FP1 Versuch E121: Elektro-optischer Effekt

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I. PRELIMINARY REMARK

This is version 3.1 of the experimental instructions. Corrections and suggestions for the experiment and these experimental instructions are warmly welcome.

II. INTRODUCTION

Certain materials change their optical properties when exposed to an electric field. These materials are generally crystals or liquid crystals. This change is due to the (electrical) forces that modify the position, orientation and/or shape of the molecules or atoms from which the material is made up. The electro-optical effect is the resulting change in the refractive index when a dc or low-frequency electric field is applied. In anisotropic materials, a change in the refractive index generally causes not only a phase change but also an influence on the polarization state of the transmitted light. One speaks of a linear electro-optical effect (**Pockels effect**) when the change in the refractive index is proportional to the electric field. In the case of quadratic dependence, one speaks of the **Kerr effect**.

The relative change in refractive index due to the electro-optical effect is generally very small, typically of the order of 10^{-5}. However, when the (laser) light passes through a crystal, this effect adds up quickly and you get a phase shift of 2π after just a few cm for normal laboratory field strengths.

The electro-optical effect makes it possible to realize electrically controllable optical components. Some important ones are mentioned in the list below:

- A lens made of electro-optically active material allows the focal length to be changed [1].
- A prism made of electro-optically active material serves as an optical scanner [2].
- An electro-optical crystal can be used to phase modulate the light (electro-optical modulator, EOM). In particular, such an EOM, when placed in one arm of a Mach-Zehnder interferometer, allows the realization of a fast optical intensity modulator (i.e. an optical switch) [3], chapter 20.
- An anisotropic electro-optical crystal can be used as an electrically controllable wave plate. Again, such a wave plate, this time placed between two crossed polarizers, allows the realization of a fast optical intensity modulator [3], chapter 20.
- Liquid crystals that have an electro-optical effect allow the realization of displays [4].

In particular, the components that allow the (fast) modulation of light have become an integral part of optical communication technology [5].

In this exercise you will use the EOM to realize amplitude modulation in two different ways. When the electro-optical crystal is placed in the arm of a Mach-Zehnder interferometer one can modulate the output power. This allows a measurement of the electro-optical coefficients of the crystal for the two polarization axes. The operation of a switch can be demonstrated. Then, you will realize an optical switch in a different way by placing the crystal in between two polarizers.

III. SAFETY INSTRUCTIONS

A HeNe laser is used in this practical experiment. Although this has a low power (approx. 1 mW), it still poses a **danger** if the light gets into the eye. Therefore, caution should be exercised when setting it up: The laser beam should always be blocked when not in use for a long period of time. The main beam, branched beams or reflections must not leave the optical table. All beams must always be guided parallel to the plane of the optical table. It is also important to ensure that no reflective surfaces (such as a screwdriver) accidentally come into the beam.

Furthermore, the high-voltage power supply of the HeNe laser poses a risk of electric shock. The laser housing must therefore be handled with appropriate care and must not be opened.

The high-voltage amplifier also poses a risk of electric shock. Make sure that the device is correctly grounded and that the high voltage is routed correctly.

A. Pockels- and Kerr-effect

When an electro-magnetic wave, with electric field E_L propagates through a non-magnetic dielectric material, it causes the charges to oscillate around their equilibrium positions. This, in turn, causes the emission of additional electromagnetic waves. The overall field response of the electrons is usually described in terms of the dielectric polarization density P. In linear materials, which we will consider in this document, the polarization density is proportional to the electric field: $P = \epsilon_0 \chi E_L$, where χ is the electric susceptibility. The total resulting field in the material is the sum of the external field and the response of the material. We describe this by the electric displacement $D = \epsilon_0 E_L + P$. We can define the electric permittivity ϵ by $D = \epsilon E_L$, such that $\epsilon = \epsilon_0(1+\chi)$. The phase velocity of the wave in the material is given by $c/n = 1/\sqrt{\epsilon\mu_0}$, where n is the refraction index. In the presence of an additional applied static electric field E [11], the equilibrium position of the charges can change slightly. As a result, the refractive index of an electro-optical medium is a function $n(E)$ of the applied electric field E. Since this functional dependence is only weak, it makes sense to expand the refractive index into a Taylor series around E=0 (for now, without taking into account the vectorial character of E and the possible anisotropy of the medium):

$$n(E) = n_0 + a_1 E + \frac{1}{2} a_2 E^2 + ..., \qquad (1)$$

where a_1 and a_2 are obtained by one-time and two-time derivatives, respectively. As we will see, it makes sense to introduce two new coefficients: $r = -2a_1/n_0^3$ and $s = -a_2/n_0^3$. Then it results:

$$n(E) = n_0 - \frac{1}{2} r n_0^3 E - \frac{1}{2} s n_0^3 E^2 + ... \qquad (2)$$

