



All About Bragg Angle Errors in AO Modulators & Deflectors

Application Note IM1022

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1. Introduction

In this application note, we address two questions, namely

- ... What happens to the output beam direction in an acousto-optic modulator or deflector when the Bragg angle is misadjusted
- ... What happens to the output beam intensity when the Bragg condition is not satisfied

Principally we consider two different cases

- 1) An acousto-optic modulator in which the input laser beam is misaligned from the true Bragg input angle.
- 2) An acousto-optic deflector in which the input laser beam is aligned for the true Bragg condition at a single acoustic frequency but is misaligned for all other frequencies over the range of scan angle.

This discussion pertains to single-transducer, non-beam-steered devices.

2. Fundamentals

In an acousto-optic modulator or deflector, maximum intensity of diffracted light in the first order beam occurs when the Bragg condition is satisfied. Figure 1 shows the geometry of input and output laser beams relative to the acoustic column.^{1*} The Bragg condition is met when the angle of incidence θ_b , is:

$$\theta_b(\text{medium}) \approx \sin \theta_b = \frac{\lambda_m}{2\Lambda} = \frac{\lambda f}{2\eta v} \quad (1)$$

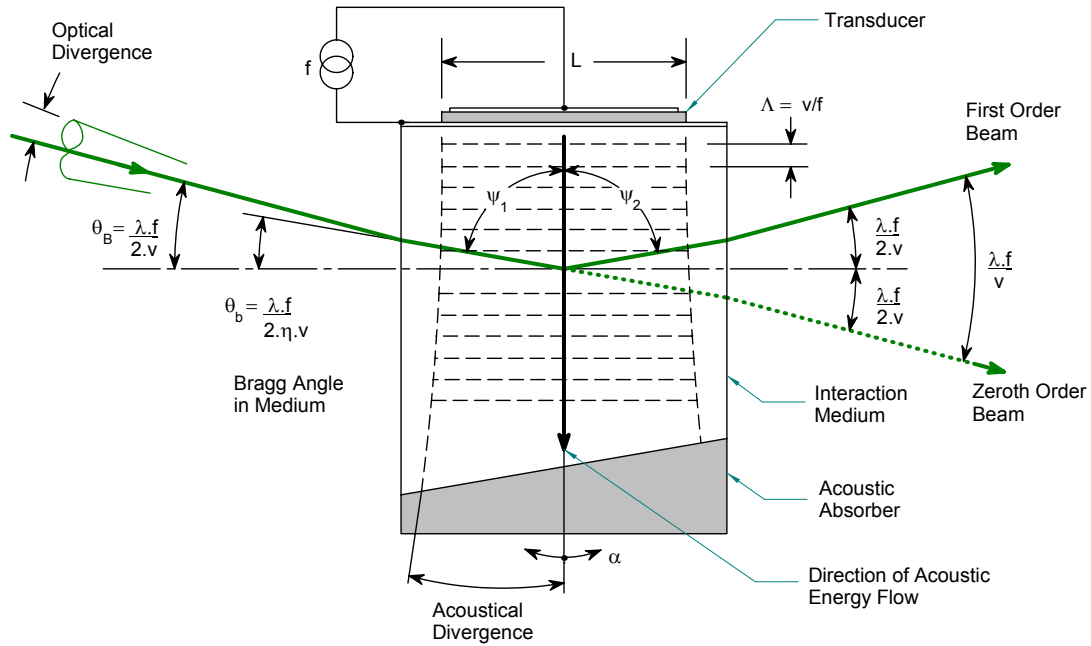
where:

λ	= laser wavelength in free space
λ_m	= laser wavelength in medium
Λ	= acoustic wavelength in medium = $\frac{v}{f}$
v	= acoustic velocity in medium
f	= acoustic frequency
η	= index of refraction

Outside the medium, the Bragg angle is simply:

$$\theta_B(\text{air}) = \frac{\lambda f}{2v}$$

¹ *For isotropic media such as Glass, Lead Molybdate, and Tellurium Dioxide in the orientations normally used for acousto-optic modulators.



When Bragg Condition is exactly satisfied, the angles ψ_1 and ψ_2 are equal

Figure 1: Geometry of Input, Output and Acoustic Beams in an Acousto-Optic Modulator / Deflector

When the Bragg condition is satisfied, the angle of incidence and the angle of diffraction are identical, and the direction of acoustic energy flow exactly bisects the incidence and exit angles.

The intensity of the first order beam is given by the equation

$$\frac{I_1}{I_0} = [\rho] \cdot \left[\sin^2 \frac{\pi}{2} \sqrt{\frac{M_2 L P_a}{2 H}} \right] \cdot \left[\text{Sinc}^2 \left(\frac{\pi L \alpha}{\Lambda} \right) \right]$$

Where:

- I_1 = Intensity of first order beam
- I_0 = Intensity of zeroth order beam when the acoustic energy in the medium is zero
- ρ = A variable <1 which is related to the acousto-optic 'Q' of the Bragg cell and to the optical beam divergence to the acoustic column divergence
- λ = Laser wavelength in free space
- M_2 = Elasto-optic figure of merit of the interaction medium
($36.3 \times 10^{-15} \text{ M}^2/\text{w}$ for lead Molybdate)

$P_a =$	Acoustic power in the interaction medium = (k) electrical input power to transducer
$L =$	Interaction length
$H =$	Width of acoustic column
$\Lambda =$	Acoustic wavelength in medium = v / f = Acoustic velocity / Acoustic frequency
$\alpha =$	Bragg angle error

The three parts of the equation have the following significance.

