# VBF Higgs production at NNLO ... and beyond

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Based on PRL115(082002) and PRL117(072001) in collaboration with Matteo Cacciari, Frédéric Dreyer, Gavin Salam & Giulia Zanderighi

... and work in progress with Barbara Jäger and Giulia Zanderighi.

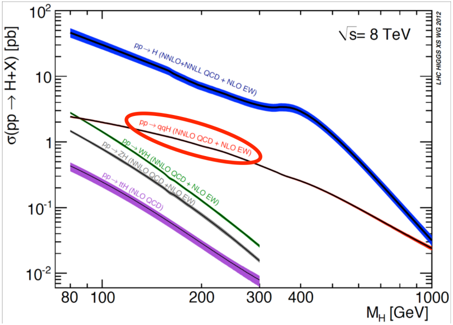
# This work

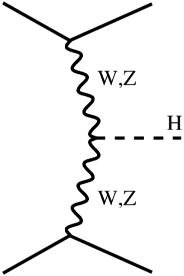
- first semi-differential N$^3$LO calculation of VBF Higgs production in the structure function approach
- first fully differential NNLO calculation of VBF Higgs production using a novel "projection-to-Born" method
- Some (very) preliminary results on MiNLO improved VBF Hjjj production

## What to expect

- N$^3$LO corrections are at the permille level and well within the scale uncertainty band of the NNLO prediction
- the associated scale uncertainty is reduced by a factor 5 at N$^3$LO
- NNLO corrections are sizeable, $\mathcal{O}(10\%)$, and outside NLO band
- the corrections are (almost everywhere) negative
- only moderate shrinkage of NNLO bands compared to NLO bands
- the "projection-to-Born" method is quite general and can be extended to other similar processes

# Reasons to study VBF




- largest cross section at tree-level and second-largest of all channels
- distinct signature of two forward jets
- tagging reduces backgrounds (eg $H \to b\bar{b}$)
- non-zero Higgs transverse momentum at lowest order
- sensitive to CP properties of the Higgs through correlation of forward jets
- sensitive to trilinear Higgs self-coupling through loop-corrections

# Reasons to study VBF

| Production process | ATLAS+CMS | ATLAS | CMS |
|---|---|---|---|
| $\mu_{\mathrm{ggF}}$ | $1.03^{+0.17}_{-0.15}$ | $1.25^{+0.24}_{-0.21}$ | $0.84^{+0.19}_{-0.16}$ |
| $\mu_{\mathrm{VBF}}$ | $1.18^{+0.25}_{-0.23}$ | $1.21^{+0.33}_{-0.30}$ | $1.13^{+0.37}_{-0.34}$ |
| $\mu_{WH}$ | $0.88^{+0.40}_{-0.38}$ | $1.25^{+0.56}_{-0.52}$ | $0.46^{+0.57}_{-0.54}$ |
| $\mu_{ZH}$ | $0.80^{+0.39}_{-0.36}$ | $0.30^{+0.51}_{-0.46}$ | $1.35^{+0.58}_{-0.54}$ |
| $\mu_{ttH}$ | $2.3^{+0.7}_{-0.6}$ | $1.9^{+0.8}_{-0.7}$ | $2.9^{+1.0}_{-0.9}$ |

- largest cross section at tree-level and second-largest of all channels
- distinct signature of two forward jets
- tagging reduces backgrounds (eg $\mathrm{H} \to \mathrm{b}\bar{\mathrm{b}}$)
- non-zero Higgs transverse momentum at lowest order
- sensitive to CP properties of the Higgs through correlation of forward jets
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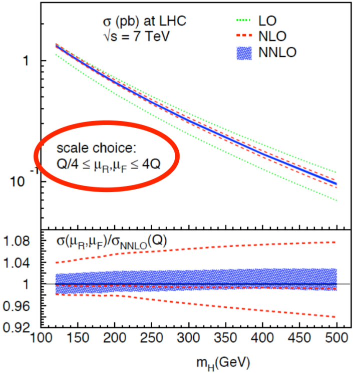
# Inclusive NNLO VBF Higgs Production

Until recently VBF Higgs production was only known inclusively at NNLO.

[Bolzoni et al. (2010)]



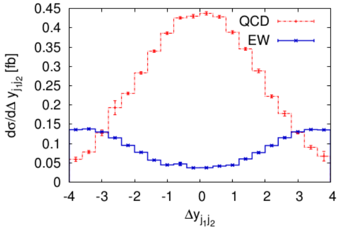

- the calculation suggests tiny renormlisation and factorisation scale variations $(\sim 1-2\%)$
- NNLO results well within NLO band
- result obtained in the structure function approach

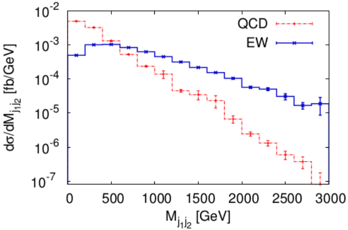
No cuts can be applied to the calculation, as it is totally inclusive over hadronic final states with the same vector boson momenta.

# VBF Cuts

Due to huge QCD backgrounds a set of very selective cuts have to be applied. Typical cuts are:

- jets defined with anti-$k_t$, $R = 0.4$ and $p_t > 25$ GeV
- two hardest jets within $|y| < 4.5$
- high dijet invariant mass, $M_{j_1 j_2} > 600$ GeV, and separation, $\Delta y_{j_1 j_2} > 4.5$
- hardest jets in opposite hemispheres, $y_{j_1} y_{j_2} < 0$

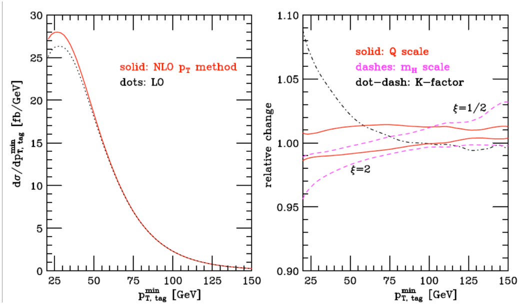



[Jäger, Zanderighi (2011)]

# Exclusive NLO VBF Higgs Production

To enable the application of realistic VBF cuts one has to be fully differential. Until recently differential VBF Higgs production was known to NLO(+PS). [Figy, Oleari, Zeppenfeld (2003)]

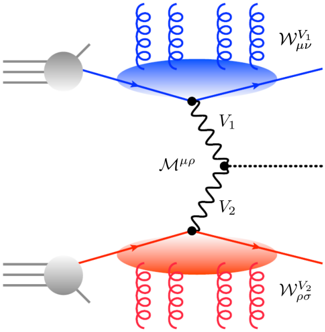



Calculation suggests small uncertainties from missing higher order corrections ($\sim 2\%$).

