

# sample\_ACVF

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## Sample Autocovariance Function

The sample autocovariance function is an important tool in time series analysis. It measures the covariance between values of a time series across different lags. The function `sample_ACVF` is an implementation of the sample autocovariance function.

The output of the sample autocovariance function is dependent on the considered time series and lag:

- If the output value is near zero then we can assume that there is a weak relation between the points  $X_t$  and  $X_{t+h}$ .
- If the output value is big and positive then we can assume that there is a strong relation between the points  $X_t$  and  $X_{t+h}$ .
- If the output value is big and negative then we can assume that there is a strong relation between the points  $X_{t+h}$  and  $X_t$ .

### Example 1: small example

`X` represents a time series and `h` is an integer subset of  $(-n, n)$ , where `n` represents the length of `X`. By calling `sample_ACVF(X, h)` the sample autocovariance function on `X` for `h` is computed.

```
X <- c(2, 4, 6, 8, 10)
h <- 2

zeitreihen::sample_ACVF(X, h)
```

```
##      2
## -0.8
```

The output is the value `-0.8`, which is the computation of the `sample_ACVF` on `X` for `h`, and has named attribute `2`, which is the corresponding lag `h`.

What happens:

- Within the function, it is checked if `X` and `h` have the required format.
- The mean of `X`, which is  $\bar{x}$ , is computed.
- The length of `X`, which is  $n$ , is extracted from `X`.
- For each  $h_i \in h$ : we compute  $n^{-1} \sum_{t=1}^{n-|h_i|} (x_{t+|h_i|} - \bar{x}) \cdot (x_t - \bar{x})$ 
  - We considered the vector, which is formed by the last  $n - h$  values from `X`, and subtract component wise the mean of `X` from it.
  - We considered the vector, which is formed by the first  $n - h$  values from `X`, and subtract component wise the mean of `X` from it.
  - We compute the dot product of the vectors
  - Last step, we divide the dot product of the vectors by  $n$
- At last, the values are returned where the `names` attribute is assigned `h`.

Required Format:

- `X` must be an atomic vector of positive length and only allows either numeric or complex values also `NA`, `Inf` or `NaN` aren't allowed.

- `h` must be is a numeric vector with unique values in interval  $(-n,n)$ , where `n` is the length of the time series `X` and `NA`, `Inf` or `NaN` are not allowed.

### Example 2: medium example

```
X <- c(2 + 1i, 2 + 4i, 6, 8i, 10)
h <- c(0, 3, 1)
```

```
zeitreihen::sample_ACVF(X, h)
```

```
##           0           3           1
## 3.360-16.80i 1.656+ 1.84i 0.392+14.48i
```

Hint: `sample_ACVF` doesn't return the values sorted on `h`. It returns the values in the order of `h`.

### Example 3: big example

```
X <- c(0:10, 20:30, 40:50, 50:58)
```

```
zeitreihen::sample_ACVF(X)
```

```
##           0           1           2           3           4           5
## 360.406463 337.432580 314.077745 290.365768 266.320457 241.965622
##           6           7           8           9          10          11
## 217.325073 192.422619 167.282070 141.927235 115.720360  89.323008
##          12          13          14          15          16          17
##  67.947765  46.643950  25.435374   4.345845 -16.600826 -37.380831
##          18          19          20          21          22          23
## -57.970360 -78.345603 -98.482750 -112.403912 -125.825073 -131.605078
##          24          25          26          27          28          29
## -136.623178 -140.855564 -144.278426 -146.867954 -148.600340 -149.451774
##          30          31          32          33          34          35
## -149.398445 -148.416545 -140.528183 -131.449344 -121.923226 -111.182823
##          36          37          38          39          40          41
## -99.204325 -85.963921 -71.437804 -55.602162 -38.433188 -19.907070
```

Calling the `sample_ACVF`, where `h` is omitted is allowed. Within `sample_ACVF` we use the `[0,n-1]` for `h`.  
Hint: The Output will get big so mind to get only the  $h_i$  that are needed.

### Example 4: Incorrect Inputs

```
X <- list(2, 4, 6, 8, 10)
h <- 2
```

```
zeitreihen::sample_ACVF(X, h)
```

```
## Error in zeitreihen::sample_ACVF(X, h): X must be an atomic vector
```

```
X <- c(2, 4, 6, NA, 10)
h <- 2
```

```
zeitreihen::sample_ACVF(X, h)
```

```
## Error in zeitreihen::sample_ACVF(X, h): X may not contain NAs
```

```
X <- c(2, 4, 6, 8, 10)
h <- 5
```

```
zeitreihen::sample_ACVF(X, h)
```

```
## Error in zeitreihen::sample_ACVF(X, h): All values of h must be from the interval (-length(X), length(X))
```