqrt{\frac{1}{\lambda_{0}^{2}} - \frac{2gdm^{2}}{h^{2}}}\right)$$

b)
$$\lambda_0 = 0.2 \text{ m}$$
; $L = 45 \text{ mm}$; $d = 24 \text{ mm}$; $\Delta \phi = 81^{\circ}$

$$\psi_{1} = \psi_{2} e^{\frac{2\Delta\phi + \pi}{2}i}$$

$$\psi_{2} = \frac{\psi_{0}}{\sqrt{2}}$$

$$\psi'_{1} = \frac{1}{\sqrt{2}}(\psi_{1} + i\psi_{2}) = \frac{\psi_{2}}{\sqrt{2}}(e^{\frac{2\Delta\phi + \pi}{2}i} + i) = \frac{\psi_{2}}{\sqrt{2}}e^{\frac{\pi}{2}i}(e^{\Delta\phi i} + 1)$$

$$\psi'_{2} = \frac{1}{\sqrt{2}}(\psi_{2} + i\psi_{1}) = \frac{\psi_{2}}{\sqrt{2}}(i e^{\frac{2\Delta\phi + \pi}{2}i} + 1) = \frac{\psi_{2}}{\sqrt{2}}(e^{(\Delta\phi + \pi)i} + 1)$$

$$I_{0} \propto |\psi_{0}|^{2} = 5000 \text{ 1/s}$$

$$I_{1} \propto |\psi'_{1}|^{2} = \left| \frac{\psi_{2}}{\sqrt{2}} e^{\frac{\pi}{2}i} (e^{\Delta\phi i} + 1) \right|^{2} = \frac{\psi_{0}^{2}}{2} |1 + \cos(\Delta\phi)| = \underline{\underline{2891 1/s}}$$

$$I_{2} \propto \left| \frac{\psi_{2}}{\sqrt{2}} (e^{(\Delta\phi + \pi)i} + 1) \right|^{2} = \frac{\psi_{0}^{2}}{2} |1 - \cos(\Delta\phi)| = \underline{\underline{2109 1/s}}$$