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A curve, C, has equation $y = 2x^2 + 8k^2x - 3$ where k is a constant.

Show that when $k = 0$, the turning point on C has coordinates (0, -3).

$y = 2x^2 + 8k^2x - 3$

$\frac{dy}{dx} = 4x + 8k^2$

$4x + 8(0)^2 = 4x = 0$

$x = 0$

$2x^2 + 8(0)^2x - 3$

$y = 2x^2 - 3 = y = -3$

b

4 marks

how that when $k \neq 0$, the turning point on C must have a negative x-coordinate.

ow did you do?

uck? View related notes

View answer

c

2 marks

hen $k \neq 0$ determine whether or not the y-coordinate of the turning point is negative.

ow did you do?

$y = 2x^2 + 8k^2 x - 3 + 8k^2(-2k^2) - 3$

$y = 2(-2k^2)^2 + -16k^4 - 3$

$8k^4$

$y = -8k^4 - 3 < 0$

A curve, C, has equation $y = 2x^2 + 8k^2x - 3$ where k is a constant.

Show that when $k = 0$, the turning point on C has coordinates (0, -3).

$\frac{dy}{dx} = 4x + 8k^2 = 0$

$4x = -8k^2$

$x = \frac{-8k^2}{4} = \underline{-2k^2}$

$k \neq 0 \Rightarrow k^2 > 0 \Rightarrow -2k^2 = x < 0$

$k \neq 0 \Rightarrow k^2 > 0 \Rightarrow k^4 > 0$

$k^4 > 0$

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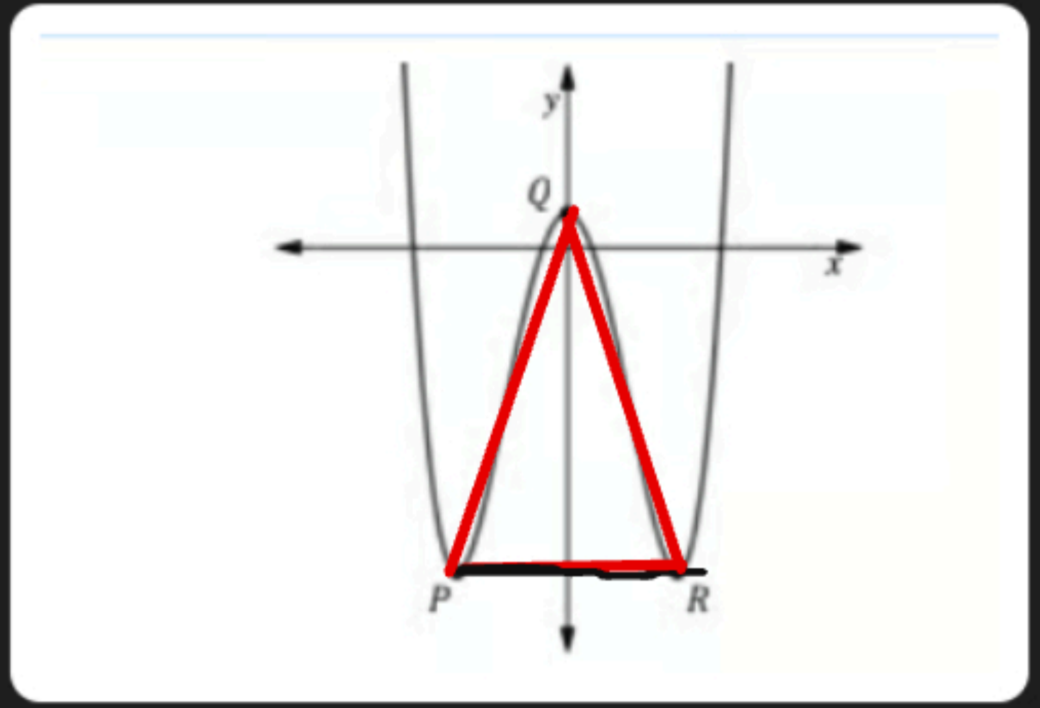
2

7 marks

Part of the graph with equation $y = 2x^4 - 16x^2 + 3$ is shown below.



