2. Feld einer Hohlkugel

$$m = 5\ \mathrm{kg};\quad k = 2\ \mathrm{N/m};\quad a = 0.1\ \mathrm{m};\quad h = 0.4\ \mathrm{m};\quad l = 0.1\ \mathrm{m}$$

a) $\rho = \frac{Q}{V}$

$$V = \int_V dV = \int_0^{\pi}\int_0^{2\pi}\int_{R_i}^{R_a} r^2 \sin(\theta)\,\mathrm{d}r\mathrm{d}\varphi\mathrm{d}\theta$$

$$= \tfrac{4}{3}\pi(R_a^3 - R_i^3)$$

$$\rho = \underline{\underline{\tfrac{3Q}{4\pi}(R_a^3 - R_i^3)}}$$

b) $F = -m\omega^2 x;\quad \Delta x = x_1 - x_2$

$$m\ddot{x}_1 = -m{\omega_a}^2 x_1 - k(x_1 - x_2)$$

$$m\ddot{x}_2 = -m{\omega_a}^2 x_2 - k(x_2 - x_1)$$

$$\Rightarrow\ m(\ddot{x}_1 - \ddot{x}_2) = -m{\omega_a}^2(x_1 - x_2) - 2k(x_1 - x_2) = -(m{\omega_a}^2 + 2k)(x_1 - x_2)$$

$$\Delta\ddot{x} = -\underbrace{({\omega_a}^2 + \tfrac{2k}{m})}_{=\,\omega_b}\Delta x$$

$$\omega_b = \sqrt{\omega_a^2 + \tfrac{2k}{m}} = \underline{\underline{6.43\ \mathrm{rad/s}}}$$

c) $\delta\omega = \omega_b - \omega_a$

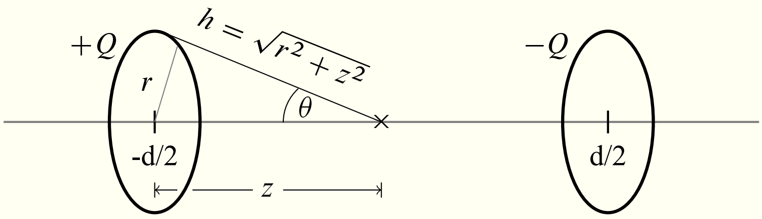
$$0 = \cos\left(\tfrac{1}{2}\delta\omega t\right)$$

$$t = \frac{2\arccos(0)}{\delta\omega} = \underline{\underline{50.29\ \mathrm{s}}}$$

3. Zwei ringförmige Ladungsträger

$$k = \frac{1}{4\pi\epsilon_0}$$

a) $h = \sqrt{r^2 + z^2}$; $\cos(\theta) = \frac{z}{h}$; $\mathrm{d}Q = \lambda \mathrm{d}r$; $\mathrm{d}r = r\mathrm{d}\varphi$; $Q = 2r\pi\lambda$



Due to the symmetric nature of this problem we can neglect vertical components of forces

$$\mathrm{d}E_1 = k\,\frac{\mathrm{d}Q}{h^2}\cos(\theta) = k\,\frac{\lambda \mathrm{d}r}{r^2+z^2}\,\frac{z}{\sqrt{r^2+z^2}} = k\,\frac{\lambda r z}{(r^2+z^2)^{3/2}}\,\mathrm{d}\varphi$$

$$E_1 = \int \mathrm{d}E_1 = k\,\frac{\lambda r z}{(r^2+z^2)^{3/2}}\int_0^{2\pi}\mathrm{d}\varphi = k\,\frac{\lambda 2\pi r z}{(r^2+z^2)^{3/2}} = k\,\frac{Qz}{(r^2+z^2)^{3/2}}$$

b) $F = -m\omega^2 x$; $\Delta x = x_1 - x_2$

$$m\ddot{x}_1 = -m\omega_a{}^2 x_1 - k(x_1 - x_2)$$
$$m\ddot{x}_2 = -m\omega_a{}^2 x_2 - k(x_2 - x_1)$$
$$\Rightarrow\ m(\ddot{x}_1 - \ddot{x}_2) = -m\omega_a{}^2(x_1 - x_2) - 2k(x_1 - x_2) = -(m\omega_a{}^2 + 2k)(x_1 - x_2)$$
$$\Delta\ddot{x} = -\underbrace{(\omega_a{}^2 + \tfrac{2k}{m})}_{=\,\omega_b}\Delta x$$

$$\omega_b = \sqrt{\omega_a^2 + \tfrac{2k}{m}} = \underline{\underline{6.43\ \mathrm{rad/s}}}$$

c) $\delta\omega = \omega_b - \omega_a$

$$0 = \cos\left(\tfrac{1}{2}\delta\omega t\right)$$
$$t = \frac{2\arccos(0)}{\delta\omega} = \underline{\underline{50.29\ \mathrm{s}}}$$