1) $S = k_B \ln\left(\frac{N!}{n!(N-n)!}\right) \approx \cancel{N} k_B \left(N\ln N - N - n\ln n + n - (N-n)\ln N-n + N-n\right) =$

$= k_B\left(N\ln\frac{N}{N-n} + n\ln\frac{N-n}{\cancel{N}n}\right)$

$p_1 = \frac{n}{N},\ p_2 = 1-p_1$

$S_g = -k_B\sum_i p_i \ln p_i = -k_B(p_1\ln p_1 + p_2\ln p_2) = -k_B\left(\frac{n}{N}\ln\left(\frac{n}{N}\right) + \frac{N-n}{N}\ln\left(\frac{N-n}{N}\right)\right) =$

$= -k_B\left(\frac{n}{N}\ln\left(\frac{n}{N-n}\right) + \ln\left(\frac{N-n}{N}\right)\right) = \cancel{\frac{k_B}{N}\left(n\ln\left(\frac{n}{N-n}\right) + N\ln\right)}$

$= \frac{k_B}{N}\left(n\ln\frac{N-n}{n} + N\ln\left(\frac{N}{N-n}\right)\right) = \underline{\underline{\frac{S}{N}}}$

b)

$\Omega(E) = \sum_{a_1=0}^{1} \cdots \sum_{a_N=0}^{1} \delta(E - \mu B\sum_i^N a_i) = \sum_{a_1=0}^{1}\cdots\sum_{a_N=0}^{1} \int\frac{dk}{2\pi} e^{ik(E-\mu B\sum_i^N a_i)} =$

$= \int\frac{dk}{2\pi} e^{ikE} \sum_{a_1}^{N}\cdots\sum_{a_N}^{N} e^{-ik\mu B\sum_i^N a_i} = \int\frac{dk}{2\pi} e^{ikE} \sum_{a_1}^{N}\cdots\sum_{a_N}^{N}\prod_{a_i}^{N} e^{-ik\mu B a_i} =$

$= \int\frac{dk}{2\pi} e^{ikE}\left(1+e^{-ik\mu B}\right)^N = \int\frac{dk}{2\pi} e^{N\left(ik\frac{E}{N} + \ln(1+e^{-ik\mu B})\right)}$

c) $I = \lim_{N\to\infty}\int e^{N\mathcal{E}(x)}dx = \lim_{N\to\infty} e^{N\mathcal{E}(x_0)}\sqrt{\frac{2\pi}{N|\mathcal{E}''(x_0)|}}$

$\mathcal{E}(k) = ik\frac{E}{N} + \ln(1+e^{-ik\mu B})$

$\mathcal{E}'(k) = i\frac{E}{N} + \frac{1}{1+e^{-ik\mu B}} e^{-ik\mu B}(-iB\mu)$

$\mathcal{E}''(k) = (-iB\mu)^2\frac{e^{-ik\mu B}}{1+e^{-ik\mu B}} + (-iB\mu)^2\frac{(e^{-ik\mu B})^2}{(1+e^{-ik\mu B})^2} =$

$= -(B\mu)^2\frac{1}{4\cos^2\left(\frac{Bk\mu}{2}\right)} = -\left(\frac{B\mu}{2\cos\left(\frac{Bk\mu}{2}\right)}\right)^2$

$\to \Omega(E) = \int\frac{dk}{2\pi} e^{N\left(ik\frac{E}{N} + \ln(1+e^{-ik\mu B})\right)}$

$\approx \frac{e^{N\left(ik_0\frac{E}{N} + \ln(1+e^{-ik_0\mu B})\right)}}{2\pi}\sqrt{\frac{2\pi}{N\left(\frac{B\mu}{2\cos(k_0\frac{B\mu}{2})}\right)^2}}$

$= e^{N\left(ik_0\frac{E}{N} + \ln(1+e^{-ik_0\mu B})\right)}\frac{2\cos\left(k_0\frac{B\mu}{2}\right)}{B\mu}\sqrt{\frac{1}{2\pi N}}$

$k_0: \mathcal{E}'(k) \overset{!}{=} 0 \Rightarrow k_0 = \frac{i\left(2i\pi m + \ln\left(-\frac{E}{E-BN\mu}\right)\right)}{B\mu}$

$\Omega(E)\big|_{k=k_0} = \frac{1}{BN\mu}\sqrt{\frac{N}{2\pi E(E-B\mu N)}}\left(BN\mu\ln\left(\frac{BN\mu}{BN\mu-E}\right) - E\ln\left(\frac{E}{BN\mu-E}\right)\right)$ (Mathematica)

