87. Ein sehr elastischer Ball

a) $h = 0.5$ m; $\quad m_1 \ll m_2$

$$z(t) = 0.5 - \tfrac{g}{2}t^2 \quad \Rightarrow \quad t_1 = 0.32 \text{ s}$$
$$v(t_1) = 3.13 \text{ m/s}$$

Nach dem Stoß:

$$v_1 = \underline{\underline{9.4 \text{ m/s}}}$$
$$v_2 = \underline{\underline{3.13 \text{ m/s}}}$$

b)

$$z_1(t) = 9.4t - \tfrac{g}{2}t^2$$
$$v_1(t) = 9.4 - gt$$
$$0 = 9.4 - gt \quad \Rightarrow \quad t_1 = 0.96 \text{ s}$$
$$z_1(t_1) = \underline{\underline{4.5 \text{ m}}}$$

96. Zwei Himmelskörper

$$\vec{v}_1 = \begin{pmatrix} v_1 \\ 0 \end{pmatrix} \qquad \vec{v}_2 = \begin{pmatrix} 0 \\ v_2 \end{pmatrix} \qquad \vec{v}_E = \tfrac{1}{m_1+m_2}\begin{pmatrix} m_1 v_1 \\ m_2 v_2 \end{pmatrix}$$
$$|v_1| = v_1 \qquad |v_2| = v_2 \qquad |v_E| = \frac{\sqrt{(m_1 v_1)^2 + (m_2 v_2)^2}}{m_1 + m_2}$$

a) $\underline{\underline{\alpha = \arctan\left(\frac{m_2 v_2}{m_1 v_1}\right)}}$

b) $K_1 := E_{kin}$ vor dem Stoß ; $\quad K_2 := E_{kin}$ nach dem Stoß

$$K_1 = K_2 + \Delta E$$
$$\tfrac{m_1 v_1^2}{2} + \tfrac{m_2 v_2^2}{2} = \tfrac{1}{2}(m_1 + m_2)\left(\frac{\sqrt{(m_1 v_1)^2 + (m_2 v_2)^2}}{m_1 + m_2}\right)^2 + \Delta E$$
$$\Delta E = \frac{(m_1 v_1)^2 + (m_2 v_2)^2 + m_1 m_2 v_1^2 + m_1 m_2 v_2^2 - (m_1 v_1)^2 - (m_2 v_2)^2}{2(m_1 + m_2)}$$
$$\Delta E = \underline{\underline{\frac{m_1 m_2 \left(v_1^2 + v_2^2\right)}{2(m_1 + m_2)}}}$$

c) Für $m_1 = m_2 = m$

$$\underline{\underline{\Delta E = \tfrac{1}{4} m \left(v_1^2 + v_2^2\right)}}$$

Für $m_1 \gg m_2$

$$\underline{\underline{\Delta E = \tfrac{1}{2} m_2 \left(v_1^2 + v_2^2\right)}}$$

102. Trägheitsmoment eines Hohlzylinders

a) $V = ({r_1}^2 - {r_0}^2)\pi z$

$$I = \int r^2\,dm = \rho \int_V r^2\,dV = \rho \int_0^z \int_0^{2\pi} \int_{r_0}^{r_1} r^3 dr\,d\varphi\,dz = \tfrac{{r_1}^4 - {r_0}^4}{2}\pi z \rho =$$
$$= \underline{\underline{\tfrac{1}{2}M({r_1}^2 + {r_0}^2)}}$$

113. Getränkedosen auf schiefer Ebene

$r = 0.03$ m; $\alpha = 25°$; $l = 1.5$ m; $h = l\sin(\alpha)$

a) $m = 0.06$ kg

$$mgh = \tfrac{I\omega^2 + m{v_s}^2}{2}$$
$$mgh = \tfrac{m{v_s}^2 + m{v_s}^2}{2}$$
$$v_s = \sqrt{gh}$$
$$= \underline{\underline{2.49 \text{ m/s}}}$$

b) $m_w = 0.33$ kg; $M = m + m_w = 0.39$ kg

$$Mgh = \tfrac{I\omega^2 + M{v_s}^2}{2}$$
$$Mgh = \tfrac{m{v_s}^2 + M{v_s}^2}{2}$$
$$v_s = \sqrt{\tfrac{2ghM}{2m + m_w}}$$
$$= \underline{\underline{3.28 \text{ m/s}}}$$

c)

$$Mgh = \tfrac{I_w\omega^2 + I\omega^2 + M{v_s}^2}{2}$$
$$Mgh = \tfrac{0.5m_w{v_s}^2 + m{v_s}^2 + M{v_s}^2}{2}$$
$$v_s = \sqrt{\tfrac{4Mgh}{3M + m}}$$
$$= \underline{\underline{2.81 \text{ m/s}}}$$