CSci 243 Homework 1

Due: 10:00 am, Wednesday, Sep 9
Alexander Powell

1. Construct a truth table for each of these compound propositions.

   (a) (Rosen 1.1/31(c), 3 points) $(p \vee \neg q) \rightarrow q$

p	q	$\neg q$	$p \vee \neg q$	$(p \vee \neg q) \rightarrow q$
T	T	F	T	T
T	F	T	T	F
F	T	F	F	T
F	F	T	T	F

   (b) (7 points) $(p \leftrightarrow q) \rightarrow (r \leftrightarrow s)$

p	q	r	s	$p \leftrightarrow q$	$r \leftrightarrow s$	$(p \leftrightarrow q) \rightarrow (r \leftrightarrow s)$
T	T	T	T	T	T	T
T	T	T	F	T	F	F
T	T	F	T	T	F	F
T	T	F	F	T	T	T
T	F	T	T	F	T	T
T	F	T	F	F	F	T
T	F	F	T	F	F	T
T	F	F	F	F	T	T
F	T	T	T	F	T	T
F	T	T	F	F	F	T
F	T	F	T	F	F	T
F	T	F	F	F	T	T
F	F	T	T	T	T	T
F	F	T	F	T	F	F
F	F	F	T	T	F	F
F	F	F	F	T	T	T

2. Using logical identities and laws, show the logic equivalence of

   (a) (Rosen 1.3/16, 5 points) $p \leftrightarrow q$ and $(p \wedge q) \vee (\neg p \wedge \neg q)$

   Proof.

$$
\begin{aligned}
& p \leftrightarrow q \\
& \equiv (p \rightarrow q) \wedge (q \rightarrow p) \\
& \equiv (\neg q \vee p) \wedge (\neg p \vee q) \\
& \equiv \neg q \wedge (\neg p \vee q) \vee p \wedge (\neg p \vee q) \\
& \equiv (p \wedge q) \vee (\neg p \wedge \neg q) \vee (p \wedge \neg p) \vee (q \wedge \neg q) \\
& \equiv (p \wedge q) \vee (\neg p \wedge \neg q)
\end{aligned}
$$

$\square$

(b) (Rosen 1.3/28, 5 points) $p \leftrightarrow q$ and $\neg p \leftrightarrow \neg q$

Proof.

$$
\begin{aligned}
& p \leftrightarrow q \\
& \equiv (q \to p) \wedge (p \to q) \\
& \equiv (p \vee \neg q) \wedge (q \vee \neg p) \\
& \equiv (\neg(\neg p) \vee \neg q) \wedge (\neg(\neg q) \vee \neg p) \\
& \equiv (\neg p \to \neg q) \wedge (\neg q \to \neg p) \\
& \equiv \neg p \leftrightarrow \neg q
\end{aligned}
$$

□

3. Understanding quantified predicates.

(a) (3 points) English to quantified predicates: Use predicates, quantifiers, logical and mathematical operators to express statement: "The difference of two negative integers is not necessarily negative".

Solution:

$\exists n < 0, \exists m < 0\ (n - m \geq 0)$

(b) (7 points) Quantified predicate to English: Give the truth value of each of these statement if the domain of all variables consists of all real numbers.

i. $\forall n \exists m (n^2 < m)$
For any number n, there exists some other number m such that m is greater than n^2.

ii. $\exists n \forall m (n < m^2)$
There exists some number n such that for all m, n is less than m^2.

iii. $\forall n \exists m (n + m = 0)$
For any number n there exists some number m such that the sum of n and m equals 0.

iv. $\exists n \forall m (nm = m)$
There exists some number n such that for all m the product of n and m equals m.

v. $\exists n \exists m (n^2 + m^2 = 4)$
There exist two numbers, n and m such that the sum of their squares equals 4.

vi. $\exists n \exists m (n + m = 4 \wedge n - m = 1)$
There exist two numbers, n and m such that the sum of n and m equals 4 and n minus m equals 1.

vii. $\forall n \forall m \exists p (p = (m + n)/2)$
For any number n and for any number m there exists some other number p such that p equals the sum of m and n divided by 2.

4. (Rosen 1.5/30, 10 points) Rewrite each of these statements so that negations appear only within predicates, i.e., so that no negation is outside a quantifier or an expression involving logical operators.

   (a)
$$\neg\forall x\forall y P(x,y)$$
$$\equiv \exists x\exists y\neg P(x,y)$$

   (b)
$$\neg\forall y\exists x P(x,y)$$
$$\equiv \exists y\forall x\neg P(x,y)$$

   (c)
$$\neg\forall y\forall x(P(x,y)\vee Q(x,y))$$
$$\equiv \exists y\exists x\neg(P(x,y)\vee Q(x,y))$$
$$\equiv \exists y\exists x(\neg P(x,y)\wedge\neg Q(x,y))$$

   (d)
$$\neg(\exists x\exists y\neg P(x,y)\wedge\forall x\forall y Q(x,y))$$
$$\equiv \neg(\exists x\exists y\neg P(x,y))\vee\neg(\forall x\forall y Q(x,y))$$
$$\equiv (\forall x\forall y\neg(\neg P(x,y)))\vee(\exists x\exists y\neg Q(x,y))$$
$$\equiv (\forall x\forall y P(x,y))\vee(\exists x\exists y\neg Q(x,y))$$

   (e)
$$\neg\forall x(\exists y\forall z P(x,y,z)\wedge\exists z\forall y P(x,y,z))$$
$$\equiv \exists x\neg(\exists y\forall z P(x,y,z)\wedge\exists z\forall y P(x,y,z))$$
$$\equiv \exists x\neg(\exists y\forall z P(x,y,z))\vee\neg(\exists z\forall y P(x,y,z))$$
$$\equiv \exists x((\forall y\exists z\neg P(x,y,z))\vee(\forall z\exists y\neg P(x,y,z)))$$