



# VBF Higgs production at NNLO ... and beyond

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Based on [PRL115\(082002\)](#) and [PRL117\(072001\)](#) in collaboration with  
Matteo Cacciari, Frédéric Dreyer, Gavin Salam & Giulia Zanderighi

... and work in progress with Barbara Jäger and Giulia Zanderighi.

# This work

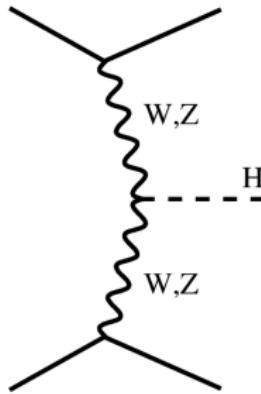
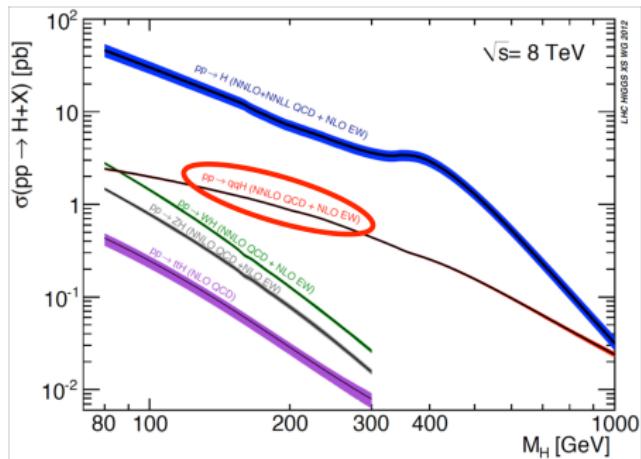
- first semi-differential  $N^3LO$  calculation of VBF Higgs production in the **structure function approach**
- first fully differential NNLO calculation of VBF Higgs production using a novel “**projection-to-Born**” method
- Some (very) preliminary results on MiNLO improved VBF  $H_{jjj}$  production

## What to expect

- $N^3LO$  corrections are at the permille level and well within the scale uncertainty band of the NNLO prediction
- the associated scale uncertainty is reduced by a factor 5 at  $N^3LO$
- NNLO corrections are sizeable,  $\mathcal{O}(10\%)$ , and outside NLO band
- the corrections are (almost everywhere) negative
- only moderate shrinkage of NNLO bands compared to NLO bands
- the “**projection-to-Born**” method is quite general and can be extended to other similar processes



# Reasons to study VBF



- largest cross section at tree-level and second-largest of all channels
- distinct signature of two forward jets
- tagging reduces backgrounds (eg  $H \rightarrow b\bar{b}$ )
- non-zero Higgs transverse momentum at lowest order
- sensitive to CP properties of the Higgs through correlation of forward jets
- sensitive to trilinear Higgs self-coupling through loop-corrections



# Reasons to study VBF

Production process	ATLAS+CMS	ATLAS	CMS
$\mu_{ggF}$	$1.03^{+0.17}_{-0.15}$	$1.25^{+0.24}_{-0.21}$	$0.84^{+0.19}_{-0.16}$
$\mu_{VBF}$	$1.18^{+0.25}_{-0.23}$	$1.21^{+0.33}_{-0.30}$	$1.13^{+0.37}_{-0.34}$
$\mu_{WH}$	$0.88^{+0.40}_{-0.38}$	$1.25^{+0.56}_{-0.52}$	$0.46^{+0.57}_{-0.54}$
$\mu_{ZH}$	$0.80^{+0.39}_{-0.36}$	$0.30^{+0.51}_{-0.46}$	$1.35^{+0.58}_{-0.54}$
$\mu_{ttH}$	$2.3^{+0.7}_{-0.6}$	$1.9^{+0.8}_{-0.7}$	$2.9^{+1.0}_{-0.9}$

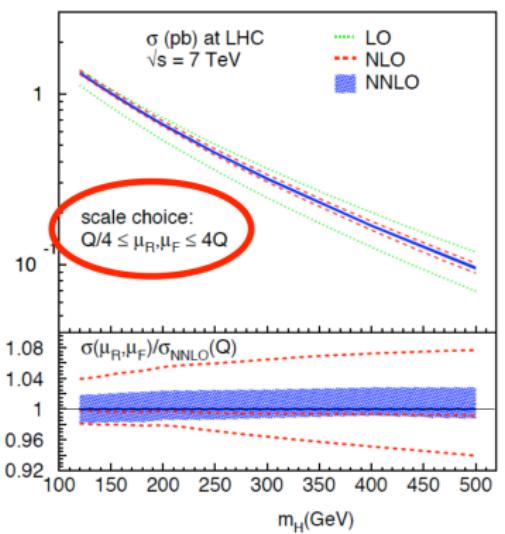
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# Inclusive NNLO VBF Higgs Production

Until recently VBF Higgs production was only known inclusively at NNLO.

[Bolzoni et al. (2010)]



- the calculation suggests tiny renormalisation and factorisation scale variations ( $\sim 1 - 2\%$ )
- NNLO results well within NLO band
- result obtained in the structure function approach

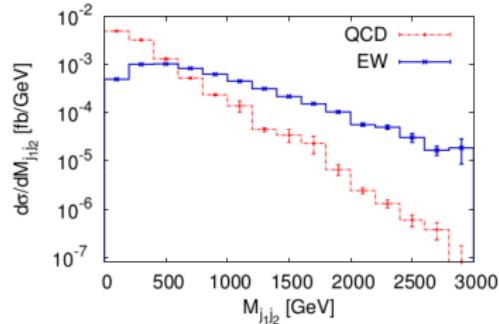
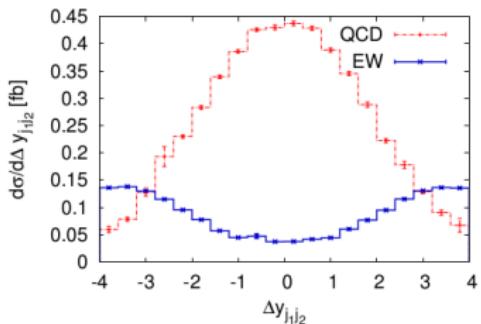
No cuts can be applied to the calculation, as it is totally inclusive over hadronic final states with the same vector boson momenta.



# VBF Cuts

Due to huge QCD backgrounds a set of very selective cuts have to be applied. Typical cuts are:

- jets defined with anti- $k_t$ ,  $R = 0.4$  and  $p_t > 25 \text{ GeV}$
- two hardest jets within  $|y| < 4.5$
- high dijet invariant mass,  $M_{j_1 j_2} > 600 \text{ GeV}$ , and separation,  $\Delta y_{j_1 j_2} > 4.5$
- hardest jets in opposite hemispheres,  $y_{j_1} y_{j_2} < 0$



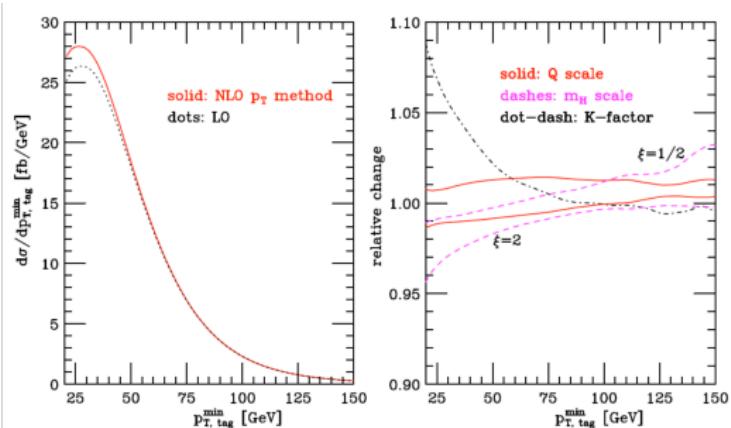
[Jäger, Zanderighi (2011)]



# Exclusive NLO VBF Higgs Production

To enable the application of **realistic VBF cuts** one has to be fully differential. Until recently differential VBF Higgs production was known to **NLO(+PS)**.

[Figy, Oleari, Zeppenfeld (2003)]



Calculation suggests small uncertainties from missing higher order corrections ( $\sim 2\%$ ).



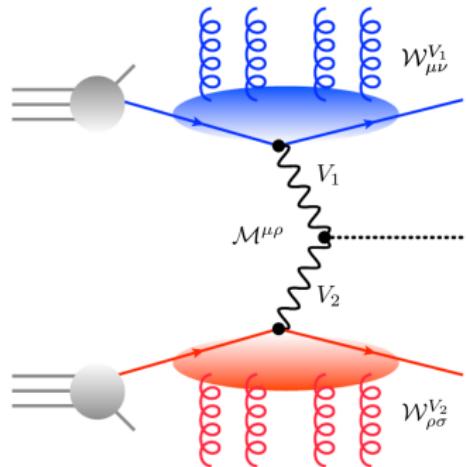
# Structure Function Approach

One can think of VBF Higgs production as a double Deep Inelastic Scattering (**DIS** $\times$ **DIS**) with no cross-talk between the upper and lower sectors.

[Han, Valencia, Willenbrock (1992)]

- this picture is accurate to more than 1%

[Bolzoni et al. (2012)], [Ciccolini, Denner, Dittmaier (2008)], [Harlander et al. (2008)], [Andersen et al. (2008)]



- the factorisation of the two sectors is exact if one imagines two copies of QCD,  $\text{QCD}_1$  and  $\text{QCD}_2$ , respectively for the upper and lower sectors.
- all DIS coefficients are known to to  $\text{N}^3\text{LO}$ .
- as the DIS coefficients are inclusive over the hadronic final state, **the calculation cannot provide differential results**.

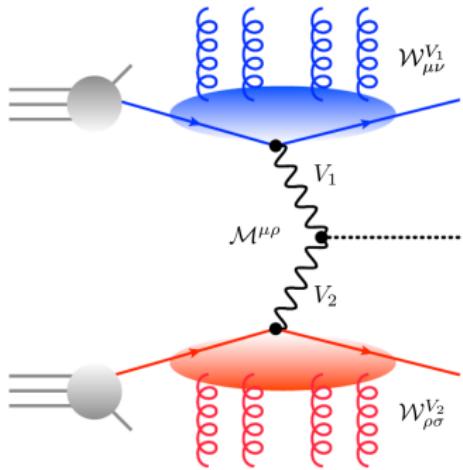


# Structure Function Approach

$$d\sigma_{VBF} = \frac{G_F^2}{s} M_{V_1}^2 M_{V_2}^2 \Delta_{V_1}^2(Q_1^2) \Delta_{V_2}^2(Q_2^2) \times \\ \mathcal{W}_{\mu\nu}^{V_1}(x_1, Q_1^2) \mathcal{M}^{\mu\rho} \mathcal{M}^{*\nu\sigma} \mathcal{W}_{\rho\sigma}^{V_2}(x_2, Q_2^2) d\Omega_{VBF}$$

and

$$\mathcal{M}^{\mu\nu} = 2\sqrt{\sqrt{2}G_F} M_V^2 g^{\mu\nu}$$



## Hadronic Tensor

$$\mathcal{W}_{\mu\nu}^V(x_i, Q_i^2) = \\ \left( -g_{\mu\nu} + \frac{q_{i,\mu} q_{i,\nu}}{q_i^2} \right) F_1^V(x_i, Q_i^2) \\ + \frac{\hat{P}_{i,\mu} \hat{P}_{i,\nu}}{P_i \cdot q_i} F_2^V(x_i, Q_i^2) \\ + i \epsilon_{\mu\nu\rho\sigma} \frac{P_i^\rho q_i^\sigma}{2P_i \cdot q_i} F_3^V(x_i, Q_i^2)$$



