1) $\cancel{\gamma = \frac{F}{2A}}$ $\cancel{A = \dots}$

$dW = \gamma dA = -p dV$ $\qquad dV = 4\pi r^2 dr$

$dA = 8\pi r dr$

$\Rightarrow p = -\frac{\gamma 2}{r}$

b) $dU = dW + dQ = \gamma dA + T dS$

c) $F = U - TS$

$dF = dU - TdS - SdT = \gamma dA + TdS - TdS - SdT = \gamma dA - SdT$

d) $\left(\frac{\partial F}{\partial A}\right)_T = \gamma \qquad \left(\frac{\partial F}{\partial T}\right)_A = -S$

$\Rightarrow \left(\frac{\partial S}{\partial A}\right)_T = -\left(\frac{\partial \gamma}{\partial T}\right)_A \qquad \left(\frac{\partial \gamma}{\partial T}\right)_A < 0$, weil Teilchen sich mehr bewegen

$\Rightarrow \frac{\partial S}{\partial A} > 0$

e) $\gamma = \left(\frac{\partial F}{\partial A}\right)_T = \left(\frac{\partial U}{\partial A}\right)_T - \left(\frac{T\partial S}{\partial A}\right)_T = \left(\frac{\partial U}{\partial A}\right)_T + T\left(\frac{\partial \gamma}{\partial T}\right)_A$

$\Rightarrow \left(\frac{\partial U}{\partial A}\right)_T = \gamma - T\left(\frac{\partial \gamma}{\partial T}\right)_A$

f) $\left(\frac{\partial U}{\partial A}\right)_T = \left(\frac{dQ}{dA}\right)_T + \left(\frac{dW}{dA}\right)_T = -T\left(\frac{\partial \gamma}{\partial T}\right)_A + \gamma$

$\Rightarrow \left(\frac{dQ}{dA}\right)_T = -T\left(\frac{\partial \gamma}{\partial T}\right)_A$

$\Delta Q = -T\left(\frac{\partial \gamma}{\partial T}\right)_A \Delta A \qquad \Delta Q > 0$

$\Delta S = \frac{\Delta Q}{T} > 0$, größere Oberfläche $\Rightarrow$ mehr Mikrozustände $\Rightarrow$ mehr Entropie

2) $dU = T\,dS + B\,dm \qquad \chi \approx \frac{\mu_0 m}{BV} \propto \frac{1}{T} \quad \Rightarrow m \propto \frac{1}{T}$

$F = U - TS - mB \qquad C_B = T\left(\frac{\partial S}{\partial T}\right)_B$

$dF = -S\,dT - m\,dB$

$\left(\frac{\partial F}{\partial T}\right)_B = -S \qquad \left(\frac{\partial F}{\partial B}\right)_T = -m$

$\left(\frac{\partial S}{\partial B}\right)_T = \left(\frac{\partial m}{\partial T}\right)_B \Rightarrow \cancel{\partial S = \partial B}\ \Delta S = \Delta B\left(\frac{\partial m}{\partial T}\right)_B$

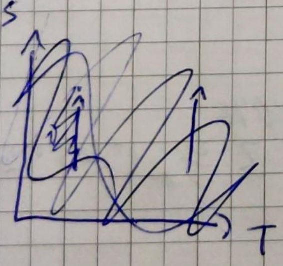
$\Delta Q = \Delta S\,T = \Delta B\,T\left(\frac{\partial m}{\partial T}\right)_B = \cancel{\Delta B\,T\,\frac{1}{T^2}}$

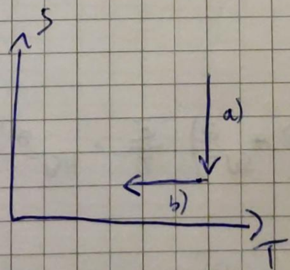
$\overset{m \propto \frac{1}{T}}{\propto} \Delta B\,T\left(-\frac{1}{T^2}\right) = \underline{\underline{-\frac{\Delta B}{T}}} < 0 \qquad (\Delta B > 0,\ T > 0)$

b) $\left(\frac{\partial T}{\partial B}\right)_S = -\frac{T}{C_B}\left(\frac{\partial m}{\partial T}\right)_B \propto +\frac{1}{C_B T}$

$\Rightarrow \Delta T = \frac{\Delta B}{C_B T} \qquad \Delta B < 0 \Rightarrow \Delta T < 0 \qquad (C_B, T > 0)$

c)





d) a) Mehr Spins ∥ zu B als im Gleichgewicht ohne B ⇒ mehr Ordnung / weniger Mikrozustände

b) Demagnetisierung erhöht # Mikrozustände, senkt aber auch Temperatur, wodurch die Besetzung geändert wird

3) $H = -J \sum_{i}^{N} \sigma_{i1}\sigma_{i2}$ $\quad E_{\uparrow\uparrow} = \cancel{-2J}$ $\quad E_{\uparrow\downarrow}\ \cancel{2J}$ $\quad E_{\downarrow\downarrow} = 2J$

$\cancel{Z = \sum_{i} e^{-\beta(-J\sum_{j}^{N}\sigma_{j1}\sigma_{j2})}}$ $\quad \sigma_1\sigma_2 = 1 \quad E_{\uparrow\uparrow}/E_{\downarrow\downarrow} = -\cancel{2}J$