$\frac{E}{\mu B} = n$

$= \frac{n}{NE}\sqrt{\frac{N}{2\pi E^2(1-\frac{N}{n})}}\left(\frac{NE}{n}\ln\left(\frac{\frac{EN}{n}}{\frac{EN}{n}-E}\right) - E\ln\left(\frac{E}{\frac{EN}{n}-E}\right)\right) =$

$= \frac{n}{N}\sqrt{\frac{N}{2\pi E^2(1-\frac{N}{n})}}\left(\frac{N}{n}\ln\left(\frac{N}{N-n}\right) - \ln\left(\frac{n}{N-n}\right)\right)$

$k_B =$

$= \frac{\sqrt{\frac{N}{2\pi E^2(1-\frac{N}{n})}}}{N}\left(n\ln\left(\frac{N-n}{n}\right) + N\ln\left(\frac{N}{N-n}\right)\right) = \frac{S_g}{k_B}$

$N \to \infty$. $\Omega(E) \approx e^{N(ck_0\frac{E}{N} + \ln(1+e^{-k_0 c\mu B}))}$

$\Rightarrow \log(\Omega(E)|_{k=k_0}) = N\left(ck_0\frac{E}{N} + \ln(1+e^{-k_0 c\mu B})\right) =$

$= N\left(\frac{E}{N}\log\left(-\frac{E-B\mu N}{E}\right)\frac{1}{\mu B} + \ln\left(1+e^{\mu B \ln\left(-\frac{E-B N\mu}{E}\right)\frac{1}{\mu B}}\right)\right) =$

$= N\left(p_+ \ln\left(\frac{1}{p_+}-1\right) + \ln\left(1+\frac{1}{\frac{1}{p_+}-1}\right)\right) =$

$p_+ = \frac{E}{B\mu N}$

$= N\left(p_+\ln(1-p_+) - p_+\ln(p_+) + \ln\left(\frac{1}{1-p_+}\right)\right) =$

$= N(-p_+\ln(p_+) - (1-p_+)\ln(1-p_+))$

$= -N(p_+\ln(p_+) + p_-\ln(p_-))$

$= \frac{N S_g}{k_B}$

2) $f(\vec{x}_0+\vec{h}) - f(\vec{x}_0) \approx f(\vec{x}_0) + \nabla f(\vec{x}_0)\vec{h} - f(\vec{x}_0) = \sum_i \frac{\partial f(\vec{x}_0)}{\partial x_i}\vec{e}_{x_i} \cdot h_i \vec{e}_{x_i} = \sum_i \frac{\partial f(\vec{x}_0)}{\partial x_i} h_i$

a) $df = f(\vec{x}_0 + d\vec{x}) - f(\vec{x}_0) = \sum_i \frac{\partial f(\vec{x}_0)}{\partial x_i} dx_i \Rightarrow \frac{df}{dt} = \sum_i \frac{\partial f(\vec{x}_0)}{\partial x_i}\frac{dx_i}{dt}$.

c) ~~$I_\gamma = \int_\gamma f(x,y)\,dx + g(x,y)\,dy = \int_\gamma \ldots \vec{\nabla}\vec{F} = \int_\gamma$~~ ~~$\vec{F} = (f(x,y), g(x,y))$~~

$I_\gamma = \int_\gamma f(x,y)\,dx + g(x,y)\,dy \underset{\uparrow}{=} \int_\gamma \frac{\partial F}{\partial x}dx + \frac{\partial F}{\partial y}dy \underset{\uparrow}{=} \int_\gamma dF = F(\gamma_b) - F(\gamma_a)$

$\vec{\nabla}F = \binom{f}{g}$ b)

Assumption

Für $\gamma_a = \gamma_b \Rightarrow I_\gamma = 0$

d) $pV = Nk_B T \quad E = \frac{3}{2}Nk_B T$

$dS = \frac{dE}{T} + \frac{p\,dV}{T} = \frac{3}{2}\frac{Nk_B\,dT}{T} + \frac{Nk_B\,dV}{V}$

$\Delta S = S_a - S_b = \int_{V_1}^{2V_1} \frac{Nk_B\,dV}{V} + \int_{T_1}^{T_2} \frac{3}{2}\frac{Nk_B\,dT}{T} = Nk_B\left(\ln(2) + \frac{3}{2}\ln\left(\frac{35}{30}\right)\right) \approx 7{,}69\ \mathrm{J/K}$

3a)

```
In[1]:= V1 := 10 dm^3

V2 := 50 dm^3

T1 := 300 K

k := k

n := N_A * 1 mol

e[T_] := 3 / 2 n * k * T
ΔW =
 UnitConvert[Integrate[-2 / 3 e[T1] / V, {V, V1, V2}], "J"] // N

Out[7]= -4014.48 J
```

b)

