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Electro-optical effect

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Abstract

In this report the working principle and some properties of an electro-optical modulator (EOM) are examined. In such a device, the refractive index can be altered by ~~inducing~~ an electric field ~~in~~to the crystal. Two experimental setups are realized. Measurements using a Mach-Zehnder interferometer are taken in order to determine the half wave voltage and bandwith. Afterwards, a switch is constructed and the on/off ratio is measured.

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Contents

0 Introduction

Without lasers, the world would look quite different from the world we know. From GPS to accurate clocks, lasers are omnipresent in the modern world, essential in a big variety of applications. But not only technical applications are worth mentioning here; they also play an indispensable role in various physical experiments, shedding light on phenomena ranging from quantum mechanics to material science. In this experiment, we created setups, in which lasers are combined with an electro-optical-modulator (EOM). This component contains a crystal, which changes its optical properties when exposed to an electric field. This behaviour is called the electro-optical effect, which can be applied in many ways. For example, one can create lenses with adaptive focal length as well as optical switches. In the first setup, a Mach-Zehnder-interferometer is set up, with an EOM placed in one of its arms. The ~~interference pattern~~ of the output beam is recorded by a photodiode, enabling us to determine the half wave voltage and the bandwidth. Afterwards, an optical switch is realized in a simpler setup, where a laser beam passes an EOM and finally enters a photodiode. Before and after the EOM, waveplates and polarizers are used to inspect the behavior of different polarisation~~'~~s of the laser light.

The first chapter of this report deals with the necessary theory, followed by the experimental setup and procedure. In the next chapter, the results and data analysis are presented. Finally, the results are interpreted.

1 Theory

This section describes all relevant formulas needed to understand the experiment. The theory refers to the script [1].

1.1 Pockels- and Kerr-effect

An electro-magnetic wave, which passes non-magnetic, dielectric material, causes charges to oscillate. This behaviour changes optical properties like the refractive index n. The exact behaviour is complicated, but since the n changes little with respect to E, we can expand it into a Taylor series of the form

$$n(E) = n_0 - \frac{1}{2} r n_0^3 E - \frac{1}{2} s n_0^3 E^2 + \dots. \tag{1}$$

Here, $a_1 = -rn_0^3/2$ and $a_2 = -sn_0^3$ are the series coefficients obtained via first and second order derivatives and n_0 represents the refractive index without external fields. Expressing the coefficients a_1 and a_2 in this way allows the electric impermeability $\eta = \epsilon_0/\epsilon = 1/n^2$ to adopt the simple form

$$\eta(E) = \eta_0 + rE + sE^2 + \dots. \tag{2}$$

Usually, the second order term is already tiny and can be neglected. Refractive index and impermeability then follow a linear trend with increasing E. This effect is called the Pockels-effect with the corresponding Pockels coefficient r.

For materials with inversion symmetry, the first order term in the expansion has to equal zero since the polarity of E does not matter. Thus, ~~the~~ the second order term has to be considered where refractive index is proportional to the square of the electric field. This effect, typically much lower than Pockels-effect, is called Kerr-effect.

1.2 Anisotropic materials

When studying the Pockels-effect, anisotropic materials have to be used. However this means, that material properties like n, ϵ and η are no longer independent of the orientation of the material. They have to be extended to 3×3 matrices, in a basis of our choice. We choose a basis in which ϵ is diagonal, with eigenvalues ϵ_i. This basis defines three directions, also known as optical axes, in which the polarization of propagating light does not change. The axes have distinct refractive indices $n_i^2 = \epsilon_i/\epsilon_0$. Any incident light can be decomposed onto the optical axes.

Before studying the properties of anisotropic materials, we have to generalise Equation (2) to comply with the extension to matrices. The new relation

$$\eta_{ij}(\vec{E}) = (\eta_0)_{ij} + \sum_k r_{ijk} E_k + \sum_{kl} s_{ijkl} E_k E_l + \dots \tag{3}$$

contains a vectorial electric field $\vec{E}$ and tensorial generalisations of both Pockels (r_{ijk}) and Kerr (s_{ijkl}) coefficients. These contain 3^3 and 3^4 components respectively, but symmetry arguments reduce the total degrees of freedom.