Part 1: The portion of light, ρ , that is diffracted into the first order beam depends on (1) the acousto-optic 'Q' of the Bragg cell and (2) the ratio of optical beam divergence to the acoustic column divergence. We express ρ as a product of two factors:

$$\rho = (K_Q)(K_\beta) \quad (4)$$

K_Q is a non-linear function of acousto-optic 'Q' which in turn is a fixed parameter of the Bragg cell pertaining to its interaction length, center frequency, and acoustic velocity. For practical Bragg cells such as the Isomet Models 1201, 1205 and 1206, $Q = 12$ and $K_Q = 0.96$. K_β is a non-linear function of β , which in turn is a variable operating parameter of the Bragg cell selected by the user and pertaining to the laser beam divergence. When the laser beam divergence is small (1mm laser beam diameter in practical modulators) nearly all of the light is diffracted, i.e. $K_\beta > 0.9$. When the laser is

highly focused for fast rise time, then a smaller portion of the light is diffracted. Figure 2 is a plot of K_β vs. the parameter β where:

$$\beta = \frac{\pi}{2} \cdot \frac{\text{optical divergence in medium, full angle}}{\text{acoustical divergence, full angle}}$$

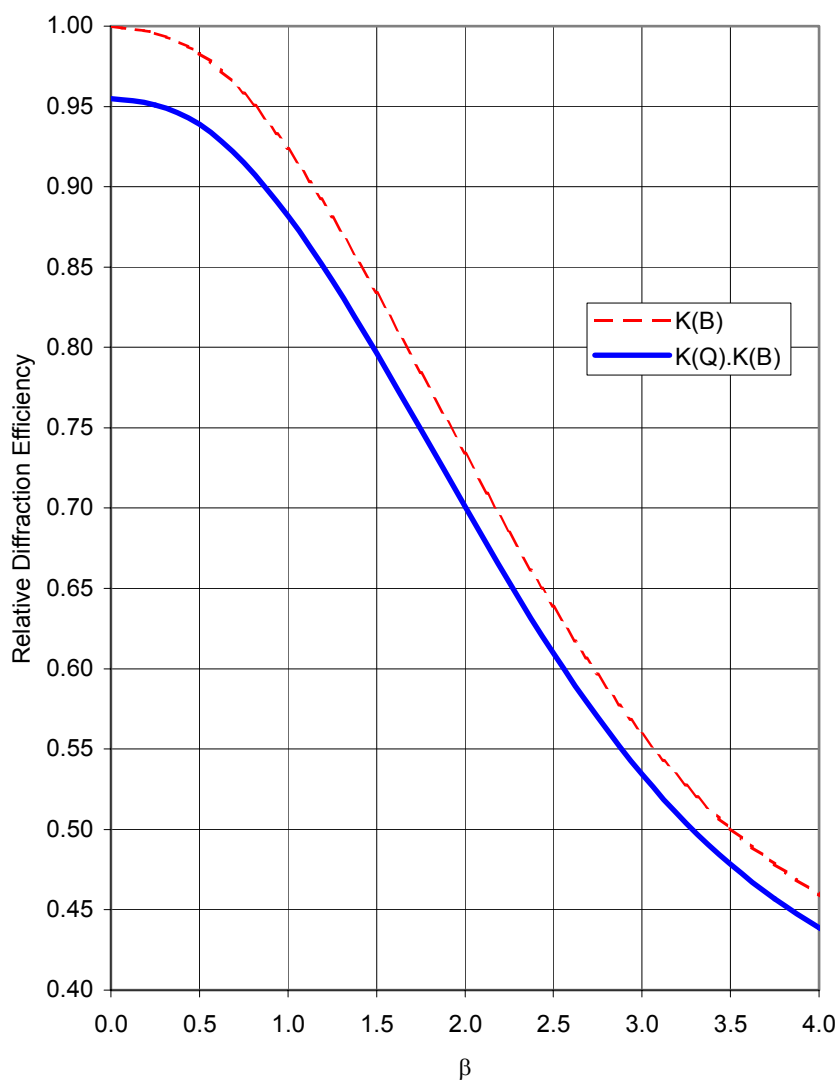
Also shown in Figure 2 is a plot of $\rho = (K_Q)(K_\beta)$ as a function of β for $Q = 12$.

Part 2: The familiar transfer function of light intensity vs. acoustic drive power is given in this part of the equation. When P_a is zero no light is diffracted. When $P_a = \frac{\lambda^2 H}{2M_2 L}$ the modulator reaches saturation and nearly all of the light is diffracted (diminished by the effect of Part 1 and Part 3).

Part 3: This part of the equation accounts for the effect of Bragg angle misadjustment through the radiation pattern of the transducer. The transducer is uniformly illuminated aperture and its acoustic radiation pattern is sinc(x) function. The intensity of the diffracted beam attributable to the radiation pattern is

$$I(\alpha) = \frac{\text{Sin}^2\left(\frac{\pi L \alpha}{\Lambda}\right)}{\left(\frac{\pi L \alpha}{\Lambda}\right)^2} = \frac{\text{Sin}^2\left(\frac{\pi L f \alpha}{v}\right)}{\left(\frac{\pi L f \alpha}{v}\right)^2} \quad (5)$$

Where α is the angular deviation off the direction of acoustic propagation in the medium. At $\alpha = 0$, the true Bragg condition exists and $I(\alpha) = 1$.



$$\beta = \frac{\pi \cdot \text{Optical Divergence}}{2 \text{ Acoustical Divergence}}$$

Figure 2: Variation of intensity in an Acousto-Optic Modulator as a function of Optical and Acoustical Divergence

3. Misaligned Modulator

In an acousto-optic modulator, the center acoustic frequency is held constant; the acoustic drive power is varied between zero and saturation so as to control the intensity of the first order beam. Figure 1 shows the relationship between the input, zeroth order output, and first order output beams relative to the direction of acoustic energy flow ($\alpha = 0$) for the true Bragg condition where $f = f_0$ is the center acoustic frequency.