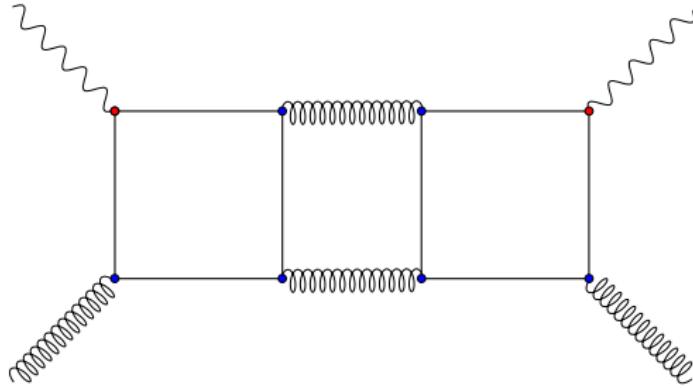
# Structure Function Approach

The Structure Functions can be expressed as convolutions of the short distance DIS coefficient functions and PDFs

$$F_i^V = \sum_{a=q,g} C_{i,a}^V \otimes f_a, \quad i=2,L,3, \quad V=Z,W^+,W^-,$$

where DIS coefficient functions are known to the third order in  $\alpha_s$ . [Moch, Rogal, Vermaseren, Vogt (2005-2008)]

Conceptually N<sup>3</sup>LO not more complicated than NNLO, but non-trivial flavour structures start appearing at N<sup>3</sup>LO



# What about missing N<sup>3</sup>LO PDFs?

All approximations aside, we can't truly claim N<sup>3</sup>LO accuracy without also having N<sup>3</sup>LO PDFs available.

Naive estimate

$$\delta_A^{\text{PDF}} = \frac{1}{2} \left| \frac{\sigma_{\text{NNLO-PDF}}^{\text{NNLO}} - \sigma_{\text{NLO-PDF}}^{\text{NNLO}}}{\sigma_{\text{NNLO-PDF}}^{\text{NNLO}}} \right| = 1.1\%,$$

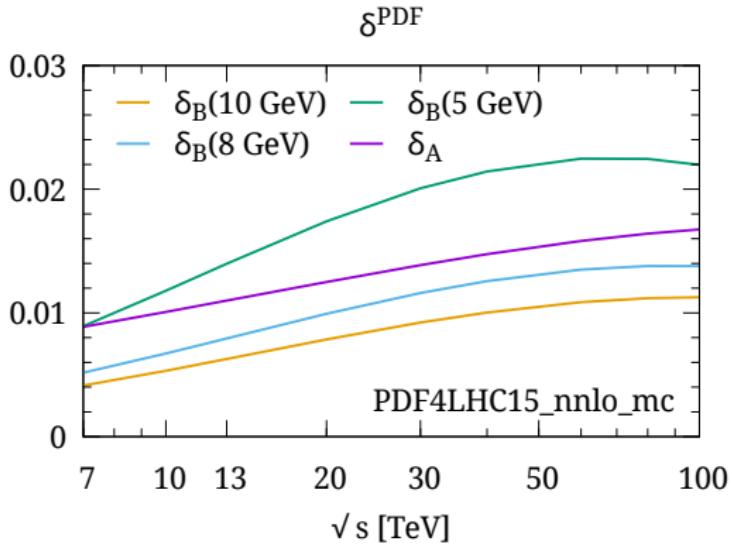
and slightly more sophisticated ( $Q_0 = 8 \text{ GeV}$ )

$$f^{\text{N}^3\text{LO,approx.}}(x, Q) = f^{\text{NNLO}}(x, Q) \frac{F_2^{\text{NNLO}}(x, Q_0)}{F_2^{\text{N}^3\text{LO}}(x, Q_0)},$$

$$\delta_B^{\text{PDF}}(Q_0) = \left| \frac{\sigma^{\text{N}^3\text{LO}} - \sigma_{\text{rescaled}}^{\text{N}^3\text{LO}}(Q_0)}{\sigma^{\text{N}^3\text{LO}}} \right| = 7.9\%.$$



# What about missing N<sup>3</sup>LO PDFs?



- the value of  $Q_0 = 8 \text{ GeV}$  is found by requiring that the method is reliable in estimating NNLO corrections to PDFs
  - the uncertainty associated with varying the scale in the DGLAP evolutions is found to be below the permille level
- could obtain approximate N<sup>3</sup>LO PDFs using just N<sup>3</sup>LO coefficient functions



# Scale variations at N<sup>3</sup>LO

We use RGE methods to evaluate the structure functions at arbitrary renormalisation scales

$$\alpha_S(Q) = \alpha_S(\mu_R) + \alpha_S^2(\mu_R)\beta_0 L_{RQ} + \alpha_S^3(\mu_R)(\beta_0^2 L_{RQ}^2 + \beta_1 L_{RQ}) + \mathcal{O}(\alpha_S^4(\mu_R))$$

$$L_{RQ} = \ln\left(\frac{\mu_R^2}{Q^2}\right), \quad L_{FQ} = \ln\left(\frac{\mu_F^2}{Q^2}\right)$$

$$\begin{aligned} C_i &= \sum_{k=0} \left( \frac{\alpha_S(Q)}{2\pi} \right)^k C_i^{(k)} = \\ &C_i^{(0)} + \frac{\alpha_S(\mu_R)}{2\pi} C_i^{(1)} + \\ &\left( \frac{\alpha_S(\mu_R)}{2\pi} \right)^2 \left( C_i^{(2)} + 2\pi\beta_0 C_i^{(1)} L_{RQ} \right) + \\ &\left( \frac{\alpha_S(\mu_R)}{2\pi} \right)^3 \left[ C_i^{(3)} + 4\pi\beta_0 C_i^{(2)} L_{RQ} \right. \\ &\left. + 4\pi^2 C_i^{(1)} L_{RQ} (\beta_1 + \beta_0^2 L_{RQ}) \right] + \mathcal{O}(\alpha_S^4) \end{aligned}$$

$$\begin{aligned} f(x, Q) &= f(x, \mu_F) \left( 1 - \frac{\alpha_S(\mu_R)}{2\pi} L_{FQ} P^{(0)} \right. \\ &- \left( \frac{\alpha_S(\mu_R)}{2\pi} \right)^2 L_{FQ} \left[ P^{(1)} - \frac{1}{2} L_{FQ} (P^{(0)})^2 \right. \\ &- \left. \pi\beta_0 P^{(0)} (L_{FQ} - 2L_{RQ}) \right] \\ &- \left( \frac{\alpha_S(\mu_R)}{2\pi} \right)^3 L_{FQ} \left[ P^{(2)} - \frac{1}{2} L_{FQ} (P^{(0)} P^{(1)} \right. \\ &+ P^{(1)} P^{(0)}) + \pi\beta_0 (L_{FQ} - 2L_{RQ}) \times \\ &(L_{FQ} (P^{(0)})^2 - 2P^{(1)}) + \frac{1}{6} L_{FQ}^2 (P^{(0)})^3 \\ &+ 4\pi^2 \beta_0^2 P^{(0)} (L_{RQ}^2 - L_{FQ} L_{RQ} + \frac{1}{3} L_{FQ}^2) \\ &\left. \left. - 2\pi^2 \beta_1 P^{(0)} (L_{FQ} - 2L_{RQ}) \right] + \mathcal{O}(\alpha_S^4) \right) \end{aligned}$$



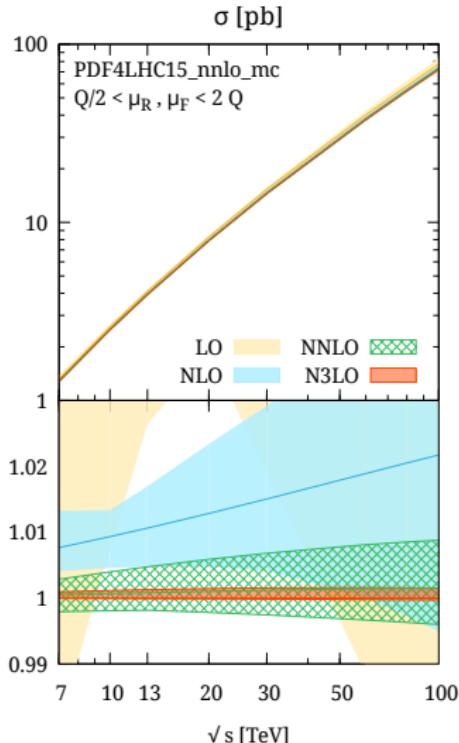
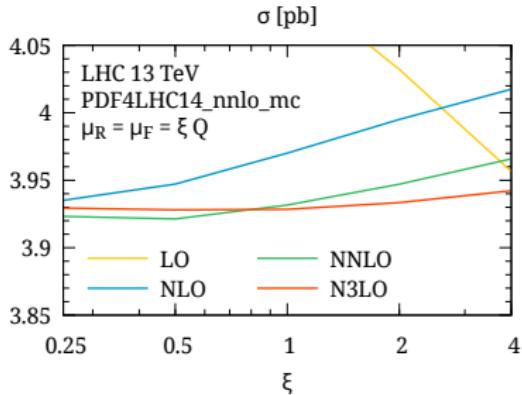
# N<sup>3</sup>LO results

	$\sigma^{(13 \text{ TeV})} [\text{pb}]$	$\sigma^{(14 \text{ TeV})} [\text{pb}]$	$\sigma^{(100 \text{ TeV})} [\text{pb}]$
LO	$4.099^{+0.051}_{-0.067}$	$4.647^{+0.037}_{-0.058}$	$77.17^{+6.45}_{-7.29}$
NLO	$3.970^{+0.025}_{-0.023}$	$4.497^{+0.032}_{-0.027}$	$73.90^{+1.73}_{-1.94}$
NNLO	$3.932^{+0.015}_{-0.010}$	$4.452^{+0.018}_{-0.012}$	$72.44^{+0.53}_{-0.40}$
N <sup>3</sup> LO	$3.928^{+0.005}_{-0.001}$	$4.448^{+0.006}_{-0.001}$	$72.34^{+0.11}_{-0.02}$

- We study pp collisions with PDF4LHC\_nnlo\_mc and electroweak parameters fixed to their PDG values
- the central renormalisation and factorisation scale is set equal to  $Q_1, Q_2$  and varied independently by a factor 2 up and down
- N<sup>3</sup>LO corrections are tiny ( $\sim 2\%$ ) but predictions well within NNLO scale uncertainty



# $N^3\text{LO}$ results

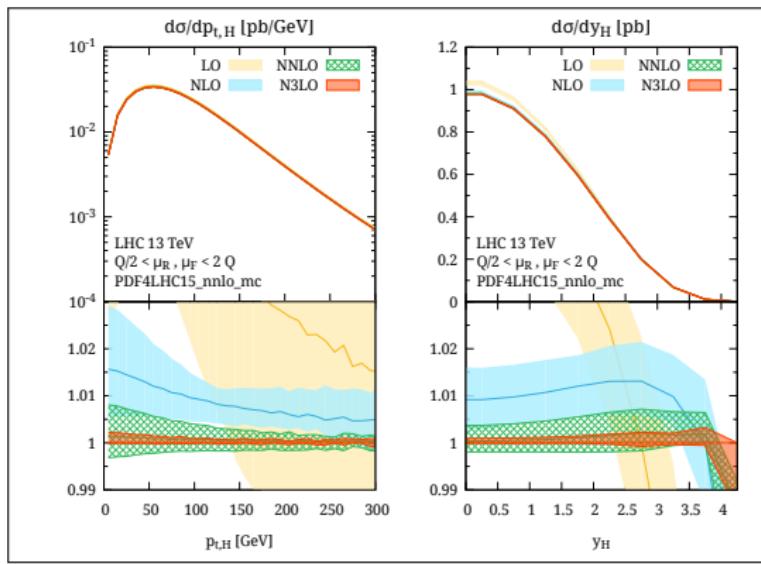


- the  $N^3\text{LO}$  corrections are tiny over a large range of energies and stay well within the scale uncertainty band of the NNLO prediction
- cross section becomes extremely stable under the variation of renormalisation and factorisation scales



# $N^3LO$ results

From the knowledge of  $Q_1$  and  $Q_2$  it is trivial to reconstruct the momentum of the Higgs. The calculation is therefore fully differential in the Higgs kinematics.



