

1. Potentialverlauf einer geladenen Hohlkugel

$$k = \frac{1}{4\pi\epsilon_0}, V = \int_{V_i}^{\infty} \vec{E} \, d\vec{s}$$

$$E(r) = \begin{cases} 0 & \text{for } 0 < r \leq R_i, \\ \frac{kQ}{R_a^3 - R_i^3} \left(r - \frac{R_i^3}{r^2} \right) & \text{for } R_i < r < R_a, \\ kQ \frac{1}{r^2} & \text{for } R_a \leq r. \end{cases}$$

For $R_a \leq r$:

$$E(r) = kQ \frac{1}{r^2}$$

$$V_1(r) = \int_r^{\infty} \vec{E}(r) \, d\vec{r} = kQ \int_r^{\infty} \frac{1}{r^2} \, dr = kQ \frac{1}{R_a}$$

$$V_1(r) = kQ \frac{1}{r}$$

For $R_i < r < R_a$:

$$E(r) = \frac{kQ}{R_a^3 - R_i^3} \left(r - \frac{R_i^3}{r^2} \right)$$

$$V_2(r) = \int_r^{\infty} \vec{E}(r) \, d\vec{r} = kQ \left(\int_r^{R_a} \frac{r^3 - R_i^3}{r^2(R_a^3 - R_i^3)} \, dr + \underbrace{\int_{R_a}^{\infty} \frac{1}{r^2} \, dr}_{= V_1(R_a)} \right) =$$

$$= kQ \left(-\frac{r^3 R_a - r R_a^3 + 2r R_i^3 + 2R_a R_i^3}{2r R_a^4 - 2r R_a R_i^3} + \frac{1}{R_a} \right)$$

$$V_2(r) = -kQ \frac{r^3 - 3r R_a^2 + 2R_i^3}{2r R_a^3 - 2r R_i^3}$$

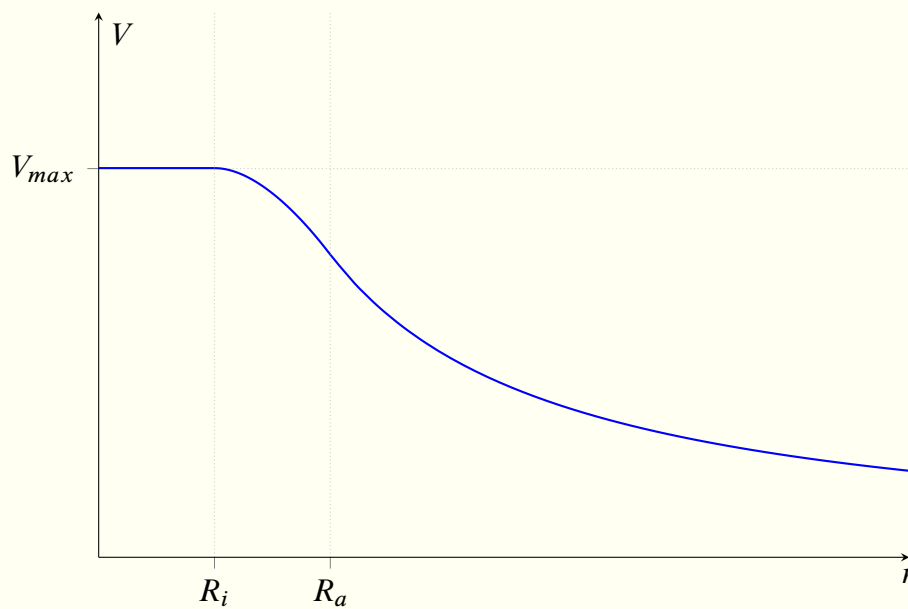
For $r \leq R_i$:

$$E(r) = 0$$

$$V_3(r) = \int_r^\infty \vec{E}(r) \, d\vec{r} = kQ \left(\underbrace{\int_r^{R_i} 0 \, dr}_{=0} + \underbrace{\int_{R_i}^\infty \frac{r^3 - R_i^3}{r^2(R_a^3 - R_i^3)} \, dr}_{=V_2(R_i)} \right) =$$

$$V_3(r) = -kQ \frac{3(R_a + R_i)}{2(R_a^2 + R_a R_i + R_i^2)}$$

$$V(r) = \begin{cases} -kQ \frac{3(R_a + R_i)}{2(R_a^2 + R_a R_i + R_i^2)} & \text{for } 0 < r \leq R_i, \\ -kQ \frac{r^3 - 3rR_a^2 + 2R_i^3}{2rR_a^3 - 2rR_i^3} & \text{for } R_i < r < R_a, \\ kQ \frac{1}{r} & \text{for } R_a \leq r. \end{cases}$$