When the input laser beam is intentionally misaligned from its true Bragg position by angle δ and the modulator is held fixed in position, both the zeroth and first order beams are shifted by an identical angle δ (and in the same clockwise or counter clockwise direction) from either their true Bragg positions. This is shown in Figure 3.

For the condition shown in Figure 3, all three beams are misaligned by the angle δ outside the interaction medium and by the angle $\frac{\delta}{\eta}$ inside the medium. The bisector of input and first order output

beams is also shifted by $\frac{\delta}{\eta}$ from the direction of acoustic energy flow. It is the angle α between the bisector and the direction of acoustic energy flow that is used to calculate the loss of intensity due to misalignment in equation (5). The relation between α and δ is given by:

$$\alpha = \frac{\delta}{\eta} \quad (6)$$

It is convenient to show the variation in intensity as a function of the characteristic interaction length

$$L_o = \frac{\eta}{\lambda} \cdot (\Lambda_o)^2 = \frac{\eta}{\lambda} \left(\frac{v}{f_o} \right)^2 \quad (7)$$

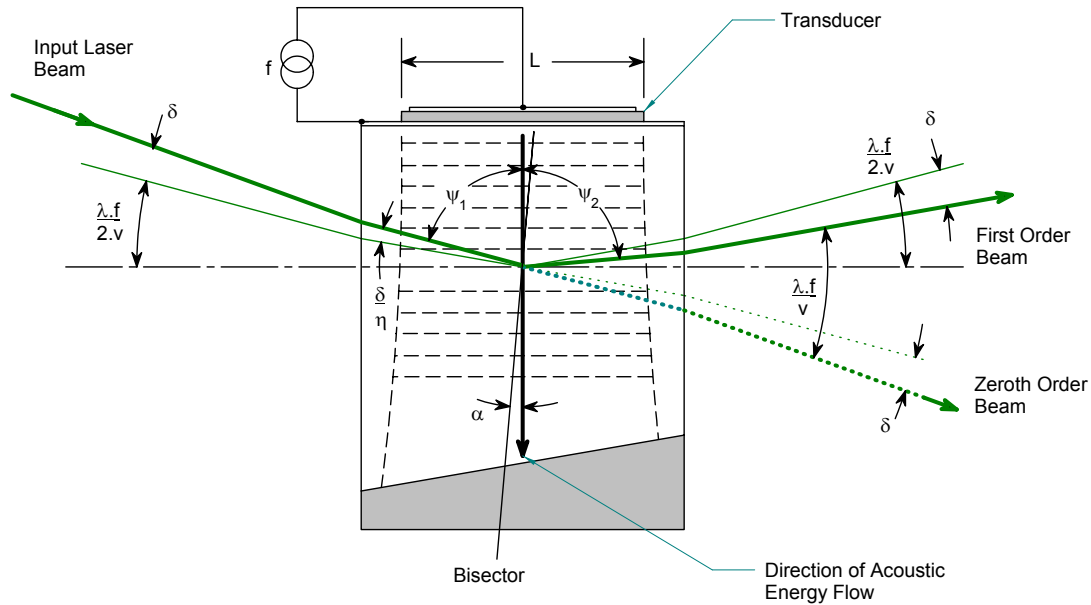
and normalized to the input Bragg angle at the center frequency f_o . Substituting equation (6) in (5), we obtain:

$$I(\alpha) = \frac{\sin^2 \left(\frac{\pi L f_o \delta}{\eta v} \right)}{\left(\frac{\pi L f_o \delta}{\eta v} \right)^2} \quad (8)$$

which is identically equal to

$$I(\alpha) = \frac{\sin^2 \left(\frac{\pi}{2} \cdot \frac{L}{L_o} \cdot \frac{\delta}{\theta_B} \right)}{\left(\frac{\pi}{2} \cdot \frac{L}{L_o} \cdot \frac{\delta}{\theta_B} \right)^2} \quad (9)$$

where θ_B is the Bragg angle in air.



The entry angle of the input laser beam is intentionally misaligned from the true Bragg position by the angle δ . The First order beam is also misaligned from the true Bragg position by the angle δ and is reduced in intensity. The reduction in intensity is a function of the Bragg angle error, α , where $\alpha = \delta/\eta$. For the case shown $\psi_1 \neq \psi_2$

Figure 3: Bragg Angle Error in an Acousto-Optic Modulator

Equation (9) is plotted in Figure 4 for various values of the ratio L/L_0 . The abscissa values are expressed in units of δ/θ_B . $\delta = 0$ corresponds to the true Bragg condition with no misalignment. It is seen that the curves are symmetric about the true Bragg position; that is a Bragg angle misalignment in the positive direction causes the exact same reduction in intensity as the identical misalignment in the negative direction. It is also seen that the shorter the interaction length L , the less pronounced is the reduction of intensity due to misalignment. Of course, the shorter the interaction length, the higher the required drive power for saturation.

A good compromise in choice of interaction length is $L/L_0 = 2$. Both the Isomet Models 1205 and 1206 Acousto-Optic Modulators are designed for this nominal value. Both models are highly tolerant of misalignment in Bragg angle adjustment. A 12% error in Bragg angle causes only a 5% reduction in intensity.