The higher terms of this development are generally orders of magnitude smaller and are therefore negligible. One can now rewrite this expansion in terms of the electrical impermeability $\eta = \epsilon_0/\epsilon = 1/n^2$. From $\Delta\eta = (d\eta/dn)\Delta n = (-2/n_0^3)(-\frac{1}{2} r n_0^3 E - \frac{1}{2} s n_0^3 E^2)$ is obtained

$$\eta(E) = \eta_0 + rE + sE^2 + ... \qquad (3)$$

As we will see, r and s depend on the direction of the electric field and the polarization of the light.

For the Pockels effect, the third term in the development of the refractive index is negligible. So it turns out

$$n(E) \approx n_0 - \frac{1}{2} r n_0^3 E \qquad (4)$$

The coefficient r is called the Pockels coefficient. Typical values are in the range from 1 to 100 pm/V. Voltages of 100 V, applied over a distance of 1 mm, result in relatively small refractive index changes of around 10^{-7} to 10^{-5}. In our experiment we will use a $LiNbO_3$ crystal (lithium niobate) as the electro-optical crystal. This has a very high Pockels coefficient of $r \approx 31$ pm/V (if the field and polarization directions to the optical axis of the crystal are chosen correctly). Quartz, for example, has a Pockels coefficient that is approximately 100 times lower. Another typical electro-optical crystal is LiTaO3 (lithium tantalate) with $r \approx 30$ pm/V. If a medium has central inversion symmetry (as in the case of gases, liquids and certain crystals), then $n(E)$ must be a symmetric function since it must be invariant under the direction reversal of E. So the Pockels coefficient r disappears, whereby

$$n(E) \approx n_0 - \frac{1}{2} s n_0^3 E^2. \qquad (5)$$

Such a medium then exhibits the (usually much weaker) Kerr effect. Typical values range from 10^{-18} to 10^{-14} m^2/V^2 for crystals and 10^{-22} to 10^{-19} m^2/V^2 for liquids. For more details about this, see [3], chapter 20.

B. Anisotropic materials

As we have just discussed, in order to get a non-zero Pockels coefficient and therefore a significant change of refractive index, we have to employ anisotropic materials lacking inversion symmetry. In these, the response of the electrons and the driving field are not necessarily aligned. You can understand this by considering the forces exerted by neighboring atoms on each other. These create preferred oscillation directions, in such a way that when forcing the charges in the direction of $\vec{E}$, the response $\vec{P}$ might not be in the same direction. We describe this by allowing χ, ϵ and η to be general 3x3 matrices. In many materials, these matrices are symmetric, e.g. $\epsilon_{i,j} = \epsilon_{j,i}$. This symmetry is violated for chiral materials possessing optical activity, like liquid crystals and quartz and generally in the presence of magnetic fields [3]. For the crystals used in this experience, we will only work with symmetric matrices. The phase velocity of the waves was defined by $c/n = 1/\sqrt{\epsilon\mu_0}$. In order to avoid dealing with square roots of matrices, we square this relation, giving $n^2 = \epsilon/\epsilon_0$.

Here we see that in general, the refractive index will also be a matrix. This matrix describes how, apart from the phase velocity, the polarization of the electric field can change during propagation through the medium. To simplify the description, we will choose a basis in which ϵ is diagonal, with eigenvalues ϵ_i. This defines special modes of oscillation of the electromagnetic field, that propagate through the material without changing polarization. The index of refraction of these waves will be given by $n_i^2 = \epsilon_i/\epsilon_0$. We can see here that if we want the refraction index to be real (no absorption), we need all the eigenvalues of ϵ to be positive. The directions of oscillation of the eigenmodes define special crystallographic directions, called optical axes.

Let's consider what would happen if the polarization of the electromagnetic wave was initally not aligned along one of the optical axis, say at 45 degrees between the x and y axis. In this case, we can decompose the field along the two crystallographic directions and remember that these two field components will propagate with different phase velocity inside the material. As a result, the polarization of the field will become elliptic during propagation. We will make use of this effect in one of the setups described in the following.

The crystal that we will use is a uniaxial crystal. This means that two of the eigenvalues are identical $n_1 = n_2 = n_o$, these two directions are called 'ordinary'. The third direction defines the extraordinary axis, also called the optical axis $n_3 = n_e$. Note: please don't confuse the ordinary refraction index n_o and n_0, the index of refraction for E=0.