Some crystals (uniaxial crystals) have only two distinct refractive indices with $n_1 = n_2 = n_o$, the refractive index of the ordinary axis, and $n_3 = n_e$ the extraordinary refractive index. In the case of an uniaxial crystal of the trigonal 3m group (such as $LiNbO_3$), the pockel tensor can be reduced to a 6×3 matrix by mapping the first two indices i, j onto one index I. The mapping rules are listed in Table 1.

Table 1: The mapping rules from double index (i, j) onto single index I are listed for a pockels coefficient of a uniaxial trigonal 3m crystal.

	Index pair (i,j)					
	$(1,1)$	$(2,2)$	$(3,3)$	$(2,3)$ or $(3,2)$	$(1,3)$ or $(3,1)$	$(1,2)$ or $(2,1)$
Index I	1	2	3	4	5	6

After remapping the indices, the pockels coefficient can be displayed as

$$r_{IK} = \begin{pmatrix} 0 & -r_{22} & r_{13} \\ 0 & r_{22} & r_{13} \\ 0 & 0 & r_{33} \\ 0 & r_{51} & 0 \\ r_{51} & 0 & 0 \\ -r_{22} & 0 & 0 \end{pmatrix}. \tag{4}$$

We will now consider an electric field parallel to the extraordinary axis $\vec{E} = (0, 0, E)$. In such a case, the sums in Equation (3) can be omitted and the refractive index (or impermeability for that matter) for the ordinary and extraordinary axis decouple. We are left with the following equations

$$\frac{1}{n_o^2(E)} = \frac{1}{n_o^2} + r_{13}E$$
$$\frac{1}{n_e^2(E)} = \frac{1}{n_e^2} + r_{33}E$$

, which can be brought into the approximate form

$$n_o(E) \approx n_o \cancel{+} \frac{1}{2} n_o^3 r_{13} E \tag{5}$$
$$n_e(E) \approx n_e \cancel{+} \frac{1}{2} n_e^3 r_{33} E. \tag{6}$$

Here n_o and n_e are refractive indices without external field, whilst when considering nonzero E it is explicitly mentioned ($n_o(E)$ and $n_e(E)$). Both indices depend on different elements of the pockel matrix r_{13} and r_{33}.

1.3 Phase and intensity modulation

When a light wave tra~~ns~~verses medium, its electric field E_L experiences a phase shift. This phase shift depends on the length of the Pockels crystal, the vacuum wavelength of the laser, the external electric field E, the Pockels coefficient r and ~~die~~ refractive index without external field n_0. The phase shift ϕ is given by

$$\phi = \frac{2\pi n_0 L}{\lambda_0} - \pi \frac{r n_0^3 E L}{\lambda_0}.$$

The electric field is created by a applying a voltage V to two parallel metal plates mounted on opposite sides of the crystal acting as a plate capacitor, separated by the crystals width d. Using $E = V/d$, we can define the half-wave-voltage, which is the voltage needed to induce a phase shift of π. The half-wave-voltage then is

$$V_\pi = \frac{d}{L} \frac{\lambda_0}{r n_0^3}, \tag{7}$$

which simplifies the term for the phase shift to

$$\phi = \frac{2\pi n_0 L}{\lambda_0} - \pi \frac{V}{V_\pi}. \tag{8}$$

If an EOM is placed in one arm of a Mach-Zehnder interferometer, the phase shift only occurs in one beam. In the output beam, the two waves interfere. When changing the phase difference by varying the voltage, constructive or destructive interference can be achieved. The transmittance T is given by

$$T(V) = \cos^2\left(\alpha - \frac{\pi}{2}\frac{V}{V_\pi}\right). \tag{9}$$

In this relation, α is a fixed phase. If α=0 is reached in an experimental setup, the interferometer functions as an optical switch, by changing the input voltage from 0 to V_π. This, however, only works if 50/50 beam splitters are used.

1.4 Modulation of a ~~circularly~~ polarised laser beam

Upon examination of a circularly polarised laser beam that is transmitted through the EOM, it becomes evident that the electric field exhibits a horizontal and vertical component, which follows the ordinary and extraordinary optical axes, respectively. In the case of circular polarisation, the intensity is distributed equally on both axes. In order to predict the V_π for this case, it is possible to utilise the $V_{\pi,o}$ and $V_{\pi,e}$. These are the half-wave voltages for the ordinary and extraordinary optical axis. In accordance with the definition of the have wave voltage, the total phase shift must be equal to π. This implies that

$$\phi = \phi_0 - \pi\frac{V}{V_\pi} = \phi_o + \phi_e = \phi_{0,o} - \pi\frac{V}{V_{\pi,o}} + \phi_{0,e} - \pi\frac{V}{V_{\pi,e}}$$

leading to

$$\frac{1}{V_\pi} = \frac{1}{V_{\pi,e}} \cancel{+} \frac{1}{V_{\pi,e}}$$

. This conclusion can be reached simply by comparing the coefficients of efficiency.