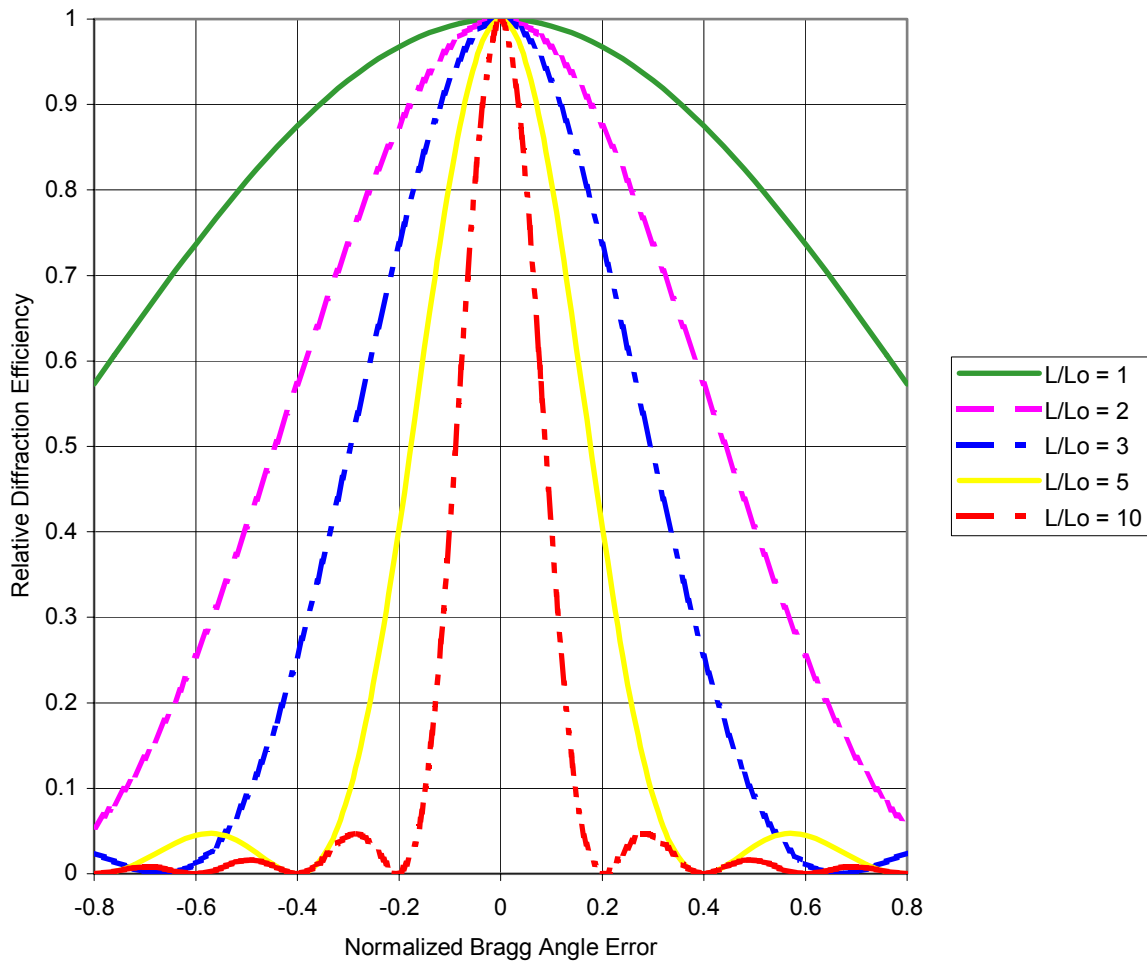


Figure 4: Intensity Variation vs. Bragg Angle Error in an Acousto-Optic Modulator

4. Acousto-Optic Deflector, Bragg Angle Adjusted at f_0

In an Acousto-Optic Deflector, the drive power is held constant and the drive frequency is varied over a range of frequency Δf centered at f_0 . At f_0 the Bragg condition is satisfied; Figure 1 shows the input and output angles relative to the direction of acoustic power propagation.

At f_0 the deflector is identical to the modulator previously described. In the deflector however, we hold the input beam position fixed at its Bragg angle for f_0 and we vary the frequency. The zeroth order beam exiting from the deflector also remains fixed in position, but the first order beam varies in position according to equation (2). Figure 5 shows the geometric relationship of the input, output, and acoustic beams at f_0 and the lower (f_L) and upper (f_H) frequency limits. At f_0 , the Bragg condition is satisfied. Above and below f_0 , the fixed entry angle deviates from the true Bragg condition. Note also in Figure 5 the change of the acoustic beam shape with frequency. At the upper frequency limit, f_H , the acoustic beam is less divergent than at the lower frequency limit, f_L . This has an important bearing on the variation in intensity as will be explained later.

The equation for variation of intensity as a function of frequency for the deflector case can be derived from equation (5). In this instance however, it is more convenient for us to express the regular deviation α as a function of normalized frequency rather than a function of normalized Bragg angle. In Figure 5, the deviation of the output beam from the true f_0 Bragg position is:

$$\delta(air) = \frac{\lambda(f_0 - f)}{v} \quad (10)$$

and in the medium

$$\delta(medium) = \frac{\lambda(f_0 - f)}{\eta v} \quad (11)$$

As in the prior analysis the error angle α is the angle between the direction of the acoustic beam and the bisector of the input and first-order output laser beams. However, in this case only the first order output beam has shifted relative to the direction of acoustic energy flow, so the error angle α is