- the corrections are almost flat throughout the entire spectrum
- the  $N^3LO$  prediction completely contained within the scale uncertainty band of the NNLO prediction
- only differential in the momenta of the proton remnants, and hence no real information on the tagging jets



# Limitations of the structure function approach

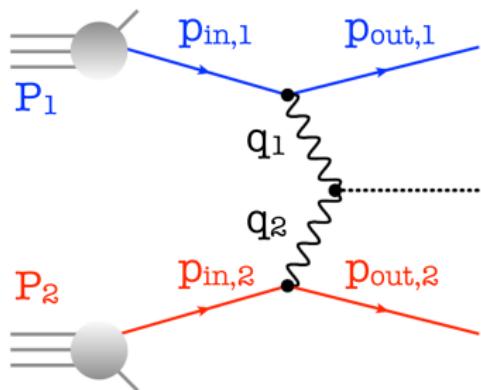
- gluon exchanges between the upper and lower hadronic sectors, which appear at NNLO, but are kinematically and colour suppressed; These contributions along with the heavy-quark loop induced contributions have been estimated to contribute at the permille level [Bolzoni et al. (2012)]
- t-/u-channel interference which are known to contribute  $\mathcal{O}(5\%)$  at the fully inclusive level and  $\mathcal{O}(0.5\%)$  after VBF cuts have been applied [Ciccolini, Denner, Dittmaier (2008)]
- contributions from s-channel production, which have been calculated up to NLO. At the inclusive level these contributions are sizeable but they are reduced to  $\mathcal{O}(5\%)$  after VBF cuts [Ciccolini, Denner, Dittmaier (2008)]
- single-quark line contributions, which contribute to the VBF cross section at NNLO. At the fully inclusive level these amount to corrections of  $\mathcal{O}(1\%)$  but are reduced to the permille level after VBF cuts have been applied [Harlander et al. (2008)]
- loop induced interference between VBF and ggH production. These contributions have been shown to be much below the permille level [Andersen et al. (2008)]



# Beyond the Structure Function Approach

**This work:** We eliminate the limitations of the Structure Function Approach.

If the scattering is Born like, then the vector boson momenta  $q_i$ , and on-shell conditions, fix the incoming and outgoing parton momenta:



$$p_{in,i} = x_i P_i$$

$$p_{out,i} = x_i P_i - q_i$$

$$x_i = \frac{q_i^2}{2q_i P_i}$$

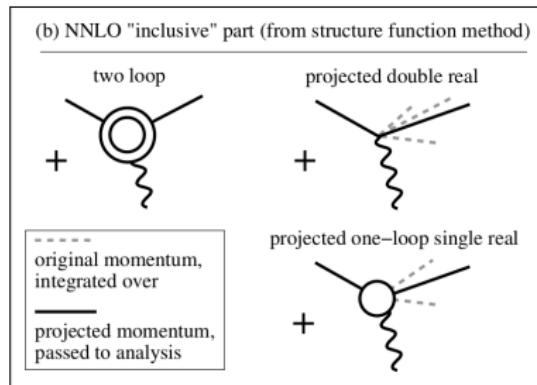


# Beyond the Structure Function Approach

The calculation is based on **two ingredients**:

## 1. An “inclusive” contribution

- use the Structure Function Approach and use four-vectors  $q_1, q_2$  to assign Born-like kinematics using the equations below
- use the projected Born-like momenta to compute differential distributions



$$p_{in,i} = x_i p_i$$

$$p_{out,i} = x_i p_i - q_i$$

$$x_i = \frac{q_i^2}{2q_i p_i}$$



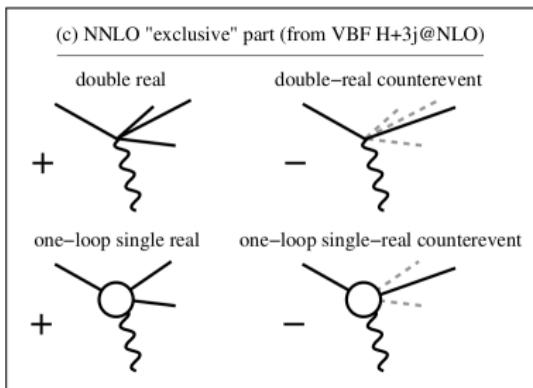
# Beyond the Structure Function Approach

The calculation is based on **two ingredients**:

## 2. An “exclusive” contribution

- use the electroweak  $H + jjj$  NLO calculation in the factorized approximation

[Figy et al. (2007)], [Jäger et al. (2014)]



- for each parton, keep track of whether it belongs to the upper or lower sector, and compute vector-boson momenta  $q_1, q_2$
- for each event add **counter-event** with projected Born kinematics and opposite weight

The counter-events **cancel** identically with the projected terms from the “inclusive” contribution.



# Beyond the Structure Function Approach

Schematically we express the “projection-to-Born” (P2B) method as

$$\begin{aligned} d\sigma &= \int d\Phi_B (B + V) + \int d\Phi_R R \\ &= \underbrace{\int d\Phi_B (B + V) + \int d\Phi_R R_{P2B}}_{\text{“inclusive” contribution}} + \underbrace{\int d\Phi_R R - \int d\Phi_R R_{P2B}}_{\text{“exclusive” contribution}} \end{aligned}$$

- from the “exclusive” ingredient we get the full double-real and one-loop single-real contributions.
- when integrated over phase-space, the counter-events cancel the projected double-real and one-loop single-real contributions from the “inclusive” ingredient

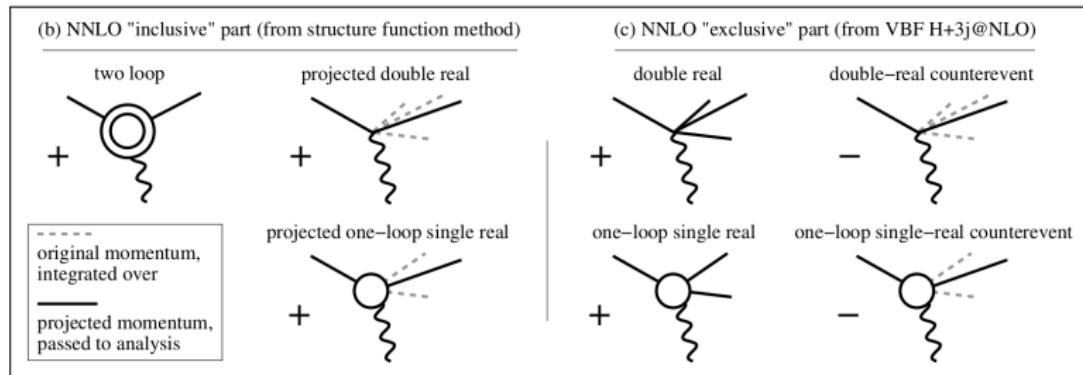
Hence the sum of the two contributions gives the complete, **fully differential** NNLO result.



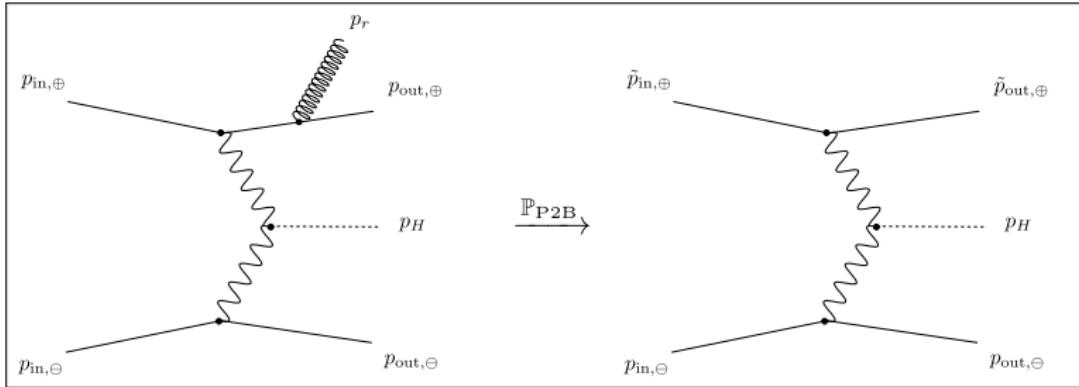
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# The projection



$$\Omega_R = (p_{in,\oplus}, p_{in,\ominus}, p_H, p_{out,\oplus}, p_{out,\ominus}, p_r)$$

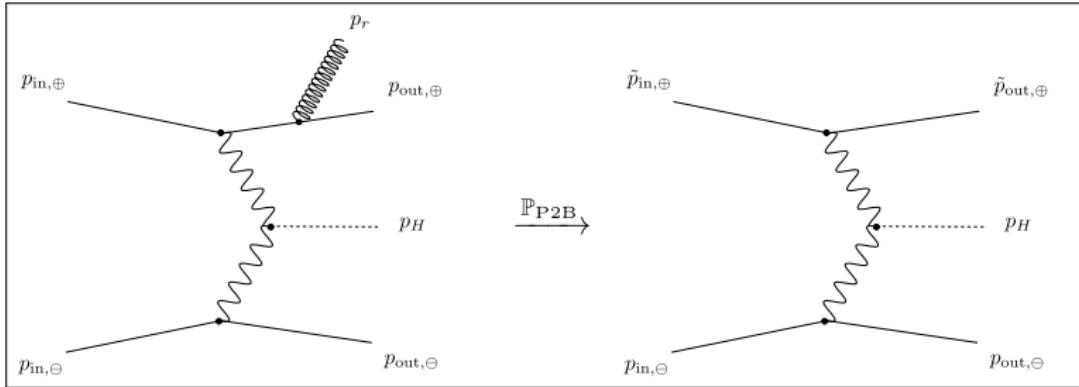
$$\begin{aligned}\mathbb{P}_{P2B} \Omega_R &= \Omega_{P2B} \\ &= (\tilde{p}_{in,\oplus}, \tilde{p}_{in,\ominus}, p_H, \tilde{p}_{out,\oplus}, \tilde{p}_{out,\ominus})\end{aligned}$$

Use lightcone coordinates:

$$p = (p^x, p^y, p^-, p^+) \quad \text{with} \quad p^\pm = \frac{1}{\sqrt{2}}(p^E \pm p^z)$$



