184. Resonanzverhalten einer Stimmgabel

$f_0 = 440$ Hz; $\omega_0 = 2\pi f_0$; $x(\tau) = x_m \mathrm{e}^{-\beta\tau/2}$; $x(5) = \frac{1}{10}x(0)$

a)

$$\frac{x_m}{10} = x_m \mathrm{e}^{-\beta 5/2}$$
$$\beta = -\frac{2}{5}\ln(0.1)$$
$$\tau = \frac{1}{\beta} = \underline{\underline{1.09 \text{ s}}}$$

b) $\omega = \sqrt{\omega_0^2 - \frac{\beta^2}{4}}$

$$\omega = 2764.6 \text{ Hz} \qquad (\omega_0 \approx \omega)$$
$$f = \frac{\omega}{2\pi} = 440 \text{ Hz} \qquad (f_0 \approx f)$$
$$f * \tau = \underline{\underline{477.7}}$$

c) $Q = \frac{\omega}{\beta}$

$$Q = \underline{\underline{3001.63}}$$

d) $\delta f = \frac{f}{Q}$

$$\delta f = \underline{\underline{0.15 \text{ Hz}}}$$

e) $x_m(f) = \frac{2\pi f_0}{\sqrt{16\pi^2(f-f_0)+\beta^2}}$; $E(f) = \frac{(\pi f_0)^2}{4\pi^2(f-f_0)^2+(\beta/2)^2}$; $f_1 = 439.5$ Hz

$$x_m(f_0) = \frac{2\pi f_0}{\sqrt{16\pi^2(f_0-f_0)+\beta^2}} = 3001.63$$
$$x_m(f_1) = \frac{2\pi f_0}{\sqrt{16\pi^2(f_1-f_0)+\beta^2}} = 435.348$$
$$x_{rel} = \frac{x_m(f_1}{x_m(f_0)} = \underline{\underline{0.15}}$$

$$E(f_0) = \frac{(\pi f_0)^2}{4\pi^2(f-f_0)^2+(\beta/2)^2} = 1.9 * 10^5$$
$$E(f_1) = \frac{(\pi f_0)^2}{4\pi^2(f-f_1)^2+(\beta/2)^2} = 9.0 * 10^6$$
$$E_{rel} = \frac{E(f_1)}{E(f_0)} = \underline{\underline{0.021}}$$