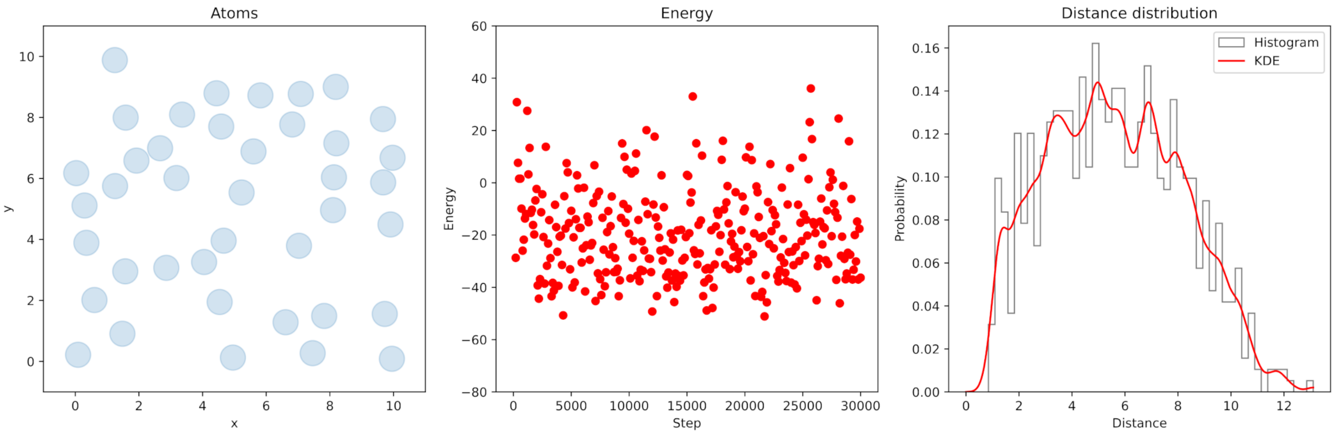
```
In[8]:= γAr := 5 / 3
T2 := T1 (V1 / V2) ^ (γAr - 1)
ΔW = UnitConvert[Integrate[e[T] / T, {T, T1, T2}], "J"] // N

Out[10]= -2461.93 J
```

c)

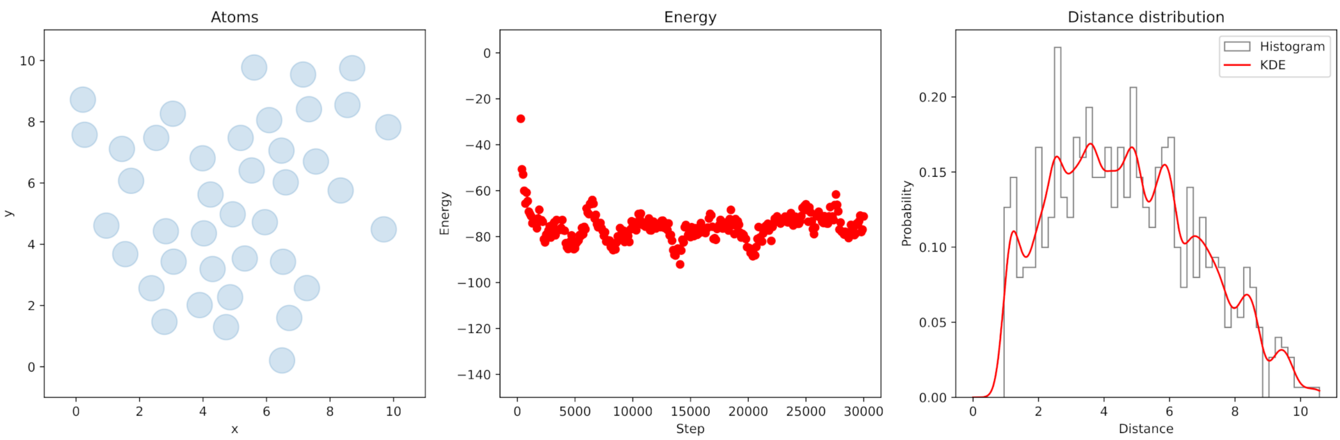
```
In[95]:= a := 0.1363 m^3 J

b := 0.0000322 m^3

eqn := n * k * T == (p + a / V ^ 2) (V - b)
ΔW = UnitConvert[Integrate[-p /. Solve[eqn, p] /. T → T1, {V, V1, V2}], "J"][[1]]

Out[98]= -4010.02 J
```

In a) wird am meisten Arbeit geleistet, weil keine Energieumwandlung in Wärme berücksichtigt wird, während in b) Energieverlust durch Erhitzen des Gases miteinbezogen wird. In Realität lässt sich Argon sehr gut als ideales Gas nähern, weshalb c) dem Wert aus a) ähnelt.

Gas:



Flüssig:



Fest: