Erhaltungssätze und das Σ-Teilchen

a)

	Q	L	B	J
$n \to p + \pi^-$	0/0	0/0	1/1	$\frac{1}{2}/\frac{1}{2}$
$p \to \pi^0 + e^+$	1/1	0/1	1/0	$1/\frac{1}{2}$
$\pi^+ \to e^+ + \nu_e + \gamma$	1/1	0/0	1/1	0/2
$\nu_e + p \to n + e^+$	1/1	1/1	1/1	1/1
$\nu_e + n \to p + e^-$	0/0	1/1	1/1	1/1

b) $K^- + p \longrightarrow \pi^- + \Sigma$

	$K^- + p$	π^-	Σ
B	1	0	1
S	-1	0	-1
Q	0	-1	1

Isospin-Eigenzustände

a) $p = \left|\frac{1}{2}, \frac{1}{2}\right\rangle, \quad n = \left|\frac{1}{2}, -\frac{1}{2}\right\rangle, \quad \pi^0 = |1,0\rangle, \quad \pi^+ = \pi^- = |1,1\rangle$

b)

p	n
$\pi^+: \quad \left\|\frac{1}{2}, \frac{1}{2}\right\rangle \otimes \|1,1\rangle = \left\|T, \frac{3}{2}\right\rangle$	$\left\|\frac{1}{2}, -\frac{1}{2}\right\rangle \otimes \|1,1\rangle = \left\|T, \frac{1}{2}\right\rangle$
$\pi^0: \quad \left\|\frac{1}{2}, \frac{1}{2}\right\rangle \otimes \|1,0\rangle = \left\|T, \frac{1}{2}\right\rangle$	$\left\|\frac{1}{2}, -\frac{1}{2}\right\rangle \otimes \|1,0\rangle = \left\|T, -\frac{1}{2}\right\rangle$
$\pi^-: \quad \left\|\frac{1}{2}, \frac{1}{2}\right\rangle \otimes \|1,-1\rangle = \left\|T, -\frac{1}{2}\right\rangle$	$\left\|\frac{1}{2}, -\frac{1}{2}\right\rangle \otimes \|1,-1\rangle = \left\|T, -\frac{3}{2}\right\rangle$

c)

$$|\pi^+ p\rangle = |1,1\rangle \otimes \left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle = \left|\tfrac{3}{2},\tfrac{3}{2}\right\rangle$$

$$|\pi^- p\rangle = |1,-1\rangle \otimes \left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle = \sqrt{\tfrac{1}{3}}\left|\tfrac{3}{2},-\tfrac{1}{2}\right\rangle - \sqrt{\tfrac{2}{3}}\left|\tfrac{1}{2},-\tfrac{1}{2}\right\rangle$$

$$|\pi^0 n\rangle = |1,0\rangle \otimes \left|\tfrac{1}{2},-\tfrac{1}{2}\right\rangle = \sqrt{\tfrac{2}{3}}\left|\tfrac{3}{2},-\tfrac{1}{2}\right\rangle + \sqrt{\tfrac{1}{3}}\left|\tfrac{1}{2},-\tfrac{1}{2}\right\rangle$$

d)

$$|\pi^+ p\rangle \otimes |\pi^0\rangle = \left|\tfrac{3}{2},\tfrac{3}{2}\right\rangle \otimes |1,0\rangle = \sqrt{\tfrac{2}{5}}\left|\tfrac{5}{2},\tfrac{3}{2}\right\rangle + \sqrt{\tfrac{3}{5}}\left|\tfrac{3}{2},\tfrac{3}{2}\right\rangle$$

Wirkungsquerschnitt bei der Pion-Proton-Streuung

a) $\Delta^{++} = \left|\tfrac{3}{2},\tfrac{3}{2}\right\rangle, \quad \Delta^{+} = \left|\tfrac{3}{2},\tfrac{1}{2}\right\rangle, \quad \Delta^{0} = \left|\tfrac{3}{2},-\tfrac{1}{2}\right\rangle, \quad \Delta^{-} = \left|\tfrac{3}{2},-\tfrac{1}{2}\right\rangle$

b)

Für $\pi^+ p$: $\sigma_{\text{total}} = \sigma_{\text{elastic}} \approx 200\ \text{mb}$

