1. Potentialverlauf einer geladenen Hohlkugel

$$k = \frac{1}{4\pi\epsilon_0}, V = \int_{V_i}^{\infty} \vec{E}\, \mathrm{d}\vec{s}$$

$$\boxed{E(r) = \begin{cases} 0 & \text{for } 0 < r \leq R_i, \\ \frac{kQ}{R_a^3 - R_i^3}\left(r - \frac{R_i^3}{r^2}\right) & \text{for } R_i < r < R_a, \\ kQ\,\frac{1}{r^2} & \text{for } R_a \leq r. \end{cases}}$$

<u>For $R_a \leq r$:</u>

$$E(r) = kQ\,\frac{1}{r^2}$$

$$V_1(r) = \int_r^{\infty} \vec{E}(r)\,\mathrm{d}\vec{r} = kQ \int_r^{\infty} \frac{1}{r^2}\,\mathrm{d}r = kQ\,\frac{1}{R_a}$$

$$V_1(r) = kQ\,\frac{1}{r}$$

<u>For $R_i < r < R_a$:</u>

$$E(r) = \frac{kQ}{R_a^3 - R_i^3}\left(r - \frac{R_i^3}{r^2}\right)$$

$$V_2(r) = \int_r^{\infty} \vec{E}(r)\,\mathrm{d}\vec{r} = kQ\left(\int_r^{R_a} \frac{r^3 - R_i^3}{r^2(R_a^3 - R_i^3)}\,\mathrm{d}r + \underbrace{\int_{R_a}^{\infty} \frac{1}{r^2}\,\mathrm{d}r}_{= V_1(R_a)}\right) =$$

$$= kQ\left(-\frac{r^3 R_a - rR_a^3 + 2rR_i^3 + 2R_a R_i^3}{2rR_a^4 - 2rR_a R_i^3} + \frac{1}{R_a}\right)$$

$$V_2(r) = -kQ\,\frac{r^3 - 3rR_a^2 + 2R_i^3}{2rR_a^3 - 2rR_i^3}$$

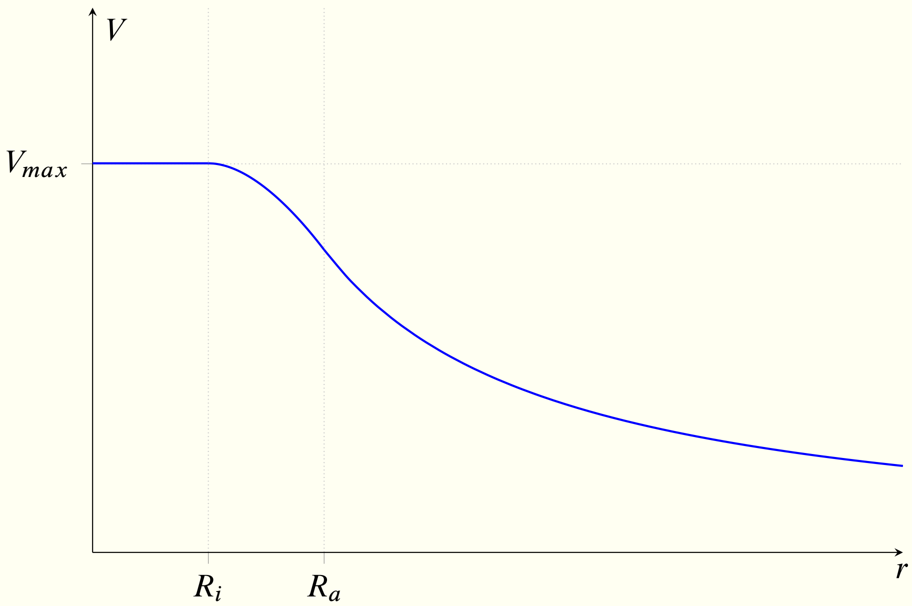
For $r \leq R_i$:

$$E(r) = 0$$

$$V_3(r) = \int_r^\infty \vec{E}(r)\,\mathrm{d}\vec{r} = kQ\left(\underbrace{\int_r^{R_i} 0\,\mathrm{d}r}_{=0} + \underbrace{\int_{R_i}^\infty \frac{r^3-R_i^3}{r^2(R_a^3-R_i^3)}\,\mathrm{d}r}_{=V_2(R_i)}\right) =$$

$$V_3(r) = -kQ\frac{3(R_a+R_i)}{2(R_a^2+R_aR_i+R_i^2)}$$

$$\boxed{V(r) = \begin{cases} -kQ\frac{3(R_a+R_i)}{2(R_a^2+R_aR_i+R_i^2)} & \text{for } 0 < r \leq R_i, \\ -kQ\frac{r^3-3rR_a^2+2R_i^3}{2rR_a^3-2rR_i^3} & \text{for } R_i < r < R_a, \\ kQ\frac{1}{r} & \text{for } R_a \leq r. \end{cases}}$$



