

Materiewellen und Formfaktoren

a) $R_{\text{Atom}} \approx 1.73 \text{ \AA}; \quad R_{\text{Kern}} \approx 7.5 \text{ fm}$

$$\lambda_{\text{Atom}} = \frac{h}{\sqrt{2E_{\text{Atom}}m_{\alpha}}} \stackrel{!}{=} R_{\text{Atom}} \Rightarrow E_{\text{Atom}} = \frac{2m_{\alpha}R_{\text{Atom}}^2}{h^2} = \underline{\underline{6. \text{ eV}}}$$

$$\lambda_{\text{Kern}} = \frac{h}{\sqrt{2E_{\text{Kern}}m_{\alpha}}} \stackrel{!}{=} R_{\text{Kern}} \Rightarrow E_{\text{Kern}} = \frac{2m_{\alpha}R_{\text{Kern}}^2}{h^2} = \underline{\underline{3.67 \text{ MeV}}}$$

b)

$$\begin{aligned} F(\mathbf{q}^2) &= \int_{\mathbb{R}^3} e^{\frac{i\mathbf{q}\mathbf{x}}{\hbar}} f(\mathbf{x}) d\mathbf{x} = \int_0^{2\pi} \int_0^{\pi} \int_0^{\infty} e^{\frac{iqr\cos(\theta)}{\hbar}} f(r)r^2 \sin(\theta) dr d\theta d\varphi \\ &= 2\pi \int_0^{\infty} f(r)r^2 \int_0^{\pi} e^{\frac{iqr\cos(\theta)}{\hbar}} \sin(\theta) d\theta dr = 2\pi \int_0^{\infty} f(r)r^2 \int_{-1}^1 e^{\frac{iqr u}{\hbar}} du dr \\ &= 4\pi \int_0^{\infty} f(r)r^2 \left(e^{\frac{iqr}{\hbar}} - e^{-\frac{iqr}{\hbar}} \right) \frac{\hbar}{2iqr} dr = 4\pi \int_0^{\infty} \frac{\sin\left(\frac{qr}{\hbar}\right)}{\frac{qr}{\hbar}} f(r)r^2 dr \end{aligned}$$

c) $\langle r^2 \rangle = 4\pi \int_0^{\infty} r^4 f(r) dr; \quad \frac{\sin\left(\frac{qr}{\hbar}\right)}{\frac{qr}{\hbar}} \approx 1 + \frac{r^2 q^2}{6\hbar^2}$

$$\begin{aligned} F(\mathbf{q}^2) &\approx 4\pi \int_0^{\infty} f(r) \left(r^2 + \frac{r^4 q^2}{6\hbar^2} \right) dr = 4\pi \int_0^{\infty} f(r)r^2 dr + \langle r^2 \rangle \frac{6\hbar^2}{q^2} = 1 + \langle r^2 \rangle \frac{6\hbar^2}{q^2} \\ &\Rightarrow \langle r^2 \rangle = \underline{\underline{\frac{(F(\mathbf{q}^2) - 1) q^2}{6\hbar^2}}} \end{aligned}$$

d) $f(r) = f_0 e^{-ar}$

$$\int_{\mathbb{R}^3} f(\mathbf{x}) d\mathbf{x} \stackrel{!}{=} 1 \iff \int_0^{\infty} f_0 e^{-ar} r^2 dr = \frac{1}{4\pi} \iff \underline{\underline{f_0 = \frac{a^3}{8\pi}}}$$

e) $F(q^2) = (1 + \alpha^2)^{-2}$

$$F(q^2) = 4\pi \int_0^\infty \frac{\sin\left(\frac{qr}{\hbar}\right)}{\frac{qr}{\hbar}} f(r) r^2 dr = \frac{\hbar a^3}{2q} \int_0^\infty \sin\left(\frac{qr}{\hbar}\right) e^{-ar} r dr = \frac{a^4 \hbar^4}{(a^2 \hbar^2 + q^2)^2}$$

$$\Rightarrow \alpha(q, a) = \frac{q}{a\hbar}$$

Diese Ladungsverteilung beschreibt Protonen.

f) $f(r) = \begin{cases} \frac{3}{4\pi R_0^3} & \text{für } 0 \leq r \leq R_0 \\ 0 & \text{für } R_0 < r \end{cases}$

$$F(q^2) = 4\pi \int_0^\infty \frac{\sin\left(\frac{qr}{\hbar}\right)}{\frac{qr}{\hbar}} f(r) r^2 dr = 3 \frac{\hbar}{q R_0^3} \int_0^{R_0} \sin\left(\frac{qr}{\hbar}\right) r dr =$$

$$= 3 \frac{\sin\left(\frac{qR_0}{\hbar}\right) - \frac{qR_0}{\hbar} \cos\left(\frac{qR_0}{\hbar}\right)}{\frac{q^3 R_0^3}{\hbar^3}} = 3 \frac{\sin(x) - x \cos(x)}{x^3}, \text{ mit } x(q) = \frac{qR_0}{\hbar}$$

