

1. Influenz in einer Metallplatte

$$k = \frac{1}{4\pi\epsilon_0}; \quad r^2 = x^2 + y^2; \quad \sigma = \epsilon_0 E$$

$$E_1(x, y, z) = k \frac{Q}{x^2 + y^2 + (z+R)^2}$$

$$E_2(x, y, z) = E_1(x, y, z - 2R) = k \frac{Q}{x^2 + y^2 + (z-R)^2}$$

$$E_{ges}(x, y, z) = E_1 + E_2 = kQ \left(\frac{1}{x^2 + y^2 + (z+R)^2} + \frac{1}{x^2 + y^2 + (z-R)^2} \right)$$

$$\begin{aligned} \sigma(r) &= \epsilon_0 E_{ges}(x, y, z) = \epsilon_0 kQ \left(\frac{1}{x^2 + y^2 + (z+R)^2} + \frac{1}{x^2 + y^2 + (z-R)^2} \right) \\ &= \underline{\underline{\frac{Q}{2\pi} \frac{1}{r^2 + R^2}}} \end{aligned}$$

2. Feld einer Hohlkugel

$$k = \frac{1}{4\pi\epsilon_0}$$

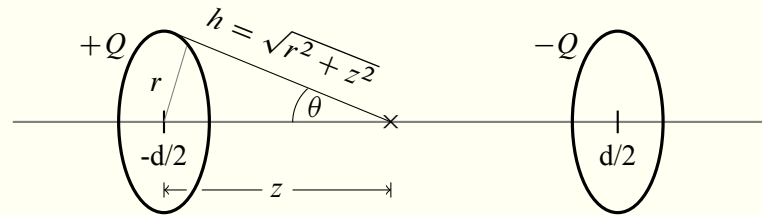
a. $\rho = \frac{Q}{V}$

$$\begin{aligned} V &= \int_V dV = \int_0^\pi \int_0^{2\pi} \int_{R_i}^{R_a} r^2 \sin(\theta) \, dr \, d\varphi \, d\theta \\ &= \frac{4}{3} \pi (R_a^3 - R_i^3) \\ \rho &= \underline{\underline{\frac{3Q}{4\pi} \frac{1}{R_a^3 - R_i^3}}} \end{aligned}$$

3. Zwei ringförmige Ladungsträger

$$k = \frac{1}{4\pi\epsilon_0}$$

a. $h = \sqrt{r^2 + z^2}; \quad \cos(\theta) = \frac{z}{h}; \quad dQ = \lambda dr; \quad dr = r d\varphi; \quad Q = 2r\pi\lambda$



Due to the symmetric nature of this problem we can neglect vertical components of forces

$$dE_1(z) = k \frac{dQ}{h^2} \cos(\theta) = k \frac{\lambda dr}{r^2 + z^2} \frac{z}{\sqrt{r^2 + z^2}} = k \frac{\lambda r z}{(r^2 + z^2)^{3/2}} d\varphi$$

$$E_1(z) = \int dE_1 = k \frac{\lambda r z}{(r^2 + z^2)^{3/2}} \int_0^{2\pi} d\varphi = k \frac{\lambda 2\pi r z}{(r^2 + z^2)^{3/2}} = k \frac{Qz}{(r^2 + z^2)^{3/2}}$$

$$E_2(z) = -E_1(z - d) = -k \frac{Q(z - d)}{(r^2 + (z - d)^2)^{3/2}}$$

$$\begin{aligned} E_{ges}(z) &= E_1 + E_2 = k \frac{Qz}{(r^2 + z^2)^{3/2}} - k \frac{Q(z - d)}{(r^2 + (z - d)^2)^{3/2}} \\ &= kQ \left(\frac{z + \frac{d}{2}}{\left(r^2 + \left(z + \frac{d}{2}\right)^2\right)^{3/2}} + \frac{\frac{d}{2} - z}{\left(r^2 + \left(z - \frac{d}{2}\right)^2\right)^{3/2}} \right) \end{aligned}$$

b. $F = -m\omega^2 x; \quad \Delta x = x_1 - x_2$

$$\begin{aligned} \int_{-d/2}^{d/2} qE(z) dz &= kQq \left(\int_{-d/2}^{d/2} \frac{z + \frac{d}{2}}{\left(r^2 + \left(z + \frac{d}{2}\right)^2\right)^{3/2}} dz + \int_{-d/2}^{d/2} \frac{\frac{d}{2} - z}{\left(r^2 + \left(z - \frac{d}{2}\right)^2\right)^{3/2}} dz \right) \\ &= kQq \left(\int_0^d \frac{u}{(r^2 + u^2)^{3/2}} du + \int_{-d}^0 \frac{v}{(r^2 + v^2)^{3/2}} dv \right) \end{aligned}$$

c. $\delta\omega = \omega_b - \omega_a$

$$0 = \cos\left(\frac{1}{2}\delta\omega t\right)$$

$$t = \frac{2 \arccos(0)}{\delta\omega} = \underline{\underline{50.29 \text{ s}}}$$