Finite Automata Homework 10

Due: Thursday, Nov 19
Alexander Powell

1. Prove that $A = \{w_{2i} \mid w_{2i} \notin L(M_i)\}$ is not Turing-recognizable, where w_{2i} is the $2i$th string in the lexicographicorder of binary strings and M_i is the TM whose binary code is w_i.

   Solution:

   Proof. If $A = \{w_{2i} \mid w_{2i} \notin L(M_i)\}$, then we can also write $A = \{< M > | M$ is a TM which doesn't accept $< M >\}$. Let's assume that A is Turing recognizable. Then there exists a TM M such that $A = L(M_i)$, for each i. From the definition of A, we know $w_{2i} \in A$ when $w_{2i} \notin L(M_i)$, which is the case when $w_{2i} \in L(M_i)$. This is a contradiction to our assumption. Therefore, we can conclude that A is not Turing recognizable.

   $\square$

2. Prove that $ES_{TM} = \{$code of M: TM M accepts the empty string$\}$ is undecidable. (Hint: Reduction from A_{TM}.)

   Solution:

   Proof. We begin by reducing from A_{TM}. Let's assume, to the contrary, that M can decide ES_{TM}. Then we can construct a Turing machine D that uses M to show A_{TM} is Turing decideable. The machine D works as follows: D takes in the input $< M, w >$. Inside D, the machine will process each string. M runs on w; if M accepts w, then D accepts, else D rejects.

   Since this machine decides A_{TM}, we have a contradiction to the statement that A_{TM} is non Turing decideable. Therefore, we can conclude that our assumption is false and this ES_{TM} is not decideable.

   $\square$

3. Let $T = \{< M > \mid M$ is a TM that accepts w^R whenever it accepts w $\}$. Show that T is undecidable.

   Solution:

   Proof. We will use a proof by contradiction. Let's assume, to the contrary, that T is decideable. Then there exists some Turing machine D that decides T. We can reduce A_{TM} to T, where

   $$A_{TM} = \{< M, w > \mid M \text{ is a TM that accepts } w\},$$

   and we know A_{TM} is undecidable. Also, we will denote the machine that accepts A_{TM} as A. The description of A is as follows:
   It will take in an input of $< M, w >$ (M is a Turing machine and w is a string).

   (a) First, A will contruct TM R. On input x:
   (b) If x is in a reversible form, then R accepts. If not, then w is run through R and either accepts of rejects.
   (c) Then T is run on the code of $< R >$. If T accepts then A accepts, and if T rejects then A rejects.

   Thus, we have shown that A_{TM} is decidable, which is a contradiction to our original assumption. Therefore, we can conclude that T is undecidable. $\square$