## 177. Stabpendel mit Kugeln

$$L = 0.3 \text{ m}; \qquad r_1 = 0.02 \text{ m}; \qquad r_2 = 0.03 \text{ m}$$
$$m_0 = 0.1 \text{ kg}; \qquad m_1 = 0.1 \text{ kg}; \qquad m_2 = 0.3 \text{ kg}$$

a) $I_{ges} = I_{Stab} + I_{Kugel_1} + I_{Kugel_2}$

$I_{Stab} = \frac{1}{12} m_0 L^2 = 7.5 * 10^{-4} \text{ kg m}^2$

$I_{Kugel_1} = \frac{2}{5} m_1 r_1^2 + m_1 (\frac{L}{2} + r_1)^2 = 2.9 * 10^{-3} \text{ kg m}^2$

$I_{Kugel_2} = \frac{2}{5} m_2 r_2^2 + m_2 (\frac{L}{2} + r_2)^2 = 9.8 * 10^{-3} \text{ kg m}^2$

$\Rightarrow I_{ges} = \underline{\underline{1.3 * 10^{-2} \text{ kg m}^2}}$

b)

$x_{Stab_{sp}} = 0 \text{ m}$

$x_{Kugeln_{sp}} = \dfrac{m_1 \left(-\frac{2r_1 + L}{2}\right) + m_2 \left(\frac{2r_2 + L}{2}\right)}{m_1 + m_2} = 0.0925 \text{ m}$

$x_{sp} = \dfrac{x_{Stab_{sp}} * m_0 + x_{Kugeln_{sp}} * (m_1 + m_2)}{m_0 + m_1 + m_2} = \underline{\underline{0.074}} \text{ m}$

c) $T = 2\pi \sqrt{\frac{I}{Mgl}}$

$T = 2\pi \sqrt{\dfrac{I}{(m_0 + m_1 + m_2) \, g \, x_{sp}}}$

$= \underline{\underline{1.2 \text{ s}}}$

## 184. Resonanzverhalten einer Stimmgabel

$$f_0 = 440 \text{ Hz}; \quad \omega_0 = 2\pi f_0; \quad x(\tau) = x_m e^{-\beta \tau / 2}; \quad x(5) = \frac{1}{10} x(0)$$

a)

$$\frac{x_m}{10} = x_m e^{-\beta 5/2}$$

$$\beta = -\frac{2}{5} \ln(0.1)$$

$$\tau = \frac{1}{\beta} = \underline{\underline{1.09 \text{ s}}}$$

b) $\omega' = \sqrt{\omega_0^2 - \frac{\beta^2}{4}}$

$$\omega' = 2764.6 \text{ Hz} \qquad\qquad (\omega_0 \approx \omega')$$

$$f' = \frac{\omega'}{2\pi} = 440 \text{ Hz} \qquad\qquad (f_0 \approx f')$$

$$f' * \tau = \underline{\underline{477.7}}$$

c) $Q = \frac{\omega_0}{\beta}$

$$Q = \underline{\underline{3002}}$$

d) $\delta f = \frac{f'}{Q}$

$$\delta f = \underline{\underline{0.15 \text{ Hz}}}$$

e) $x_m(f) = \frac{2\pi f_0}{\sqrt{16\pi^2(f-f_0)^2+\beta^2}}; \quad E(f) = \frac{(\pi f_0)^2}{4\pi^2(f-f_0)^2+(\beta/2)^2}; \quad f_1 = 439.5 \text{ Hz}$

$$x_m(f_0) = \frac{2\pi f_0}{\sqrt{16\pi^2(f_0-f_0)^2+\beta^2}} = 3001.63$$

$$x_m(f_1) = \frac{2\pi f_0}{\sqrt{16\pi^2(f_1-f_0)^2+\beta^2}} = 435.348$$

$$x_{rel} = \frac{x_m(f_1)}{x_m(f_0)} = \underline{\underline{0.15}}$$

$$E(f_0) = \frac{(\pi f_0)^2}{4\pi^2(f_0-f_0)^2+(\beta/2)^2} = 9.0 * 10^6$$

$$E(f_1) = \frac{(\pi f_0)^2}{4\pi^2(f_1-f_0)^2+(\beta/2)^2} = 1.9 * 10^5$$

$$E_{rel} = \frac{E(f_1)}{E(f_0)} = \underline{\underline{0.021}}$$

## 185. Resonante Anregung eines Federpendels

$$m = 0.1 \text{ kg}; \quad k = 40 \text{ N/m}$$

a) $\omega_0 = \sqrt{\frac{k}{m}} = 20 \text{ rad/s}$

$\frac{x_m}{2} = x_m * \mathrm{e}^{-\beta t/2}$

$\beta = -\frac{\ln(0.5)2}{t} = \underline{\underline{0.14 \text{ s}^{-1}}}$

$\omega' = \sqrt{\omega_0{}^2 - \frac{\beta^2}{4}} = \underline{\underline{19.99988 \text{ rad/s}}}$

b)

$Q = \frac{\omega_0}{\beta} = \underline{\underline{144.3}}$

c) $\hat{y}_{res} = \frac{\omega_0}{\beta} y_0; \quad \hat{y}_{res} = 0.012 \text{ m}$

$y_0 = \hat{y}_{res} \frac{\beta}{\omega_0} = \underline{\underline{8.32 * 10^{-5} \text{ m}}}$

d) $y_2 = y_0 \frac{\omega_0}{\sqrt{4(\omega_1 - \omega_0)^2 + \beta^2}}; \quad \omega_1 = \omega_0 \pm 0.2\pi$

$y_2 = \underline{\underline{0.0013 \text{ m}}}$