The graph has three stationary points, indicated on the graph by points P, Q and R.
Find the area of the triangle PQR.

$$\frac{dy}{dx} = 8x^3 - 32x$$

$$8x^3 - 32x = 0$$

$$8x(x^2 - 4) = 0$$

$$8x = 0 \Rightarrow x = 0$$

$$x^2 - 4 = 0 \Rightarrow x^2 = 4$$

$$x = \pm 2$$

$$P = 2(-2)^4 - 16(-2)^2 + 3 = 32 - 64 + 3 = -29$$

$$Q = 2(0)^4 - 16(0)^2 + 3 = 3$$

$$R = 2(2)^4 - 16(2)^2 + 3 = 32 - 64 + 3 = -29$$

$P = (-2, -29)$

$Q = (0, 3)$

$R = (2, -29)$

$\frac{1}{2} \times 4 \times 34 = 68$ units^2

e point A is the only stationary point on the curve with equation $y = kx^2 + \frac{16}{x}$ where k is a

nstant.

en that the coordinates of A are $\left(\frac{2}{3}, a\right)$

d the value of a.

ow your working clearly.

a = ...

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$$\frac{a}{\left(x/y\right)} = a \times \frac{y}{x}$$

e.g. $\frac{1}{5/8} = \frac{8}{5}$

$$y = kx^2 + \frac{16}{x}$$

$$= kx^2 + 16x^{-1}$$

$x = \frac{2}{3}$

$$\frac{dy}{dx} = 2kx + -16x^{-2}$$

$$= 2k\left(\frac{2}{3}\right) + -16 \times \left(\frac{2}{3}\right)^{-2} =$$

$$\left(\frac{2}{3}\right)^{-2} = \frac{1}{\left(\frac{2}{3}\right)^2} = \frac{1}{\left(\frac{4}{9}\right)} = \frac{9}{4}$$

$16\left(\frac{9}{4}\right)$

$$\frac{4}{3}k - 36 = 0$$

$$\frac{4}{3}k = 36$$

$$4k = 108$$

$$y = 27x^2 + \frac{16}{x}$$

$$y = 27\left(\frac{2}{3}\right)^2 + \frac{16}{\left(\frac{2}{3}\right)} =$$

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7

6 marks

The curve **C** has equation $y = ax^3 + bx^2 - 12x + 6$ where a and b are constants.

The point A with coordinates (2, −6) lies on **C**.

The gradient of the curve at A is 16.

Find the y coordinate of the point on the curve whose x coordinate is 3.
Show clear algebraic working.

$y = ax^3 + bx^2 - 12x + 6$

$\frac{dy}{dx} = 3ax^2 + 2bx - 12$

$3a(2)^2 + 2b(2) - 12 = 16$

$12a + 4b - 12 = 16$

$12a + 4b = 28$

$3a + b = 7$

$-6 = a(2)^3 + b(2)^2 - 12(2) + 6$

$8a + 4b - 24 + 6 = -6$

$8a + 4b - 18 = -6$

$8a + 4b = 12$

$2a + b = 3$

$3a + b = 7$

$2a + b = 3$

$a = 4$

$3(4) + b = 7$

$\cancel{2(4) + b = 3}$

$12 + b = 7 \longrightarrow b = -5$

$y = 4(3)^3 + -5(3)^2 - 12(3) + 6$

$108 - 45 - 36 + 6$

$y = 33$

64

8

5 marks

A particle P is moving along a straight line.

The fixed point O lies on the line.

At time t seconds ($t \geqslant 0$), the displacement of P from O is s metres where

$$s = t^3 - 9t^2 + 33t - 6$$

Find the minimum speed of P.

