2. Feld einer Hohlkugel

$$k = \frac{1}{4\pi\epsilon_0}$$

a. $\rho = \frac{Q}{V}$

$$V = \int_V dV = \int_0^{\pi}\int_0^{2\pi}\int_{R_i}^{R_a} r^2 \sin(\theta)\,\mathrm{d}r\mathrm{d}\varphi\mathrm{d}\theta$$
$$= \frac{4}{3}\pi(R_a^3 - R_i^3)$$
$$\underline{\underline{\rho = \frac{3Q}{4\pi}\frac{1}{R_a^3 - R_i^3}}}$$

For $r \geq R_a$:

$$q_{in} = Q$$
$$\frac{q_{in}}{\epsilon_0} = \oint \vec{E}(\vec{r})\,\mathrm{d}\vec{A} = E(r)\oint \mathrm{d}A = E(r)4\pi r^2$$
$$E(r) = kQ\frac{1}{r^2}$$

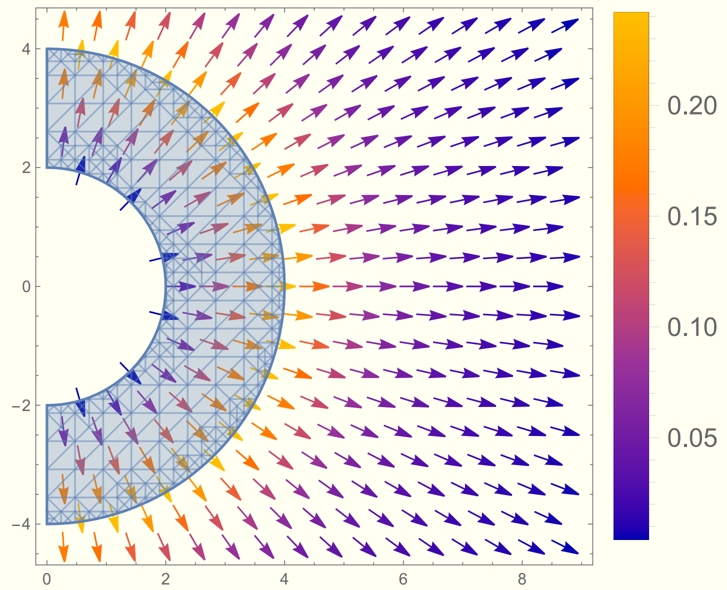
For $R_i < r < R_a$:

$$q_{in} = \rho V_{in} = Q\frac{r^3 - R_i^3}{R_a^3 - R_i^3}$$
$$\frac{q_{in}}{\epsilon_0} = \oint \vec{E}(\vec{r})\,\mathrm{d}\vec{A} = E(r)\oint \mathrm{d}A = E(r)4\pi r^2$$
$$E(r) = kQ\frac{r^3 - R_i^3}{R_a^3 - R_i^3}\frac{1}{r^2}$$

For $r \leq R_i$:

$$q_{in} = 0$$
$$E(r) = 0$$

$$\boxed{E(r) = \begin{cases} 0 & \text{for } 0 < r \leq R_i, \\ \frac{kQ}{R_a^3 - R_i^3}\left(r - \frac{R_i^3}{r^2}\right) & \text{for } R_i < r < R_a, \\ kQ\frac{1}{r^2} & \text{for } R_a \leq r. \end{cases}}$$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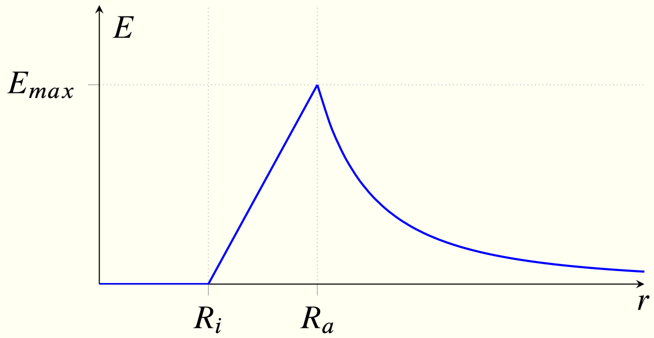
E
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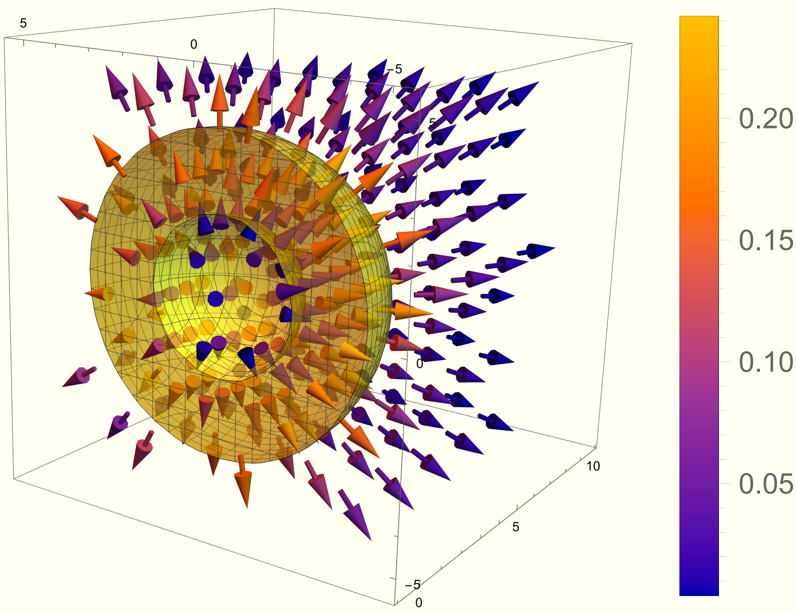
R_i
R_a
r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