1. Magnetfeld eines asymmetrischen Leiters

a) $r = \sqrt{x^2 + y^2}$

$$
\begin{aligned}
I_+ &= I(1 + \tfrac{a^2}{R^2}) \\
I_- &= -I\,\tfrac{a^2}{R^2} \\
B_+ &= \tfrac{\mu_0 I_+}{2r\pi} = \tfrac{\mu_0 I_+}{2\pi\sqrt{x^2+y^2}} \\
B_- &= \tfrac{\mu_0 I_-}{2r\pi} = \tfrac{\mu_0 I_-}{2\pi\sqrt{(x-b)^2+y^2}} \\
B &= B_+ + B_- = \tfrac{\mu_0 I_+}{2\pi\sqrt{x^2+y^2}} + \tfrac{\mu_0 I_-}{2\pi\sqrt{(x-b)^2+y^2}} = \\
&= \tfrac{\mu_0 I}{2\pi}\left[\tfrac{1}{\sqrt{x^2+y^2}} + \tfrac{a^2}{R^2}\left(\tfrac{1}{\sqrt{x^2+y^2}} - \tfrac{1}{\sqrt{(x-b)^2+y^2}}\right)\right] \\
B(2R,0) &= \underline{\underline{\tfrac{\mu_0 I}{4R\pi}\left[1 + \tfrac{a^2}{R}\left(\tfrac{1}{R} - \tfrac{1}{R-b}\right)\right]}}
\end{aligned}
$$

b)

$$
B(0,2R) = \underline{\underline{\tfrac{\mu_0 I}{2R\pi}\left[\tfrac{1}{2} + \tfrac{a^2}{R}\left(\tfrac{1}{2R} - \tfrac{1}{\sqrt{b^2+4R^2}}\right)\right]}}
$$

2. Induktion

$$
\vec{B} = B_x \hat{x}
$$

a)

$$
\Phi(t) = \int \vec{B}\,\mathrm{d}\vec{A} = B\cos(\omega t)\int 1\,\mathrm{d}\vec{A} = r^2\pi B\cos(\omega t)
$$

$$
U(t) = -\frac{\mathrm{d}\Phi}{\mathrm{d}t} = \underline{\underline{r^2\pi\omega B\sin(\omega t)}}
$$

b) For $\alpha = \frac{\pi}{4}$ current I will flow counterclockwise to increase the B-Field
For $\alpha = \frac{3\pi}{4}$ current I will flow clockwise to counter the B-Field

c)

$$I = \frac{U}{R}$$
$$\frac{\mathrm{d}Q}{\mathrm{d}t} = -\frac{1}{R}\frac{\mathrm{d}\Phi}{\mathrm{d}t}$$
$$\Delta Q = -\frac{r^2\pi B}{R}\int_0^{\pi/2\omega}\cos(\omega t)\,\mathrm{d}t = \underline{\underline{\frac{r^2\pi B}{R}}}$$

3. Indunktionsspannung - Stab

$$I := I_{\mathrm{Bat}}$$

a)

$$\Phi = Blx$$
$$U = -Blv$$
$$I_{\mathrm{Ind}} = -\frac{Blv}{R}$$
$$\vec{F}_{\mathrm{Ind}} = I_{\mathrm{Ind}}\vec{l}\times\vec{B} = -\frac{B^2l^2v}{R}\hat{x}$$
$$\vec{F}_L = I\vec{l}\times\vec{B} = IlB\hat{x}$$
$$\vec{F}_{\mathrm{ges}}(v) = \vec{F}_{\mathrm{Ind}} + \vec{F}_L = \underline{\underline{lB(I - \frac{Blv}{R})\hat{x}}}$$

b)

$$m\frac{\mathrm{d}v}{\mathrm{d}t} = lB\left(I - \frac{Bl}{R}v(t)\right)$$
$$v(t) = \frac{IR}{Bl}\left(1 - \mathrm{e}^{-\frac{B^2l^2}{mR}t}\right)$$
$$\lim_{t\to\infty} v(t) = \underline{\underline{\frac{IR}{Bl}}}$$

c)

$$I_{\mathrm{ges}} = I + I_{\mathrm{Ind}} = I - \frac{Blv}{R} = I - \frac{BlRI}{BlR} = \underline{\underline{0}}$$