Die Friedmann–Gleichung

$$H(t)^2 = \left(\frac{\dot{a}}{a}\right)^2 = H_0^2\left(\frac{\Omega_r}{a^4} + \frac{\Omega_m}{a^3} + \Omega_\Lambda - \frac{\Omega_0 - 1}{a^2}\right)$$

a)
- Ω_0 . . . total density
- Ω_r . . . radiation density
- Ω_m . . . matter density (Dark + Baryonic)
- Ω_Λ . . . cosmological constant (vacuum density)

b)

$$H(t)^2 = \left(\frac{\dot{a}}{a}\right)^2 = \left(\frac{\frac{da}{dt}}{a}\right)^2 \quad \Rightarrow \quad a^2 H^2(t) = \left(\frac{da}{dt}\right)^2 \quad \Rightarrow \quad da = \frac{dt}{aH(t)}$$

$$\int 1 \, dt = \int \frac{1}{aH(a)} \, da$$

c)
- $\Omega_0 \propto a^{-2}$
- $\Omega_r \propto a^{-4}$
- $\Omega_m \propto a^{-3}$
- $\Omega_\Lambda \propto$

d) Radiation dominated: $H(t) = H_0\sqrt{\frac{1}{a(t)^4}}$

$$\int_0^t 1 \, d\tilde{t} = \int_0^{a(t)} \frac{1}{\tilde{a}H(\tilde{a})} \, d\tilde{a} = \int_0^{a(t)} \frac{\tilde{a}}{H_0} \, d\tilde{a} = \frac{a(t)^2}{2H_0}$$

$$a(t) = \underline{\underline{\sqrt{2H_0 t}}}$$

Matter dominated: $H(t) = H_0\sqrt{\frac{1}{a(t)^4}}$

$$\int_0^t 1\,\mathrm{d}\tilde{t} = \int_0^{a(t)} \frac{1}{\tilde{a}H(\tilde{a})}\,\mathrm{d}\tilde{a} = \int_0^{a(t)} \frac{\sqrt{\tilde{a}}}{H_0}\,\mathrm{d}\tilde{a} = \frac{2\sqrt{a(t)^3}}{3H_0}$$

$$a(t) = \underline{\underline{\left(\frac{3H_0t}{2}\right)^{\frac{2}{3}}}}$$

Cosmological Constant: $H(t) = H_0$

$$\int_0^t 1\,\mathrm{d}\tilde{t} = \int_c^{a(t)} \frac{1}{\tilde{a}H(\tilde{a})}\,\mathrm{d}\tilde{a} = \int_c^{a(t)} \frac{1}{\tilde{a}H_0}\,\mathrm{d}\tilde{a} = \frac{\ln(a(t))}{H_0}$$

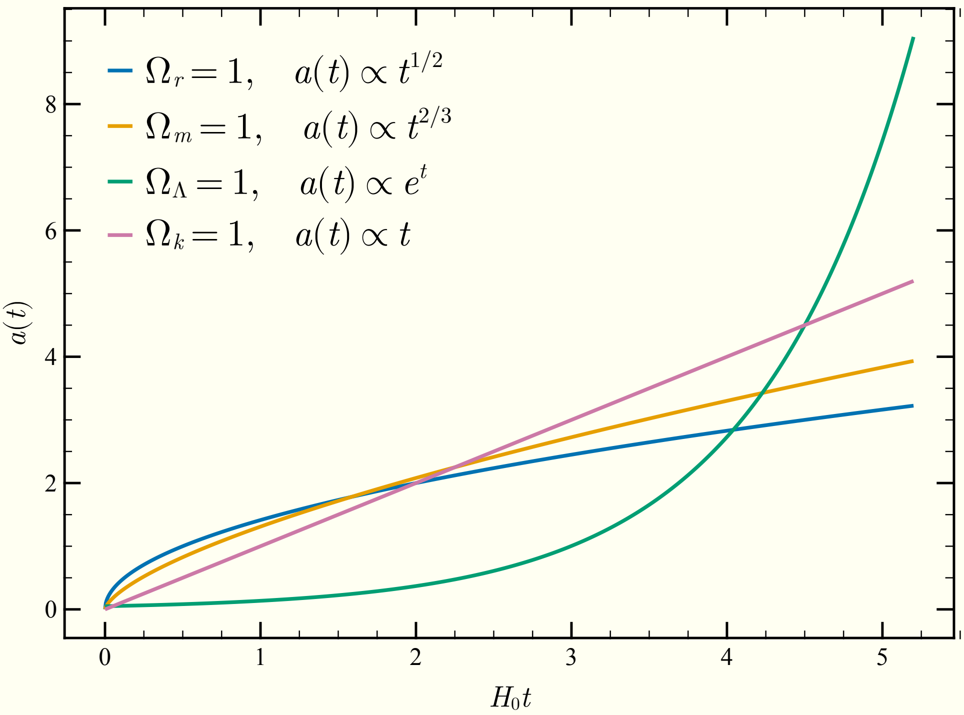
$$a(t) = \underline{\underline{\mathrm{e}^{H_0t+c}}}$$

Curvature dominated: $H(t) = H_0\sqrt{\frac{1}{a(t)^2}}$

$$\int_0^t 1\,\mathrm{d}\tilde{t} = \int_0^{a(t)} \frac{1}{\tilde{a}H(\tilde{a})}\,\mathrm{d}\tilde{a} = \int_0^{a(t)} \frac{1}{H_0}\,\mathrm{d}\tilde{a} = \frac{a(t)}{H_0}$$

$$a(t) = \underline{\underline{H_0t}}$$

e)


$\Omega_r = 1, \quad a(t) \propto t^{1/2}$
$\Omega_m = 1, \quad a(t) \propto t^{2/3}$
$\Omega_\Lambda = 1, \quad a(t) \propto e^t$
$\Omega_k = 1, \quad a(t) \propto t$
$a(t)$
$H_0 t$
0
2
4
6
8
0
1
2
3
4
5