2. Elektrisches Feld und Potential eines geladenen Stabes

$$k = \frac{1}{4\pi\epsilon_0}; \quad \mathrm{d}Q = \lambda \mathrm{d}z; \quad d = \sqrt{r_P^2 + (z_P - z)^2}$$

a)

$$\mathrm{d}V = k\,\frac{1}{d}\,\mathrm{d}Q = k\lambda\,\frac{1}{\sqrt{r_P^2+(z_P-z)^2}}\,\mathrm{d}z$$

$$V = k\lambda \int\limits_{-l/2}^{l/2} \frac{1}{\sqrt{r_P^2+(z_P-z)^2}}\,\mathrm{d}z =$$

$$\underline{\underline{= k\lambda \ln\left(\frac{\sqrt{\left(\frac{l}{2}+z_P\right)^2 + r_P^2} + \frac{l}{2} + z_P}{\sqrt{\left(-\frac{l}{2}+z_P\right)^2 + r_P^2} - \frac{l}{2} + z_P}\right)}}$$

b) $z_P = 0; \quad r_P = x_P; \quad l \ll x_P$

$$V = k\lambda \ln\left(\frac{\sqrt{l^2 + 4r_P^2} + l}{\sqrt{l^2 + 4r_P^2} - l}\right)$$

$$= k\lambda \ln\left(\frac{\sqrt{l^2 + 4x_P^2} + l}{\sqrt{l^2 + 4x_P^2} - l}\right)$$

$$\approx k\lambda \ln\left(\frac{2x_P + l}{2x_P - l}\right) \approx k\lambda \ln(1)$$

$$= \underline{\underline{0}}$$

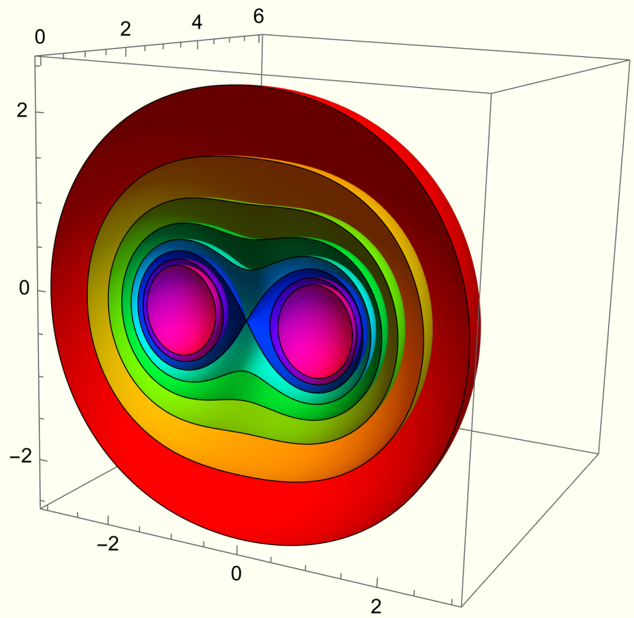
$$\boxed{\begin{aligned} \sqrt{l^2 + 4x_P^2} &\approx 2x_P \\ 2x_P \pm l &\approx 2x_P \end{aligned}}$$

3. Elektrische Ladung zwischen Kugelladungen

$$k = \frac{1}{4\pi\epsilon_0}$$

a) $\vec{F} = \vec{E}_2(9.5d) * q = \underline{\underline{k \frac{qQ}{(9.5d)^2} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}}}$

b)



c)

$$K = W$$

$$\frac{1}{2}mv^2 = \int_{9.5d}^{9d} \vec{F}\,\mathrm{d}\vec{r} = kqQ\left(\frac{1}{9.5d} - \frac{1}{9d}\right) = kqQ\,\frac{1}{171d}$$

$$v = \underline{\underline{\sqrt{\frac{2kqQ}{171dm}}}}$$

d)

$$
\begin{aligned}
kqQ \int\limits_{9.5d}^{9d} \tfrac{1}{r^2}\,\mathrm{d}r &= kqQ \int\limits_{d}^{x} \tfrac{1}{(r-10d)^2} - \tfrac{1}{r^2}\,\mathrm{d}r \\
\tfrac{1}{171d} &= \tfrac{1}{10d-x} - \tfrac{10}{9d} + \tfrac{1}{x} \\
x(10d-x) &= \tfrac{1710d^2}{191} \\
x_{1,2} &= \tfrac{1}{955}\left(191d \pm \sqrt{585415}\,d\right) \\
x_1 &= \tfrac{1}{955}\left(191d - \sqrt{585415}\,d\right) \approx 0.99d \\
d + x_1 &= \underline{\underline{1.99d}}
\end{aligned}
$$