.... m/s

$v = 3t^2 - 18t + 33$

$\frac{dv}{dt} = 6t - 18 = 0$

$t = 3$

$v = 3(3)^2 - 18(3) + 33$

$v = 27 - 54 + 33$

$v = 6$ m/s

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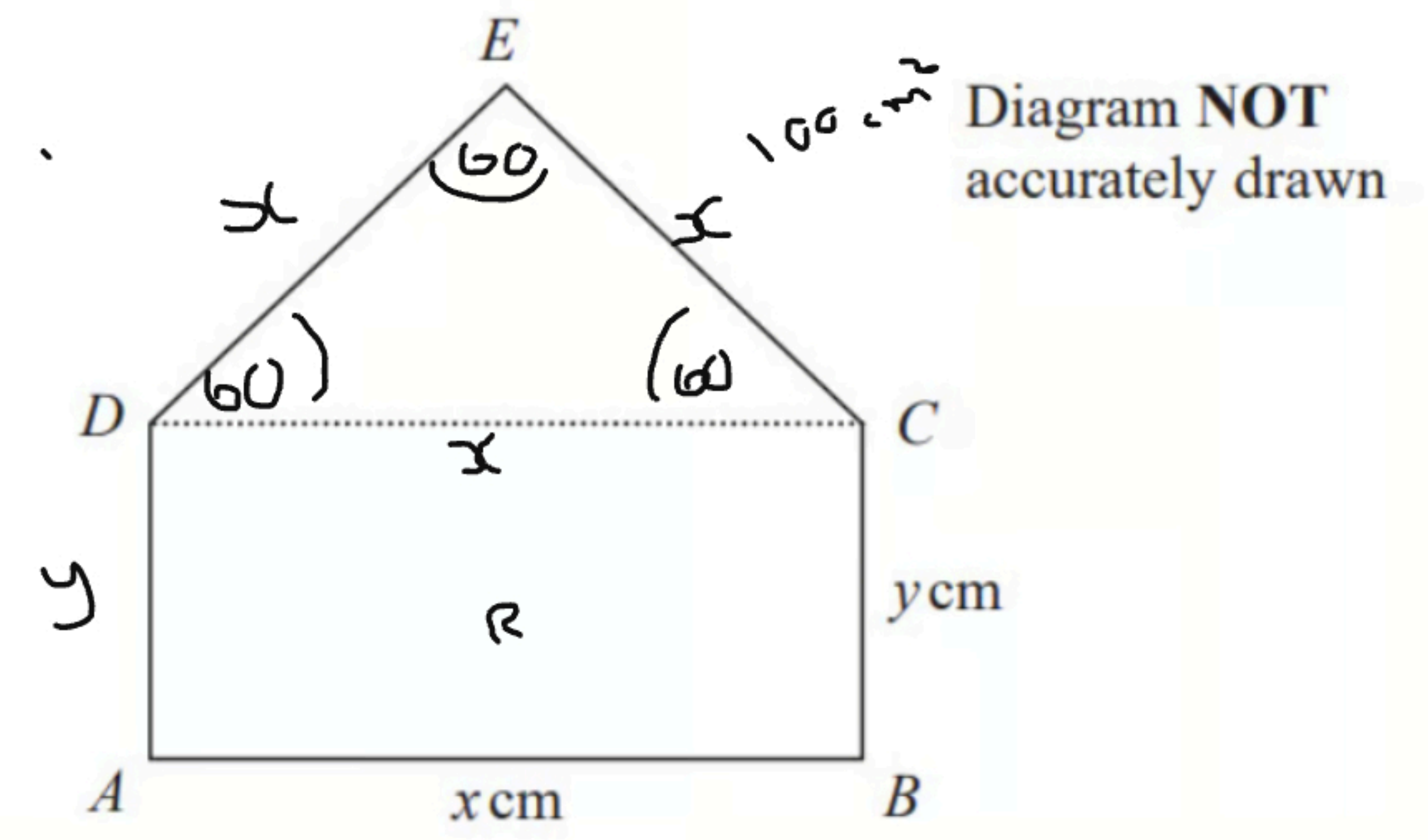
a

3 marks

$BCED$ is a five-sided shape.



BCD is a rectangle.
CD is an equilateral triangle.

$B = x$ cm $BC = y$ cm

e perimeter of $ABCED$ is 100 cm.
e area of $ABCED$ is R cm^2

ow that $R = \frac{x}{4}\left(200 - [6 - \sqrt{3}]x\right)$

$x + x + y + y + x = 3x + 2y = 100$

$2y = 100 - 3x$

$y = \frac{100 - 3x}{2}$

$R = \frac{100 - 3x}{2} \times x + \frac{1}{2}x^2 \sin(60)$

$\frac{100 - 3x}{2} \times x + \frac{1}{2}$

$\frac{\sqrt{3}}{2}$

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9b

3 marks

(i) Find the value of x for which R has its maximum value.

Give your answer in the form $\dfrac{p}{q-\sqrt{3}}$ where p and q are integers.

x = ... [2]

(ii) Explain why the maximum value of R is given by this value of x.

[1]

6 marks

article moves along a straight line.
e fixed point O lies on this line.
e displacement of the particle from O at time t seconds , $t \geqslant 0$, is s metres where

$$s = t^3 + 4t^2 - 5t + 7$$

ime T seconds the velocity of P is V m/s where $V \geqslant -5$

d an expression for T in terms of V.

e your expression in the form $\dfrac{-4 + \sqrt{k + mV}}{3}$ where k m and are integers to be found.

$T =$ ....

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