1. Magnetfeld eines asymmetrischen Leiters

a) $r = \sqrt{x^2 + y^2}$

$$I_+ = I(1 + \tfrac{a^2}{R^2})$$

$$I_- = -I\,\tfrac{a^2}{R^2}$$

$$B_+ = \frac{\mu_0 I_+}{2r\pi} = \frac{\mu_0 I_+}{2\pi\sqrt{x^2+y^2}}$$

$$B_- = \frac{\mu_0 I_-}{2r\pi} = \frac{\mu_0 I_-}{2\pi\sqrt{(x-b)^2+y^2}}$$

$$B = B_+ + B_- = \frac{\mu_0 I_+}{2\pi\sqrt{x^2+y^2}} + \frac{\mu_0 I_-}{2\pi\sqrt{(x-b)^2+y^2}} =$$

$$= \frac{\mu_0 I}{2\pi}\left[\frac{1}{\sqrt{x^2+y^2}} + \frac{a^2}{R^2}\left(\frac{1}{\sqrt{x^2+y^2}} - \frac{1}{\sqrt{(x-b)^2+y^2}}\right)\right]$$

$$B(2R,0) = \frac{\mu_0 I}{4R\pi}\left[1 + \frac{a^2}{R}\left(\frac{1}{R} - \frac{1}{R-b}\right)\right]$$

b)

$$B(0,2R) = \frac{\mu_0 I}{2R\pi}\left[\frac{1}{2} + \frac{a^2}{R}\left(\frac{1}{2R} - \frac{1}{\sqrt{b^2+4R^2}}\right)\right]$$

2. Induktion

$$\vec{B} = B_x\hat{x}$$

a)

$$\Phi(t) = \int \vec{B}\,\mathrm{d}\vec{A} = B\cos(\omega t)\int 1\,\mathrm{d}\vec{A} = r^2\pi B\cos(\omega t)$$

$$U(t) = -\frac{\mathrm{d}\Phi}{\mathrm{d}t} = r^2\pi\omega B\sin(\omega t)$$

b) For $\alpha = \frac{\pi}{4}$ current I will flow counterclockwise to increase the B-Field
For $\alpha = \frac{3\pi}{4}$ current I will flow clockwise to counter the B-Field

c)

$$I = \frac{U}{R}$$

$$\frac{\mathrm{d}Q}{\mathrm{d}t} = -\frac{1}{R}\frac{\mathrm{d}\Phi}{\mathrm{d}t}$$

$$\Delta Q = -\frac{r^2\pi B}{R}\int_0^{\pi/2\omega}\cos(\omega t)\,\mathrm{d}t = \underline{\underline{\frac{r^2\pi B}{R}}}$$

3. Indunktionsspannung - Stab

$$I := I_{\text{Bat}}$$

a)

$$\Phi = Blx$$

$$U = -Blv$$

$$I_{\text{Ind}} = -\frac{Blv}{R}$$

$$\vec{F}_{\text{Ind}} = I_{\text{Ind}}\vec{l}\times\vec{B} = -\frac{B^2l^2v}{R}\hat{x}$$

$$\vec{F}_L = I\vec{l}\times\vec{B} = IlB\hat{x}$$

$$\vec{F}_{\text{ges}}(v) = \vec{F}_{\text{Ind}} + \vec{F}_L = \underline{\underline{lB(I - \frac{Blv}{R})\hat{x}}}$$

b)

$$m\frac{\mathrm{d}v}{\mathrm{d}t} = lB\left(I - \frac{Bl}{R}v(t)\right)$$

$$v(t) = \frac{IR}{Bl}\left(1 - \mathrm{e}^{-\frac{B^2l^2}{mR}t}\right)$$

$$\lim_{t\to\infty} v(t) = \underline{\underline{\frac{IR}{Bl}}}$$

c)

$$I_{\text{ges}} = I + I_{\text{Ind}} = I - \frac{Blv}{R} = I - \frac{BlRI}{BlR} = \underline{\underline{0}}$$