# Structure Function Approach

One can think of VBF Higgs production as a double Deep Inelastic Scattering (DIS×DIS) with no cross-talk between the upper and lower sectors. [Han, Valencia, Willenbrock (1992)]

- this picture is accurate to more than 1%
  [Bolzoni et al. (2012)], [Ciccolini, Denner, Dittmaier (2008)], [Harlander et al. (2008)], [Andersen et al. (2008)]



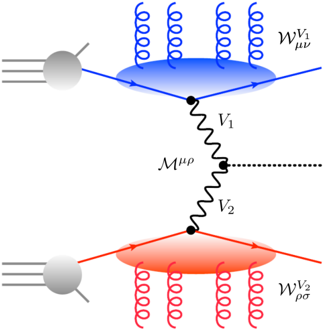

- the factorisation of the two sectors is exact if one imagines two copies of QCD, $\text{QCD}_1$ and $\text{QCD}_2$ , respectively for the upper and lower sectors.
- all DIS coefficients are known to to $\text{N}^3\text{LO}$.
- as the DIS coefficients are inclusive over the hadronic final state, the calculation cannot provide differential results.

# Structure Function Approach

$$d\sigma_{\text{VBF}} = \frac{G_F^2}{s} M_{V_1}^2 M_{V_2}^2 \Delta_{V_1}^2(Q_1^2)\Delta_{V_2}^2(Q_2^2) \times \mathcal{W}_{\mu\nu}^{V_1}(x_1, Q_1^2)\mathcal{M}^{\mu\rho}\mathcal{M}^{*\nu\sigma}\mathcal{W}_{\rho\sigma}^{V_2}(x_2, Q_2^2)\, d\Omega_{\text{VBF}}$$

and

$$\mathcal{M}^{\mu\nu} = 2\sqrt{\sqrt{2}G_F} M_V^2 g^{\mu\nu}$$




## Hadronic Tensor

$$\mathcal{W}_{\mu\nu}^{V}(x_i, Q_i^2) = \left(-g_{\mu\nu} + \frac{q_{i,\mu}q_{i,\nu}}{q_i^2}\right) F_1^V(x_i, Q_i^2) + \frac{\hat{P}_{i,\mu}\hat{P}_{i,\nu}}{P_i \cdot q_i} F_2^V(x_i, Q_i^2) + i\epsilon_{\mu\nu\rho\sigma}\frac{P_i^\rho q_i^\sigma}{2P_i \cdot q_i} F_3^V(x_i, Q_i^2)$$

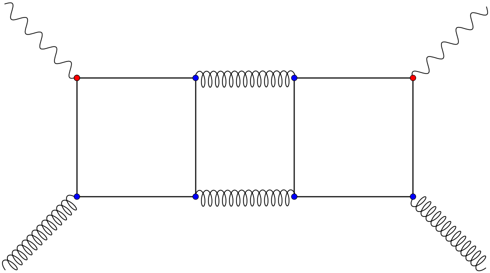
# Structure Function Approach

The Structure Functions can be expressed as convolutions of the short distance DIS coefficient functions and PDFs

$$F_i^V = \sum_{a=q,g} C_{i,a}^V \otimes f_a \,, \quad i = 2, L, 3\,, \quad V = Z, W^+, W^- \,,$$

where DIS coefficient functions are known to the third order in $\alpha_s$. [Moch, Rogal, Vermaseren, Vogt (2005-2008)]

Conceptually N$^3$LO not more complicated than NNLO, but non-trivial flavour structures start appearing at N$^3$LO



# What about missing N$^3$LO PDFs?

All approximations aside, we can't truely claim N$^3$LO accuracy without also having N$^3$LO PDFs available.
Naive estimate

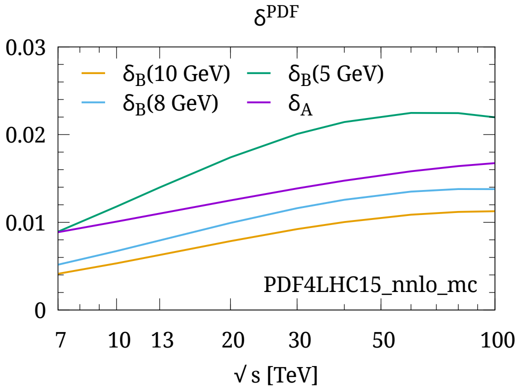
$$\delta_A^{\mathrm{PDF}} = \frac{1}{2}\left|\frac{\sigma^{\mathrm{NNLO}}_{\mathrm{NNLO\text{-}PDF}} - \sigma^{\mathrm{NNLO}}_{\mathrm{NLO\text{-}PDF}}}{\sigma^{\mathrm{NNLO}}_{\mathrm{NNLO\text{-}PDF}}}\right| = 1.1\%\,,$$

and slightly more sophisticated ($Q_0 = 8$ GeV)

$$f^{\mathrm{N^3LO,approx.}}(x,Q) = f^{\mathrm{NNLO}}(x,Q)\,\frac{F_2^{\mathrm{NNLO}}(x,Q_0)}{F_2^{\mathrm{N^3LO}}(x,Q_0)}\,,$$

$$\delta_B^{\mathrm{PDF}}(Q_0) = \left|\frac{\sigma^{\mathrm{N^3LO}} - \sigma^{\mathrm{N^3LO}}_{\mathrm{rescaled}}(Q_0)}{\sigma^{\mathrm{N^3LO}}}\right| = 7.9‰\,.$$

# What about missing N$^3$LO PDFs?




- the value of $Q_0 = 8$ GeV is found by requiring that the method is reliable in estimating NNLO corrections to PDFs
- the uncertainty associated with varying the scale in the DGLAP evolutions is found to be below the permille level

→ could obtain approximate N$^3$LO PDFs using just N$^3$LO coefficient functions

# Scale variations at $N^3LO$

We use RGE methods to evaluate the structure functions at arbitrary renormalisation scales

$$\alpha_S(Q) = \alpha_S(\mu_R) + \alpha_S^2(\mu_R)\beta_0 L_{RQ} + \alpha_S^3(\mu_R)(\beta_0^2 L_{RQ}^2 + \beta_1 L_{RQ}) + \mathcal{O}(\alpha_S^4(\mu_R))$$

$$L_{RQ} = \ln\left(\frac{\mu_R{}^2}{Q^2}\right), \quad L_{FQ} = \ln\left(\frac{\mu_F{}^2}{Q^2}\right)$$

$$C_i = \sum_{k=0} \left(\frac{\alpha_S(Q)}{2\pi}\right)^k C_i^{(k)} =$$
$$C_i^{(0)} + \frac{\alpha_S(\mu_R)}{2\pi} C_i^{(1)} +$$
$$\left(\frac{\alpha_S(\mu_R)}{2\pi}\right)^2 \left(C_i^{(2)} + 2\pi\beta_0 C_i^{(1)} L_{RQ}\right) +$$
$$\left(\frac{\alpha_S(\mu_R)}{2\pi}\right)^3 \Big[C_i^{(3)} + 4\pi\beta_0 C_i^{(2)} L_{RQ}$$
$$+ 4\pi^2 C_i^{(1)} L_{RQ}(\beta_1 + \beta_0^2 L_{RQ})\Big] + \mathcal{O}(\alpha_S^4)$$