When unpolarized light propagate through such a system, it experiences birefringence. The most intuitive description of this phenomenon is given in terms of the index ellipsoid, defined by the surface satisfying:

$$\sum_{i,j=1,2,3} \eta_{i,j} x_i x_j = 1 \tag{6}$$

see [3] and the document: *FP1 EOM extra notes by Bo Huang* on OLAT for more details on the derivation. When we use the coordinate system defined by the optical axes, we get

$$\sum_{i,j=1,2,3} x_i^2/n_i^2 = 1 \tag{7}$$

which shows that the principal axes of the ellipsoid have a length given by n_i. When light is propagating through

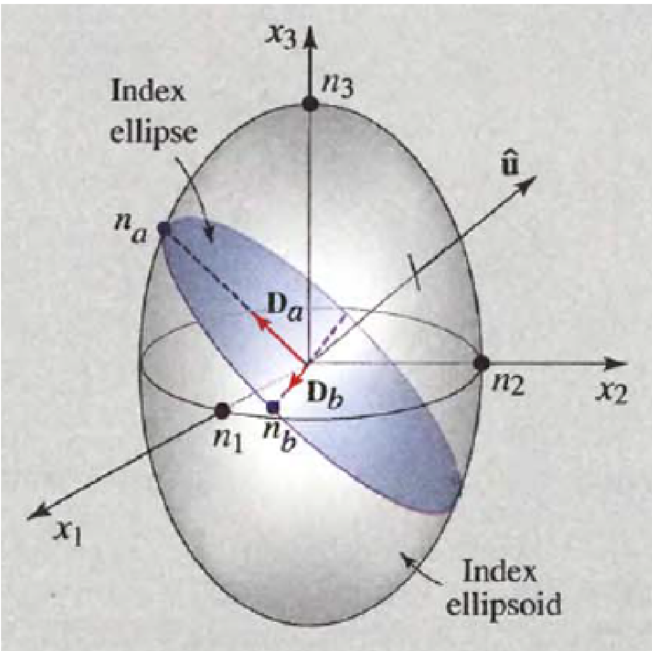

FIG. 1. Geometric construction to determine the relevant refraction indices for light propagating in the direction defined by $\hat{u} = \vec{k}/|k|$. The transverse plane to $\hat{u}$ contains all possible polarizations of the light (directions of $\vec{D}$). The plane intercepts the index ellipsoid in an ellipse, called index ellipse. The lengths of the major and minor axis of the index ellipse define $n_{a,b}$. Figure taken from [3]

the material at an arbitrary angle, the relevant indices $n_{a,b}$ for the two possible polarization directions are found using the geometric construction in Fig. 1. In the special case of uniaxial crystals ($n_1 = n_2 = n_o$), n_b is always equal to n_o, independently of the propagation direction. This defines the ordinary wave, whose polarization is orthogonal to both $\vec{k}$ and the optical axis. For the ordinary wave $\vec{E}$ is parallel to $\vec{D}$. The extraordinary wave has an index of refraction given by $1/n_a^2 = \cos(\theta)^2/n_o^2 + \sin(\theta)^2/n_e^2$ where θ is the angle between the propagation vector and the optical axis. n_a varies between n_e and n_o depending on the propagation direction. For the extraordinary wave, $\vec{D}$ and $\vec{E}$ are generally not aligned, with $\vec{E}$ taking a non zero component along $\vec{k}$. This has important consequences in term of energy propagation. In particular the Poynting vector $\vec{S} = 1/2\vec{E} \times \vec{H}$ is not aligned with $\vec{k}$ for the extraordinary wave. The direction of energy flow is therefore not orthogonal to the phase fronts of the wave.

Finally, let's consider refraction in such a system. When light passes through an interface between two media, it experiences refraction. The new propagation direction can be found by Snell's law: $n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$, where region 1 is outside the material and region 2 is inside. The angles $\theta_{1,2}$ are the angles between $\vec{k}$ and the normal to the surface. In anistropic media, this law is to be understood in terms of the k vector, predicting the new direction of $\vec{k}$ inside the material. Birefringence (or double refraction) happens because in general the two polarizations of the light have different refraction indices and therefore they will give rise to two different directions of $\vec{k}$ inside the material. Moreover, since $\vec{S}$ is not necessarily aligned with $\vec{k}$ inside the material, finding the direction a beam of light will take has to also take this into account.