1.5 Optical contrast

Lastly, we will introduce the optical contrast, which is a measure for how well one can distinguish high from low intensities in a signal. It's formal definition

$$\text{contrast} = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} \cdot 100\% \tag{10}$$

is given by the ratio between intensity difference between maximum and minimum intensity $I_{\max} - I_{\min}$ and the summed intensity $I_{\max} + I_{\min}$.

2 Setup and procedure

In this chapter, the execution of the experiments is described. In all following experiments a HeNe laser with a wavelength of $\lambda = 633$ nm is used. In the first part of the experiment a Mach-Zehnder interferometer is constructed. Firstly, the laser beam is directed through a polarizer and two mirrors before it is split by a 50/50 beam splitter. Both beams are redirected via a mirror, and pointed again onto a 50/50 beam splitter. Afterwards, the beam passes a mirror, an intensity filter and a collimating lens, before hitting a photodiode, connected to an oscilloscope. All optical elements have to be carefully placed, to have parallel beams exiting the interferometer, leading to interference along the following optical path. Rough alignment is best done by projecting the laser beams onto a far object, like a wall, and adjusting the mirrors position and inclination angles to get the laser spots to align at all distances. Afterwards, fine adjustments can be made by beam walking the laser, whilst using the oscilloscopes minimum and maximum readings and Equation (10) to maximize the contrast. Optimally, the contrast should be more than 90 %.

After this alignment, an elecro-optical modulator (EOM) containing a $LiNbO_3$ crystal with length 2,45(1) cm and height 1,8(1) mm is placed in one arm of the interferometer and connected to an amplifier and function generator. At the function generator, a frequency of 30 Hz and an amplitude of 3 V, which is amplified 100 times by the amplifier, is inserted into the EOM. Now, several measurements (for statistical reasons) are done by saving both~~,~~ signal of the function generator and the recorded intensity from the oscilloscope. After these measurements, a $\lambda/2$ waveplate is placed after the polarizer to rotate the polarization by 90° and traces from function generator and photodiode are saved multiple times. After this, the frequency is raised from 30 Hz to 100 Hz and from here in steps of 100 Hz until 500 Hz are reached. For every step, maximum and minimum intensity are recorded. The setup whole setup can be seen in Figure 1.