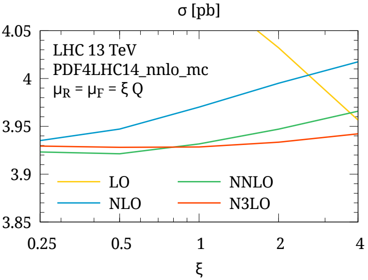
$$f(x,Q) = f(x,\mu_F)\Bigg(1 - \frac{\alpha_S(\mu_R)}{2\pi} L_{FQ} P^{(0)}$$
$$- \left(\frac{\alpha_S(\mu_R)}{2\pi}\right)^2 L_{FQ}\Big[P^{(1)} - \frac{1}{2} L_{FQ}(P^{(0)})^2$$
$$- \pi\beta_0 P^{(0)}(L_{FQ} - 2L_{RQ})\Big]$$
$$- \left(\frac{\alpha_S(\mu_R)}{2\pi}\right)^3 L_{FQ}\Big[P^{(2)} - \frac{1}{2} L_{FQ}(P^{(0)}P^{(1)}$$
$$+ P^{(1)}P^{(0)}) + \pi\beta_0(L_{FQ} - 2L_{RQ})\times$$
$$(L_{FQ}(P^{(0)})^2 - 2P^{(1)}) + \frac{1}{6} L_{FQ}^2 (P^{(0)})^3$$
$$+ 4\pi^2\beta_0^2 P^{(0)}(L_{RQ}^2 - L_{FQ}L_{RQ} + \frac{1}{3}L_{FQ}^2)$$
$$- 2\pi^2\beta_1 P^{(0)}(L_{FQ} - 2L_{RQ})\Big] + \mathcal{O}(\alpha_S^4)\Bigg)$$

# N$^3$LO results

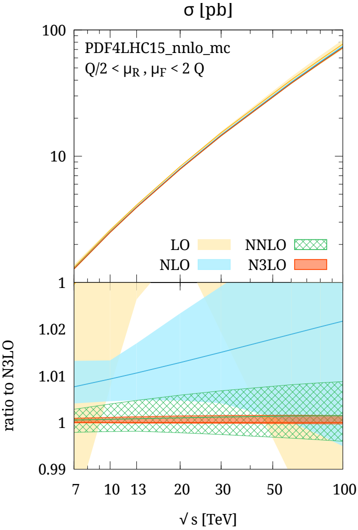
| | $\sigma^{(13\text{ TeV})}$ [pb] | $\sigma^{(14\text{ TeV})}$ [pb] | $\sigma^{(100\text{ TeV})}$ [pb] |
|---|---|---|---|
| LO | $4.099^{+0.051}_{-0.067}$ | $4.647^{+0.037}_{-0.058}$ | $77.17^{+6.45}_{-7.29}$ |
| NLO | $3.970^{+0.025}_{-0.023}$ | $4.497^{+0.032}_{-0.027}$ | $73.90^{+1.73}_{-1.94}$ |
| NNLO | $3.932^{+0.015}_{-0.010}$ | $4.452^{+0.018}_{-0.012}$ | $72.44^{+0.53}_{-0.40}$ |
| N$^3$LO | $3.928^{+0.005}_{-0.001}$ | $4.448^{+0.006}_{-0.001}$ | $72.34^{+0.11}_{-0.02}$ |

- We study pp collisions with PDF4LHC_nnlo_mc and electroweak parameters fixed to their PDG values
- the central renormalisation and factorisation scale is set equal to $Q_1, Q_2$ and varied independently by a factor 2 up and down
- N$^3$LO corrections are tiny $(\sim 2‰)$ but predictions well within NNLO scale uncertainty

# N$^3$LO results

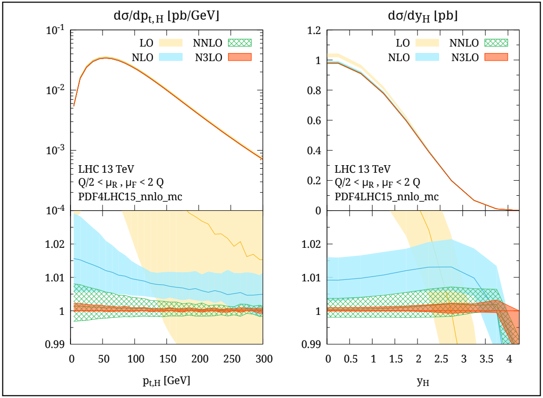



- the N$^3$LO corrections are tiny over a large range of energies and stay well within the scale uncertainty band of the NNLO prediction
- cross section becomes extremely stable under the variation of renormalisation and factorisation scales



# $N^3LO$ results

From the knowledge of $Q_1$ and $Q_2$ it is trivial to reconstruct the momentum of the Higgs. The calculation is therefore fully differential in the Higgs kinematics.




- the corrections are almost flat throughout the entire spectrum
- the $N^3LO$ prediction completely contained within the scale uncertainty band of the NNLO prediction
- only differential in the momenta of the proton remnants, and hence no real information on the tagging jets

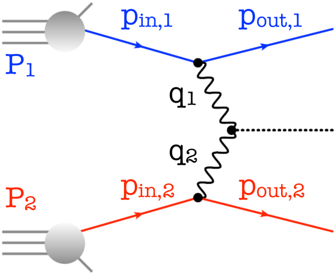
# Limitations of the structure function approach

- gluon exchanges between the upper and lower hadronic sectors, which appear at NNLO, but are kinematically and colour suppressed; These contributions along with the heavy-quark loop induced contributions have been estimated to contribute at the permille level [Bolzoni et al. (2012)]
- t-/u-channel interference which are known to contribute $\mathcal{O}(5‰)$ at the fully inclusive level and $\mathcal{O}(0.5‰)$ after VBF cuts have been applied [Ciccolini, Denner, Dittmaier (2008)]
- contributions from s-channel production, which have been calculated up to NLO. At the inclusive level these contributions are sizeable but they are reduced to $\mathcal{O}(5‰)$ after VBF cuts [Ciccolini, Denner, Dittmaier (2008)]
- single-quark line contributions, which contribute to the VBF cross section at NNLO. At the fully inclusive level these amount to corrections of $\mathcal{O}(1\%)$ but are reduced to the permille level after VBF cuts have been applied [Harlander et al. (2008)]
- loop induced interference between VBF and ggH production. These contributions have been shown to be much below the permille level [Andersen et al. (2008)]

# Beyond the Structure Function Approach

Next: We eliminate the limitations of the Structure Function Approach.