For the experiment, we will always enter the uniaxial crystal at right angle with respect to the optical axis, giving $n_a = n_o$ and $n_b = n_e$. The input surface of the crystal is also cut at right angle with the optical axis, see Fig. 2. This makes the input angle $\theta_1 = 0$. For this specific situation, birefringence does not occur and both ordinary and extraordinary waves propagate in the same direction.

C. The electro-optic effect in anisotropic materials

When we apply a DC electric field $\vec{E}$ to the material, equation 3 generalizes to

$$\eta_{ij}(\vec{E}) = (\eta_0)_{ij} + \sum_k r_{ijk} E_k + \sum_{kl} s_{ijkl} E_k E_l + ...,$$

where η_0 is the impermeability tensor for $\vec{E} = 0$ and $r_{ijk} = \partial \eta_{i,j}/\partial E_k$ and $2s_{ijkl} = \partial^2 \eta_{i,j}/\partial E_k \partial E_l$ are the tensorial generalizations of the Pockels and Kerr coefficients, with in principle $3^3 = 27$ and $3^4 = 81$ components. Since η is symmetric in i, j and s_{ijkl} is a Hessian matrix in indices k, l, r_{ijk} and s_{ijkl} only have a maximum of

$6*3$ or $6*6$ independent components. By convention, one writes r and s as $6*3$ and $6*6$ matrices r_{Ik} and s_{IK}, respectively. The indices are summarized as follows: $(i,j)=(1,1)\Rightarrow I=1$, $(i,j)=(2,2)\Rightarrow I=2$, $(i,j)=(3,3)\Rightarrow I=3$, $(i,j)=(2,3)$ or $(3,2)\Rightarrow I=4$, $(i,j)=(1,3)$ or $(3,1)\Rightarrow I=5$ and $(i,j)=(1,2)$ or $(2,1)\Rightarrow I=6$, analogously for K.

The crystal symmetries further constrain the entries of r and s. For example, for the important uniaxial ($n_1=n_2=n_o$, ordinary refractive index, and $n_3=n_e$, extraordinary refractive index) crystal group *trigonal 3m* (which applies e.g. to $LiNbO_3$ and $LiTaO_3$) one has the following general form of the $6*3$ matrix for r_{Ik}

$$\begin{pmatrix} 0 & -r_{22} & r_{13} \\ 0 & r_{22} & r_{13} \\ 0 & 0 & r_{33} \\ 0 & r_{51} & 0 \\ r_{51} & 0 & 0 \\ -r_{22} & 0 & 0 \end{pmatrix}$$

For $LiNbO_3$, at our operating wavelength and temperature, the values of the parameters are known. For the index of refraction, at standard room temperature and pressure, the Sellmeier equation gives $n_e=2.2022$, $n_o=2.2865$. The non-zero coefficients of the Pockels tensor are $r_{33}=30.8(2)$ pm/V, $r_{13}=8.6(2)$ pm/V, $r_{22}=3.4(2)$ pm/V, $r_{51}=28.0(2)$ pm/V. In the following, we consider the case relevant to this experiment, in which the electrode surfaces are perpendicular to the extraordinary optical axis of a crystal of the 'Trigonal 3m' type. The electric field $\vec{E}$, therefore points in the direction of the extraordinary optical axis $\vec{E}=(0,0,E)$.

The changes in the ordinary and extraordinary refractive index decouple and are given by

$$\frac{1}{n_o^2(E)}=\frac{1}{n_o^2}+r_{13}E$$

and

$$\frac{1}{n_e^2(E)}=\frac{1}{n_e^2}+r_{33}E.$$

This can be further manipulated into an approximate equation for the ordinary refractive index

$$n_o(E)=n_o-\frac{1}{2}n_o^3r_{13}E \tag{8}$$

and analogously for the extraordinary refractive index

$$n_e(E)=n_e-\frac{1}{2}n_e^3r_{33}E. \tag{9}$$

Note the analogy to equation (4). When an electric field is applied along the extrordinary optical axis, the crystal remains uniaxial with the same main axes, but the main refractive indices change according to equations (8) and (9). In the more general case in which the field would be applied in an arbitrary direction, the crystal might not remain uniaxial.

D. Phase modulator

When the laser light passes through the crystal, its electric field $\vec{E}_{\mathrm{L}}$ gets phase shifted due to the propagation in the medium. If the length of the Pockels crystal is L, the light experiences a phase shift $\phi=n(E)k_0L=2\pi n(E)L/\lambda_0$, where λ_0 is the vacuum wavelength. This gives

$$\phi=\phi_0-\pi\frac{rn_0^3EL}{\lambda_0}$$

with $\phi_0=2\pi n_0L/\lambda_0$. If the static electric field is generated by a voltage V which is applied to two opposite surfaces of the crystal at a distance of d (see Fig.2), i.e. $E=V/d$, then

$$\phi=\phi_0-\pi\frac{V}{V_\pi},$$

where

$$V_\pi=\frac{d}{L}\frac{\lambda_0}{rn_0^3}$$

is defined as the half-wave voltage, i.e. the voltage necessary to induce a π phase shift.

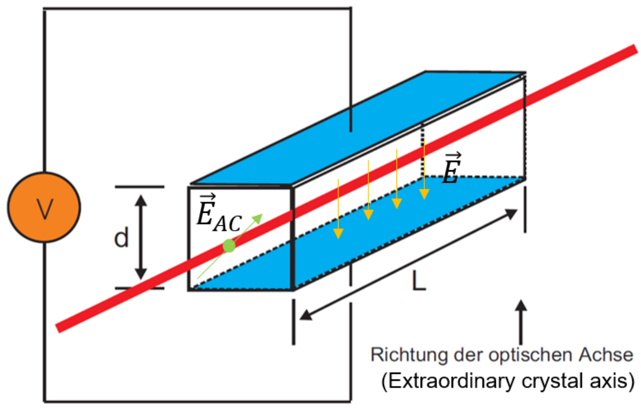

FIG. 2. Transversal electro-optical modulator of length L and height d. The optical crystal axis is perpendicular to the electrode surfaces selected as indicated. The Light propagates along the direction given by L. The electric field $\vec{E}$ applied by the electrodes is in the direction of the extraordinary axis. The polarization of the light is indicated by $\vec{E}_{\mathrm{L}}\equiv\vec{E}_{\mathrm{AC}}$ and will be changed in the experiment. In the experiment, the light is propagating horizontally (parallel to the optical table) and the electrodes create an horizontal field, perpendicular to the propagation direction.

E. Intensity modulator

A phase modulator, which is inserted into one arm of an interferometer, can serve as an intensity modulator (see Fig.3). The interference between the beams leads to an intensity of the light at the detector I_o given by

$$I_o \propto |E_1 + E_2|^2 \tag{10}$$

where $E_{1,2}$ are the electric fields from the different possible light paths arriving at the detector. In general, the beams can be out of phase with each other, $E_2 = E_1 e^{i\varphi}$ and φ is the differential phase shift between the two paths the light can take. The phase difference depends on the electric field applied to the EOM as only one path goes through the EOM. The transmittance $T = I_o/I_i$ of such a setup, depending on the applied voltage V for an ideal interferometer with 50/50 beam splitters is given by (show this!)

$$T(V) = \cos^2\left(\alpha - \frac{\pi}{2}\frac{V}{V_\pi}\right), \tag{11}$$

where α is an (unimportant) fixed phase, which depends on the optical path length difference. If you could set the optical path lengths in the interferometer so that α were a multiple of 2π, then $T(0) = 1$ and $T(V_\pi) = 0$, i.e. the interferometer can be used as an optical switch if you change the voltage from 0 to V_π.

To derive Eq. (11), consider the amplitude of the electric field travelling in the interferometer as shown in Fig.3, taking into account all possible paths to the detector. The 50/50 beam splitter is simply a coated piece of glass. The coating is such that the transmission is 50% and it follows Fresnel's rules of transmission and reflection regarding the phase of the electric field. The polarization of the light is not important, as it remains constant through the interferometer. Don't forget that reflections from low to high index of refraction cause a π phase shift.

For practical and commercial use of the amplitude modulator with this strategy, the interferometer is usually realized on a microscopic waveguide, using integrated/fiber optics. This greatly improves stability, but makes the device less accessible for learning purposes. As you will experience in the lab, when realized in free space, this device is extremely sensitive to environmental noise in the form of acoustic vibrations. Another possible way to realize a modulator or a switch for optical light makes use of polarizing optics and it is this general strategy that is most commonly utilized in LCD screens. This is the second setup that you will have to build and characterize. See Fig.4 for the basic geometry. In this case, the EOM is in between two polarizing waveplates and polarizers. The polarizers let through horizontally polarized light and reflect vertically polarized light. The waveplates are thin slabs of anisotropic dielectric material and allow you to rotate the polarization of the laser by an adjustable amount. After the first polarizer, the light will be horizontally polarized. You have to adjust the waveplates such that the light polarization is rotated by a 45° angle. This is achieved using the rotation mounts in which the waveplates are held.

When 45° polarized light goes through the EOM, as it's not aligned along one of the optical axis, we have to decompose it into vertical and horizontal components. The horizontal component is aligned with the optical (extraordinary) axis of the EOM, and will therefore experience an index of refraction as given in Eq. (9). The vertical component is instead aligned with the ordinary crystal axis, so it will propagate according to the index of refraction given in Eq. (8). This means that, after propagation, the two polarization components of the light will be phase shifted. For example, if the starting polarization is $\hat{x} + \hat{y}$, it might change into $\hat{x} - \hat{y}$ if the phase shift is π. The second waveplate rotates the polarization again by 45°, such that the light will transmit through the second polarizer, if the phase shift is zero. If the phase shift is π, the light does not transmit at the second polarizer. This gives rise to a change in transmission as a function of the applied voltage, analogous to what observed with the interferometer.

IV. EXPERIMENTAL SETUP

The experimental setup for the electro-optical effect is shown schematically in Fig.3. The aim is to measure the phase shift generated by the change in refractive index.

For this purpose, the light from an (unpolarized) HeNe laser [6] is sent through a polarizer (consisting of a $\lambda/2$ wave plate and a polarizing beam splitter cube) to a Mach-Zehnder interferometer. In one arm of the interferometer there is an electro-optical crystal ($LiNbO_3$, length and height of the crystal (with error bar) can be estimated by observing the pictures of the crystal on OLAT. Make sure you use the pictures corresponding to your station). This crystal sits between two capacitor plates so that when a (high) voltage is applied, an electric field is generated inside the crystal. The field direction is aligned with the direction of the extraordinary optical axis of the crystal. This is the situation described by Eq. (8) and (9). At the output of the interferometer, the light is sent to a photodetector and the intensity can be visualized on an oscilloscope as a photovoltage. The applied high-voltage is generated by a signal generator together with a high-voltage amplifier.

Your first task is to build the optical setup of Fig.3. Obtaining high contrast of the interference trace requires careful alignment of the optical elements. The instructor can help you with that. Once the setup is completed, you should record a trace on the scope, giving you the amount of light as a function of the applied voltage to the EOM. With this you should be able to extract the value of V_π. Repeat the measurement several times to gather statistics. Perform the measurement for vertical as well as horizontal polarization of the laser (the polarization can be adjusted by use of a waveplate inserted after the polarizer). The data can be analysed to extract the value of r_{13} and r_{33} typical of the material used in the experiment. In your second task, you should build the setup

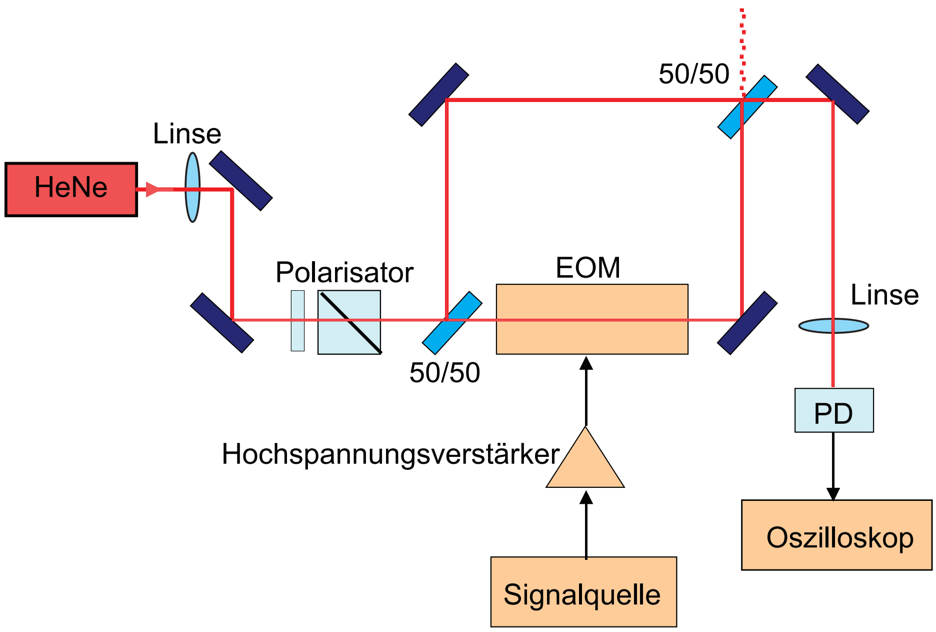

FIG. 3. Experimental setup for the electro-optical effect. The electro-optical crystal (EOM) sits in one arm of a Mach-Zehnder interferometer. This allows the phase swing to be measured as a function of the applied high voltage. The light from the HeNe laser is adjusted with the use of some optics (as an example a lens and polarizer in the picture). The EOM is driven by an amplifier, feeded by a signal generator with adjustable frequency and amplitude. The output signal is collected by a photodiode, connected to an oscilloscope. Changing the applied voltage to the EOM over time changes the amount of light transmitted as recorded by the oscilloscope trace.

in Fig.4 and gather the trace of intensity versus voltage, as done in the first task. The alignment of this setup is much simpler and the stability of the signal should be greatly improved. Based on the previous measurement, come up with a prediction for the V_π you should measure in this case and discuss the agreement obtained.

Aligning the interferometer will require you to perform the technique of beam walk. If you have never beam walked a laser beam, then you will learn it here. The contrast of the interferometer will give you a good signal to optimize the beam overlap.

V. TOOLS AND ACCESSORIES

HeNe laser with external power supply, electro-optical modulator (EOM), frequency source with high-voltage amplifier, aluminum deflection mirror, various optical and opto-mechanical elements, photodetector, digital storage oscilloscope, adjustable multiple power supply for power supply.

Note: The optical components must be handled with the necessary care. The surfaces of the aluminum mirrors must not be touched. If this happens, these surfaces should be cleaned immediately with methanol (the assistant can show you how to do this). In addition, the EOM is a delicate and very expensive device (if hit hard, the crystal can break). Therefore, the EOM with holder must be firmly screwed to the table at all times.

VI. PREPARATION OF THE EXPERIMENT

It is recommended that you carefully study these instructions and the literature on the electro-optical effect. You should sit down with your fellow experimenter in good time before the experiment and discuss the physics of this experiment in detail. This cannot be accomplished in 1 to 2 hours, but requires considerably more time! Here are some questions you should be able to answer:

- What is a Mach-Zehnder interferometer? What is the difference to a Michelson interferometer?

- What is the contrast of an interferometer and how do you measure it? Derive the equation (11) for the transmittance of the interferometer. What happens if the beam splitters are no longer 50/50 but, for example, 40/60? For this, matrix methods might help. You can set up the transfer matrix of the interferometer, by considering the relation between the electric field amplitudes of the two inputs and the two outputs beams

$$\begin{pmatrix} E_1^o \\ E_2^o \end{pmatrix} = \begin{pmatrix} M_{1,1} & M_{1,2} \\ M_{2,1} & M_{2,2} \end{pmatrix} \begin{pmatrix} E_1^i \\ E_2^i \end{pmatrix} \tag{12}$$

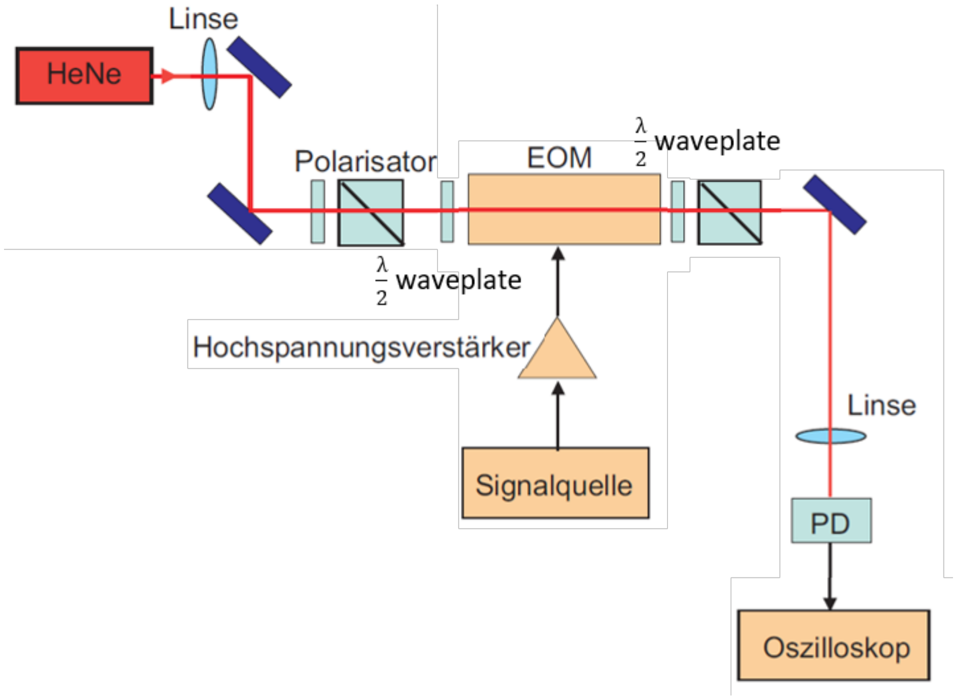

FIG. 4. Second setup to be realized in the experiment. In this case the intensity modulation is generated by making use of the two polarizations of the light, experiencing different refraction indices.

To guarantee energy conservation, $|E_1^o|^2 + |E_2^o|^2 = |E_1^i|^2 + |E_2^i|^2$, we require the matrix M to be unitary. Matrices like this can be multiplied, start by finding the matrix of the first beam splitter, than the propagation through the 2 paths and finally the last beam splitter.

- What exactly is birefringence, i.e. what is the difference between ordinary and extraordinary rays? More specifically: What is the index ellipsoid and how do you use it to determine the refractive index for a given propagation direction and polarization? How do the electric field $\vec{E}$ and the electric displacement density $\vec{D}$ relate to each other in an anisotropic crystal? And what about the wave vector $\vec{k}$ and the Poynting vector $\vec{S}$? You can restrict yourself to uniaxial crystals.

- How does the electro-optical effect generally affects the index ellipsoid? (The effect in our case is described in Eq. (9)) It can be helpful to think about this microscopically. What is the approximate effect of the electric field on the positions of the atoms of the crystal? Think about this also in the limit of very large field.

- What is the expected half-wave voltage, for the different situations relevant to the experiment? Calculate this before the experiment!

- What is the best way to measure half-wave voltage using an interferometer?

- What exactly does the photodiode measure? What temporal and spatial averaging does the photodiode produce? And what kind of signal does it give out?

VII. EXECUTION OF THE EXPERIMENT

The optical setup is first implemented with the help of the assistant, according to Fig.3. To do this, the light from the HeNe laser is coupled through a collimating lens (look at the far field of the laser!) and via 2 mirrors and the polarizer into a Mach-Zehnder interferometer. First, examine the interference on a white card or on the photodiode without the electro-optical crystal in the beam path. Only then do you introduce the electro-optical crystal into one arm of the interferometer. Optimize the contrast (it should be more than 90%). Care must be taken to ensure a clean (central) adjustment of the transmitted light through the crystal. Is the polarization direction chosen correctly? Then you optimize the contrast of the interferometer. To do this, observe one output of the interferometer via a lens on a photodiode using an oscilloscope. Use the oscilloscope to measure the contrast. Then connect the signal source to the EOM via the high-voltage amplifier (caution: high voltages!). The high voltage can be observed at the same time on an oscilloscope. Modulate the applied high voltage sinusoidally at approx. 30 Hz, optimize the contrast of the interferometer again and then measure the half-wave voltage by acquiring the oscillation from the scope. Several measurements are carried out to test the repeatability of the method. Compare the values obtained with the expected ones, for the ordinary and extraordinary axis. You can change the polarization of the light at the interferometer input with a waveplate to do this. In your second task, you should build the setup in 4 and gather the trace of intensity versus voltage, as done in the first task. The alignment of this setup is much simpler and the stability of the signal should be greatly improved. Based on the previous measurement, come up with a prediction for the V_π you should measure in this case and discuss the agreement obtained.

Now you increase the frequency of the signal source in order to determine qualitatively (i.e. not with an exact measurement) the bandwidth of the optical modulation. In this context, we define bandwidth in terms of the time response of the measurement apparatus. When trying to modulate faster and faster, the system will eventually be unable to follow, giving a constant response. The bandwidth is, loosely speaking, the frequency for which the interference contrast will drop significantly (for example a factor of two or more) with respect to the contrast at low frequency. What limits the bandwidth with this setup? Try to consider all significant elements that could influence the speed of the system and make an educated guess. Finally, you apply a square-wave pulse the EOM in order to realize an optical switch. Measure the on/off ratio at low frequency. What happens to the on/off ratio depending on the frequency of the square wave pulses?

VIII. PROTOCOL AND EVALUATION

As usual, the protocol should contain a detailed description of the tasks carried out in the experiment including the measured data. Please pay attention to a sensible structure. This means that the protocol has a short introduction, followed by a theoretical part (with only the most important results relevant to the experiment) and a detailed description of the experiment and its implementation. Finally, you should report the experimental results including evaluation, discussion and outlook. A sensible error calculation must be carried out. Illustrations must be provided with meaningful captions so that the reader can immediately recognize what is to be shown. Refer to the general guidelines on OLAT.

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[11] Please note that here we are talking about a large, slowly varying electric field, as produced by electrodes for example. This has to be distinguished from the electric field of optical electromagnetic waves traveling through the material, whose propagation is influenced by the value of the index of refraction n. We will use the symbol E_L for the second kind of electric field