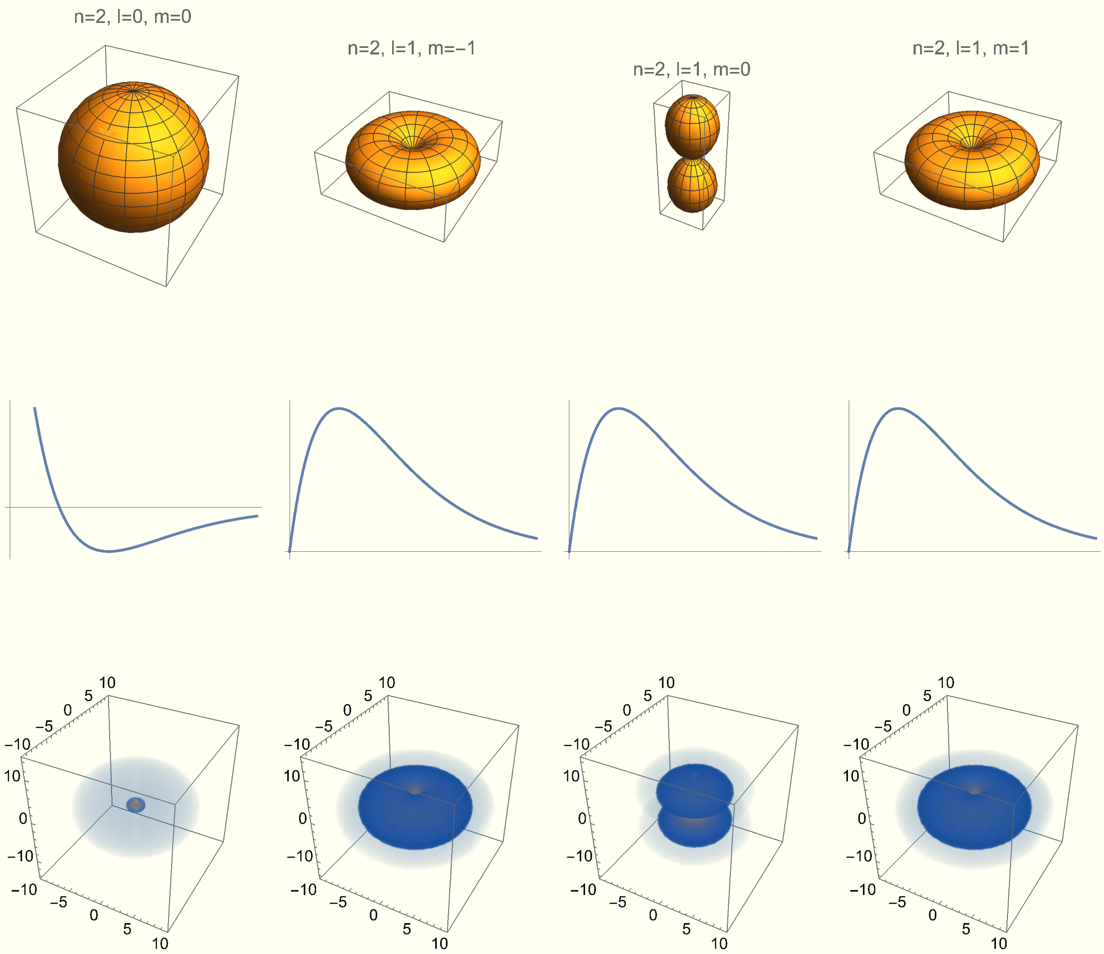
Wasserstoffatom: Aufenthaltswahrscheinlichkeiten

a)



On top you can see the spherical and radial probability of an electron in a hydrogen Atom for different Quantum numbers. Below is a density plot of the square modulus of the whole wave function (as product of the upper 2 plots). If you sum all 4 states up you can kind of recognize a radial symmetry, as the $l = 0, m = 0$ state is already symmetric and the other three apparently add up to a sphere.

b)

$$\langle \hat{r} \rangle = \int_V r|\psi_{2,1,1}|^2 \,\mathrm{d}V = \int_0^{2\pi}\int_0^{\pi}\int_0^{\infty} \frac{1}{24a_0^5}\,\mathrm{e}^{-\frac{r}{a}} r^2 \frac{3}{8\pi}\sin(\theta)^2 r r^2 \sin(\theta)\,\mathrm{d}r\,\mathrm{d}\theta\,\mathrm{d}\phi =$$

$$= \frac{1}{32a_0^5}\int_0^{\infty} r^5\,\mathrm{e}^{-\frac{r}{a_0}}\,\mathrm{d}r \int_0^{\pi}\sin(\theta)^3\,\mathrm{d}\theta = \frac{1}{32a_0^5}\,120a_0^6\,\frac{4}{3} = \underline{\underline{5a_0}}$$

$$\langle \hat{r}^2 \rangle = \int_V r^2|\psi_{2,1,1}|^2\,\mathrm{d}V = \frac{1}{32a_0^5}\int_0^{\infty} r^6\,\mathrm{e}^{-\frac{r}{a_0}}\,\mathrm{d}r \int_0^{\pi}\sin(\theta)^3\,\mathrm{d}\theta = \underline{\underline{30a_0^2}}$$

$$\langle \Delta\hat{r}^2 \rangle = \langle \hat{r}^2 \rangle - \langle \hat{r} \rangle^2 = 30a_0^2 - 25a_0^2 = \underline{\underline{5a_0^2}}$$

c) $d = 1.75$ fm

$$P = \int_V |\psi_{2,0,0}|^2\,\mathrm{d}V = \int_0^{2\pi}\int_0^{\pi}\int_0^{d} \frac{1}{4a_0^3}\,\mathrm{e}^{-\frac{r}{a}}\left(1 - \frac{r}{a_0} + \frac{r^2}{4a_0^2}\right)\frac{1}{4\pi} r^2 \sin(\theta)\,\mathrm{d}r\,\mathrm{d}\theta\,\mathrm{d}\phi =$$

$$= \frac{1}{2a_0^3}\int_0^{d} \mathrm{e}^{-\frac{r}{a}}\left(1 - \frac{r}{a_0} + \frac{r^2}{4a_0^2}\right) r^2\,\mathrm{d}r = \underline{\underline{6.11\times 10^{-15}}}$$