2. Elektrisches Feld und Potential eines geladenen Stabes

$$k = \frac{1}{4\pi\epsilon_0}; \quad dQ = \lambda dz; \quad d = \sqrt{r_P^2 + (z_P - z)^2}$$

a)

$$\begin{aligned} dV &= k \frac{1}{d} dQ = k\lambda \frac{1}{\sqrt{r_P^2 + (z_P - z)^2}} dz \\ V &= k\lambda \int_{-l/2}^{l/2} \frac{1}{\sqrt{r_P^2 + (z_P - z)^2}} dz = \\ &= k\lambda \ln \left(\frac{\sqrt{\left(\frac{l}{2} + z_P\right)^2 + r_P^2} + \frac{l}{2} + z_P}{\sqrt{\left(-\frac{l}{2} + z_P\right)^2 + r_P^2} - \frac{l}{2} + z_P} \right) \end{aligned}$$

b) $z_P = 0; \quad r_P = x_P; \quad l \ll x_P$

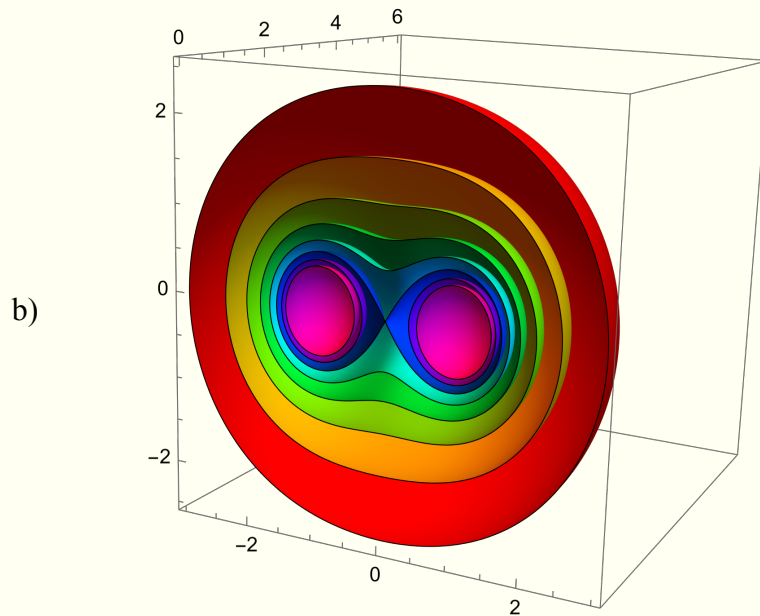
$$\begin{aligned} V &= k\lambda \ln \left(\frac{\sqrt{l^2 + 4r_P^2} + l}{\sqrt{l^2 + 4r_P^2} - l} \right) \\ &= k\lambda \ln \left(\frac{\sqrt{l^2 + 4x_P^2} + l}{\sqrt{l^2 + 4x_P^2} - l} \right) \\ &\approx k\lambda \ln \left(\frac{2x_P + l}{2x_P - l} \right) \approx k\lambda \ln(1) \\ &= \underline{\underline{0}} \end{aligned}$$

$$\begin{aligned} \sqrt{l^2 + 4x_P^2} &\approx 2x_P \\ 2x_P \pm l &\approx 2x_P \end{aligned}$$

3. Elektrische Ladung zwischen Kugelladungen

$$k = \frac{1}{4\pi\epsilon_0}$$

$$\text{a) } \underline{\underline{\vec{F} = \vec{E}_2(9.5d) * q = k \frac{qQ}{(9.5d)^2} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}}}$$



c)

$$K = W$$

$$\frac{1}{2}mv^2 = \int_{9.5d}^{9d} \vec{F} \, d\vec{r} = kqQ \left(\frac{1}{9.5d} - \frac{1}{9d} \right) = kqQ \frac{1}{171d}$$

$$\underline{\underline{v = \sqrt{\frac{2kqQ}{171dm}}}}$$

d)

$$kqQ \int_{9.5d}^{9d} \frac{1}{r^2} dr = kqQ \int_d^x \frac{1}{(r-10d)^2} - \frac{1}{r^2} dr$$

$$\frac{1}{171d} = \frac{1}{10d-x} - \frac{10}{9d} + \frac{1}{x}$$

$$x(10d - x) = \frac{1710d^2}{191}$$

$$x_{1,2} = \frac{1}{955} \left(191d \pm \sqrt{585415} d \right)$$

$$x_1 = \frac{1}{955} \left(191d - \sqrt{585415} d \right) \approx 0.99d$$

$$d - x_1 = 0.01d$$

$$d + (d - x_1) = \underline{\underline{1.01d}}$$