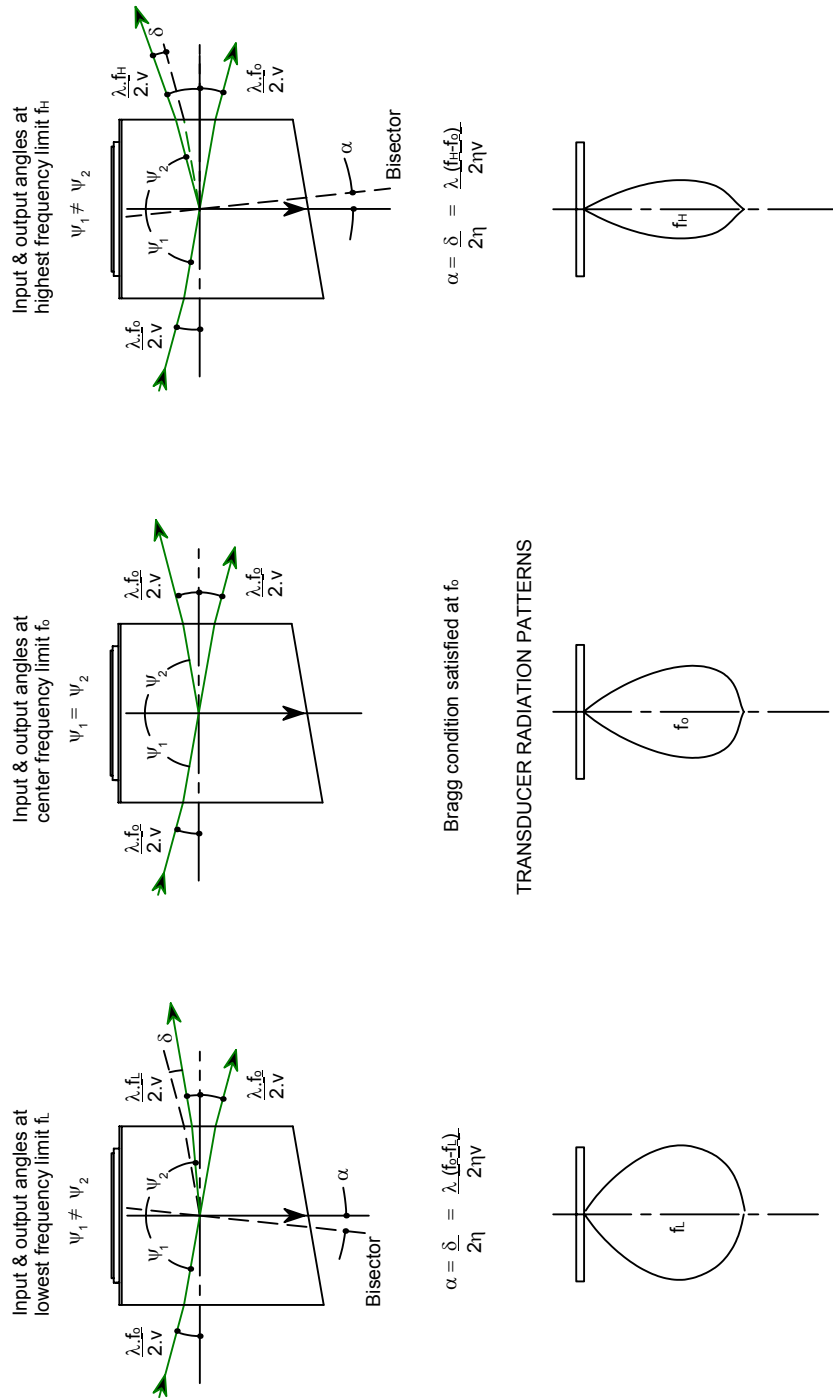
$$\alpha = \frac{\delta}{2\eta} = \frac{\lambda(f_0 - f)}{2\eta v} \quad (12)$$

We proceed to derive an expression for intensity variation as a function of frequency by substituting equation (12) in equation (5) which yields

$$I(\alpha) = \text{Sinc}^2 \left(\frac{\pi L f}{v} \cdot \frac{\lambda(f_0 - f)}{2\eta v} \right) \quad (13)$$

Also we substitute the value of L_0 from equation (7) in equation (13) giving

$$I(\alpha) = \text{Sinc}^2 \left[\frac{\pi}{2} \frac{L}{L_0} \left(\frac{f}{f_0} \right) \left(1 - \frac{f}{f_0} \right) \right] \quad (14)$$



The entry angle of the input laser beam remains fixed at the center frequency (f_0) Bragg position. The zeroth order beam also remains fixed position. The first order beam moves as a function of frequency such that the deviation from the true Bragg condition is $\alpha = \delta/2\eta = \lambda\Delta f/2\eta v$. The acoustical divergence also varies with frequency.

Figure 5: Bragg Angle Error in an Acousto-Optic Deflector

Finally we use the normalized notation for frequency $F = \frac{f}{f_0}$ in equation (14). Thus

$$I(a) = \text{Sinc}^2 \left[\frac{\pi}{2} \frac{L}{L_0} (F)(1-F) \right] \quad (15)$$

with:

(F) = Normalized frequency (divergence factor)

(F-1) = Normalized Frequency Deviation from true Bragg

Although equation (15) has the same general structure as equation (9), there is an important difference, namely the curve $I(a)$ is not symmetric about the center frequency, f_0 . A given percentage deviation in frequency below f_0 causes a smaller roll off in intensity than does the same percentage deviation in frequency above f_0 . This is clearly shown in Figure 6, which is a plot of equation (15) for several values L/L_0 . The fact that the intensity versus normalized frequency curve is non-symmetric is the result of the change in acoustic beam divergence with frequency term, (F), in equation (15). At frequencies below f_0 , there is less roll off because the acoustic beam is more divergent; at frequencies above f_0 there is more roll off because the acoustic beam is less divergent.^{2*}

