Magnetisches Moment von Kernen

a) $\boldsymbol{j} = \boldsymbol{l} + \boldsymbol{s}$

$$\boldsymbol{j}^2 = (\boldsymbol{l} + \boldsymbol{s})^2 = \boldsymbol{l}^2 + \boldsymbol{s}^2 + 2\boldsymbol{l} \cdot \boldsymbol{s} = \boldsymbol{l}^2 + \boldsymbol{s}^2 + 2\boldsymbol{l} \cdot (\boldsymbol{j} - \boldsymbol{l}) = \boldsymbol{s}^2 - \boldsymbol{l}^2 + 2\boldsymbol{j} \cdot \boldsymbol{l}$$

$$\Rightarrow \boldsymbol{j} \cdot \boldsymbol{l} = \frac{1}{2}\left(\boldsymbol{j}^2 + \boldsymbol{l}^2 - \boldsymbol{s}^2\right)$$

b)

$$\langle j m_j | g_l \boldsymbol{j}^2 | j m_j \rangle = j(j+1)$$

$$\begin{aligned}\langle j m_j | g_l \boldsymbol{j} \cdot \boldsymbol{l} | j m_j \rangle &= \langle j m_j | g_l \frac{1}{2}\left(\boldsymbol{j}^2 + \boldsymbol{l}^2 - \boldsymbol{s}^2\right) | j m_j \rangle = \\ &= \frac{g_l}{2}\Big[j(j+1) + l(l+1) - s(s+1)\Big]\end{aligned}$$

$$\begin{aligned}\langle j m_j | g_s \boldsymbol{s} \cdot \boldsymbol{j} | j m_j \rangle &= \langle j m_j | g_l \frac{1}{2}\left(\boldsymbol{j}^2 + \boldsymbol{s}^2 - \boldsymbol{l}^2\right) | j m_j \rangle = \\ &= \frac{g_s}{2}\Big[j(j+1) + s(s+1) - l(l+1)\Big]\end{aligned}$$

$$\langle j m_j | g_l \boldsymbol{j} \cdot \boldsymbol{l} + g_s \boldsymbol{s} \cdot \boldsymbol{j} | j m_j \rangle = \langle j m_j | g_l \boldsymbol{j} \cdot \boldsymbol{l} | j m_j \rangle + \langle j m_j | g_s \boldsymbol{s} \cdot \boldsymbol{j} | j m_j \rangle =$$
$$= \frac{1}{2}\left(g_l\Big[j(j+1) + l(l+1) - s(s+1)\Big] + g_s\Big[j(j+1) + s(s+1) - l(l+1)\Big]\right)$$

$$\begin{aligned}g_{\text{Kern}} &= \frac{\langle j m_j | g_l \boldsymbol{j} \cdot \boldsymbol{l} + g_s \boldsymbol{s} \cdot \boldsymbol{j} | j m_j \rangle}{\langle j m_j | g_l \boldsymbol{j}^2 | j m_j \rangle} = \\ &= \frac{g_l\big[j(j+1) + l(l+1) - s(s+1)\big] + g_s\big[j(j+1) + s(s+1) - l(l+1)\big]}{2j(j+1)}\end{aligned}$$

c) $\Delta_{lsj} := j(j+1) + s(s+1) - l(l+1)$

$$\begin{aligned}g_{\text{Kern}} &= g_{\text{Kern}} + g_l - g_l = g_l + \frac{g_l(-\Delta_{lsj} + 2j(j+1)) + g_s\Delta_{lsj} - g_l 2j(j+1)}{2j(j+1)} \\ &= g_l + \frac{g_l(-\Delta_{lsj}) + g_s\Delta_{lsj}}{2j(j+1)} = g_l + (g_s - g_l)\frac{\Delta_{lsj}}{2j(j+1)}\end{aligned}$$

d) $j = l \pm \frac{1}{2}; \quad s = \frac{1}{2}; \quad s(s+1) = \frac{3}{4}; \quad j(j+1) =$

$$g_{\text{Kern}} = g_l \pm (g_s - g_l)$$

e)

Der Isospin des Deuterons

a)

b)

c)

d)

e)

f)

g)

Quadrupolmoment der Kerne

$$x = ar\sin(\theta)\cos(\varphi)$$
$$y = ar\sin(\theta)\sin(\varphi)$$
$$z = br\cos(\theta)$$

a) $\rho_{\text{el}} = \frac{Ze}{V} = \frac{3Ze}{4\pi a^2 b}$; $\quad \|\boldsymbol{r}\|^2 = r^2 \left(a^2 \sin(\theta)^2 + b^2 \cos(\theta)^2\right)$

$$Q = \int_V \rho_{\text{el}}(\boldsymbol{r}) \left[3z^2 - \|\boldsymbol{r}\|^2\right] \mathrm{d}V = \int\limits_0^1 \int\limits_0^{2\pi} \int\limits_0^{\pi} \rho_{\text{el}} a^2 b r^2 \sin(\theta)(3z^2 - r^2)\, \mathrm{d}\theta\, \mathrm{d}\varphi\, \mathrm{d}r$$

$$= 2\pi \rho_{\text{el}} a b^2 \int\limits_0^1 \int\limits_0^{\pi} r^2 \sin(\theta)(3r^2 \cos(\theta)^2 - r^2)\, \mathrm{d}\theta\, \mathrm{d}r$$

b) $Q(\text{Ta}) = 6 \cdot 10^{-24} e \ \text{cm}^2$; $\quad Q(\text{Sb}) = -1.2 \cdot 10^{-24} e \ \text{cm}^2$

$$a = R(1+\epsilon)$$
$$b = \frac{R}{\sqrt{1+\epsilon}}$$

$$\Rightarrow a^2 - b^2 = (1+\epsilon)^2 R^2 - \frac{R^2}{1+\epsilon} \approx 3R^2\epsilon$$