177. Stabpendel mit Kugeln

$$L = 0.3 \text{ m}; \qquad r_1 = 0.02 \text{ m}; \qquad r_2 = 0.03 \text{ m}$$
$$m_0 = 0.1 \text{ kg}; \qquad m_1 = 0.1 \text{ kg}; \qquad m_2 = 0.3 \text{ kg}$$

a) $I_{ges} = I_{Stab} + I_{Kugel_1} + I_{Kugel_2}$

$$I_{Stab} = \tfrac{1}{12} m_0 L^2 = 7.5 * 10^{-4} \text{ kg m}^2$$
$$I_{Kugel_1} = \tfrac{2}{5} m_1 r_1^2 + m_1 (\tfrac{L}{2} + r_1)^2 = 2.9 * 10^{-3} \text{ kg m}^2$$
$$I_{Kugel_2} = \tfrac{2}{5} m_2 r_2^2 + m_2 (\tfrac{L}{2} + r_2)^2 = 9.8 * 10^{-3} \text{ kg m}^2$$

$$\Rightarrow I_{ges} = \underline{\underline{1.3 * 10^{-2} \text{ kg m}^2}}$$

b)

$$x_{Stab_{sp}} = 0 \text{ m}$$
$$x_{Kugeln_{sp}} = \frac{m_1 \left(-\frac{2r_1 + L}{2}\right) + m_2 \left(\frac{2r_2 + L}{2}\right)}{m_1 + m_2} = 0.0925 \text{ m}$$

$$x_{sp} = \frac{x_{Stab_{sp}} * m_0 + x_{Kugeln_{sp}} * (m_1 + m_2)}{m_0 + m_1 + m_2} = \underline{\underline{0.074}} \text{ m}$$

c) $T = 2\pi \sqrt{\frac{I}{Mgl}}$

$$T = 2\pi \sqrt{\frac{I}{(m_0 + m_1 + m_2)\, g \left(\frac{2r_2 + L}{2}\right)}}$$
$$= \underline{\underline{1.2 \text{ s}}}$$

184. Resonanzverhalten einer Stimmgabel

$f_0 = 440 \text{ Hz}; \quad \omega_0 = 2\pi f_0; \quad x(\tau) = x_m \mathrm{e}^{-\beta\tau/2}; \quad x(5) = \frac{1}{10} x(0)$

a)

$$\tfrac{x_m}{10} = x_m \mathrm{e}^{-\beta 5/2}$$
$$\beta = -\tfrac{2}{5} \ln(0.1)$$

$$\tau = \tfrac{1}{\beta} = \underline{\underline{1.09 \text{ s}}}$$

b) $\omega = \sqrt{\omega_0^2 - \frac{\beta^2}{4}}$

$$\omega = 2764.6 \text{ Hz} \qquad (\omega_0 \approx \omega)$$
$$f = \tfrac{\omega}{2\pi} = 440 \text{ Hz} \qquad (f_0 \approx f)$$
$$f * \tau = \underline{\underline{477.7}}$$

c) $Q = \frac{\omega}{\beta}$

$Q = \underline{3001.63}$

d) $\delta f = \frac{f}{Q}$

$\delta f = \underline{0.15 \text{ Hz}}$

e) $x_m(f) = \frac{2\pi f_0}{\sqrt{16\pi^2(f-f_0)^2+\beta^2}}; \quad E(f) = \frac{(\pi f_0)^2}{4\pi^2(f-f_0)^2+(\beta/2)^2}; \quad f_1 = 439.5 \text{ Hz}$

$x_m(f_0) = \frac{2\pi f_0}{\sqrt{16\pi^2(f_0-f_0)^2+\beta^2}} = 3001.63 \text{ m}$

$x_m(f_1) = \frac{2\pi f_0}{\sqrt{16\pi^2(f_1-f_0)^2+\beta^2}} = 435.348 \text{ m}$

$x_{rel} = \frac{x_m(f_1)}{x_m(f_0)} = \underline{0.15}$

$E(f_0) = \frac{(\pi f_0)^2}{4\pi^2(f_0-f_0)^2+(\beta/2)^2} = 1.9 * 10^5 \text{ J}$

$E(f_1) = \frac{(\pi f_0)^2}{4\pi^2(f_1-f_0)^2+(\beta/2)^2} = 9.0 * 10^6 \text{ J}$

$E_{rel} = \frac{E(f_1)}{E(f_0)} = \underline{0.021}$

185. Resonante Anregung eines Federpendels

$m = 0.1 \text{ kg}; \quad k = 40 \text{ N/m}$

a) $\omega_0 = \sqrt{\frac{k}{m}} = 20 \text{ rad/s}; \quad \omega_0 = \sqrt{\frac{k}{m} - \frac{\beta^2}{4m^2}}$

$\frac{x_m}{2} = x_m * e^{-\beta t/2}$

$\beta = -\frac{\ln(0.5)2}{t} = \underline{0.14 \text{ kg/s}}$

$\omega_1 = \sqrt{\frac{k}{m} - \frac{\beta^2}{4}} = \underline{19.99988 \text{ rad/s}}$

b)

$Q = \omega_1 \frac{m}{\beta} = \underline{144.269}$

c) $x_m(\omega) = \frac{\omega_0}{\sqrt{(\omega-\omega_0)^2+\beta^2}}; \quad \omega_1 = \omega - 0.2\pi; \quad \hat{y}_{res} = 0.012 \text{ m}$

$x_m(\omega_0) = \frac{\omega_0}{\sqrt{(\omega_0-\omega_0)^2+\beta^2}}$

$x_m(\omega_1) = \frac{\omega_0}{\sqrt{(\omega_1-\omega_0)^2+\beta^2}}$

$\hat{y}_{res} = \hat{y}_{res} \frac{x_m(\omega_1)}{x_m(\omega_0)} = \underline{0.0013 \text{ m}}$