Für $\pi^- p$: $\sigma_{\text{total}} \approx 80\ \text{mb}; \quad \sigma_{\text{elastic}} \approx 20\ \text{mb}$

c)

$$\text{I} : \pi^+ + p \longrightarrow \Delta^{++} \longrightarrow \pi^+ + p$$

$$\text{II} : \pi^+ + p \longrightarrow \Delta^{++} \longrightarrow \pi^+ + p$$

$$\text{III} : \pi^- + p \longrightarrow \Delta^{0} \longrightarrow \pi^- + p$$

$$\text{IV} : \pi^- + p \longrightarrow \Delta^{0} \longrightarrow \pi^0 + n$$

d)

$$\mathrm{I} : \left|\tfrac{3}{2}, \tfrac{3}{2}\right\rangle \to \left|\tfrac{3}{2}, \tfrac{3}{2}\right\rangle \to \left|\tfrac{3}{2}, \tfrac{3}{2}\right\rangle$$

$$\mathrm{II} : \left|\tfrac{3}{2}, \tfrac{3}{2}\right\rangle \to \left|\tfrac{3}{2}, \tfrac{3}{2}\right\rangle \to \left|\tfrac{3}{2}, \tfrac{3}{2}\right\rangle$$

$$\mathrm{III} : \sqrt{\tfrac{1}{3}}\left|\tfrac{3}{2}, -\tfrac{1}{2}\right\rangle - \sqrt{\tfrac{2}{3}}\left|\tfrac{1}{2}, -\tfrac{1}{2}\right\rangle \to \left|\tfrac{3}{2}, -\tfrac{1}{2}\right\rangle \to \sqrt{\tfrac{1}{3}}\left|\tfrac{3}{2}, -\tfrac{1}{2}\right\rangle - \sqrt{\tfrac{2}{3}}\left|\tfrac{1}{2}, -\tfrac{1}{2}\right\rangle$$

$$\mathrm{IV} : \sqrt{\tfrac{1}{3}}\left|\tfrac{3}{2}, -\tfrac{1}{2}\right\rangle - \sqrt{\tfrac{2}{3}}\left|\tfrac{1}{2}, -\tfrac{1}{2}\right\rangle \to \left|\tfrac{3}{2}, -\tfrac{1}{2}\right\rangle \to \sqrt{\tfrac{2}{3}}\left|\tfrac{3}{2}, -\tfrac{1}{2}\right\rangle + \sqrt{\tfrac{1}{3}}\left|\tfrac{1}{2}, -\tfrac{1}{2}\right\rangle$$

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e) $\hat{T}\left|T, T_z\right\rangle = T\left|T, T_z\right\rangle$

$$\sigma_{\mathrm{I}} \propto \left|\left\langle \tfrac{3}{2}, \tfrac{3}{2}\right| T \left|\tfrac{3}{2}, \tfrac{3}{2}\right\rangle\right|^2 = \left|\tfrac{3}{2}\left\langle \tfrac{3}{2}, \tfrac{3}{2}\middle|\tfrac{3}{2}, \tfrac{3}{2}\right\rangle\right|^2 = \tfrac{9}{4}$$

$$\sigma_{\mathrm{II}} = \sigma_{\mathrm{I}} \propto \tfrac{9}{4}$$

$$\begin{aligned}\sigma_{\mathrm{III}} &\propto \left|\tfrac{1}{3}\left\langle \tfrac{3}{2}, -\tfrac{1}{2}\right| T \left|\tfrac{3}{2}, -\tfrac{1}{2}\right\rangle + \tfrac{2}{3}\left\langle \tfrac{1}{2}, -\tfrac{1}{2}\right| T \left|\tfrac{1}{2}, -\tfrac{1}{2}\right\rangle\right|^2 \\ &= \left|\tfrac{1}{2}\left\langle \tfrac{3}{2}, -\tfrac{1}{2}\middle|\tfrac{3}{2}, -\tfrac{1}{2}\right\rangle + \tfrac{1}{3}\left\langle \tfrac{1}{2}, -\tfrac{1}{2}\middle|\tfrac{1}{2}, -\tfrac{1}{2}\right\rangle\right|^2 = \left|\tfrac{1}{2} + \tfrac{1}{3}\right|^2 = \tfrac{5}{6}\end{aligned}$$

$$\begin{aligned}\sigma_{\mathrm{IV}} &\propto \left|\sqrt{\tfrac{2}{3}}\sqrt{\tfrac{1}{3}}\left\langle \tfrac{3}{2}, -\tfrac{1}{2}\right| T \left|\tfrac{3}{2}, -\tfrac{1}{2}\right\rangle - \sqrt{\tfrac{2}{3}}\sqrt{\tfrac{1}{3}}\left\langle \tfrac{1}{2}, -\tfrac{1}{2}\right| T \left|\tfrac{1}{2}, -\tfrac{1}{2}\right\rangle\right|^2 \\ &= \left|\tfrac{3}{2}\sqrt{\tfrac{2}{9}}\left\langle \tfrac{3}{2}, -\tfrac{1}{2}\middle|\tfrac{3}{2}, -\tfrac{1}{2}\right\rangle - \tfrac{1}{2}\sqrt{\tfrac{2}{9}}\left\langle \tfrac{1}{2}, -\tfrac{1}{2}\middle|\tfrac{1}{2}, -\tfrac{1}{2}\right\rangle\right|^2 = \left|\tfrac{3}{2}\sqrt{\tfrac{2}{9}} - \tfrac{1}{2}\sqrt{\tfrac{2}{9}}\right|^2 = \tfrac{2}{9}\end{aligned}$$