# The projection



$$\Omega_R = (p_{in,\oplus}, p_{in,\ominus}, p_H, p_{out,\oplus}, p_{out,\ominus}, p_r)$$

$$\begin{aligned} \mathbb{P}_{P2B} \Omega_R &= \Omega_{P2B} \\ &= (\tilde{p}_{in,\oplus}, \tilde{p}_{in,\ominus}, p_H, \tilde{p}_{out,\oplus}, \tilde{p}_{out,\ominus}) \end{aligned}$$

Such that

$$p_{in} = (0, 0, 0, p_{in}^+)$$

$$\tilde{p}_{in} = (0, 0, 0, \tilde{p}_{in}^+)$$

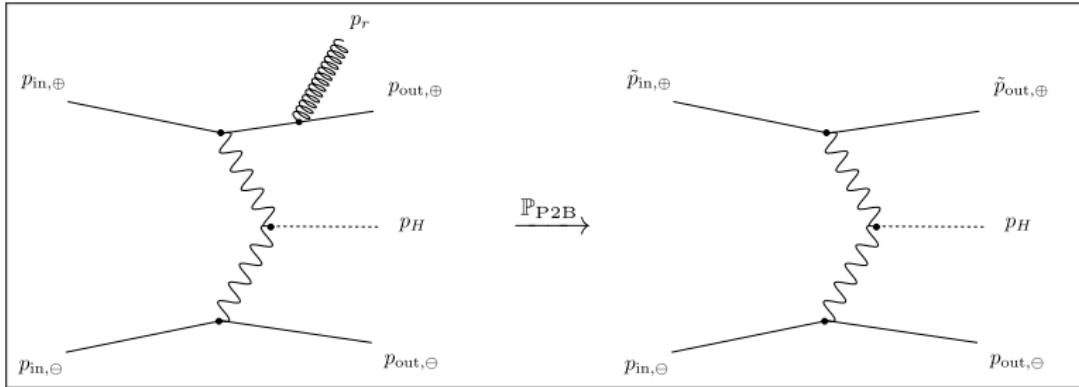
$$p_{out} = (p_{out}^x, p_{out}^y, p_{out}^-, p_{out}^+)$$

$$\tilde{p}_{out} = (\tilde{p}_{out}^x, \tilde{p}_{out}^y, \tilde{p}_{out}^-, \tilde{p}_{out}^+)$$

$$p_r = (p_r^x, p_r^y, p_r^-, p_r^+)$$



# The projection



By momentum conservation we have

$$p_{in} - p_{out} - p_r = \tilde{p}_{in} - \tilde{p}_{out} \Rightarrow \begin{cases} \tilde{p}_{out}^x = p_{out}^x + p_r^x \\ \tilde{p}_{out}^y = p_{out}^y + p_r^y \\ \tilde{p}_{out}^- = p_{out}^- + p_r^- \\ \tilde{p}_{in}^+ = p_{in}^+ - p_{out}^+ - p_r^+ + \tilde{p}_{out}^+ . \end{cases}$$

$$\begin{aligned} (\tilde{p}_{out})^2 &= 0 \\ \Rightarrow (\tilde{p}_{out}^x)^2 + (\tilde{p}_{out}^y)^2 - 2\tilde{p}_{out}^-\tilde{p}_{out}^+ &= 0 \\ \Rightarrow \tilde{p}_{out}^+ &= \frac{(p_{out}^x + p_r^x)^2 + (p_{out}^y + p_r^y)^2}{2(p_{out}^- + p_r^-)} , \end{aligned}$$



# Implementation

## 1. “inclusive” code

- matrix elements coded with structure functions using parametrised versions of the DIS coefficient functions evaluated by HOPPET
- phase-space taken from POWHEG’s VBF\_H generator

## 2. “exclusive” code

- start with the VBF\_HJJJ calculation in POWHEG (based on vbfmlo)
- extend POWHEG’s tags to uniquely associate radiation with each sector (upper or lower line)
- for each event map the kinematics onto Born-like kinematics and determine the vector-boson momenta  $q_1, q_2$  using the equations on p.7.

- we have tested the “inclusive” code against a private version of the structure function calculation (thanks to Marco Zaro) and the structure functions themselves against APFEL 2.4.1.
- we have tested that the “exclusive” code reproduces the original VBF\_HJJJ result. The sum of “inclusive” and “exclusive” at NLO agrees with VBF\_H
- tagging tested by checking that the probability of assigning a parton to the wrong sector decreases as the rapidity between the two hardest jets increases



# Phenomenology

We study 13 TeV LHC collisions with  $M_H = 125$  GeV and NNPDF3.0\_nnlo\_as118. We use the following VBF cuts:

- Jets defined with anti- $k_t$ ,  $R = 0.4$  and  $p_t > 25$  GeV
- Two hardest jets within  $|y| < 4.5$
- High dijet invariant mass,  $M_{j_1 j_2} > 600$  GeV, and separation,  $\Delta y_{j_1 j_2} > 4.5$
- Hardest jets in opposite hemispheres,  $y_{j_1} y_{j_2} < 0$

We choose a central scale which approximates well  $\sqrt{Q_1 Q_2}$  and symmetrically vary by a factor 2 up and down

$$\mu_0^2(p_{t,H}) = \frac{M_H}{2} \sqrt{\left(\frac{M_H}{2}\right)^2 + p_{t,H}^2}$$



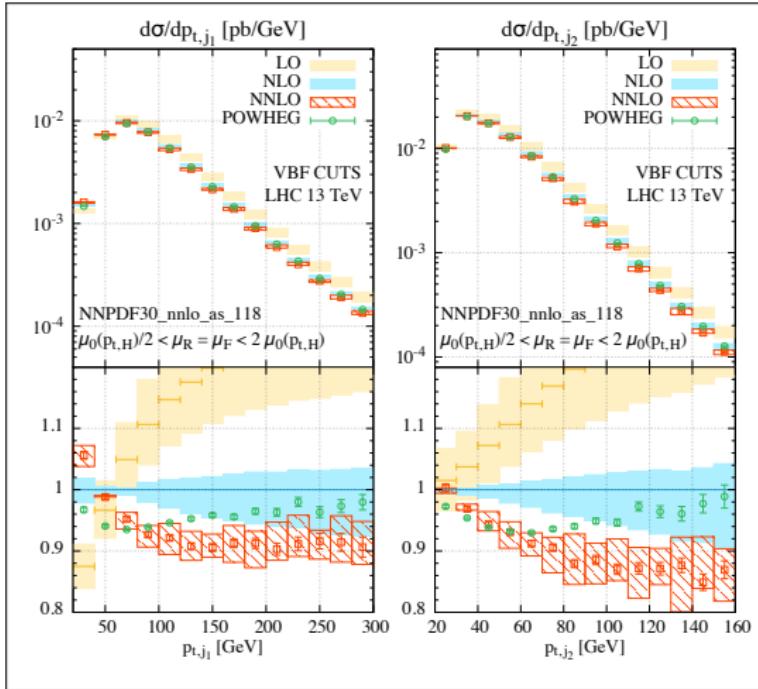
# Phenomenology

	$\sigma^{(\text{no cuts})}$ [pb]	$\sigma^{(\text{VBF cuts})}$ [pb]
LO	$4.032^{+0.057}_{-0.069}$	$0.957^{+0.066}_{-0.059}$
NLO	$3.929^{+0.024}_{-0.023}$	$0.876^{+0.008}_{-0.018}$
NNLO	$3.888^{+0.016}_{-0.012}$	$0.826^{+0.013}_{-0.014}$

- NNLO corrections tiny ( $\sim 1\%$ ) without cuts and sizeable with VBF cuts ( $\sim 5\%$ )
- NNLO results outside NLO band (also true when using NLO PDFs)
- corrections tend to be dominated by the extra real radiation. The effect is softer jets and hence fewer events pass the cuts



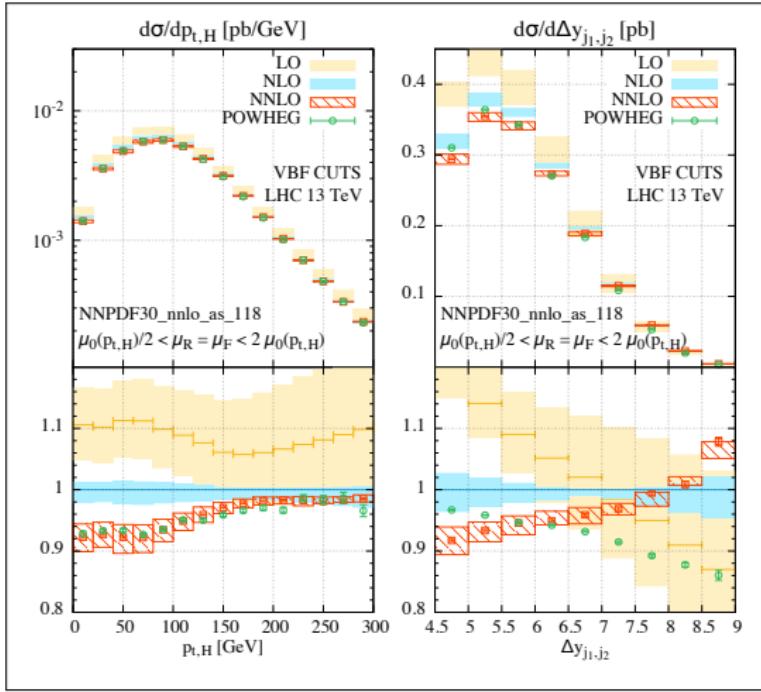
# Phenomenology



- NNLO corrections can be **large**  $\mathcal{O}(10\%)$  and are often outside the NLO band
- the NNLO corrections tend to be dominated by extra real radiation. These appear to make the **jets softer**
- NOTE: NNLO PDF used everywhere. Similar results hold when using LO/NLO PDFs
- expanding the scale variation from 3-point to 7-point doesn't change the size of the NLO bands noticeably



# More Phenomenology



- in some cases **NLO+PS agrees very well with the NNLO result** (in particular  $p_{t,H}$ ,  $M_{jj}$  and  $\phi_{jj}$ )
- in some cases **not** ( $\Delta y_{j_1,j_2}$  and  $H_t$ )
- in general only modest shrinkage of bands from NLO to NNLO
- non-trivial** kinematic dependence on **k-factors** (both LO/NLO and NNLO/NLO)

# LHCHXWG YR4 results

We study LHC collisions with  $M_H = 125$  GeV and PDF4LHC15\_nnlo\_100. We use the following setup:

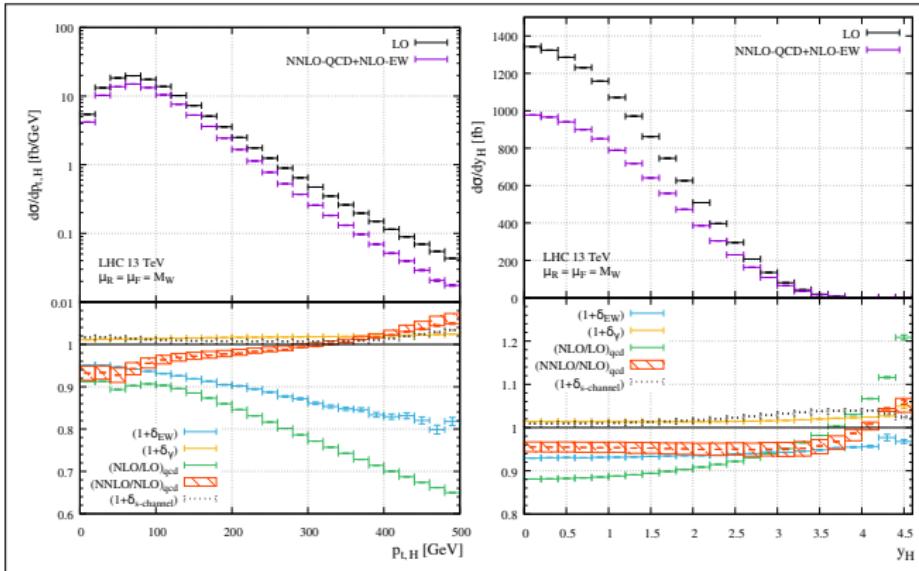
- Jets defined with anti- $k_t$ ,  $R = 0.4$  and  $p_t > 20$  GeV
- Two hardest jets within  $|y| < 5$
- Dijet invariant mass,  $M_{j_1 j_2} > 130$  GeV, and separation,  $\Delta y_{j_1 j_2} > 3$
- $\mu_R = \mu_F = M_W$  (varied a factor 2 up and down)
- Electroweak corrections obtained with HAWK [Ciccolini, Denner, Dittmaier (2008)]
- Photon PDF obtained from NNPDF2.3QED
- $\sigma^{VBF} = \sigma_{\text{NNLOQCD}}^{\text{DIS}} (1 + \delta_{\text{EW}}) + \sigma_\gamma$

$\sqrt{s}$ [TeV]	$\sigma^{VBF}$ [fb]	$\Delta_{\text{scale}} [\%]$	$\Delta_{\text{PDF}/\alpha_s/\text{PDF} \oplus \alpha_s} [\%]$	$\sigma_{\text{NNLOQCD}}^{\text{DIS}}$ [fb]	$\delta_{\text{EW}} [\%]$	$\sigma_\gamma$ [fb]	$\sigma_{s\text{-channel}}$ [fb]
7	1241	$^{+0.19}_{-0.21}$	$\pm 2.1 / \pm 0.4 / \pm 2.2$	1281	-4.4	17.1	584.5(3)
8	1601	$^{+0.25}_{-0.24}$	$\pm 2.1 / \pm 0.4 / \pm 2.2$	1656	-4.6	22.1	710.4(3)
13	3782	$^{+0.43}_{-0.33}$	$\pm 2.1 / \pm 0.5 / \pm 2.1$	3939	-5.3	51.9	1378.1(6)
14	4278	$^{+0.45}_{-0.34}$	$\pm 2.1 / \pm 0.5 / \pm 2.1$	4461	-5.4	58.5	1515.9(6)

$\sqrt{s}$ [TeV]	$\sigma^{VBF}$ [fb]	$\Delta_{\text{scale}} [\%]$	$\Delta_{\text{PDF}/\alpha_s/\text{PDF} \oplus \alpha_s} [\%]$	$\sigma_{\text{NNLOQCD}}^{\text{DIS}}$ [fb]	$\delta_{\text{EW}} [\%]$	$\sigma_\gamma$ [fb]	$\sigma_{s\text{-channel}}$ [fb]
7	602.4(5)	$^{+1.3}_{-1.6}$	$\pm 2.3 / \pm 0.3 / \pm 2.3$	630.8(5)	-6.1	9.9	8.2
8	795.9(6)	$^{+1.3}_{-1.5}$	$\pm 2.3 / \pm 0.3 / \pm 2.3$	834.8(7)	-6.2	13.1	11.1
13	1975(1)	$^{+1.3}_{-1.2}$	$\pm 2.1 / \pm 0.4 / \pm 2.2$	2084(1)	-6.8	32.3	29.0
14	2236(3)	$^{+1.5}_{-1.3}$	$\pm 2.1 / \pm 0.4 / \pm 2.1$	2362(3)	-6.9	36.7	33.1



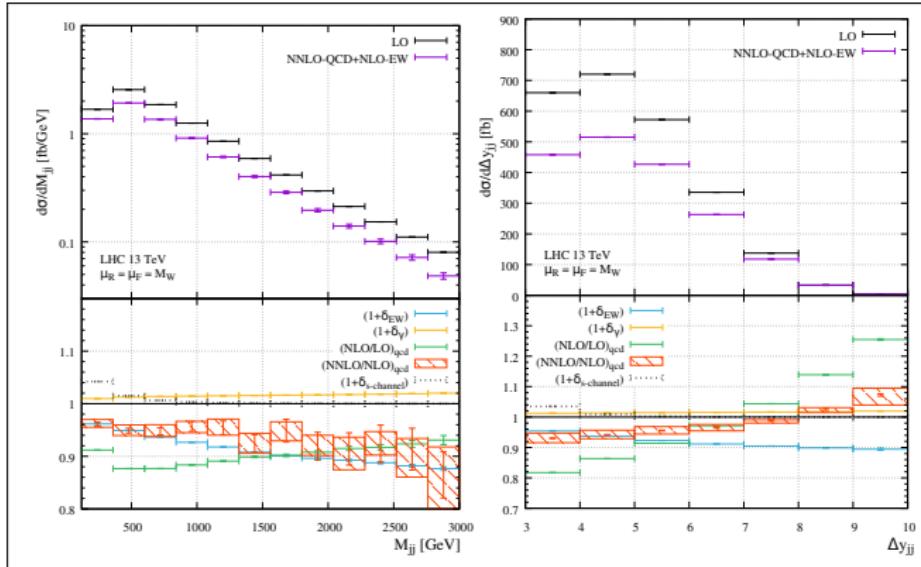
# LHCHXSWG YR4 results



- NLO-QCD corrections can be very large (notice scale choice)
- NLO-EW corrections can be sizeable in the tails of distributions
- NNLO-QCD corrections of the same order as NLO-EW corrections
- Photon induced contribution very flat



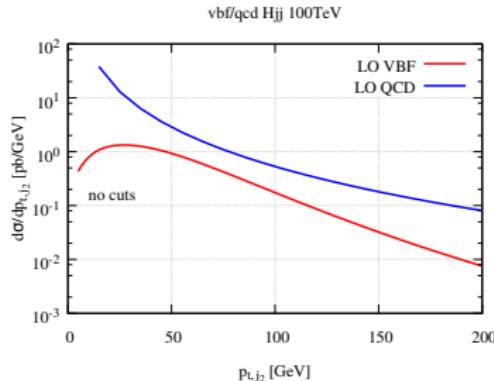
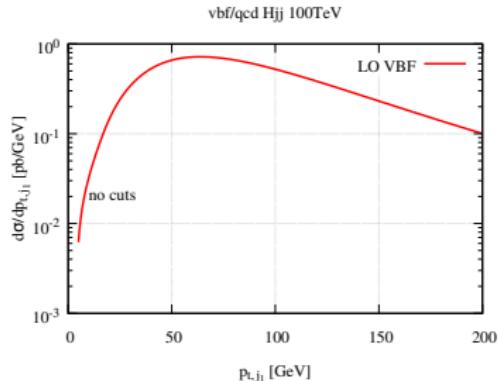
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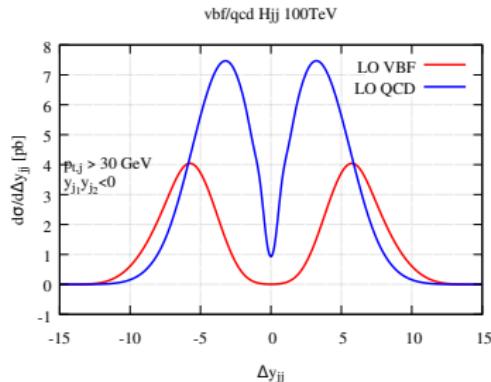
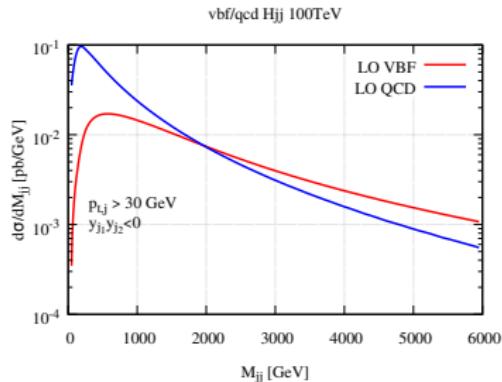
# VBF @ 100 TeV



- due to the EW production mechanism of VBF, the jet spectra are mostly unchanged going from 14 TeV to 100 TeV
- in particular, the two tag jets have transverse momenta set by the vector boson mass whereas the QCD jets tend to peak at much lower transverse momenta
- VBF cuts should therefore still efficiently suppress the QCD background even at higher energies



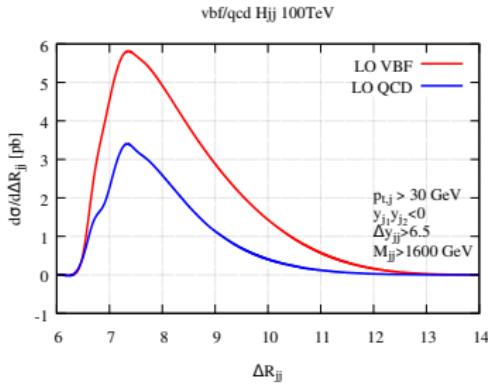
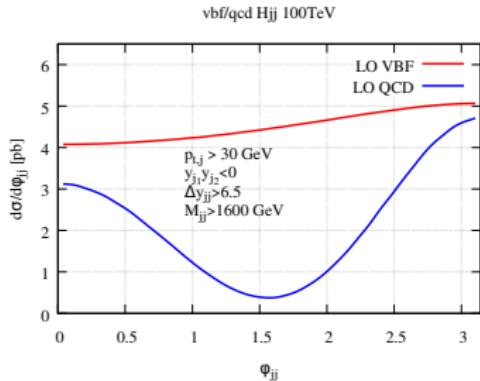
# VBF @ 100 TeV



- after applying a soft transverse momentum cut on the tagging jets and requiring the jets to be in opposite detector hemispheres, we may try to optimise our VBF cuts
- it is clear that VBF production starts dominating for  $\|\Delta y_{jj}\| > 6.5$
- after imposing this cut, the  $M_{jj}$  peak is shifted to around 2400 GeV
- in order to sufficiently suppress the QCD background while not cutting away the VBF peak we impose  $M_{jj} > 1600 \text{ GeV}$



# VBF @ 100 TeV



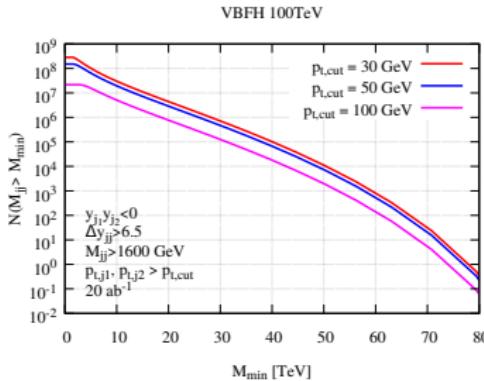
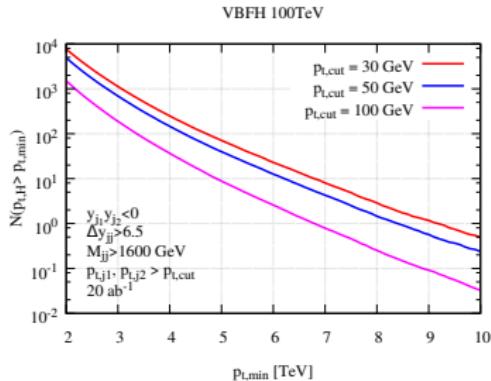
- further reduction possible by cutting in  $\phi_{jj}$  whereas a cut in  $R_{jj}$  only minimally reduces the QCD background

