Finite Automata Homework 6

Due: Thursday, Oct 22
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1. Give a description and the state diagram of the PDA for the language

$$A = \{a^i b^j c^k | i = j \text{ or } j = k \text{ where } i, j, k \geq 0\}$$

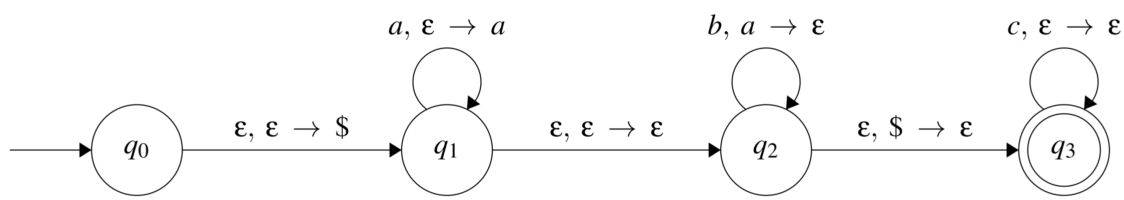
Solution:

We can describe the language A as the language containing all strings of an equal number of $a's$ and $b's$ followed by some number of $c's$ and all strings containing some number of $a's$ followed by $b's$ and then $c's$, where the number of $b's$ is equal to the number of $c's$ (where any of these numbers can be 0).

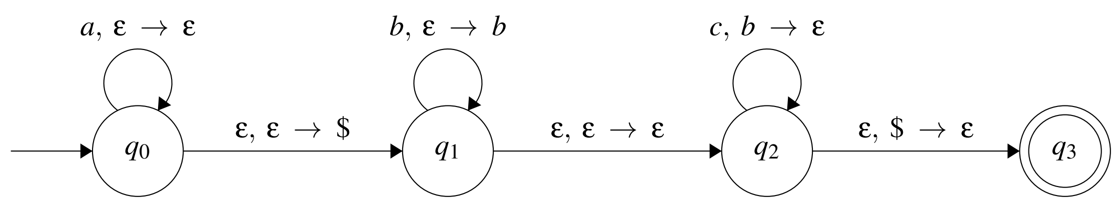
We can write A as the union of the two languages:

$$\{a^i b^i c^k | i, k \geq 0\} \cup \{a^i b^k c^k | i, k \geq 0\}$$

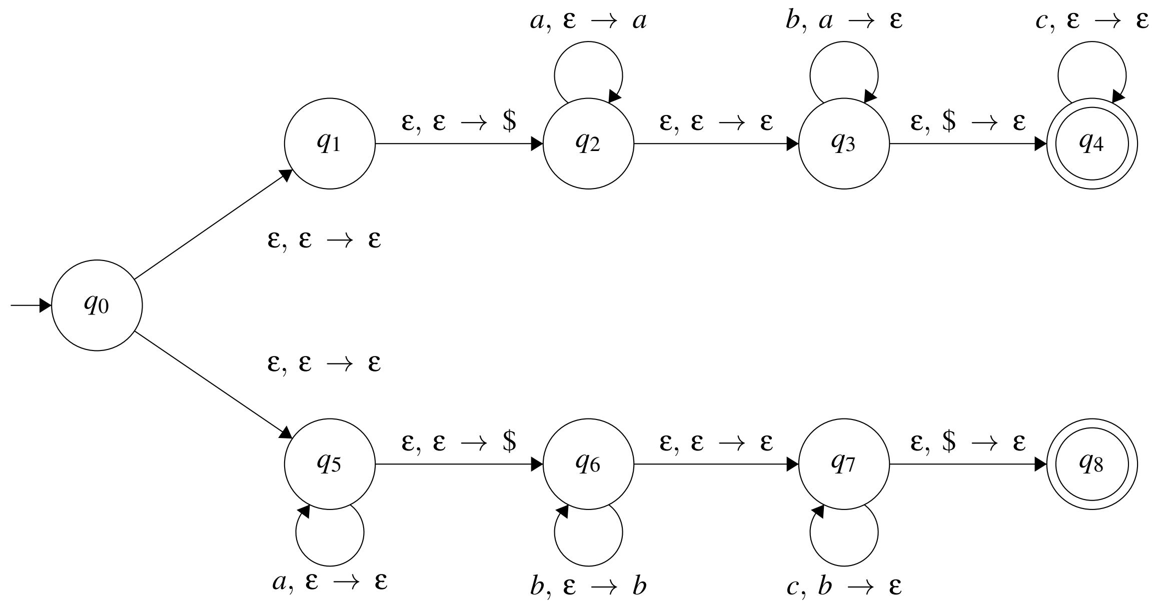
The PDA for the first language can be displayed as:



The PDA for the second language can be displayed as:



Therefore, by putting the two state diagrams together, we get the state diagram for the PDA as a whole, shown below.



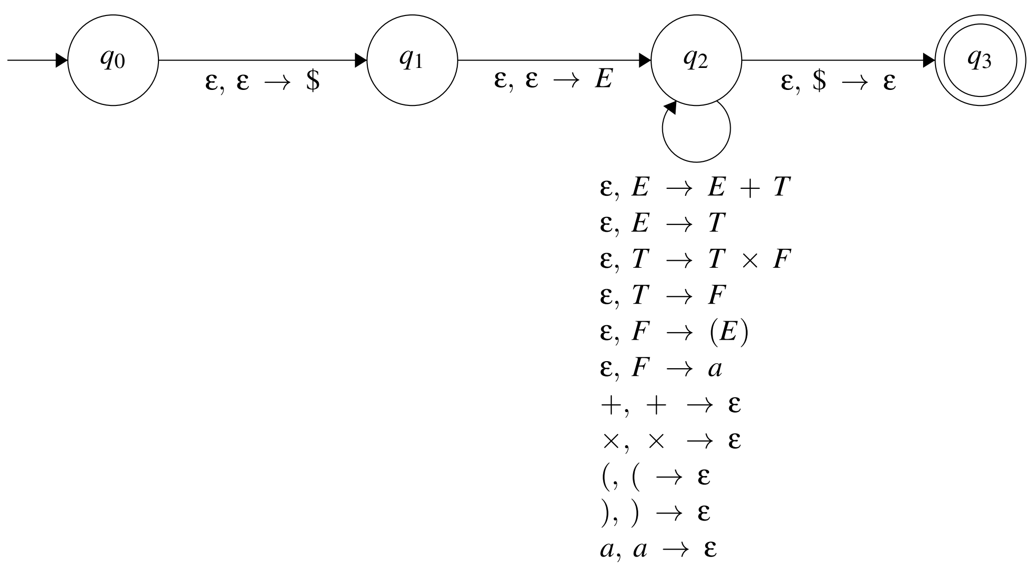
2. Convert G_4 to equivalent PDA using the procedure in theorem 2.20. G_4 is given below:

$$E \to E + T | T$$

$$T \to T \times F | F$$

$$F \to (E) | a$$

Solution:



3. Let CFG G be the grammar below:

$$S \to aSb|bY|Ya$$

$$Y \to bY|aY|\varepsilon$$

Give a simple description of the grammar and give CFG for the complement, $\overline{L(G)}$.

Solution:

$L(G)$ generates all strings over $\{a,b\}$ such that each string begins with n $a's$ and ends with n $b's$ (and n can be 0). Also, each word has either a b followed by any combination of $a's$ and $b's$ or has any combination of $a's$ and $b's$ followed by an a on the inside of the string. Therefore, the grammar for the complement of the language can be written as

$$S \to aSb|bSa|aSa|bSb|ba$$

4. Let $\Sigma = \{a,b\}$. Give the CFG generating the language of strings with twice as many $a's$ as $b's$.

Solution:

To create a grammar for this language, we simply need to make sure everytime a b is added to the string, two $a's$ are added along with it. The context free grammar can be written as:

$$S \to SS|aSaSb|aSbSa|bSaSa|\varepsilon$$

5. Let $E = \{a^i b^j | i \neq j \text{ and } 2i \neq j\}$. Show that E is a context free language.

Solution:

We can rewrite the language as the union of three languages. These are written as:

$$\{a^i b^j | i > j\} \cup \{a^i b^j | i < j \text{ and } 2i > j\} \cup \{a^i b^j | 2i < j\}$$

The first of these languages can be generated with the context free grammar below:

$$S_1 \to aA_1B_1$$

$$A_1 \to aA_1|\varepsilon$$

$$B_1 \to aB_1b|\varepsilon$$

The third language can be represented with the grammar below:

$$S_3 \to A_3B_3b$$

$$A_3 \to aA_3bb|\varepsilon$$

$$B_3 \to B_3b|\varepsilon$$

Finally, we can represent the middle grammar with the following production rules.

$$S_2 \to aA_2b$$

$$A_2 \to aA_2b|aA_2bb|abb$$

So, by putting these three grammars together to generate the union of all three languages, we get the following context free grammar.

$$S \to S_1 | S_2 | S_3$$

$$S_1 \to aA_1B_1$$

$$A_1 \to aA_1 | \varepsilon$$

$$B_1 \to aB_1b | \varepsilon$$

$$S_2 \to aA_2b$$

$$A_2 \to aA_2b | aA_2bb | abb$$

$$S_3 \to A_3B_3b$$

$$A_3 \to aA_3bb | \varepsilon$$

$$B_3 \to B_3b | \varepsilon$$