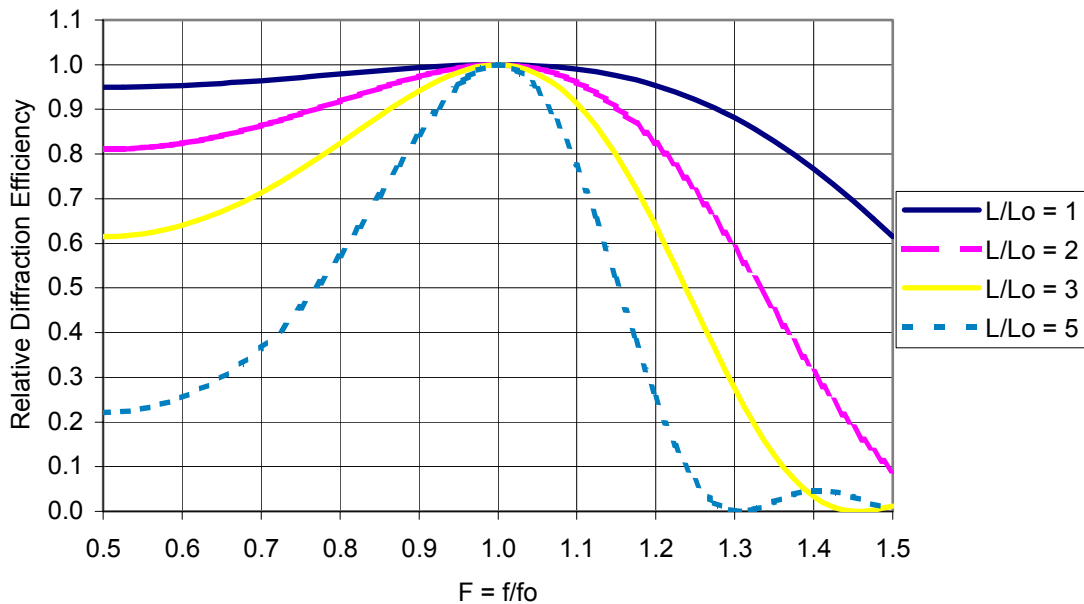


Figure 6: Intensity Variation vs. Frequency in an Acousto-Optic Deflector, Bragg adjusted at f_0

^{2*} Divergence of the uniformly illuminated acoustic transducer is

$$\alpha = \frac{1}{2} \cdot \frac{\Lambda}{L} = \frac{v}{2fL}$$

5. Acousto-Optic Deflector, Equalized Fall-off

In most deflector applications, we desire to minimize the intensity variation over the range of scan angle. One naturally concludes from the discussions in Section 4 that adjusting the Bragg angle at center frequency does not necessarily produce the best results from an intensity variation viewpoint.

As was noted in Section 4 the unequal fall-off in deflector response results from the change of acoustic divergence with frequency. We can force the deflector to exhibit equal fall-off at upper and lower frequency limits (f_H and f_L) by aligning the Bragg angle at a frequency f_m slightly higher than the center frequency f_o . The frequency f_m is easily derived from the foregoing analysis.

Since by deflection we will have adjusted the Bragg angle at a slightly higher frequency f_m above the center frequency f_o , we may substitute f_m for f_o in equation (13). This merely says we measure the deviation from true Bragg at f_m rather than f_o . Thus

$$I(\alpha) = \text{Sinc}^2 \left[\frac{\pi L f}{v} \cdot \frac{\lambda(f_m - f)}{2\eta v} \right] \quad (16)$$

After substituting the value of L_o from equation (7) in equation (16), we obtain the expression

$$I(\alpha) = \text{Sinc}^2 \left[\frac{\pi L}{2L_o} \left(\frac{f}{f_o} \right) \left(\frac{f_m - f}{f_o} \right) \right] \quad (17)$$

Which, normalized to f_o , can be restated as

$$I(\alpha) = \text{Sinc}^2 \left[\frac{\pi}{2} \frac{L}{L_o} (F)(F_m - F) \right] \quad (18)$$

This is of similar form to equation (15) in the previous section.

From equation (17) we are interested in determining the value of f_m which forces $I(\alpha)$ at the upper frequency limit to equal $I(\alpha)$ at the lower frequency limit. For this condition to prevail, obviously

$$\frac{f_H}{f_o} \left(\frac{f_H - f_m}{f_o} \right) = \frac{f_L}{f_o} \left(\frac{f_m - f_L}{f_o} \right) \quad (19)$$

and

$$f_H(f_H - f_m) = f_L(f_m - f_L) \quad (20)$$

Rearranging and solving for f_m , we obtain

$$f_m = \frac{f_H^2 + f_L^2}{f_H + f_L} \quad (21)$$

Since deflectors are characteristically specified by their nominal center frequency swing Δf (i.e. $\Delta f = f_H - f_L$), we make the following substitutions in equation (21)

$$f_H = f_o + \frac{\Delta f}{2} \quad (22)$$

$$f_L = f_o - \frac{\Delta f}{2} \quad (23)$$

So equation (21) reduces to

$$f_m = \left(f_o^2 + \frac{\Delta f^2}{4} \right) \frac{1}{f_o} = \left(1 + \frac{\Delta F^2}{4} \right) f_o \quad (24)$$

which normalized to f_o equates to

$$F_m = 1 + \frac{\Delta F^2}{4} \quad (25)$$

Either equation (24) or (25) may be used to determine f_m . To reiterate, when the Bragg angle is adjusted for peak output at f_m , the roll-off in intensity at the lowest frequency f_L will be exactly equal to the roll-off in intensity at the highest frequency f_H . Note from equation (24) that f_m is always greater than f_o .

In Figure 7, we have plotted equation (18) for an octave bandwidth deflector, that is $f_H=2f_L$, for various L/L_o ratios. Selecting an octave bandwidth deflector is significant because it is a limiting case. A greater bandwidth results in the ghost of the second-order beam at the lowest frequency overlapping the first-order beam at the highest frequency. For the octave bandwidth case,

$$\frac{\Delta f}{f_o} = \Delta F = \frac{2}{3} \quad (26)$$

and from equation (25)

$$F_m = \frac{f_m}{f_o} = 1.111 \quad (27)$$

From Figure 7 we conclude the following for the limiting octave bandwidth case:

- Setting the Bragg angle at the frequency $f_m=1.11 f_o$ does indeed equalize the intensities at f_H and f_L .
- The roll off rate above f_m is greater than the roll of rate below f_m due to the change in divergence of the transducer; the shape of the intensity curve is slightly skewed in the same way as described in Section 4.
- For $L/L_o = 2$, the intensity rolls off from unity at f_m to 0.75 at the upper and lower frequency limits.
- The variation in intensity in a deflector may be minimized by using a smaller L/L_o ratio but this dictates a higher electrical drive power which is undesirable.
- The variation in intensity may also be minimized by using less than an octave bandwidth, i.e.