VBF cuts	$\sigma(p_{t,j} > 30 \text{ GeV})$ [pb]	$\sigma(p_{t,j} > 50 \text{ GeV})$ [pb]	$\sigma(p_{t,j} > 100 \text{ GeV})$ [pb]
VBFH	14.1	7.51	1.08
QCD Hjj	5.04	1.97	0.331

No cuts	$\sigma(p_{t,j} > 30 \text{ GeV})$ [pb]	$\sigma(p_{t,j} > 50 \text{ GeV})$ [pb]	$\sigma(p_{t,j} > 100 \text{ GeV})$ [pb]
VBFH	51.3	28.5	5.25
QCD Hjj	166	78.6	23.9



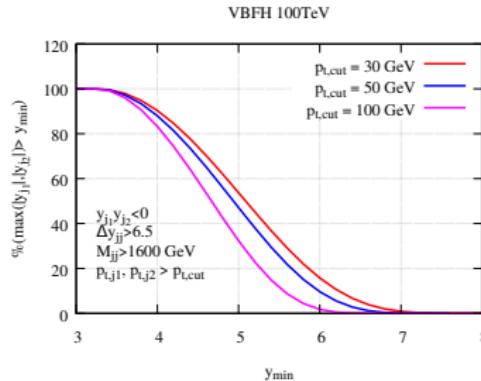
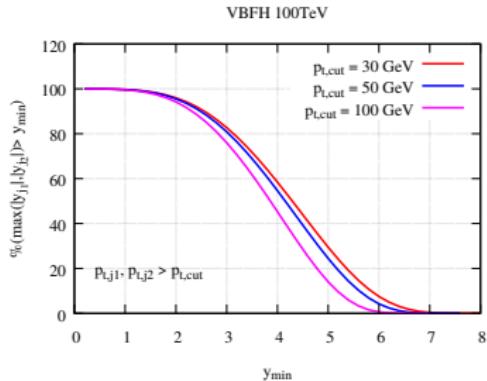
# VBF @ 100 TeV



- at a 100 TeV collider gathering  $20 \text{ ab}^{-1}$  of data, we can expect very high reach in energy
- for the transverse momentum distribution of the Higgs, we can expect a reach of almost 10 TeV
- tagging jets will be produced with an invariant mass of up to 80 TeV
- without imposing cuts the reach is of course even greater



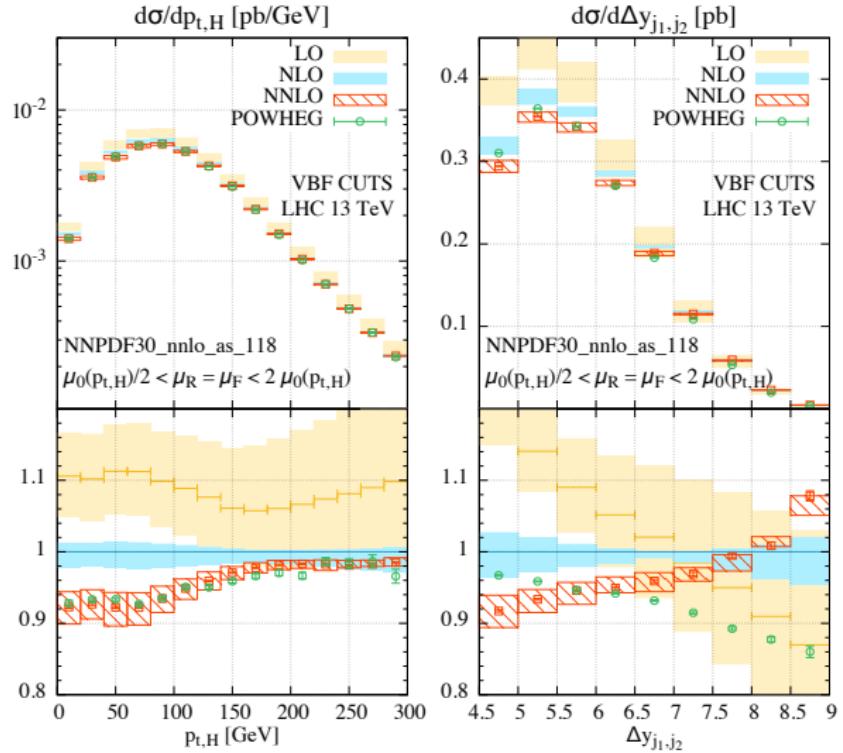
# VBF @ 100 TeV



- studying VBF at a 100 TeV collider will require impressive detectors
- before VBF cuts are applied ATLAS/CMS would lose roughly 50% of the signal
- after VBF cuts it would be as much as 80% of the signal depending on the jet definition
- in order to retain more than 90% of the signal the detector would need a rapidity reach of about 6.5



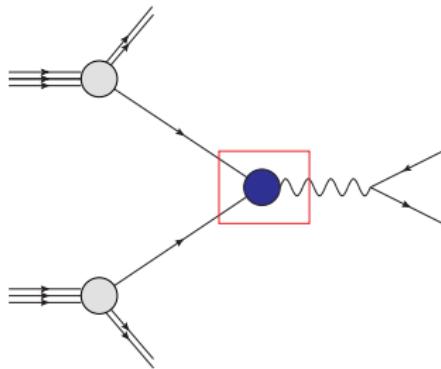
# What about parton showers?



## Parton showers

- parton shower: algorithm to resum (some classes of) collinear/soft logs in a “fully-exclusive” way.
- based on description of multiple soft-collinear real and virtual radiative corrections using a probabilistic language

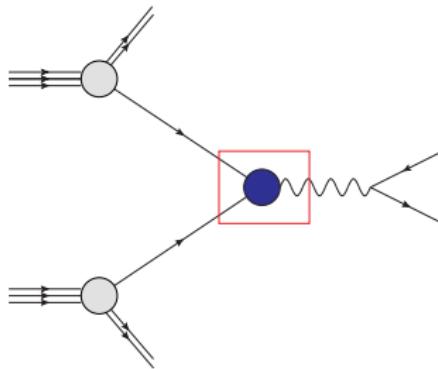
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$$d\sigma_{\text{SMC}} = \underbrace{|\mathcal{M}_B|^2 d\Phi_B}_{d\sigma_B} \left\{ \Delta(t_{\max}, t_0) \right\}$$

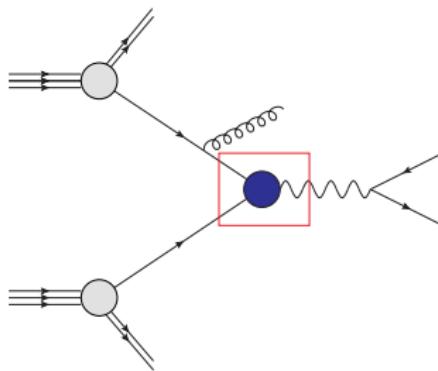


$$\Delta(t_{\max}, t) = \exp \left\{ - \int_t^{t_{\max}} d\Phi'_r \frac{\alpha_s}{2\pi} \frac{1}{t'} P(z') \right\}$$

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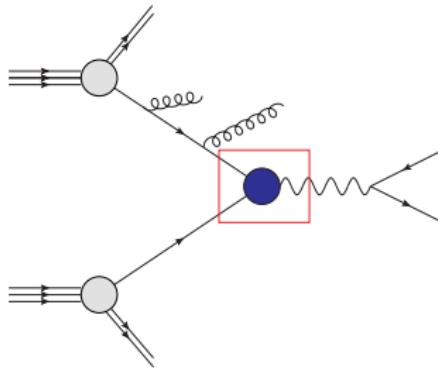


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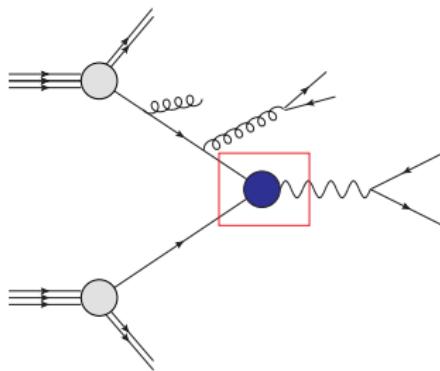


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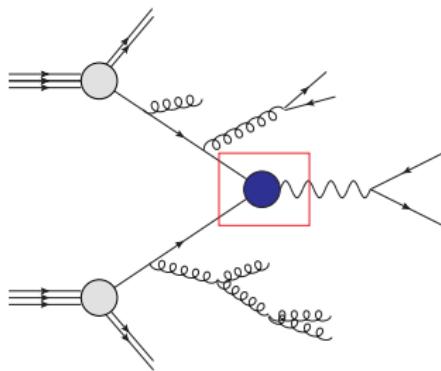


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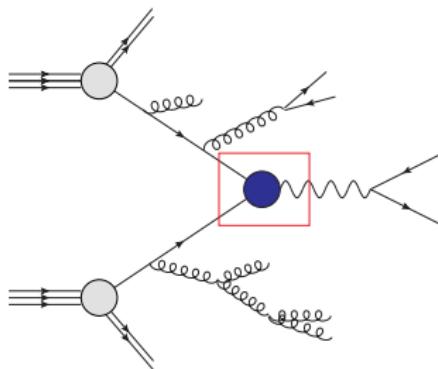


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$$\Delta(t_{\max}, t) = \exp \left\{ - \int_t^{t_{\max}} d\Phi'_r \frac{\alpha_s}{2\pi} \frac{1}{t'} P(z') \right\}$$

This is “LOPS”

- A parton shower changes shapes, not the overall normalization, which stays LO (*unitarity*)
- LL resummation is included in Sudakov form factors: easy to see that probability of having arbitrarily collinear emission becomes 0, instead of  $\infty$

- parton showers are **only LO+LL**: clearly **including NLO corrections** would be a big improvement. There are 2 methods to achieve this consistently:
- the POWHEG method:
  - do these replacement

$$B(\Phi_n) \Rightarrow \bar{B}(\Phi_n) = B(\Phi_n) + \frac{\alpha_s}{2\pi} \left[ V(\Phi_n) + \int R(\Phi_{n+1}) d\Phi_r \right]$$

$$\Delta(t_m, t) \Rightarrow \Delta(\Phi_n; k_T) = \exp \left\{ -\frac{\alpha_s}{2\pi} \int \frac{R(\Phi_n, \Phi'_r)}{B(\Phi_n)} \theta(k'_T - k_T) d\Phi'_r \right\}$$

- POWHEG “master formula” for the **hardest emission**:

$$d\sigma_{\text{POW}} = d\Phi_n \bar{B}(\Phi_n) \left\{ \Delta(\Phi_n; k_T^{\min}) + \Delta(\Phi_n; k_T) \frac{\alpha_s}{2\pi} \frac{R(\Phi_n, \Phi_r)}{B(\Phi_n)} d\Phi_r \right\}$$

[+  $p_T$ -vetoing subsequent emissions, to avoid double-counting]