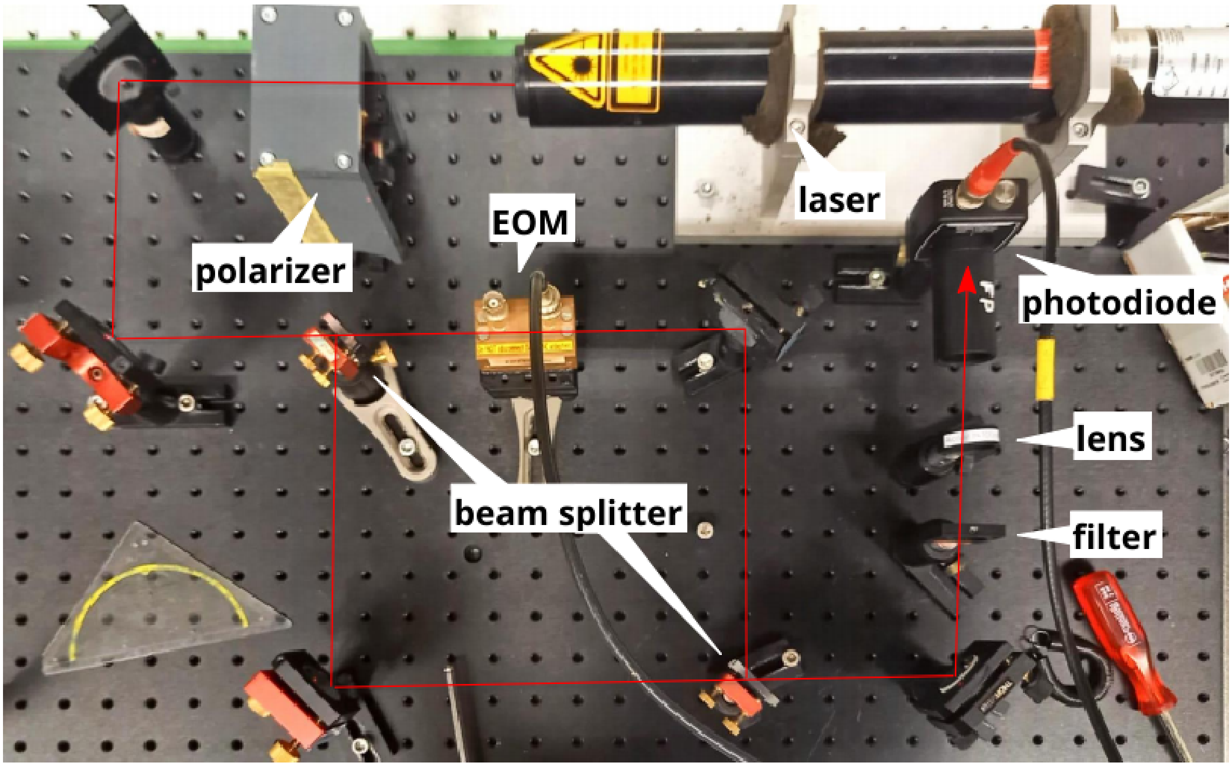

Figure 1: The setup of the first part of the experiment can be seen. Here light emitted from a HeNe laser is polarized and redirected by two mirrors onto a 50-50 beam splitter. This splits the light into two beams, which are directed by two mirrors onto another beam splitter, where, the two beams are brought to interfere. In the top arm an electro-optical modulator (EOM) is placed. In the end, the beam passes an intensity filter, a collimating lens and is recorded by a photodiode.

The second setup is a much simpler construction and can be seen in Figure 2. Again, the HeNe laser beam is collimated by a lens and redirected by two mirrors. Then, a polarizer and a $\lambda/2$ waveplate are placed, before the EOM. After propagating through the crystal it passes another $\lambda/2$ waveplate and afterwards a polarizer. After a mirror, another collimating lens is placed before the photodiode. As in the interferometer setup, a voltage of 3 V and 30 Hz is applied, and several measurements are taken. Then, the triangular voltage is switched to a square wave pulse, which lets the EOM ~~to~~ function as an optical switch. The voltage of the driving signal is first approximated from the data already taken and is further fine tuned using the oscilloscopes min/max reading to increase the on/off ratio of the output signal. Now some measurements are taken to determine the on/off ratio. Lastly, the frequency is increased to see the frequency dependent behaviour of the on/off ratio.

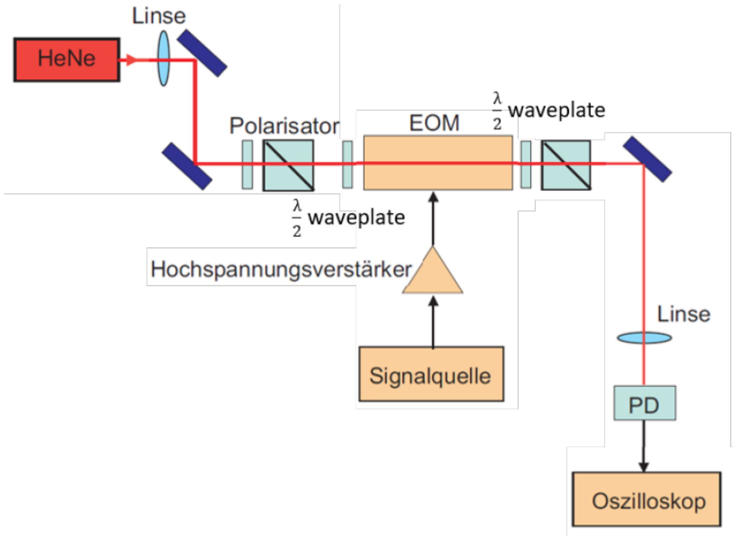

Figure 2: The setup of the second part of the experiment is shown. Here light emitted from a HeNe laser is polarized and redirected by two mirrors. Waveplates are used to modify the laser's polarization coming into the EOM. Lastly, the beam is collimated by a lens and recorded on an oscilloscope via a photodiode. Figure taken from [1].

3 Results

In order to determine the half-wave-voltage, the intensity was measured by the photodiode, with an inserted triangle voltage with amplitude 3 V and a frequency of 30 Hz. For the evaluation, the range from minimum to maximum voltage was taken. This interval of voltage is plotted against the intensity. A fit based on Equation (9) is done, as the intensity is proportional to the transmittance. There were several measurements made to gather statistics, in Figure 3, one example of a fit is shown. From this measurement, we get $V_{\pi,FG} = 2.24(1)$ V for the measurements including the $\lambda/2$-waveplate. The voltage given here refers to the voltage that has to be inserted at the function generator. At the EOM, the voltage is actually 100 times bigger, as an amplifier is used. The uncertainty of the amplifier is neglected here, as it is estimated to be very small in contrast to the determined value of the voltage. As an average value (without $\lambda/2$-waveplate) we get

$$V_{\pi,\text{mean}} = 451{,}9(5)\,\text{V}.$$

This value is already multiplied by 100, as the amplifier outputs the monitor signal at a reduced level. The uncertainty here is calculated as the standard deviation of the values for V_π. Using this value and Equation (7), the material based factor $r_{13}n_O^3$ can be determined as

$$r_{13}n_O^3 = 103(6)\,\text{pm/V}.$$

When we use the value of n_O from the script, which is given as 2.2865, we get

$$r_{13} = 8{,}9(5)\,\text{pm/V}.$$

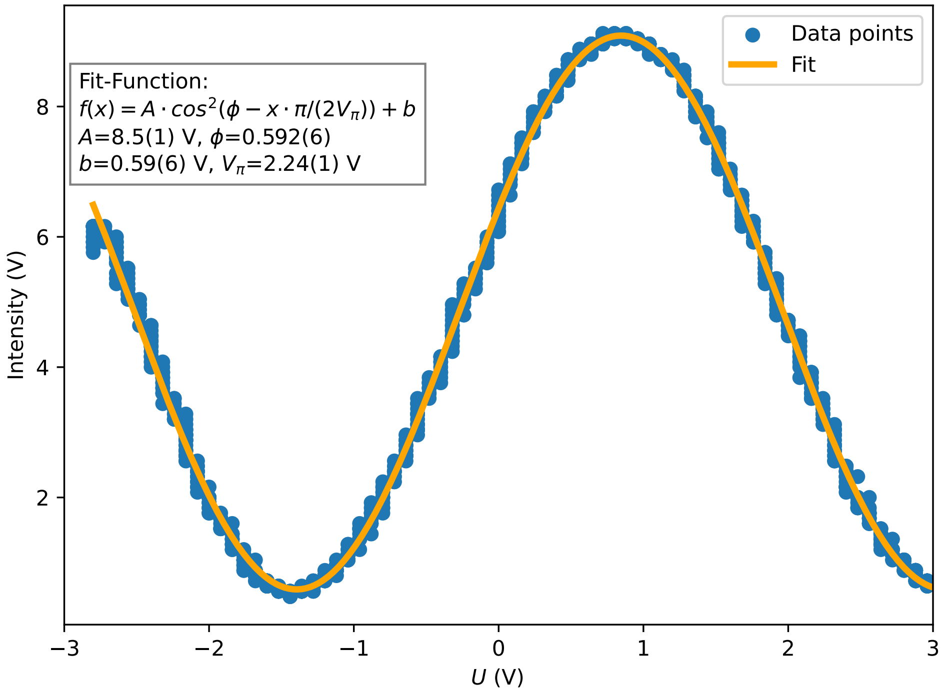

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Figure 3: The measured voltage of the photodiode is shown, depending on the input voltage U. A fit was done to determine the half wave voltage, the function and parameters are given in the plot.In this plot, only one of several measurements is shown as an example for a measurement including the $\lambda/2$-waveplate, in the later analysis, the results from many measurements are averaged.