If the scattering is Born like, then the vector boson momenta $q_i$, and on-shell conditions, fix the incoming and outgoing parton momenta:




$$p_{in,i} = x_i P_i$$

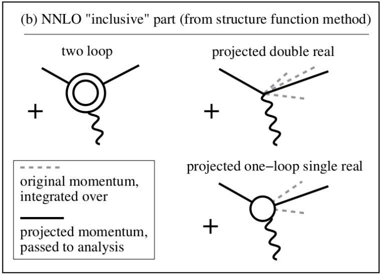
$$p_{out,i} = x_i P_i - q_i$$

$$x_i = \frac{q_i^2}{2 q_i P_i}$$

# Beyond the Structure Function Approach

The calculation is based on two ingredients:

1. *An "inclusive" contribution*
    - use the Structure Function Approach and use four-vectors $q_1$, $q_2$ to assign Born-like kinematics using the equations below
    - use the projected Born-like momenta to compute differential distributions

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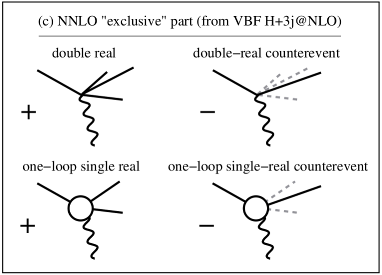
# Beyond the Structure Function Approach

The calculation is based on two ingredients:

2. *An "exclusive" contribution*
   - use the electroweak H+jjj NLO calculation in the factorized approximation [Figy et al. (2007)], [Jäger et al. (2014)]




- for each parton, keep track of whether it belongs to the upper or lower sector, and compute vector-boson momenta $q_1$, $q_2$
- for each event add counter-event with projected Born kinematics and opposite weight

The counter-events cancel identically with the projected terms from the "inclusive" contribution.

# Beyond the Structure Function Approach

Schematically we express the "projection-to-Born" (P2B) method as

$$
\begin{aligned}
d\sigma &= \int d\Phi_B (B+V) + \int d\Phi_R R \\
&= \underbrace{\int d\Phi_B (B+V) + \int d\Phi_R R_{P2B}}_{\text{“inclusive” contribution}} + \underbrace{\int d\Phi_R R - \int d\Phi_R R_{P2B}}_{\text{“exclusive” contribution}}
\end{aligned}
$$

- from the "exclusive" ingredient we get the full double-real and one-loop single-real contributions.
- when integrated over phase-space, the counter-events cancel the projected double-real and one-loop single-real contributions from the "inclusive" ingredient

Hence the sum of the two contributions gives the complete, fully differential NNLO result.

# Beyond the Structure Function Approach

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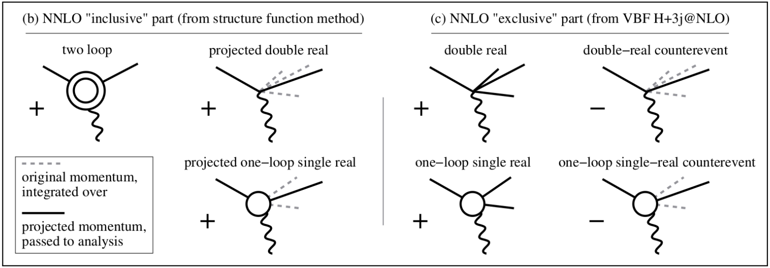
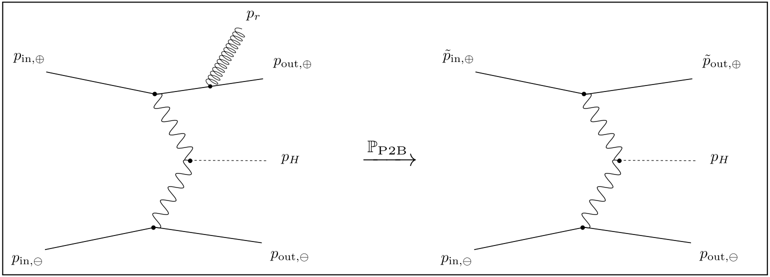
\end{aligned}
$$



# The projection



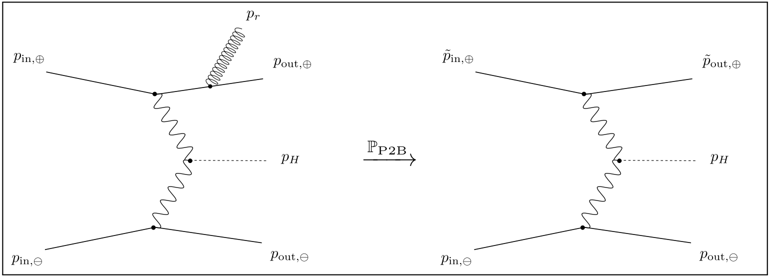

$$\Omega_R = (p_{\text{in},\oplus}, p_{\text{in},\ominus}, p_H, p_{\text{out},\oplus}, p_{\text{out},\ominus}, p_r)$$

$$\mathbb{P}_{\text{P2B}}\Omega_R = \Omega_{\text{P2B}} = (\tilde{p}_{\text{in},\oplus}, \tilde{p}_{\text{in},\ominus}, p_H, \tilde{p}_{\text{out},\oplus}, \tilde{p}_{\text{out},\ominus})$$

Use lightcone coordinates:

$$p = (p^x, p^y, p^-, p^+) \qquad \text{with} \qquad p^\pm = \frac{1}{\sqrt{2}}(p^E \pm p^z)$$

# The projection



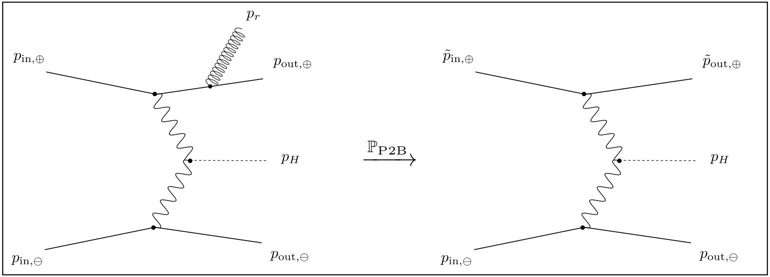

$$\Omega_R = (p_{\mathrm{in},\oplus}, p_{\mathrm{in},\ominus}, p_H, p_{\mathrm{out},\oplus}, p_{\mathrm{out},\ominus}, p_r)$$

$$\begin{aligned}\mathbb{P}_{\mathrm{P2B}}\Omega_R &= \Omega_{\mathrm{P2B}} \\ &= (\tilde{p}_{\mathrm{in},\oplus}, \tilde{p}_{\mathrm{in},\ominus}, p_H, \tilde{p}_{\mathrm{out},\oplus}, \tilde{p}_{\mathrm{out},\ominus})\end{aligned}$$