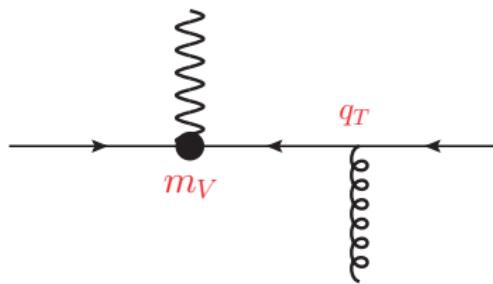
- properties:
  - inclusive observables: **@NLO**
  - first hard emission: **full tree level ME**
  - (N)LL resummation** of collinear/soft logs

- 
- NLOPS has become the standard for LHC searches (at least for SM processes)

1.  $V+j$  @ NLO,  $V+jj$  @ LO  $\Rightarrow$  use  $V+j$  @ NLOPS ( $\text{POWHEG}$ )

$$d\sigma_{\text{POWHEG}} = d\Phi_n \bar{B}_{\text{NLO}}(\Phi_n) \left\{ \Delta(\Phi_n; k_T^{\min}) + \Delta(\Phi_n; k_T) \frac{\alpha_s}{2\pi} \frac{R(\Phi_n, \Phi_r)}{B(\Phi_n)} d\Phi_r \right\}$$

$$\bar{B}_{\text{NLO}}(\Phi_n) d\Phi_n = \alpha_s(\mu_R) \left[ B + \alpha_s^{(\text{NLO})} V(\mu_R) + \alpha_s^{(\text{NLO})} \int d\Phi_r R \right] d\Phi_n$$

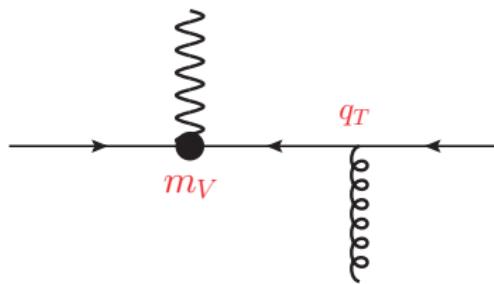


$V+j$  is a 2-scales problem ( $\rightarrow$  choice of  $\mu$  not unique)

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$V+j$  is a 2-scales problem ( $\rightarrow$  choice of  $\mu$  not unique)

- ☞ want to reach NNLO accuracy for e.g.  $y_V$ , i.e. when **fully inclusive** over QCD radiation
  - need to allow the 1st jet to become unresolved
  - the above approach needs to be modified: as it stands,  $\bar{B}_{\text{NLO}}(\Phi_n)$  is **not finite** when  $q_T \rightarrow 0!$

2. integrate over phase space regions where  $V$  is produced with arbitrarily soft/collinear jet  
(i.e. finite results when integrating over all  $q_T$  spectrum)

### MiNLO: Multiscale Improved NLO

[Hamilton,Nason,Zanderighi, 1206.3572]

- original goal: method to **a-priori** choose scales in multijet NLO computation (where hierarchy among scales can spoil accuracy since resummation of logs is missing)
- how: correct weights of different NLO terms with CKKW-inspired approach:

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- how: correct weights of different NLO terms with CKKW-inspired approach:
  - for all PS points, build the “more-likely” shower history that would have produced it (can be done by clustering kinematics with  $k_T$ -algo)
  - correct original NLO including  $\alpha_S$  couplings evaluated at nodal scales and Sudakov FFs
  - make sure that NLO accuracy is not spoiled !

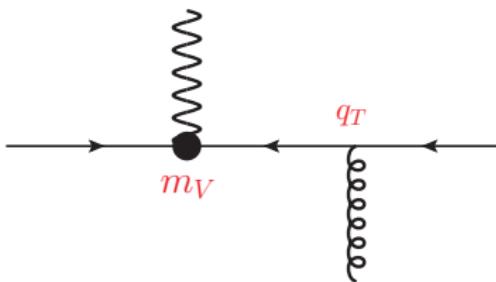
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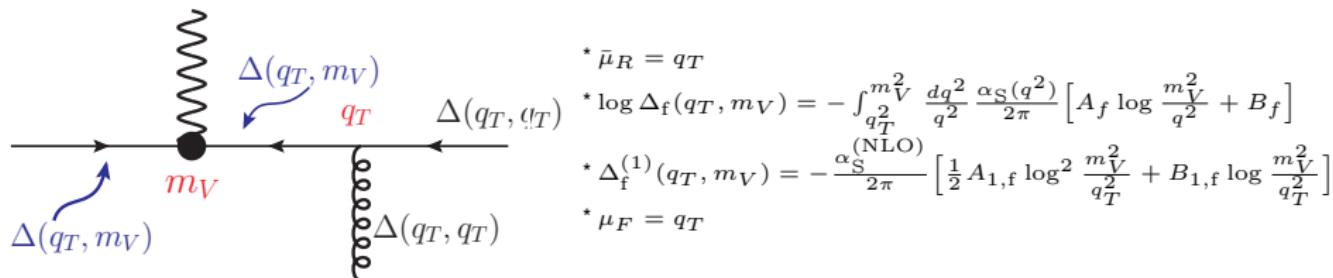
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$$\bar{B}_{\text{MiNLO}} = \alpha_s(q_T) \Delta_q^2(q_T, m_V) \left[ B \left( 1 - 2\Delta_q^{(1)}(q_T, m_V) \right) + \alpha_s^{(\text{NLO})} V(\bar{\mu}_R) + \alpha_s^{(\text{NLO})} \int d\Phi_r R \right]$$



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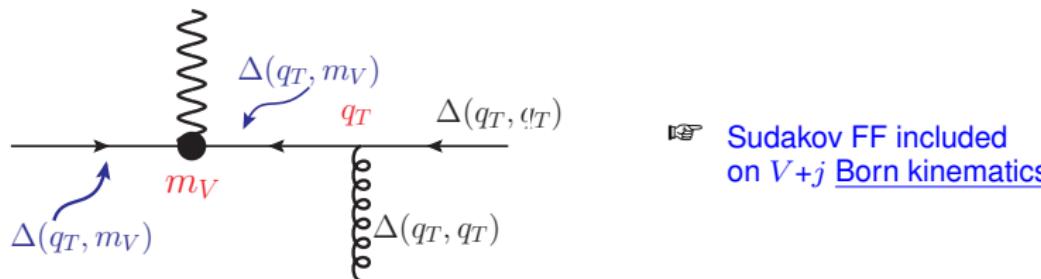
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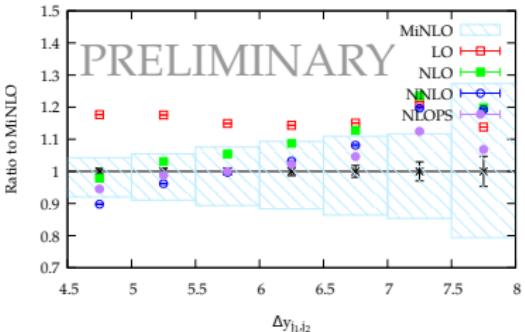
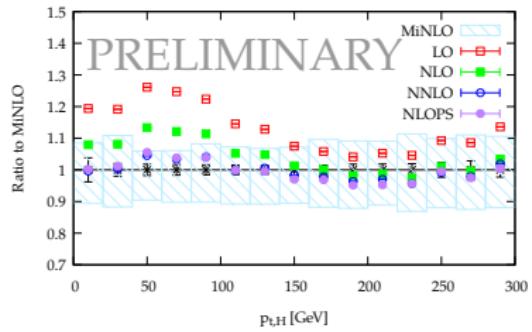
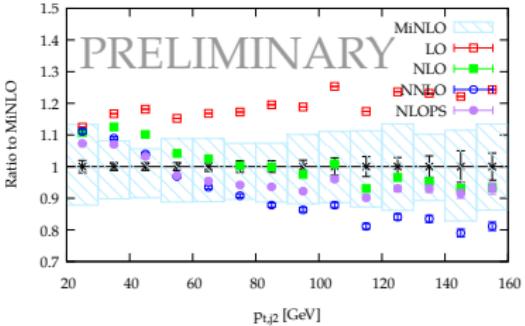
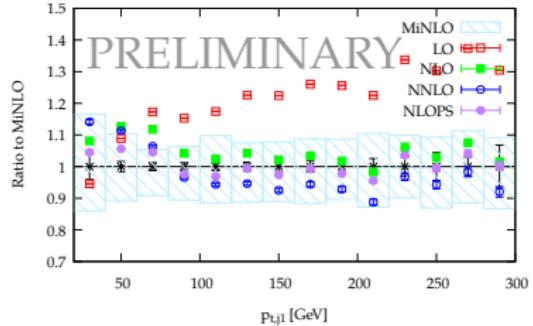
$$\bar{B}_{\text{NLO}} = \alpha_s(\mu_R) \left[ B + \alpha_s^{(\text{NLO})} V(\mu_R) + \alpha_s^{(\text{NLO})} \int d\Phi_r R \right]$$

$$\bar{B}_{\text{MiNLO}} = \alpha_s(q_T) \Delta_q^2(q_T, m_V) \left[ B \left( 1 - 2\Delta_q^{(1)}(q_T, m_V) \right) + \alpha_s^{(\text{NLO})} V(\bar{\mu}_R) + \alpha_s^{(\text{NLO})} \int d\Phi_r R \right]$$



- VJ-MiNLO yields **finite results** also when 1st jet is **unresolved** ( $q_T \rightarrow 0$ )
- $\bar{B}_{\text{MiNLO}}$  ideal to extend validity of  $V+j$  POWHEG

# MiNLO VBF H<sub>4j</sub>



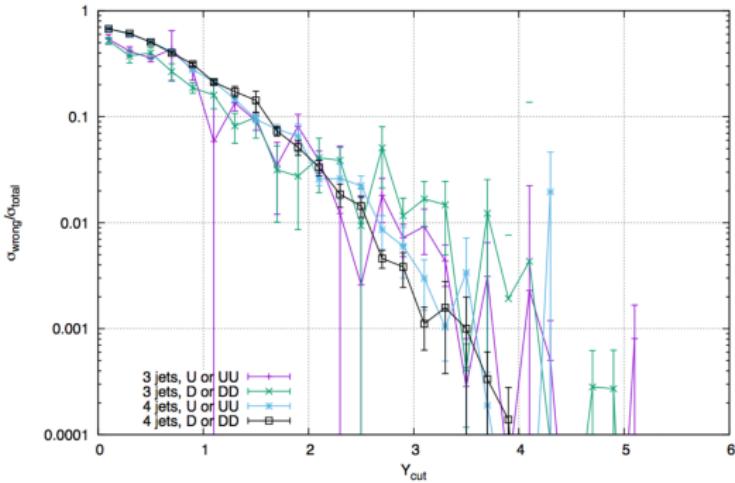
# Conclusions

- inclusive  $N^3LO$  are tiny and well within scale uncertainty band at  $NNLO$
- scale uncertainty band reduced significantly
- differential  $NNLO$  corrections are sizeable,  $\mathcal{O}(10\%)$ , and necessary for precision phenomenology
- only moderate shrinkage of  $NNLO$  bands compared to  $NLO$  bands
- $NLO$ -EW corrections are comparable to  $NNLO$ -QCD corrections and should be included
- “projection-to-Born” method can be extended to compute fully differential VBF Higgs at  $N^3LO$

A public code, `proVBFH`, will be released in the near future. Until then total cross sections and distributions with specific cuts can be provided.