When averaging over all of these measurements with the $\lambda/2$-waveplate , we get

$$V_{\pi,\text{mean,waveplate}} = 223{,}6(3)\,\text{V}.$$

Which yields to

$$r_{33}n_e^3 = 208(12)\,\text{pm/V}$$

and width the value for n_e , given in the script as 2.2022 [1], we get a value of

$$r_{33} = 19{,}4(11)\,\text{pm/V}.$$

In the next step, the bandwidth of the first setup was determined qualitatively. The bandwidth is defined as the frequency, at which the interference contrast decreases clearly, compared to the contrast at low frequency. In our case, we use the frequency, at which the contrast is half the contrast at the lowest measured frequency, which was 30 Hz. At 30 Hz, the contrast is 0.87(2), so we are looking for the frequency with a contrast of 0.434(8). To estimate this quantity, a exponential function is fitted to the data points, which are the contrast at different frequencies. The reason for the choice of an exponential is, that it describes the behaviour in the relevant measured range well. The data as well as the fit function with parameters are shown in Figure 4. Using the aforementioned definition of bandwidth and the fit parameters, we are getting a bandwith of 425(10) Hz.

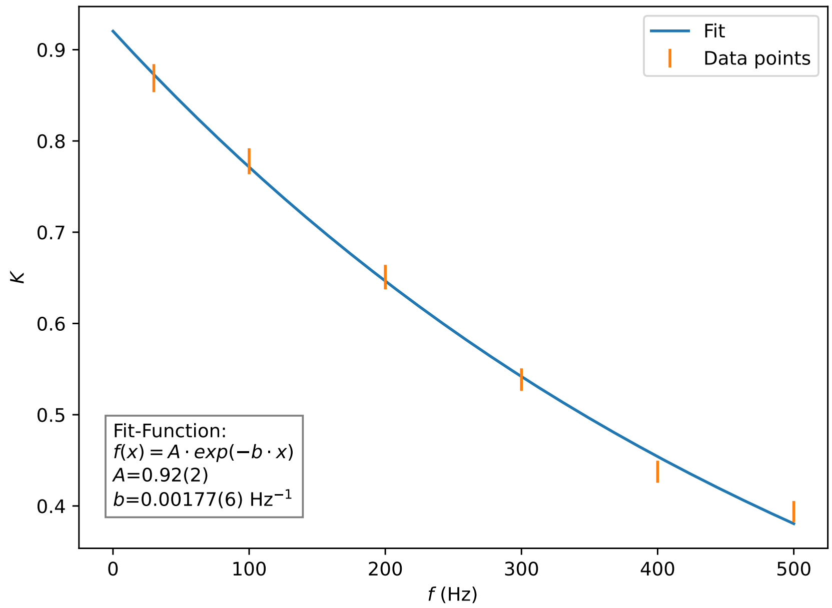

Figure 4: The measured interference contrast is shown dependent on the input frequency f. Additionally, an exponential fit is shown, parameters and function are shown in the graph.

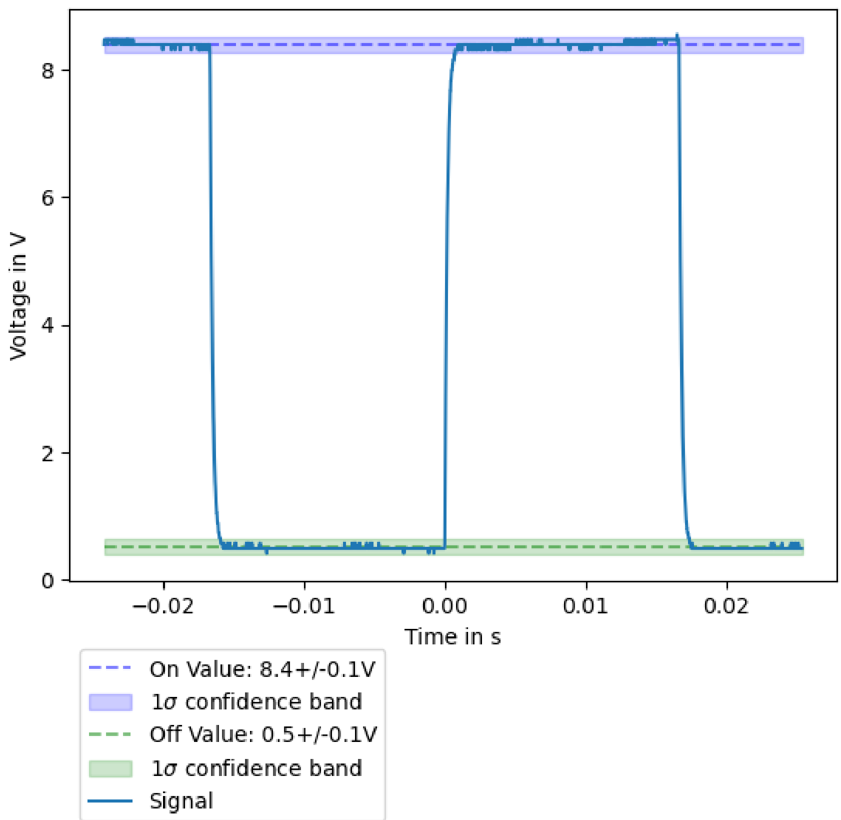

Figure 5: The EOM's behaviour as an optical switch can be seen. The average on voltage and off voltage are also shown. To determine these, the rising and falling edges were first determined. Then the system was given a settling time of 0.0003 s. The mean value up to the next edge was then determined. In each case, the standard deviation serves as uncertainty. This is visualized with a 1σ confidence band.

For the second set-up, we implemented an optical switch at the same frequency as before, but with a square-wave signal. However, it is first necessary to determine the value of $V_{\pi,2}$. This is done according to the same procedure as for the data from the first setup. The value of

$$V_{\pi,2} = 152{,}16(7)\,\mathrm{V}$$

was determined. Thereafter, the offset voltage and the amplitude of the square wave signal were manually adjusted until the maximum on/off ratio was achieved. This can be seen in Figure 5. We repeated the measurement once and determined an on/off-ratio of $V_{\mathrm{ON}}/V_{\mathrm{OFF}} = 17(4)$ and $V_{\mathrm{ON}}/V_{\mathrm{OFF}} = 18(5)$. To determine the V_{ON} and V_{Off}, we first searched for the Flank and then calculated the mean value up to the next Flank after a transient phase of 0.0003 s. We used the standard deviation as the uncertainty. The value for the transient phase was not chosen at random; it was approximately 2 times the half-life for the reaction of the system. We will discuss this further later.

To get an idea of the response time of the system, a falling edge was observed for a square wave signal. With the original setup we obtained a half-life of $1.3 \cdot 10^{-4}$ s. To bridge the internal resistance of the oscilloscope, a 50 Ω resistor was connected in parallel. Although the measured signal is much smaller than in the original setup, a half-life of $1.9 \cdot 10^{-6}$ s can be obtained. As these are qualitative and not quantitative values, no error calculation was made. For better comparison in the graph, the amplitudes have been normalised to ± 1. The final slope analyses can be seen in Figure 6. It can be seen that the half-lives differ by 2 ~~decimal points.~~