Such that

$$\begin{aligned}p_{\mathrm{in}} &= (0,0,0,p_{\mathrm{in}}^{+}) & \tilde{p}_{\mathrm{in}} &= (0,0,0,\tilde{p}_{\mathrm{in}}^{+}) \\ p_{\mathrm{out}} &= (p_{\mathrm{out}}^{x}, p_{\mathrm{out}}^{y}, p_{\mathrm{out}}^{-}, p_{\mathrm{out}}^{+}) & \tilde{p}_{\mathrm{out}} &= (\tilde{p}_{\mathrm{out}}^{x}, \tilde{p}_{\mathrm{out}}^{y}, \tilde{p}_{\mathrm{out}}^{-}, \tilde{p}_{\mathrm{out}}^{+}) \\ p_r &= (p_r^{x}, p_r^{y}, p_r^{-}, p_r^{+}) & &\end{aligned}$$

# The projection




By momentum conservation we have

$$p_{\text{in}} - p_{\text{out}} - p_r = \tilde{p}_{\text{in}} - \tilde{p}_{\text{out}} \quad \Rightarrow \quad \begin{cases} \tilde{p}^x_{\text{out}} = p^x_{\text{out}} + p^x_r \\ \tilde{p}^y_{\text{out}} = p^y_{\text{out}} + p^y_r \\ \tilde{p}^-_{\text{out}} = p^-_{\text{out}} + p^-_r \\ \tilde{p}^+_{\text{in}} = p^+_{\text{in}} - p^+_{\text{out}} - p^+_r + \tilde{p}^+_{\text{out}} . \end{cases}$$

$$\begin{aligned} (\tilde{p}_{\text{out}})^2 &= 0 \\ &\Rightarrow \quad (\tilde{p}^x_{\text{out}})^2 + (\tilde{p}^y_{\text{out}})^2 - 2\tilde{p}^-_{\text{out}}\tilde{p}^+_{\text{out}} = 0 \\ &\Rightarrow \quad \tilde{p}^+_{\text{out}} = \frac{(p^x_{\text{out}} + p^x_r)^2 + (p^y_{\text{out}} + p^y_r)^2}{2(p^-_{\text{out}} + p^-_r)} , \end{aligned}$$

# Implementation

1. "inclusive" code
   - matrix elements coded with structure functions using parametrised versions of the DIS coefficient functions evaluated by `HOPPET`
   - phase-space taken from `POWHEG`'s `VBF_H` generator
2. "exclusive" code
   - start with the `VBF_HJJJ` calculation in `POWHEG` (based on vbfnlo)
   - extend `POWHEG`'s tags to uniquely associate radiation with each sector (upper or lower line)
   - for each event map the kinematics onto Born-like kinematics and determine the vector-boson momenta $q_1$, $q_2$ using the equations on p.18.

- we have tested the "inclusive" code against a private version of the structure function calculation (thanks to Marco Zaro) and the structure functions themselves against `APFEL` 2.4.1.
- we have tested that the "exclusive" code reproduces the original `VBF_HJJJ` result. The sum of "inclusive" and "exclusive" at NLO agrees with `VBF_H`
- tagging tested by checking that the probablity of assigning a parton to the wrong sector decreases as the rapidity between the two hardest jets increases

# Phenomenology

We study 13 TeV LHC collisions with $M_H = 125$ GeV and NNPDF3.0_nnlo_as118. We use the following VBF cuts:

- Jets defined with anti-$k_t$, $R = 0.4$ and $p_t > 25$ GeV
- Two hardest jets within $|y| < 4.5$
- High dijet invariant mass, $M_{j_1 j_2} > 600$ GeV, and separation, $\Delta y_{j_1 j_2} > 4.5$
- Hardest jets in opposite hemispheres, $y_{j_1} y_{j_2} < 0$

We choose a central scale which approximates well $\sqrt{Q_1 Q_2}$ and symmetrically vary by a factor 2 up and down

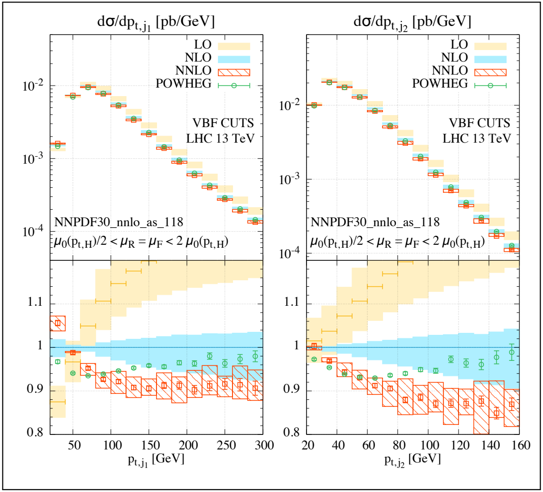
$$\mu_0^2(p_{t,H}) = \frac{M_H}{2}\sqrt{\left(\frac{M_H}{2}\right)^2 + p_{t,H}^2}$$

# Phenomenology

| | $\sigma^{(\text{no cuts})}$ [pb] | $\sigma^{(\text{VBF cuts})}$ [pb] |
|---|---|---|
| LO | $4.032^{+0.057}_{-0.069}$ | $0.957^{+0.066}_{-0.059}$ |
| NLO | $3.929^{+0.024}_{-0.023}$ | $0.876^{+0.008}_{-0.018}$ |
| NNLO | $3.888^{+0.016}_{-0.012}$ | $0.826^{+0.013}_{-0.014}$ |

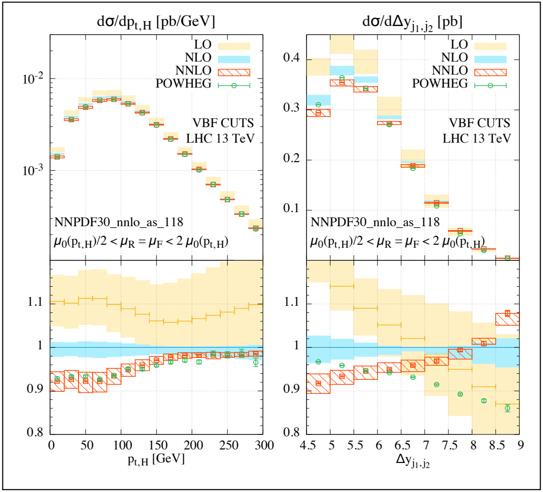
- NNLO corrections tiny $(\sim 1\%)$ without cuts and sizeable with VBF cuts $(\sim 5\%)$
- NNLO results outside NLO band (also true when using NLO PDFs)
- corrections tend to be dominated by the extra real radiation. The effect is softer jets and hence fewer events pass the cuts

# Phenomenology




- NNLO corrections can be large $\mathcal{O}(10\%)$ and are often outside the NLO band
- the NNLO corrections tend to be dominated by extra real radiation. These appear to make the jets softer
- NOTE: NNLO PDF used everywhere. Similar results hold when using LO/NLO PDFs
- expanding the scale variation from 3-point to 7-point doesn't change the size of the NLO bands noticeably

# More Phenomenology




- in some cases NLO+PS agrees very well with the NNLO result (in particular $p_{t,H}$, $M_{jj}$ and $\phi_{jj}$)
- in some cases not ($\Delta y_{j_1,j_2}$ and $H_t$)
- in general only modest shrinkage of bands from NLO to NNLO
- non-trivial kinematic dependence on k-factors (both LO/NLO and NNLO/NLO)

# LHCHXWG YR4 results

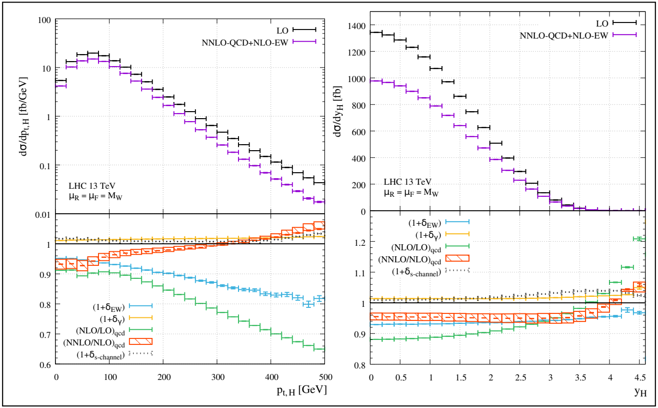
We study LHC collisions with $M_H = 125$ GeV and PDF4LHC15_nnlo_100. We use the following setup:

- Jets defined with anti-$k_t$, $R = 0.4$ and $p_t > 20$ GeV
- Two hardest jets within $|y| < 5$
- Dijet invariant mass, $M_{j_1 j_2} > 130$ GeV, and separation, $\Delta y_{j_1 j_2} > 3$
- $\mu_R = \mu_F = M_W$ (varied a factor 2 up and down)
- Electroweak corrections obtained with HAWK [Ciccolini, Denner, Dittmaier (2008)]
- Photon PDF obtained from NNPDF2.3QED
- $\sigma^{\text{VBF}} = \sigma^{\text{DIS}}_{\text{NNLOQCD}}(1 + \delta_{\text{EW}}) + \sigma_\gamma$

| $\sqrt{s}$[TeV] | $\sigma^{\text{VBF}}$[fb] | $\Delta_{\text{scale}}$[%] | $\Delta_{\text{PDF}/\alpha_s/\text{PDF}\oplus\alpha_s}$[%] | $\sigma^{\text{DIS}}_{\text{NNLOQCD}}$[fb] | $\delta_{\text{EW}}$[%] | $\sigma_\gamma$[fb] | $\sigma_{\text{s-channel}}$[fb] |
|---|---|---|---|---|---|---|---|
| 7 | 1241 | $^{+0.19}_{-0.21}$ | $\pm 2.1 / \pm 0.4 / \pm 2.2$ | 1281 | $-4.4$ | 17.1 | 584.5(3) |
| 8 | 1601 | $^{+0.25}_{-0.24}$ | $\pm 2.1 / \pm 0.4 / \pm 2.2$ | 1656 | $-4.6$ | 22.1 | 710.4(3) |
| 13 | 3782 | $^{+0.43}_{-0.33}$ | $\pm 2.1 / \pm 0.5 / \pm 2.1$ | 3939 | $-5.3$ | 51.9 | 1378.1(6) |
| 14 | 4278 | $^{+0.45}_{-0.34}$ | $\pm 2.1 / \pm 0.5 / \pm 2.1$ | 4461 | $-5.4$ | 58.5 | 1515.9(6) |

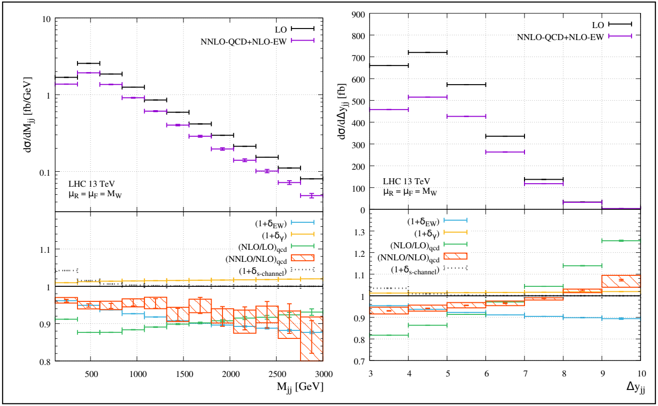
| $\sqrt{s}$[TeV] | $\sigma^{\text{VBF}}$[fb] | $\Delta_{\text{scale}}$[%] | $\Delta_{\text{PDF}/\alpha_s/\text{PDF}\oplus\alpha_s}$[%] | $\sigma^{\text{DIS}}_{\text{NNLOQCD}}$[fb] | $\delta_{\text{EW}}$[%] | $\sigma_\gamma$[fb] | $\sigma_{\text{s-channel}}$[fb] |
|---|---|---|---|---|---|---|---|
| 7 | 602.4(5) | $^{+1.3}_{-1.6}$ | $\pm 2.3 / \pm 0.3 / \pm 2.3$ | 630.8(5) | $-6.1$ | 9.9 | 8.2 |
| 8 | 795.9(6) | $^{+1.3}_{-1.5}$ | $\pm 2.3 / \pm 0.3 / \pm 2.3$ | 834.8(7) | $-6.2$ | 13.1 | 11.1 |
| 13 | 1975(1) | $^{+1.3}_{-1.2}$ | $\pm 2.1 / \pm 0.4 / \pm 2.2$ | 2084(1) | $-6.8$ | 32.3 | 29.0 |
| 14 | 2236(3) | $^{+1.5}_{-1.3}$ | $\pm 2.1 / \pm 0.4 / \pm 2.1$ | 2362(3) | $-6.9$ | 36.7 | 33.1 |

# LHCHXSWG YR4 results




- NLO-QCD corrections can be very large (notice scale choice)
- NLO-EW corrections can be sizeable in the tails of distributions
- NNLO-QCD corrections of the same order as NLO-EW corrections
- Photon induced contribution very flat

# LHCHXSWG YR4 results

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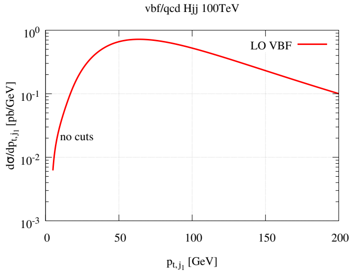
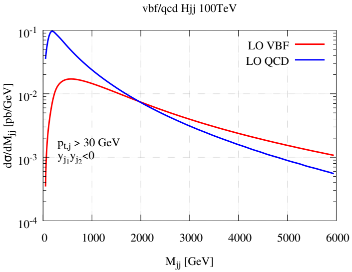
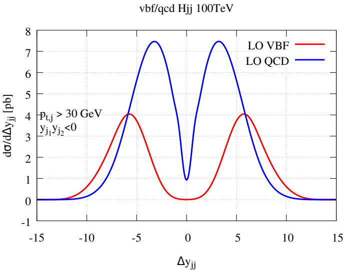
# VBF @ 100 TeV




- due to the EW production mechanism of VBF, the jet spectra are mostly unchanged going from 14 TeV to 100 TeV
- in particular, the two tag jets have transverse momenta set by the vector boson mass whereas the QCD jets tend to peak at much lower transverse momenta
- VBF cuts should therefore still efficiently suppress the QCD background even at higher energies

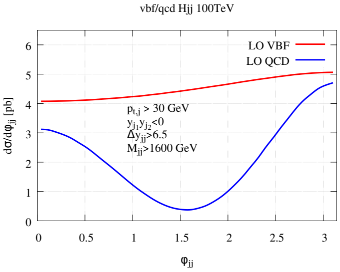
# VBF @ 100 TeV

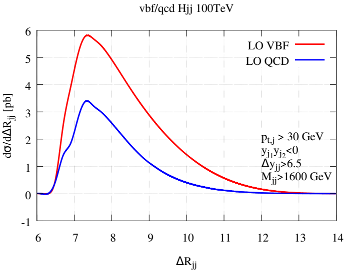



- after applying a soft transverse momentum cut on the tagging jets and requiring the jets to be in opposite dector hemispheres, we may try to optimise our VBF cuts
- it is clear that VBF production starts dominating for $\|\Delta y_{jj}\| > 6.5$
- after imposing this cut, the $M_{jj}$ peak is shifted to around 2400 GeV
- in order to sufficiently suppress the QCD background while not cutting away the VBF peak we impose $M_{jj} > 1600$ GeV

# VBF @ 100 TeV



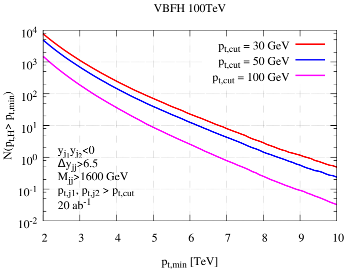



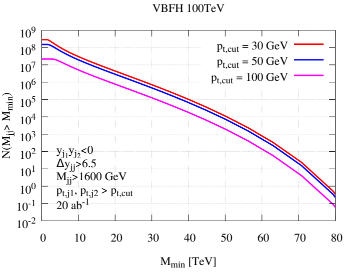

- further reduction possible by cutting in $\phi_{jj}$ whereas a cut in $R_{jj}$ only minimally reduces the QCD background

| VBF cuts | $\sigma(p_{t,j} > 30$ GeV) [pb] | $\sigma(p_{t,j} > 50$ GeV) [pb] | $\sigma(p_{t,j} > 100$ GeV) [pb] |
|---|---|---|---|
| VBFH | 14.1 | 7.51 | 1.08 |
| QCD Hjj | 5.04 | 1.97 | 0.331 |

| No cuts | $\sigma(p_{t,j} > 30$ GeV) [pb] | $\sigma(p_{t,j} > 50$ GeV) [pb] | $\sigma(p_{t,j} > 100$ GeV) [pb] |
|---|---|---|---|
| VBFH | 51.3 | 28.5 | 5.25 |
| QCD Hjj | 166 | 78.6 | 23.9 |

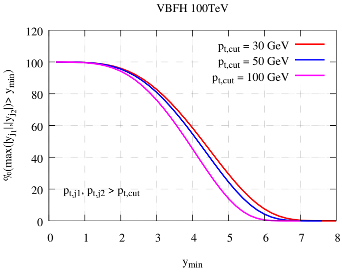
# VBF @ 100 TeV

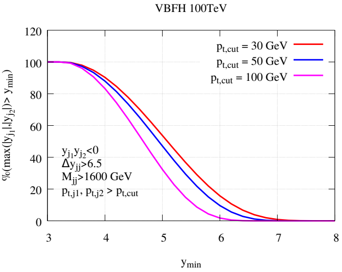



- at a 100 TeV collider gathering 20 $ab^{-1}$ of data, we can expect very high reach in energy
- for the transverse momentum distribution of the Higgs, we can expect a reach of almost 10 TeV
- tagging jets will be produced with an invariant mass of up to 80 TeV
- without imposing cuts the reach is of course even greater

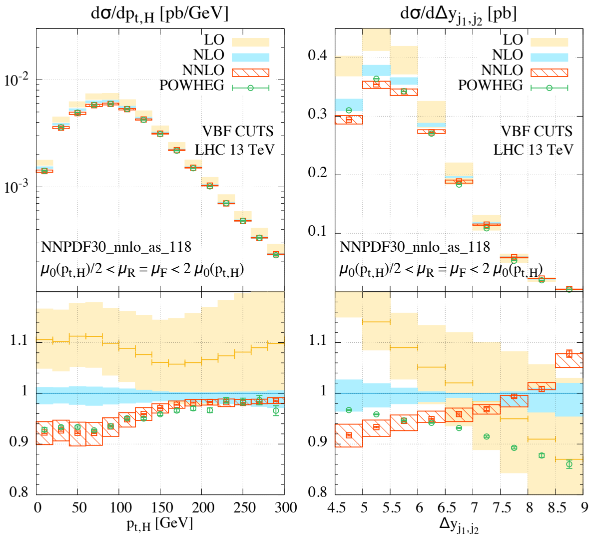
# VBF @ 100 TeV




- studying VBF at a 100 TeV collider will require impressive detectors
- before VBF cuts are applied ATLAS/CMS would lose roughly 50% of the signal
- after VBF cuts it would be as much as 80% of the signal depending on the jet definition
- in order to retain more than 90% of the signal the detector would need a rapidity reach of about 6.5

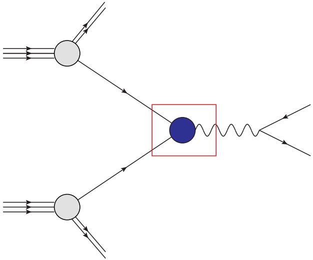
# What about parton showers?