$\sigma_1\sigma_2 = -1 \quad E_{\uparrow\downarrow}/E_{\downarrow\uparrow} = \cancel{2}J$

$$Z = 2\sum_{\sigma_1\sigma_2 = \pm 1} e^{-\beta(-J\sum_{j}^{N}\sigma_{j1}\sigma_{j2})} =$$

$$= 2\prod_{j=1}^{N}\sum_{\sigma_1\sigma_2=\pm 1} e^{\beta J\sigma_{j1}\sigma_{j2}} = 2\prod_{j=1}^{N} e^{\beta J} + e^{-\beta J} = 2\prod_{j=1}^{N} 2\cosh(\beta J) = \underline{\underline{(4\cosh(\beta J))^{N}}}$$

a) $$\langle\sigma_{i,1}|\sigma_{i,2}\rangle = \frac{1}{Z}\sum_{\uparrow\uparrow,\uparrow\downarrow,\downarrow\downarrow,\downarrow\uparrow} \sigma_{i,1}\sigma_{i,2}\, e^{-\beta H} = \frac{2}{Z}\left(e^{-\beta J} + e^{\beta J}\right) = \frac{4\sinh(\beta J)}{4\cosh(\beta J)} = \tanh(\beta J)$$

c)

$$\langle M^2\rangle = m^2N^2\langle\sigma^2\rangle = m^2N^2\lim_{N\to\infty}\langle\sigma_{i1}|\sigma_{i2}\rangle = m^2N^2\tanh(\beta J)$$

$$T\to 0:\ \langle M^2\rangle \to m^2N^2$$

d) $$S_g = -k_B\sum p_i \ln p_i = -k_B\sum_i \frac{e^{-\beta E_i}}{Z}\ln\frac{e^{-\beta E_i}}{Z} = -k_B\sum_i\left(-\frac{\beta E_i}{Z}\right)e^{-\beta E_i}\cancel{\ }\ln(Z) =$$

$$= k_B\ln(Z) + k_B\sum_i e^{-\beta E_i}\frac{\beta}{Z}E_i =$$

$$= k_B\ln(Z) + k_B\frac{4J\beta}{Z}\left(e^{-\beta J} - e^{\beta J}\right) =$$

$$= \underline{\underline{k_B\ln(Z) + k_B T\left(\frac{\partial\ln(Z)}{\partial T}\right)}}$$

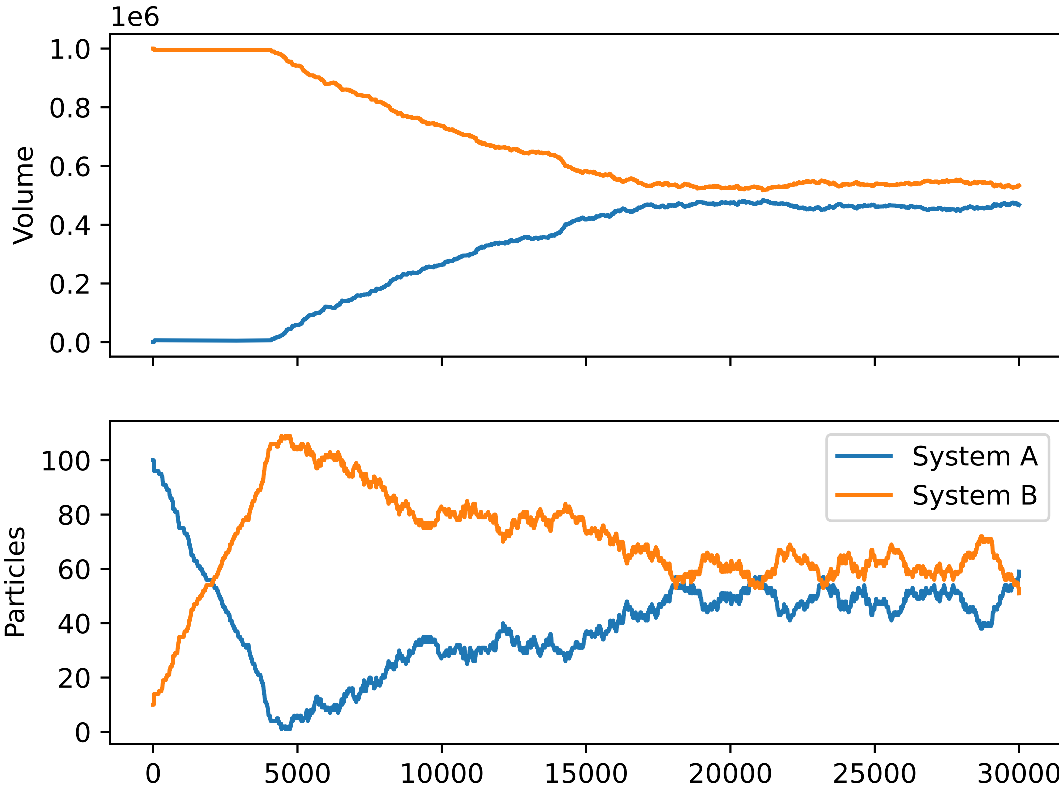
$$\frac{\partial\ln Z}{\partial T} = -\frac{1}{Z}\frac{4J\beta}{T}\sinh(J\beta)$$

e) $$S = k_B\ln\left[(4\cosh(J\beta))^N\right] + k_B T\frac{\partial\ln\left[(4\cosh(\beta J))^N\right]}{\partial T} =$$

$$= Nk_B\ln(4\cosh(J\beta)) + \frac{k_B T N}{4\cosh(\beta J)}\left(-4\sinh(\beta J)\frac{J\beta}{T}\right) =$$

$$= \underline{Nk_B\ln(4\cosh(J\beta)) - k_B\beta J\tanh(J\beta)}$$

$$T\to 0:\ S\to Nk_B\ln 2$$


1e6
1.0
0.8
0.6
0.4
0.2
0.0
Volume
100
80
60
40
20
0
Particles
System A
System B
0
5000
10000
15000
20000
25000
30000