THE DIAMETER OF QUASI-STELLAR RADIO SOURCES

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The properties of a class of optical objects whose position appears to coincide with certain radio sources in the 3C catalogue have recently been reviewed by Burbridge et al. [1] and by Greenstein [2]. The two properties that are of interest to us in this note are the suggested radius of about 10^3 light-years and an observed variation in the light intensity of about 30% in one year. (This is for 3C 48, 3C 273 appears to have a slightly longer period.)

Greenstein [2] also pointed out that it was very surprising that an object of such large diameter could fluctuate in a period as short as one year. Hoffman [3] suggested that if the fluctuation originates near the centre of the object then the whole surface brightens simultaneously and so Hoffman concluded that the observed variation in brightness did not impose any restriction on the size of the object.

The error in Hoffman's argument was pointed out by Young [4] who argued on general grounds that as some points of the object's surface were further away than others, then the increased brightness would not all be observed at the same time and so no general fluctuation would be observed. Terrell [5] has come to the same conclusions by using a more mathematical approach but seems to have ignored the actual time dependent fluctuating term. In this note we shall assume a sinusoidal fluctuation superimposed on a steady source and calculate the observed effects at a large distance, thus being more realistic than Terrell and, as we shall see, reaching fairly similar size limitations as Terrell did but also obtaining some information about the observed light variation.

Let the object, of radius a, emit amount L of light per unit area, this consisting of a steady part L_1 and a fluctuating part $L_2 \sin pt$, all points of the surface behaving similarly (as assumed by Hoffman and Terrell). The fluctuating part accounts for the variation in light thus require $p \sim 2\pi/\text{year}$. Thus $L = L_1 + L_2 \sin pt$.

Take the line joining the object centre to the observer (who is assumed to be far away) as polar axis in a spherical polar coordinate system and consider points subtending an angle between θ and $\theta + \delta\theta$ on the object's surface. All such points are $a(1 - \cos\theta)$ further away from the observer than the point, P, on the polar axis nearest the observer. Hence when light emitted at time t from P is observed, so is light emitted from this region at a time $t - a(1 - \cos\theta)/c$, c being the velocity of light. Assuming Lambert's cosine rule the amount of light received by the observer that originated from the region between θ and $\theta + \delta\theta$ is then

$$2\pi a^2 \sin\theta \cos\theta [L_1 + L_2 \sin p(t - \frac{a}{c} + \frac{a}{c}\cos\theta)]\, d\theta \ .$$

Integrating over the object surface (θ from 0 to $\frac{1}{2}\pi$) we obtain

$$\pi a^2 \left[L_1 + \frac{L_2 c^2}{a^2 p^2} \left\{ \sin pt(1 - \cos\frac{pa}{c}) - \right.\right.$$
$$\left.\left. - \cos \omega t(\frac{pa}{c} - \sin\frac{pa}{c}) \right\} \right] \qquad (2)$$

This differs quite substantially from the result obtained by Terrell for the fraction of light from the fluctuation that is observed.

If we have an object of small radius, or a long fluctuation period, that is if ap/c is small then equation (2) gives the amount of light received as $\pi a^2 [L_1 + L_2 \sin pt]$. Hence in this case, as expected, the same fluctuation is observed as was emitted.

If, as in the case of quasi-stellar radio sources, the quantity ap/c is suspected to be large, (large enough radius for the time taken by a light ray to cover its surface to exceed the fluctuation period) then eq. (2) behaves similarly to

$$\pi a^2 \left\{ L_1 - \frac{2L_2 c}{ap} \cos pt \right\} . \qquad (3)$$

A very interesting consequence of this equation, which was not obtained in any form by

Terrell, is that the observed fluctuation is proportional to cos ωt while the surface fluctuation is proportional to sin ωt and these fluctuations thus have the same period but are of opposite phase. It was not by any means obvious that the observed fluctuation should have the same period as the emitted fluctuation and the change in phase could not have been foreseen.

From (3), the maximum brightness is $\pi a^2 \{L_1 + 2 L_2 c/ap\}$ while the minimum is $\pi a^2 \{L_1 - 2 L_2 c/ap\}$. Hence the variation expressed as a percentage is $400\ L_2 c/L_1 ap$.

But Greenstein [2] gives this as 30% in a period of 1 year, hence we must have as $p = 2\pi/$ year

$$a = \frac{40\ L_2 c}{3\ L_1 p} = \frac{140\ L_2}{6\ \pi\ L_1} \text{ light year .} \qquad (4)$$

Now the surface must presumably always emit some light, hence from (1) $L_1 + L_2 \sin pt > 0$ $L_1 > L_2$. Thus (4) becomes $a \leqslant 2.1$ light years.

We should note that this is the maximum value for the radius and indeed as in practice L_1 is likely to exceed L_2 by a considerable amount the radius will probably be considerably less than this. The value is also about one order of magnitude less than the maximum radius found by Terrell and thus we can conclude that our results impose a more rigid limit on size. It thus appears that the observed variation in the brightness of 3C 48 (and the other quasi-stellar radio sources to a lesser extent) implies that the object radius must be much smaller than the suggested value of 10^3 light years.

References

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SPACE-CHARGE LIMITED PROTON CURRENTS IN ICE

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The protonic conductivity usually observed in ice is due to free protons generated by thermal dissociation of H_2O-molecules within the crystal. At temperatures sufficiently low to preclude thermal dissociation, we were able to establish space-charge limited proton currents using proton-injecting contacts and high fields. Experimental techniques were developed to cool pure ice single crystals (1.5 mm thick × 84 mm^2 ∅) as far as 77°K and to apply fields up to 450 kV/cm. The samples were prevented from cracking by use of powdered Pd electrodes electrolytically saturated with hydrogen.

The current-voltage characteristic shows clearly the shape of space-charge limited currents: an overproportional increase of current with voltage and a steep rise at a distinct voltage, as shown in fig. 1.

To explain this behaviour, some kind of proton traps must be postulated. The well known trap-filled-limit law $U_{tfl} = N_t L^2 e/2\ \epsilon\epsilon_0$ [1] yields a trap-density of about $3.5 \times 10^{12}/cm^3$ taking the dynamical value of 3.1 for ϵ.

The explanation of the traps is somewhat open to speculation. Ferroelectric domains are expected to exist in ice in this temperature region [2]. Injected protons passing along chains of hydrogen bonds cause reorientation of some of these domains. Eventually, the protons will be trapped at a Bjerrum orientational fault frozen in while the ice specimen was cooled down. Impurity states and lattice imperfections, however, may also serve as traps. At one distinct voltage all traps will be filled. Additional injection requires states in which more than one proton is associated with one trap in such a way that only the first proton is firmly bound while others are readily released. Thus, excess protons will no longer be stopped, accounting for the marked increase of the proton-current. These suggestions are supported by some experiments showing thermal release of trapped