Math 214 Homework 1

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Solve the following problems. Please remember to use complete sentences and good grammar. Each problem is 4 points.

1. Write each of the following sets as specified.

   (a) List the elements in the set $A = \{n \in \mathbb{N} : n^3 < 100\}$.

   Solution: Set A can be written as $A = \{1, 2, 3, 4\}$.

   (b) Describe the set $B = \{-3, -2, -1, 0, 1, 2, 3\}$ using the notation $\{n : p(n)\}$, where $p(n)$ specifies the property of element n.

   Solution: The set can be described as $B = \{n \in \mathbb{Z} : -3 \leq n \leq 3\}$.

2. Recall that for a set A, $\mathcal{P}(A)$ denotes the power set of A.

   (a) Find $\mathcal{P}(\mathcal{P}(\{1\}))$ and its cardinality.

   Solution: If $\mathcal{P}(A) = \{\emptyset, \{1\}\}$, then $\mathcal{P}(\mathcal{P}(A)) = \{\emptyset, \{\emptyset\}, \{\{1\}\}, \{\emptyset, \{1\}\}\}$.

   (b) Give examples of a set S such that $S \subseteq \mathcal{P}(\mathbb{N})$ and $|S| = 5$.

   Solution: The set $S = \{\{1\}, \{2\}, \{3\}, \{4\}, \{5\}\}$ has cardinality of 5 and it is a subset of $\mathcal{P}(\mathbb{N})$.

   (c) Give examples of a set S such that $S \in \mathcal{P}(\mathbb{N})$ and $|S| = 5$.

   Solution: The set $S = \{1, 2, 3, 4, 5\}$ has cardinality of 5 and belongs to $\mathcal{P}(\mathbb{N})$.

3. The following problems involve set operations.

   (a) Given an example of three sets A, B, and C such that $B \neq C$ but $B - A = C - A$.

   Solution: Let $A = \{1, 2\}$, let $B = \{3\}$ and let $C = \{2, 3\}$.
   With these three sets, $B - A = C - A = \{3\}$, and $B \neq C$.

   (b) Let $A = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}\}$. Find $\{\emptyset, \{\emptyset\}\} \cap A$.

   Solution: $\{\emptyset, \{\emptyset\}\} \cap \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}\} = \{\emptyset, \{\emptyset\}\}$

4. For a real number r, define S_r to be the interval $[r-1, r+2]$. Let $A = \{1, 3, 4\}$. Determine $\bigcup_{\alpha \in A} S_\alpha$ and $\bigcap_{\alpha \in A} S_\alpha$.

   Solution: If $A = \{1, 3, 4\}$ and S_r is the interval $[r-1, r+2]$ then the following can be calculated:

   $$S_1 = [1-1, 1+2] = [0, 3]$$

   $$S_3 = [3-1, 3+2] = [2, 5]$$

   $$S_4 = [4-1, 4+2] = [3, 6]$$

   Therefore, $\bigcup_{\alpha \in A} S_\alpha = [0,3] \cup [2,5] \cup [3,6] = [0,6]$ and $\bigcap_{\alpha \in A} S_\alpha = [0,3] \cap [2,5] \cap [3,6] = \{3\}$.

5. For two sets A and B, recall that $A \times B$ is the Cartesian product of A and B.

   (a) Let $A = \{a, b\}$. Determine $A \times \mathcal{P}(A)$.

   Solution: If $A = \{a, b\}$ and $\mathcal{P}(A) = \{\{a, b\}, \{a\}, \{b\}, \emptyset\}$ then

   $$\begin{aligned} A \times \mathcal{P}(A) &= \{a, b\} \times \{\{a, b\}, \{a\}, \{b\}, \emptyset\} \\ &= \{(a, \{a, b\}), (a, \{a\}), (a, \{b\}), (a, \emptyset), (b, \{a, b\}), (b, \{a\}), (b, \{b\}), (b, \emptyset)\} \end{aligned}$$

   (b) Let $A = \{0, 1\}$ and $B = [0, 2] \cap [1, 3]$. Describe the graph of $A \times B$.

   Solution: The graph of $A \times B$ is the union of two parallel line segments, one from $(0, 1)$ to $(0, 2)$ and the other from $(1, 1)$ to $(1, 2)$.

   (c) Let $A = \{0, 1\}$, $B = (0, 1) \cap A$ and $C = \mathbb{R}$. What is $A \times B \times C$.

6. Determine all different partitions of the set $\{1, 2, 3\}$.

   Solution: All partitions of the set $\{1, 2, 3\}$ are listed below:

   $$\{\{1\},\{2\},\{3\}\}$$

   $$\{\{1,2\},\{3\}\}$$

   $$\{\{1,3\},\{2\}\}$$

   $$\{\{1\},\{2,3\}\}$$

   $$\{\{1,2,3\}\}$$