$$\frac{\Delta f}{f_o} < \frac{2}{3}.$$

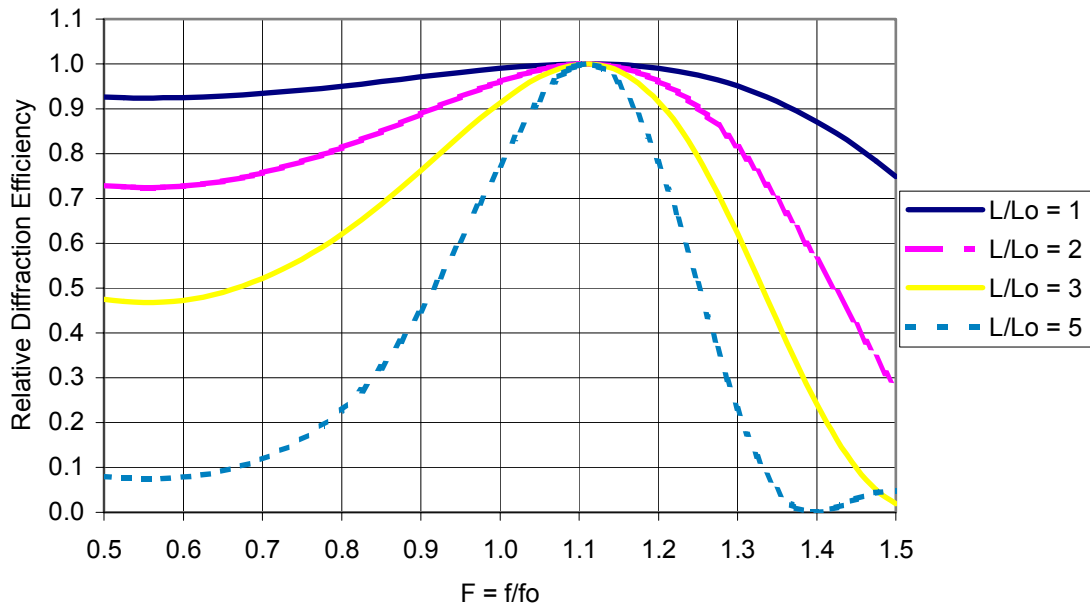


Figure 7: Intensity Variation vs. Frequency in an Octave Bandwidth Acousto-Optic Deflector, Bragg angle adjusted at $f_m = 1.11 f_o$

Finally, we consider the performance of two Isomet deflectors Models 1205C-2, 1206C and 1250C from the foregoing analysis. In these three devices, the advertised frequency swing Δf is less than an octave; therefore the optimum value of F_m is less than the 1.11 value given for the limiting case. More particularly, the $\Delta F = \frac{\Delta f}{f_o}$ for each of the deflectors is tabulated below

Model	f_m	Δf	$\Delta F = \Delta f/f_o$
1205C-2	80MHz	40MHz	0.5
1206C	110MHz	50MHz	0.455
1250C	200MHz	100MHz	0.5

From equation (24) and (25) we compute f_m and F_m for the three cases, and f_m equation (18)^{3*} we compute the roll off at the upper and lower limits. The pertinent results are listed in Table 1 below.

^{3*} $L/Lo = 2$ for all three devices

Intensity Roll-Off for 1205C-2, 1206C and 1250C Deflectors

Model	f_o MHz	Δf MHz	f_m MHz	F_m	F_H	F_L	$I(\alpha)$ at F_H, F_L	dB roll-off at F_H, F_L
1205C-2	80	40	85	1.06	1.25	0.75	0.83	-0.8dB
1206C	110	50	115.5	1.05	1.23	0.77	0.85	-0.7dB
1250C	200	100	212	1.06	1.25	0.75	0.82	-0.86dB

Figure 8 shows the calculation intensity variation across the band for the three devices.

Adjusting the deflector for equalized roll-off can be accomplished in practice in two equally satisfactory ways, either

- 1) The frequency is fixed at f_m and the Bragg angle adjusted to maximize the first order beam intensity, or
- 2) While the frequency is being swept between the prescribed limits, the Bragg angle is adjusted so that the first order beam intensity is equal at both ends of the sweep. Peak intensity will then occur at f_m .

If the second method is used then measurement of the first order beam must be done with a detector sufficiently large to cover the entire scan angle.

6. Multispot Modulator Deflector

The analysis of section 5 applies equally well to acousto-optic modulators in which multiple frequencies are separately gated to produce spatially separated, intensity-modulated spots. In this case however, the acoustic drive power can be independently adjusted at each frequency to compensate for intensity roll-off.

7. Concluding Remarks

In this application note we have described the method by which the intensity roll-off in acousto-optic modulators and deflectors due Bragg angle misalignment may be determined. In the modulator, the roll off is symmetric with respect to angular error on either side of true Bragg. In the deflector the roll-off is asymmetric with respect to change in frequency about the nominal center frequency. The asymmetry is due to the variation in acoustic divergence (of the transducer) with frequency. Roll off in intensity may be equalized at the upper and lower frequency extremes by adjusting the Bragg angle at a frequency slightly higher than the center frequency.

This analysis pertains to single transducer, non-beam-steered, acousto-optic devices. When a lesser variation in intensity vs. frequency is required, acoustic beam steering may be employed.