Drehimpulsquantenzahlen

a) $\hat{L}_z = -i\hbar\frac{\partial}{\partial\phi}; \quad \psi_{n,l,m} = a_m R_{n,l}\cos(\theta)\,\mathrm{e}^{im\phi} P_l^m$

$$\langle \hat{L}_z \rangle = \int_V \psi^* \hat{L}_z \psi\,\mathrm{d}V = -i\hbar\int_V \psi^* \frac{\partial\psi}{\partial\phi}\,\mathrm{d}V = \hbar m \int_V \psi^*\psi\,\mathrm{d}V = \underline{\underline{\hbar m}}$$

b) $\hat{\mathbf{L}}^2 Y_l^m(\theta,\phi) = l(l+1)\hbar^2 Y_l^m(\theta,\phi)$

$$\langle \hat{\mathbf{L}}^2 \rangle = \int_V \psi^* \hat{\mathbf{L}}^2 \psi\,\mathrm{d}V = \int_V \psi^* R_{n,l}\hat{\mathbf{L}}^2 Y_l^m\,\mathrm{d}V = l(l+1)\hbar^2 \int_V \psi^*\psi\,\mathrm{d}V = \underline{\underline{l(l+1)\hbar^2}}$$

c)

$$\left[\hat{\mathbf{L}}^2, \hat{L}_z\right]\psi_{n,l,m} = \hat{\mathbf{L}}^2(\hat{L}_z\psi_{n,l,m}) - \hat{L}_z(\hat{\mathbf{L}}^2\psi_{n,l,m}) =$$
$$= \hat{\mathbf{L}}^2\hbar m\psi_{n,l,m} - \hat{L}_z l(l+1)\hbar^2\psi_{n,l,m} =$$
$$= l(l+1)\hbar^3 m\psi_{n,l,m} - l(l+1)\hbar^3 m\psi_{n,l,m} = 0$$

Zeeman–Effekt

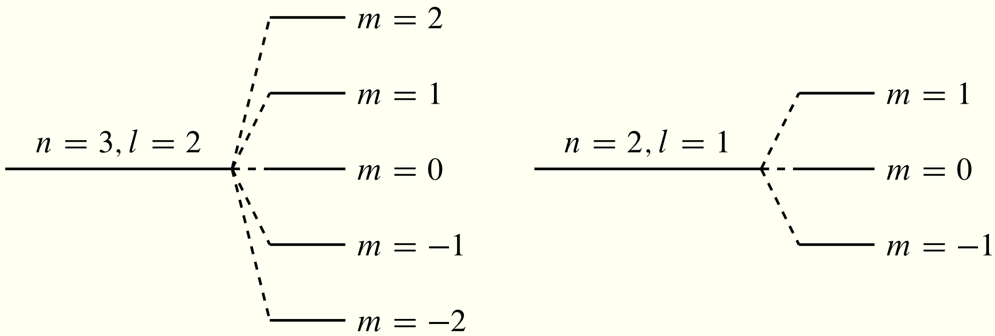
a)

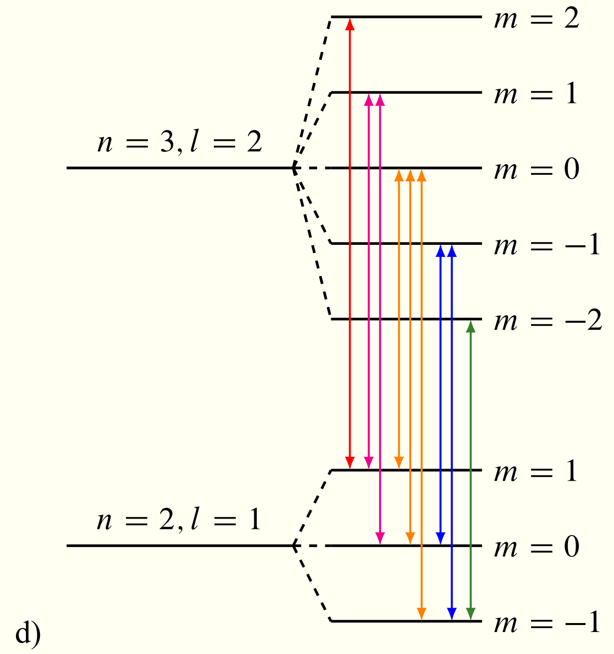
$$E = V + T = \underline{\underline{-\vec{\mu}\cdot\vec{B} + \tfrac{1}{2}m(\dot{\vec{r}})^2}}$$

b) $\vec{B} = B\vec{e}_z; \quad |L_z| = \hbar m; \quad \vec{p}_\mathrm{m} = -\frac{e}{2m_e}\vec{L}$

$$E_\text{pot, B} = -\vec{p}_\mathrm{m}\cdot\vec{B} = \tfrac{e}{2m_e}\vec{L}\cdot\vec{B} = \tfrac{e\hbar}{2m_e}mB$$
$$E_{n,l,m} = E_{n,l} + \tfrac{e\hbar}{2m_e}mB$$
$$\Delta E = \underline{\underline{\tfrac{e\hbar}{2m_e}mB}}$$

c)





$1 + 2 + 3 + 2 + 1 = \underline{\underline{9}}$