Math 214 Homework 2

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Solve the following problems. Each problem is 4 points.

1. Consider three statements:

$$P\text{: 15 is odd,} \quad Q\text{: 21 is prime,} \quad \text{and } R\text{: } \frac{1}{2} \in \mathbb{N}.$$

State each of the following in words, and determine whether they are true or false.

(a) $(\sim P) \vee Q$

Solution: 15 is not odd or 21 is prime. This statement is False.

(b) $P \wedge (\sim Q \vee R)$

Solution: 15 is odd and 21 is not prime or $\frac{1}{2}$ is a natural number. This statement is True

(c) $(P \wedge Q) \Rightarrow (\sim R)$

Solution: 15 is odd and 21 is prime implies that $\frac{1}{2}$ is not a natural number. This statement is True.

(d) $(P \vee Q) \Leftrightarrow R$

Solution: 15 is odd or 21 is prime if and only if $\frac{1}{2}$ is a natural number. This statement is False.

2. Consider a statement: "If I finish my homework, then I will go to the mall or I will play tennis unless it rains."

(a) Define statements P, Q, R, S so that the above statement is in a form $(P \wedge Q) \Rightarrow (R \vee S)$.

Solution:
P: I finish my homework
Q: It does not rain
R: I will play tennis
S: I will go to the mall

(b) Find the negation of $(P \wedge Q) \Rightarrow (R \vee S)$ by using Theorems 2.18 and 2.21, and write the negation of statement above in words.

Solution: The negation of $(P \wedge Q) \Rightarrow (R \vee S)$ is written below:

$$\sim(\sim(P \wedge Q) \vee (R \vee S))$$
$$= (\sim(\sim(P \wedge Q)) \wedge \sim(R \vee S))$$
$$= ((P \wedge Q) \wedge ((\sim R) \wedge (\sim S)))$$
$$= (P \wedge Q \wedge \sim R \wedge \sim S)$$

I will finish my homework and it will not rain and I will not play tennis and I will not go to the mall.

3. (a) For the open sentence $P(x) : 3x - 2 > 4$ over the domain $\mathbb{Z}$, determine the values of x for which $P(x)$ is true.

Solution:

$$P(x) : 3x - 2 > 4$$
$$3x > 6$$
$$x > 2$$

Therefore $P(x)$ is true for the values of $x > 2$ over the domain $\mathbb{Z}$.

Alternatively, $P(x)$ is true for $A = \{x > 2, x \in \mathbb{Z}\}$.

(b) Express the following quantified statement in logic symbols: For every integer $n \geq 2$, there exists an integer m such that $n < m < 2n$.

Solution:

$$\text{Let } S = \{n \in \mathbb{Z} : n \geq 2\}$$
$$\forall n \in S, \exists m \in \mathbb{Z}, n < m < 2n$$

4. For statements P, Q and R, use a truth table to show that $P \Rightarrow (Q \vee R)$ and $(\sim Q) \Rightarrow ((\sim P) \vee R)$ are logically equivalent. (Hint: use sample homework tex file for typing a truth table)

Solution:

P	Q	R	$Q \vee R$	$P \vee R$	$\sim P$	$(\sim P) \vee R$	$P \Rightarrow (Q \vee R)$	$\sim Q \Rightarrow ((\sim P) \vee R)$
T	T	T	T	T	F	T	T	T
T	T	F	T	T	F	F	T	T
T	F	T	T	T	F	T	T	T
T	F	F	F	T	F	F	F	F
F	T	T	T	T	T	T	T	T
F	T	F	T	F	T	T	T	T
F	F	T	T	T	T	T	T	T
F	F	F	F	F	T	T	T	T

Therefore, $P \Rightarrow (Q \vee R)$ and $\sim Q \Rightarrow ((\sim P) \vee R)$ are logically equivalent.

5. In each of the following, two open sentences $P(x)$ and $Q(x)$ over a domain S are given. For each part, determine $T = \{x \in S : P(x) \Rightarrow Q(x) \text{ is true}\}$.

(a) $P(x) : x - 3 = 4; \quad Q(x) : x \geq 8; \quad S = \mathbb{R}.$

Solution: $T = (-\infty, 7) \cup (7, \infty)$

(b) $P(x) : x \in [-1, 2]; \quad Q(x) : x^2 \leq 2; \quad S = [-1, 1].$

Solution: $T = (-\infty, \sqrt{2}] \cup (2, \infty)$

6. Consider the quantified statement: There exists an integer n such that n is odd and n^3 is even.

(a) Express the statement above in logic symbols.

Solution:

$$O = \{2n + 1 : n \in \mathbb{Z}\}$$

$$\text{And } E = \{2n : n \in \mathbb{Z}\}$$

$$(\exists n \in \mathbb{Z}) \wedge (n \in O) \wedge (n^3 \in E)$$

(b) Write the negation of the statement above in logic symbols and in words.

Solution: The negation of the statment above is:

$$\sim ((\exists n \in \mathbb{Z}) \wedge (n \in O) \wedge (n^3 \in E))$$

$$\equiv (\forall n \in \mathbb{Z}) \vee (n \notin O) \vee (n^3 \notin E)$$

In words, for every $n \in \mathbb{Z}$, n is not odd or n^3 is not even.

7. (extra credit) A very special island is inhabited only by knights and knaves. Knights always tell the truth, and knaves always lie. You meet three inhabitants: Bozo, Carl and Joe. Bozo says that Carl is a knave. Carl tells you, ‘Of Joe and I, exactly one is a knight.’ Joe claims, ‘Bozo and I are different.’Who are knights, and who are knaves? (To get full credit, you need to use a truth table)