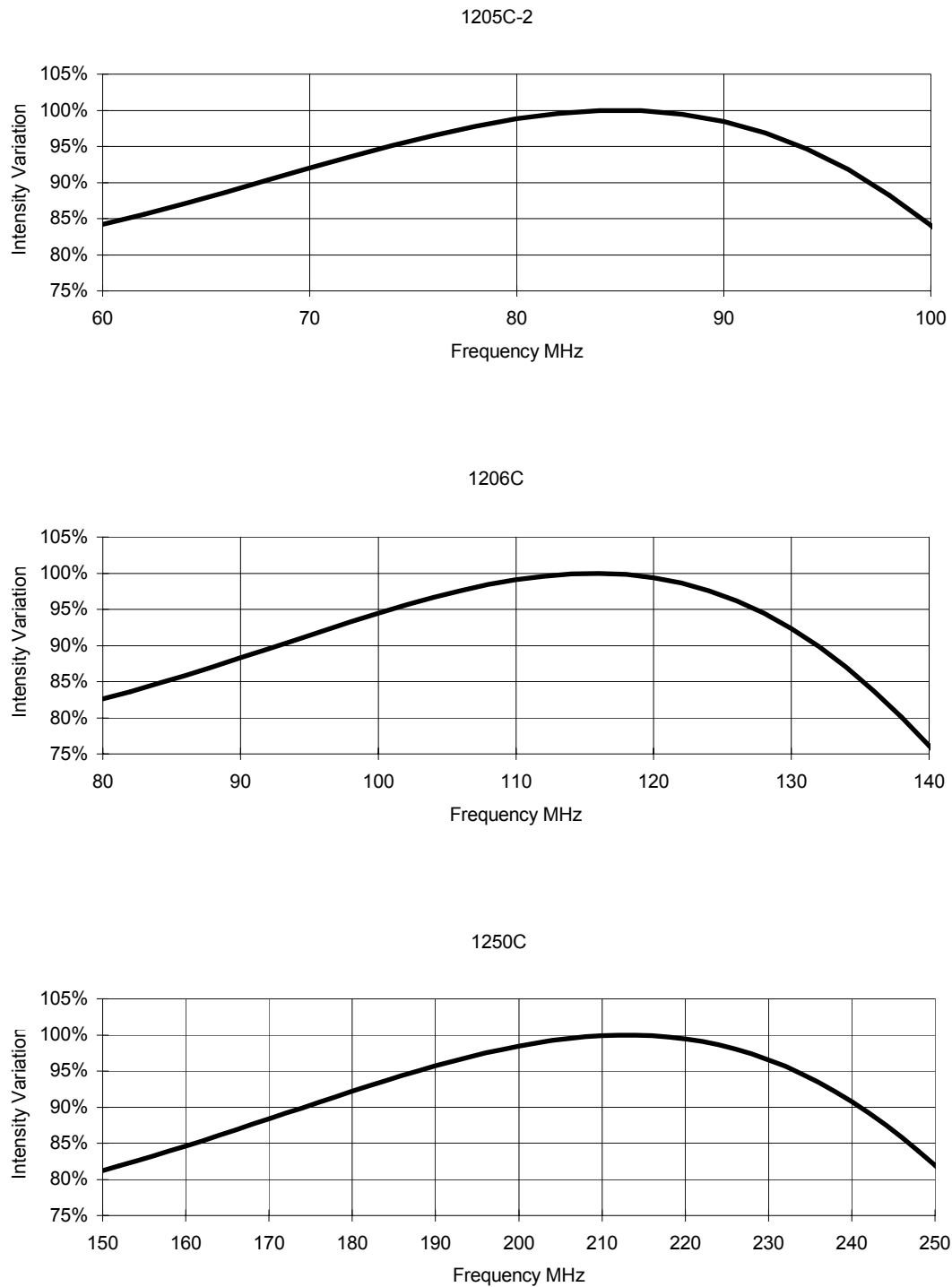


Figure 8: Intensity Variation vs. Frequency in Models 1205C-2, 1206C and 1250C Deflectors

List of Symbols and Abbreviations

F	= Instantaneous Acoustic frequency
f_H	= Upper frequency limit
f_L	= Lower frequency
f_m	= Frequency at which Bragg angle is adjusted for peak diffraction efficiency
f_o	= Center frequency
Δf	= Frequency swing $= f_H - f_L$
F	= Normalized instantaneous acoustic frequency $= f/f_o$
F_H	= Normalized upper frequency limit $= f_H/f_o$
F_L	= Normalized lower frequency limit $= f_L/f_o$
F_m	= Normalized frequency of peak diffraction efficiency $= f_m/f_o$
ΔF	= Normalized frequency swing $= \Delta f/f_o$
H	= Height of acoustic column
I_o	= Zeroth order beam intensity with no acoustic power in interaction medium
I_1	= First order beam intensity
$I(\alpha)$	= First order beam intensity as a function of Bragg angle error
L	= Interaction length
L_o	= Characteristic length $= \frac{\eta}{\lambda} \left(\frac{v}{f_o} \right)^2$
$\frac{L}{L_o}$	= Normalized interaction length
M_2	= Elasto-optic figure of merit
P_a	= Acoustic power in interaction medium
$\text{Sinc}(x)$	$= \text{Sin}(x)/(x)$

Symbols & Abbreviations (cont.)

α = Bragg angle error in medium

β = $\frac{\pi}{2} \cdot \frac{\text{(Full angle optical divergence in medium)}}{\text{(Full angle acoustical divergence in medium)}}$

$$= \frac{\lambda L f_0}{\eta v W_0}$$

Δf = Frequency swing = $f_H - f_L$

ΔF = Normalized frequency swing = $\Delta f / f_0$

δ = Angular misalignment of laser beam from true Bragg angle outside medium

η = Index of refraction

θ_B = Bragg angle in air = $\frac{\lambda f}{2v}$

θ_b = Bragg angle in medium = $\frac{\lambda f}{2\eta v}$

Λ = Acoustic wavelength = v/f

λ = Laser wavelength in air

λ_m = Laser wavelength in medium

β = $K_Q \cdot K_\beta$ = maximum value of I_1/I_0 (see Figure 2)

V = Acoustic velocity in medium

ω_0 = Laser beam waist radius in interaction medium