Diese Ladungsverteilung wird in der Natur nicht wiedergefunden, man kann schwerere Kerne aber dadurch approximieren.

g) $x(0) = 0$

$$F(0) = \lim_{x \rightarrow 0} \left(3 \frac{\sin(x) - x \cos(x)}{x^3} \right) = 3 \lim_{x \rightarrow 0} \left(\frac{1}{x^2} \right) = \text{divergiert gegen } \infty$$

Stabilstes Nuklid einer Isobare

$$M(A, Z) = Z(m_p + m_e) + (A - Z)m_n - (a_v A - a_s A^{2/3} - a_c Z^2 A^{-1/3} - a_a \frac{(A-2Z)^2}{A} + \delta a_p A^{-1/2}) c^{-2}$$

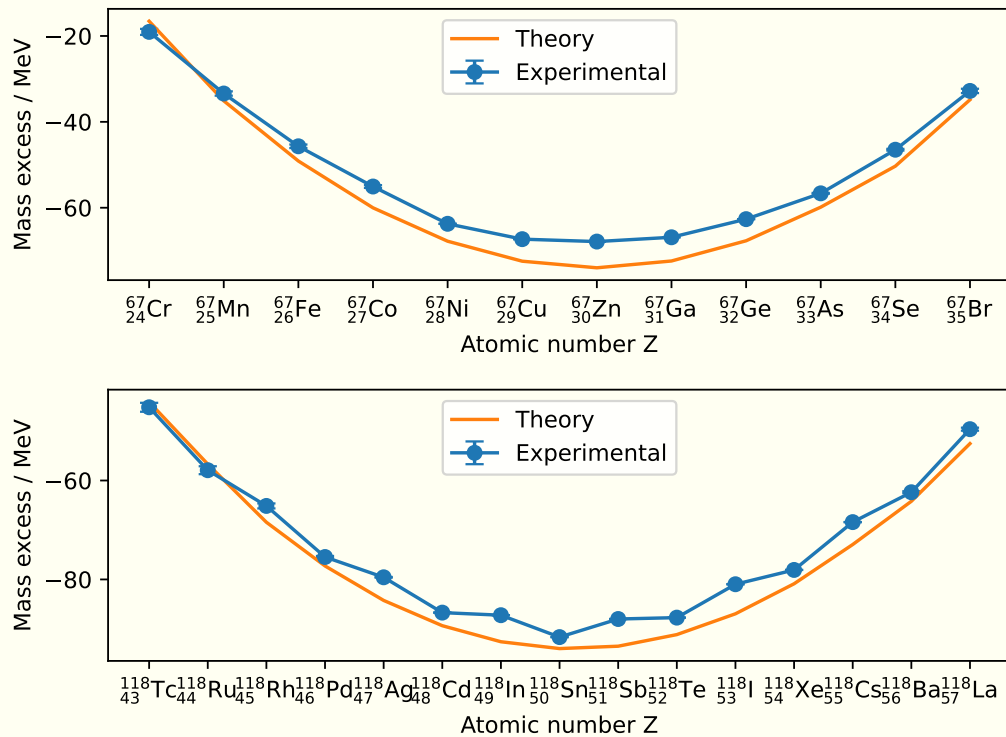
a)

$$\frac{\partial M(A, Z)}{\partial Z} = m_p + m_e - m_n - \left(\frac{4a_a(A-2Z)}{A} - \frac{2a_c Z}{A^{1/3}} \right) c^{-2} \stackrel{!}{=} 0$$

$$\Rightarrow Z_0(A) = \frac{4a_a - (m_p + m_e - m_n)c^2}{2A^{-1/3}(a_c + 4a_a A^{-2/3})}$$

b)

$$Z_0(67) = 29.9979 \approx \underline{\underline{30}} \quad Z_0(118) = 50.2517 \approx \underline{\underline{50}}$$



Luminosität des LHC

a) $\sigma = 0.07 \text{ b}$; $L_{\text{int}} \approx 200 \text{ 1/fb}$

$$L_{\text{int}} = \int \frac{1}{\sigma} \frac{dN}{dt} dt = \frac{N}{\sigma}$$

$$\Rightarrow N = L_{\text{int}} \sigma = \underline{\underline{1.4 \times 10^{16}}}$$

b) Die meisten pp-Wechselwirkungen fanden 2018 statt.