CSci 243 Homework 3

Due: 10:00 am, Wednesday, Sep 23
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1. (7 points) Determine whether these statements are true or false.

   (a) $\emptyset \in \{\emptyset\} \longrightarrow$ True
   (b) $\{\emptyset\} \in \{\{\emptyset\}\} \longrightarrow$ True
   (c) $\{\emptyset\} \subset \{\emptyset, \{\emptyset\}\} \longrightarrow$ True
   (d) $\{\{\emptyset\}\} \subset \{\emptyset, \{\emptyset\}\} \longrightarrow$ True
   (e) $\{\{\emptyset\}\} \subset \{\{\emptyset\}, \{\emptyset\}\} \longrightarrow$ True
   (f) $\{x\} \subseteq \{x\} \longrightarrow$ True
   (g) $\{x\} \in \{x\} \longrightarrow$ False

2. (6 points) Is each of these sets the power set of a set, where a and b are distinct elements? If yes, give the original set.

   (a) $\emptyset$
   No
   (b) $\{\emptyset, \{a\}\}$
   Yes, in this case the original set is $\{a\}$.
   (c) $\{\emptyset, \{a\}, \{\emptyset, a\}\}$
   No
   (d) $\{\emptyset, \{a\}, \{b\}, \{a, b\}\}$
   Yes, in this case the original set is $\{a, b\}$.

3. (4 points) Show that if $A \subseteq C$ and $B \subseteq D$ then $A \times B \subseteq C \times D$.

   Proof. To prove this, we need to show that for any element of $A \times B$, that element is also in $C \times D$. To do this, let's take some arbitary $(x, y) \in A \times B$. That is, $x \in A$ and $y \in B$. Since $A \subseteq C$ and $B \subseteq D$, then it is clear that $x \in C$ and $y \in D$ and therefore we have that $(x, y) \in C \times D$. In other words, we have proven that $A \times B \subseteq C \times D$.

   □

4. (10 points) For sets A, B, and C, prove that $(B - A) \cup (C - A) = (B \cup C) - A$

   (a) by showing each side is a subset of the other side

   Proof. Let's start by showing $(B - A) \cup (C - A) \subseteq (B \cup C) - A$. Let's take any $x \in (B - A) \cup (C - A)$. Then there are two cases: either $x \in (B - A)$ or $x \in (C - A)$. In either of those cases, x does not reside in A, but in either B or C. Then it is clear that $x \in (B \cup C) - A$.
   Going the other way, to show that $(B \cup C) - A \subseteq (B - A) \cup (C - A)$, again we take some arbitrary element $x \in (B \cup C) - A$. Then $x \in B$ or $x \in C$ but $x \notin A$. This is the equivalent to saying $x \in (B - A)$ or $x \in (C - A)$ so therefore $x \in (B - A) \cup (C - A)$. Therefore, because $(B - A) \cup (C - A) \subseteq (B \cup C) - A$ and $(B \cup C) - A \subseteq (B - A) \cup (C - A)$, then we can conclude that $(B - A) \cup (C - A) = (B \cup C) - A$.

   □

(b) by using a membership table

Proof. The membership table for $(B-A)\cup(C-A)=(B\cup C)-A$ is shown below.

A	B	C	$B-A$	$C-A$	$(B-A)\cup(C-A)$	$B\cup C$	$(B\cup C)-A$
0	0	0	0	0	0	0	0
0	0	1	0	1	1	1	1
0	1	0	1	0	1	1	1
0	1	1	1	1	1	1	1
1	0	0	0	0	0	0	0
1	0	1	0	0	0	1	0
1	1	0	0	0	0	1	0
1	1	1	0	0	0	1	0

Because the columns for $(B-A)\cup(C-A)$ is indentical to the column for $(B\cup C)-A$, we have proven that $(B-A)\cup(C-A)=(B\cup C)-A$. $\square$

5. (5 points) Find these values.

   (a) $\lfloor 1.1 \rfloor = 1$

   (b) $\lceil 1.1 \rceil = 2$

   (c) $\lfloor -0.1 \rfloor = -1$

   (d) $\lceil -0.1 \rceil = 0$

   (e) $\lfloor \frac{1}{2} + \lceil \frac{1}{2} \rceil \rfloor = \lfloor \frac{1}{2} + 1 \rfloor = 1.$

6. (8 points) Determine whether each of these functions from **Z** to **Z** is one-to-one, onto, both, or neither.

   (a) $f(n) = n-1$
   $f(n)$ is both one-to-one and onto.

   (b) $f(n) = n^2+1$.
   $f(n)$ is neither one-to-one nor onto.

   (c) $f(n) = n^3$
   $f(n)$ is both one-to-one and onto.

   (d) $f(n) = \lceil \frac{n}{2} \rceil$
   $f(n)$ is not one-to-one but it is onto.