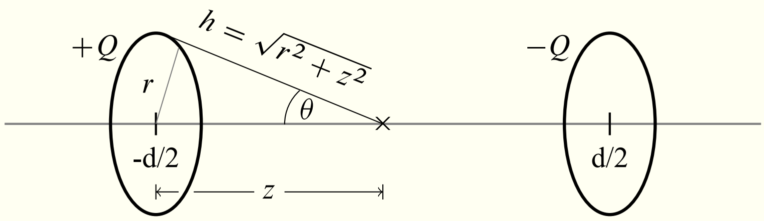
3. Zwei ringförmige Ladungsträger

$$k = \frac{1}{4\pi\epsilon_0}$$

a. $h = \sqrt{r^2+z^2}$; $\cos(\theta) = \frac{z}{h}$; $\mathrm{d}Q = \lambda \mathrm{d}r$; $\mathrm{d}r = r\mathrm{d}\varphi$; $Q = 2r\pi\lambda$



Due to the symmetric nature of this problem we can neglect vertical components of forces

$$\mathrm{d}E_1(z) = k\frac{\mathrm{d}Q}{h^2}\cos(\theta) = k\frac{\lambda \mathrm{d}r}{r^2+z^2}\frac{z}{\sqrt{r^2+z^2}} = k\frac{\lambda r z}{(r^2+z^2)^{3/2}}\mathrm{d}\varphi$$

$$E_1(z) = \int \mathrm{d}E_1 = k\frac{\lambda r z}{(r^2+z^2)^{3/2}}\int_0^{2\pi}\mathrm{d}\varphi = k\frac{\lambda 2\pi r z}{(r^2+z^2)^{3/2}} = k\frac{Qz}{(r^2+z^2)^{3/2}}$$

$$E_2(z) = -E_1(z-d) = -k\frac{Q(z-d)}{(r^2+(z-d)^2)^{3/2}}$$

$$E_{ges}(z) = E_1 + E_2 = k\frac{Qz}{(r^2+z^2)^{3/2}} - k\frac{Q(z-d)}{(r^2+(z-d)^2)^{3/2}}$$

$$= kQ\left(\frac{z+\frac{d}{2}}{\left(r^2+\left(z+\frac{d}{2}\right)^2\right)^{3/2}} + \frac{\frac{d}{2}-z}{\left(r^2+\left(z-\frac{d}{2}\right)^2\right)^{3/2}}\right)$$

b.

$$\int_{-d/2}^{d/2} qE(z)\,\mathrm{d}z = kQq\left(\int_{-d/2}^{d/2}\frac{z+\frac{d}{2}}{\left(r^2+\left(z+\frac{d}{2}\right)^2\right)^{3/2}}\mathrm{d}z + \int_{-d/2}^{d/2}\frac{\frac{d}{2}-z}{\left(r^2+\left(z-\frac{d}{2}\right)^2\right)^{3/2}}\mathrm{d}z\right)$$

$$= \frac{kQq}{2}\left(\int_{r^2}^{r^2+d^2}\frac{1}{u^{3/2}}\mathrm{d}u - \int_{r^2-d^2}^{r^2}\frac{1}{v^{3/2}}\mathrm{d}v\right)$$

$$\boxed{\begin{aligned} u &= r^2+\left(z+\tfrac{d}{2}\right)^2\\ \mathrm{d}u &= 2\left(z+\tfrac{d}{2}\right)\mathrm{d}z\\ v &= r^2+\left(z-\tfrac{d}{2}\right)^2\\ \mathrm{d}v &= 2\left(z-\tfrac{d}{2}\right)\mathrm{d}z\end{aligned}}$$

c. $\delta\omega = \omega_b - \omega_a$

$0 = \cos\left(\frac{1}{2}\delta\omega t\right)$
$t = \frac{2\arccos(0)}{\delta\omega} = \underline{\underline{50.29\ \mathrm{s}}}$