177. Stabpendel mit Kugeln

$$L=0.3 \; \mathrm{m}; \qquad r_1=0.02 \; \mathrm{m}; \qquad r_2=0.03 \; \mathrm{m}$$
 $m_0=0.1 \; \mathrm{kg}; \qquad m_1=0.1 \; \mathrm{kg}; \qquad m_2=0.3 \; \mathrm{kg}$

a)
$$\begin{split} I_{ges} &= I_{Stab} + I_{Kugel_1} + I_{Kugel_2} \\ I_{Stab} &= \frac{1}{12} m_0 L^2 = 7.5 * 10^{-4} \text{ kg m}^2 \\ I_{Kugel_1} &= \frac{2}{5} m_1 r_1^2 + m_1 (\frac{L}{2} + r_1)^2 = 2.9 * 10^{-3} \text{ kg m}^2 \\ I_{Kugel_2} &= \frac{2}{5} m_2 r_2^2 + m_2 (\frac{L}{2} + r_2)^2 = 9.8 * 10^{-3} \text{ kg m}^2 \\ &\Rightarrow I_{ges} = \underline{1}.3 * 10^{-2} \text{ kg m}^2 \end{split}$$

b)

$$x_{Stabsp} = 0 \text{ m}$$

$$x_{Kugeln_{sp}} = \frac{m_1 \left(-\frac{2r_1 + L}{2}\right) + m_2 \left(\frac{2r_2 + L}{2}\right)}{m_1 + m_2} = 0.0925 \text{ m}$$

$$x_{Stabsp} * m_0 + x_{Kugeln_{sp}} * (m_1 + m_2)$$

$$x_{sp} = \frac{x_{Stab_{sp}}*m_0 + x_{Kugeln_{sp}}*(m_1 + m_2)}{m_0 + m_1 + m_2} = \underline{0.074} \text{ m}$$

c)
$$T = 2\pi \sqrt{\frac{I}{Mgl}}$$

$$T = 2\pi \sqrt{\frac{I}{(m_0 + m_1 + m_2) g\left(\frac{2r_2 + L}{2}\right)}}$$

$$= \underline{1.2 \text{ s}}$$

184. Resonanzverhalten einer Stimmgabel

$$f_0 = 440 \text{ Hz}; \quad \omega_0 = 2\pi f_0; \quad x(\tau) = x_m e^{-\beta \tau/2}; \quad x(5) = \frac{1}{10}x(0)$$

a)

$$\frac{x_m}{10} = x_m e^{-\beta 5/2}$$
 $\beta = -\frac{2}{5} \ln(0.1)$

$$\tau = \frac{1}{\beta} = \underline{1.09 \text{ s}}$$

b)
$$\omega = \sqrt{\omega_0^2 - \frac{\beta^2}{4}}$$

$$\omega = 2764.6 \text{ Hz} \qquad (\omega_0 \approx \omega)$$

$$f = \frac{\omega}{2\pi} = 440 \text{ Hz} \qquad (f_0 \approx f)$$

$$f * \tau = \underline{477.7}$$

c)
$$Q = \frac{\omega}{\beta}$$

$$Q = 3001.63$$

d)
$$\delta f = \frac{f}{Q}$$

$$\delta f = 0.15 \text{ Hz}$$

e)
$$x_m(f) = \frac{2\pi f_0}{\sqrt{16\pi^2(f-f_0)+\beta^2}};$$
 $E(f) = \frac{f_0^2}{16(f-f_0)^2+(\beta/2)^2};$ $f_1 = 439.5 \text{ Hz}$

$$x_m(f_0) = \frac{2\pi f_0}{\sqrt{16\pi^2(f_0-f_0)+\beta^2}} = 3001.63$$

$$x_m(f_1) = \frac{2\pi f_0}{\sqrt{16\pi^2(f_1-f_0)+\beta^2}} = 435.348$$

$$x_{rel} = \frac{x_m(f_1)}{x_m(f_0)} = \underline{0.15}$$

$$E(f_0) = E_{rel}$$

185. Resonante Anregung eines Federpendels

$$m=0.1~{\rm kg}; \quad k=40~{\rm N/m}$$
 a) $\omega=\sqrt{\frac{k}{m}}=20~{\rm rad/s}; \quad \omega_0=\sqrt{\frac{k}{m}-\frac{\beta^2}{4m^2}}$

$$\frac{x_m}{2} = x_m * e^{-\beta t/2}$$

$$\beta = -\frac{\ln(0.5)2}{t} = \underline{0.14 \text{ kg/s}}$$

$$\omega_0 = \sqrt{\frac{k}{m} - \frac{\beta^2}{4}} = 19.99988 \text{ rad/s}$$

b)

$$Q = \omega_0 \frac{m}{\beta} = \underline{144.269}$$