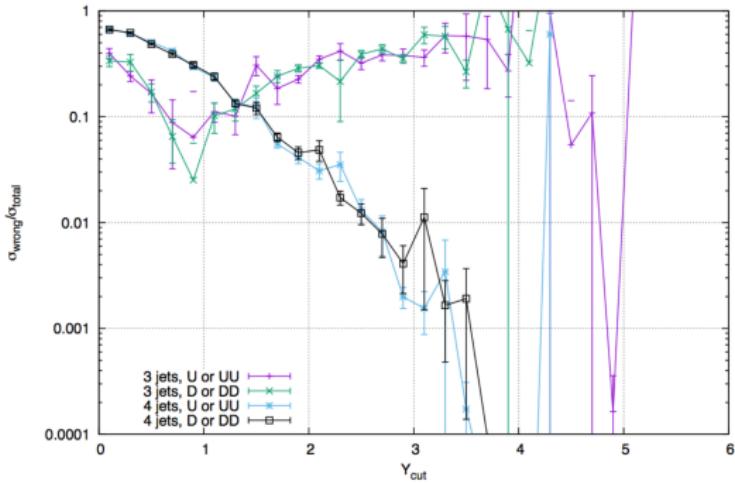
# Tagging



With no bug in the code, the probability of a tagged parton having wrong rapidity decreases with increasing rapidity separation between the two hardest jets.



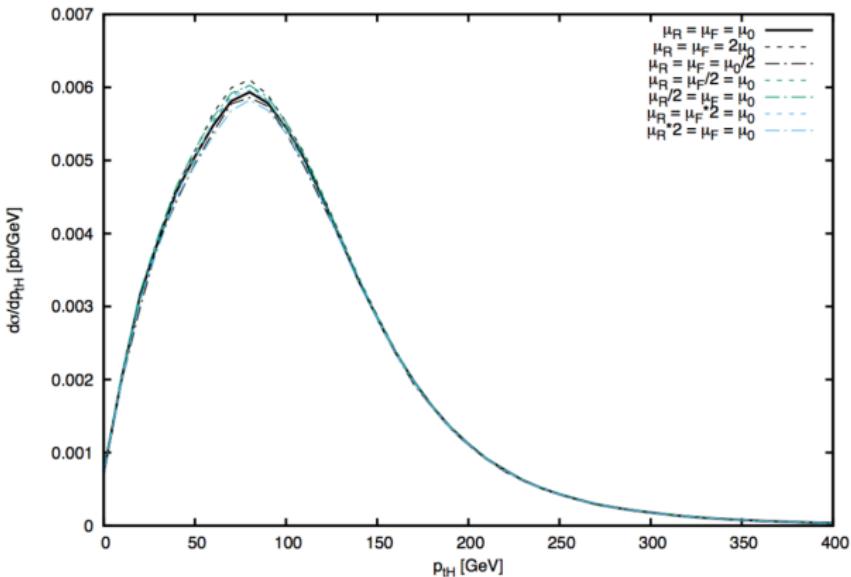
# Tagging



With an  $\mathcal{O}(1)$  bug in the code, this is clearly not the case any more.



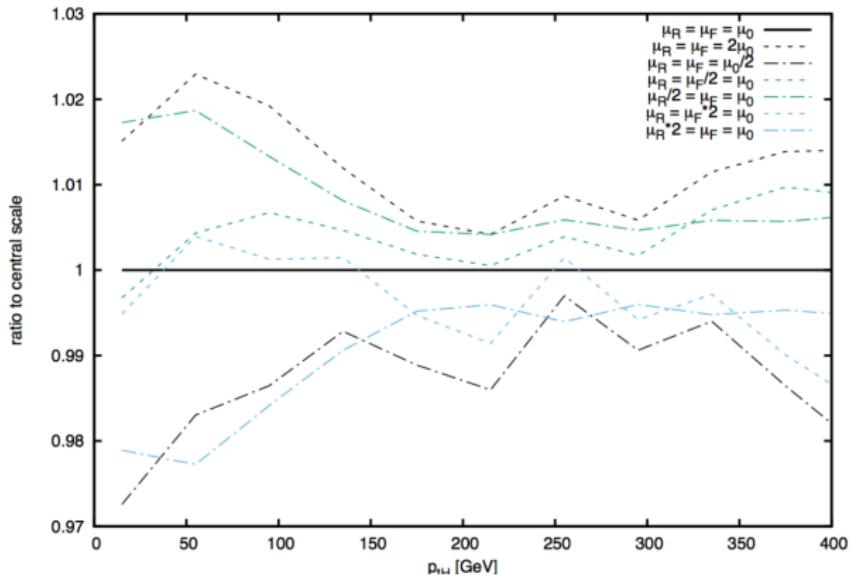
# 3-point vs 7-point scale variations



3- and 7-point scale variations are very close to eachother.



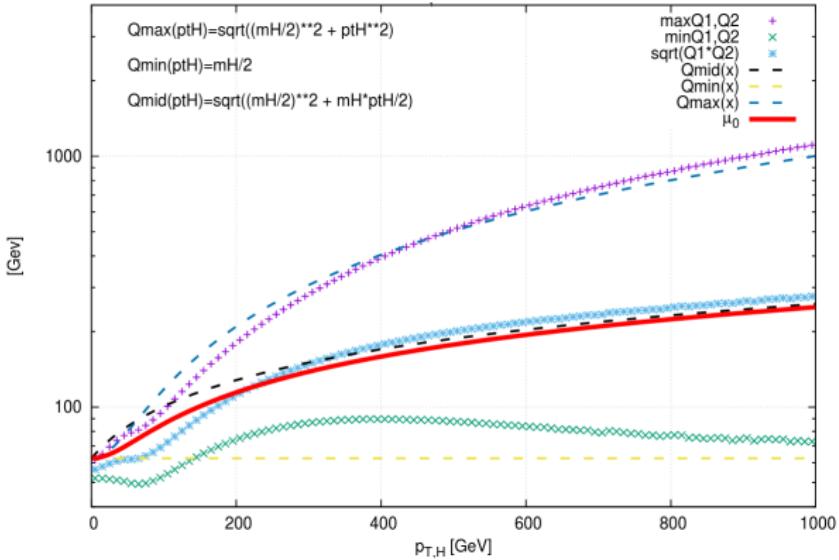
# 3-point vs 7-point scale variations



3- and 7-point scale variations are very close to eachother.



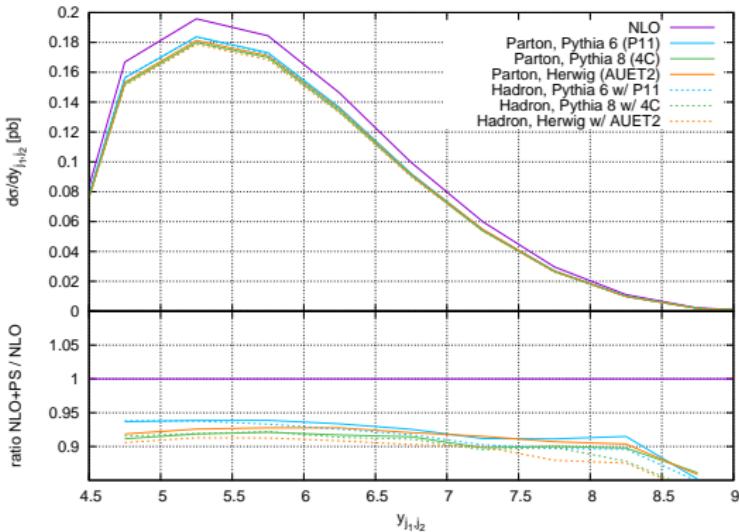
# Choice of scale



Our choice of  $\mu_0^2(p_{t,H}) = \frac{M_H}{2} \sqrt{\left(\frac{M_H}{2}\right)^2 + p_{t,H}^2}$  is very close to a choice of  $\mu = \sqrt{Q_1 Q_2}$ .



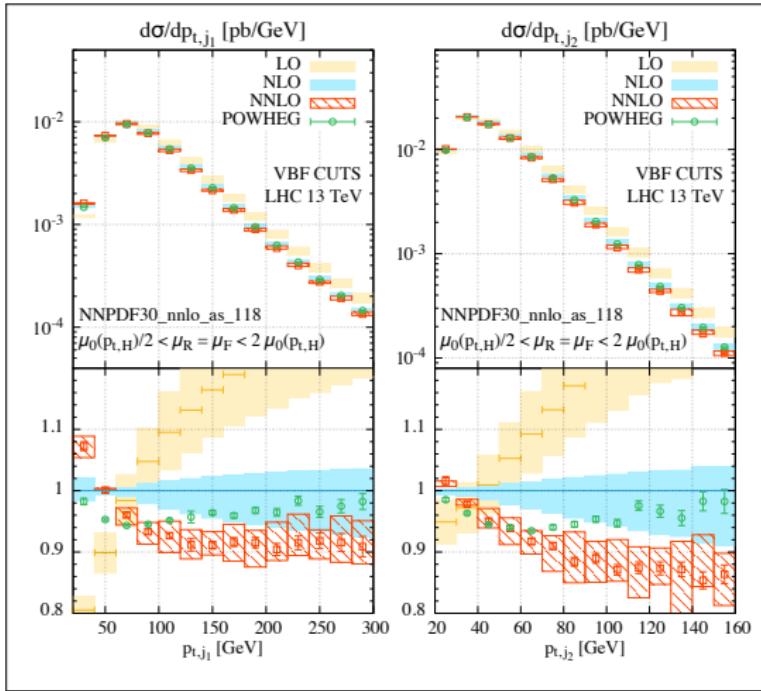
# VBF and Parton Shower



- different parton showers give relatively similar results
- hadronisation effects are consistently small



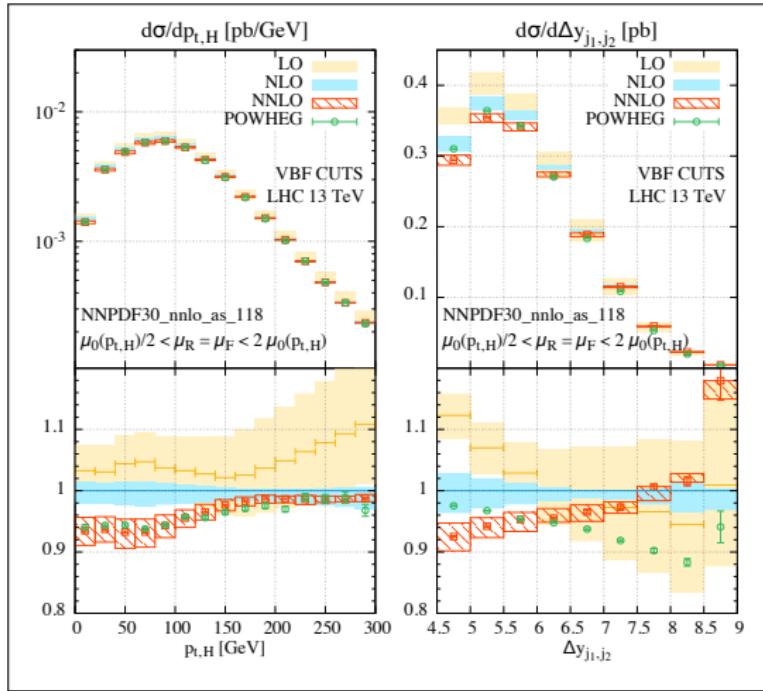
# NNLO/NLO/LO PDFs



- LO results with LO PDFs
- NLO results with NLO PDFs
- NNLO results with NNLO PDFs



# NNLO/NLO/LO PDFs



- LO results with LO PDFs
- NLO results with NLO PDFs
- NNLO results with NNLO PDFs