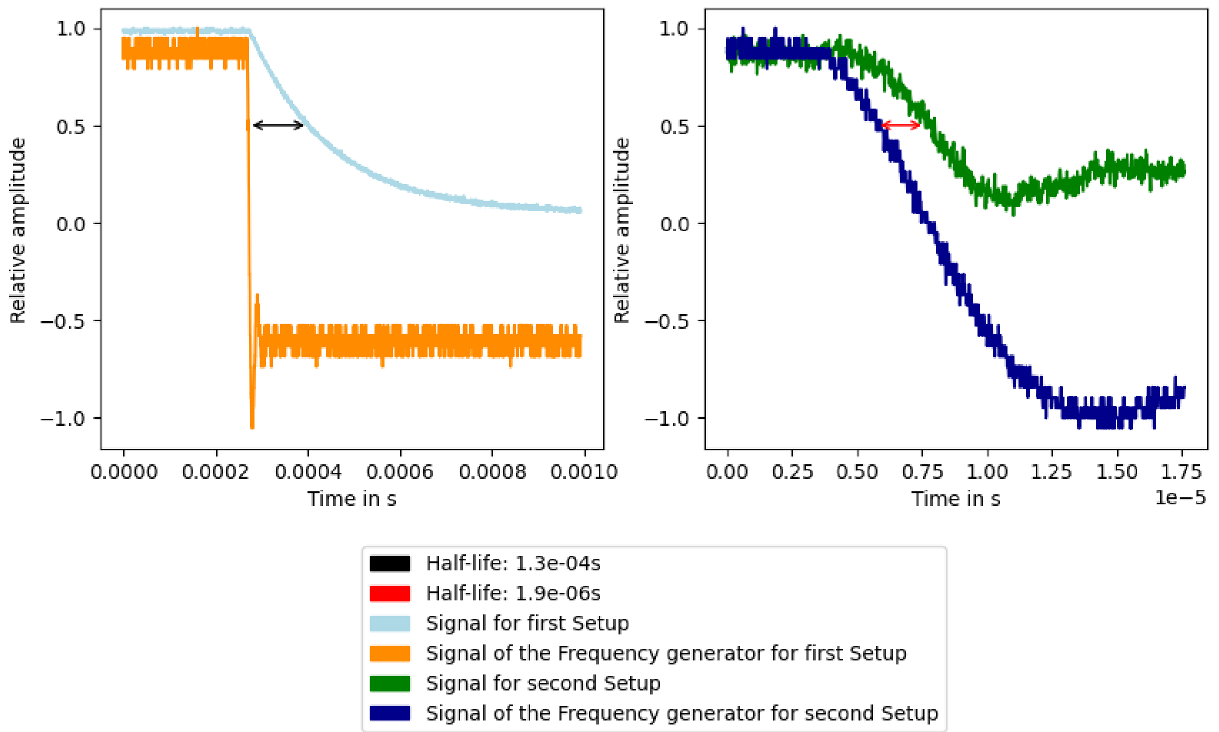

Figure 6: The response behaviour of the system can be seen. In the left picture the setup was made as befor and in the right picture a 50 Ω resistor was connected parallel to the oscilloscope to minimise the input resistance. The signals have been normalised to their maximum at ± 1. The signal from the frequency generator, the recorded signal from the laser diode and the half-life can be seen.

4 Interpretation

The first series of measurements with the Mach-Zehnder interferometer, where no waveplate is used, supports theory. Our result for the coefficient r_{13} of the pockels tensor is 8,9(5) pm/V, whereas 8,6(2) pm/V was given in the script. Thus, both values equal each other within uncertainty. When comparing the experimental value for the extraordinary axis, r_{33}, 19,4(11) pm/V, to the value in the script, 30,8(2) pm/V, a significant deviation is recognized. This could be a result of a imprecise placement of the $\lambda/2$-waveplate. Maybe it was not adjusted exactly enough to transform the beam completely from the ordinary to the extraordinary axis of the EOM, what could be a reason that r_{33} is much too small.

The measurement of contrast dependent on frequency resulted in a bandwith of 425(10) Hz. The contrast dropped approximatel exponential in the observed interval. Nevertheless, the procedure of the experiment could be improved by some changes. A more precise adjustment of all optical elements would lead to better results. If the contrast could be increased even more, the results were more accurate. As the setup of the interferometer is very sensitive, it has to be avoided to touch the table or even to speak, as this influences the result.

The second setup, in contrast, is much more stable and is not influenced that easy. With regard to the modulation of the circularly polarised laser, it can be stated that the expected half-wave voltage, evaluated with subsection 1.4, is $V_{\pi,theo}$ = 149.6(2) V, while the measured value is V_π = 152,16(7) V. The discrepancy can be attributed to the aforementioned uncertainty in determining V_π with the $\lambda/2$ waveplate. With regard to the realisation of the optical switch, it can be said that it works well. It should be noted that the on/off ratio depends significantly on V_{OFF}. The smaller it is, the larger this value is. This ratio diverges towards 0 for the limit value. The flank analysis was conducted with the objective of identifying the source of the majority of the delay. This was subsequently determined during the measurement process. It was found that the propagation velocity within the crystal plays a minor role. The value for the medium reaction of the photodiode is given by 14 ns see [2]. This is also significantly less than half the lifetime of the entire system. Another limiting factor is the function generator, which requires a certain amount of time to realise the edge of a rectangular signal. By reducing the high-impedance input resistance of the oscilloscope through the use of a resistor in parallel, the reaction time of the structure was significantly decreased.

It can therefore be concluded that this is the primary cause of the delay. To sum up, the main area for improvement is in the adjustment of the waveplate.

References

[1] Manuele Landini, Hanns-Christoph Nägerl. *FP1 Versuch E121: Elektro-optischer Effekt.* Universität Innsbruck, Innsbruck, AT: Institut für Experimentalphysik, March 21, 2024.

[2] *DET36A2 Si Biased Detector , author = Rev A, year = december 27, 2017, publisher = ThorLab.*

Erklärung

Hiermit versichern wir, dass der vorliegende Bericht selbständig verfasst wurde und alle notwendigen Quellen und Referenzen angegeben sind.

....	April 24, 2024 ....
Alexander Helbok	Date
....	April 24, 2024 ....
Jakob Hugo Höck	Date
....	April 24, 2024 ....
Max Koppelstätter	Date