# Parton showers

- parton shower: algorithm to resum (some classes of) collinear/soft logs in a "fully-exclusive" way.
- based on description of multiple soft-collinear real and virtual radiative corrections using a probabilistic language

$$d\sigma_{\rm SMC} = \underbrace{|\mathcal{M}_B|^2 d\Phi_B}_{d\sigma_B} \Big\{ \qquad \Big\}$$



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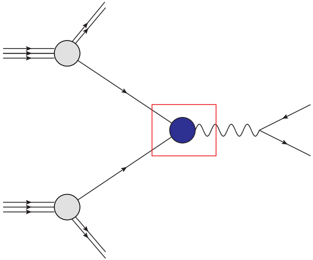
$$d\sigma_{\rm SMC} = \underbrace{|\mathcal{M}_B|^2 d\Phi_B}_{d\sigma_B} \left\{ \Delta(t_{\rm max}, t_0) \right.\qquad\qquad \left. \right\}$$



$$\Delta(t_{\rm max}, t) = \exp\left\{ - \int_t^{t_{\rm max}} d\Phi'_r \, \frac{\alpha_s}{2\pi} \frac{1}{t'} P(z') \right\}$$

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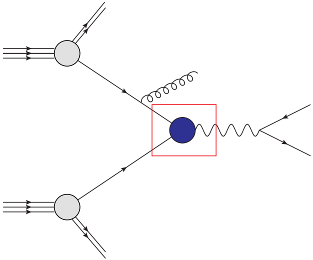

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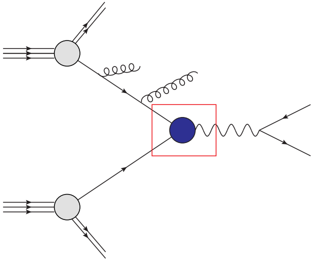

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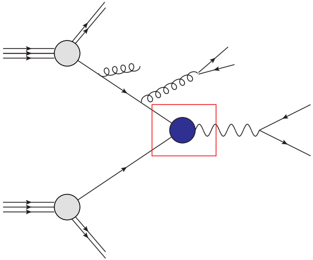

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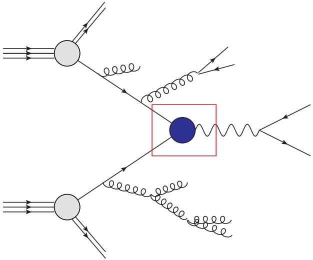

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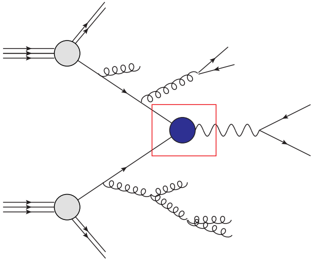


$$\Delta(t_{\max}, t) = \exp\left\{ -\int_t^{t_{\max}} d\Phi'_r \, \frac{\alpha_s}{2\pi} \frac{1}{t'} P(z') \right\}$$

This is "LOPS"

---

- A parton shower changes shapes, not the overall normalization, which stays LO (*unitarity*)
- LL resummation is included in Sudakov form factors: easy to see that probability of having arbitrarily collinear emission becomes 0, instead of $\infty$

# NLO+Parton Showers

- parton showers are only LO+LL: clearly including NLO corrections would be a big improvement. There are 2 methods to achieve this consistently:
- the POWHEG method:

1. do these replacement

$$B(\Phi_n) \Rightarrow \bar{B}(\Phi_n) = B(\Phi_n) + \frac{\alpha_s}{2\pi}\left[V(\Phi_n) + \int R(\Phi_{n+1})\, d\Phi_r\right]$$

$$\Delta(t_{\rm m}, t) \Rightarrow \Delta(\Phi_n; k_{\rm T}) = \exp\left\{-\frac{\alpha_s}{2\pi}\int \frac{R(\Phi_n, \Phi'_r)}{B(\Phi_n)}\theta(k'_{\rm T} - k_{\rm T})\, d\Phi'_r\right\}$$

2. `POWHEG` "master formula" for the hardest emission:

$$d\sigma_{\rm POW} = d\Phi_n\ \bar{B}(\Phi_n)\left\{\Delta(\Phi_n; k_{\rm T}^{\rm min}) + \Delta(\Phi_n; k_{\rm T})\frac{\alpha_s}{2\pi}\frac{R(\Phi_n, \Phi_r)}{B(\Phi_n)}\, d\Phi_r\right\}$$

[+ $p_{\rm T}$-*vetoing subsequent emissions*, to avoid double-counting]

3. properties:
   - inclusive observables: @NLO
   - first hard emission: full tree level ME
   - (N)LL resummation of collinear/soft logs

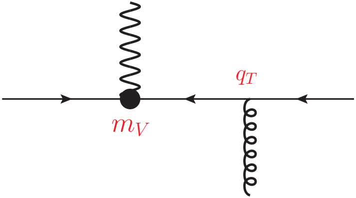
---

- NLOPS has become the standard for LHC searches (at least for SM processes)

1. $V$+$j$ @ NLO, $V$+$jj$ @ LO $\Rightarrow$ use $V$+$j$ @ NLOPS (POWHEG)

$$d\sigma_{\mathrm{POWHEG}} = d\Phi_n \, \bar{B}_{\mathrm{NLO}}(\Phi_n) \left\{ \Delta(\Phi_n; k_{\mathrm{T}}^{\mathrm{min}}) + \Delta(\Phi_n; k_{\mathrm{T}}) \frac{\alpha_s}{2\pi} \frac{R(\Phi_n, \Phi_r)}{B(\Phi_n)} \, d\Phi_r \right\}$$

$$\bar{B}_{\mathrm{NLO}}(\mathbf{\Phi}_n) \, d\mathbf{\Phi}_n = \alpha_{\mathrm{S}}(\mu_R) \left[ B + \alpha_{\mathrm{S}}^{(\mathrm{NLO})} V(\mu_R) + \alpha_{\mathrm{S}}^{(\mathrm{NLO})} \int d\Phi_{\mathrm{r}} R \right] d\mathbf{\Phi}_n$$




$V$+$j$ is a 2-scales problem ($\rightarrow$ choice of $\mu$ not unique)

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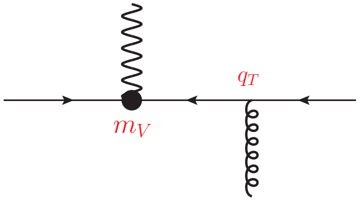


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☞ want to reach NNLO accuracy for *e.g.* $y_V$, *i.e.* when fully inclusive over QCD radiation

- need to allow the 1st jet to become unresolved
- the above approach needs to be modified: as it stands, $\bar{B}_{\mathrm{NLO}}(\Phi_n)$ is not finite when $q_T \rightarrow 0$!

2. integrate over phase space regions where $V$ is produced with arbitrarily soft/collinear jet (i.e. finite results when integrating over all $q_T$ spectrum)

MiNLO: Multiscale Improved NLO [Hamilton,Nason,Zanderighi, 1206.3572]

- original goal: method to a-priori choose scales in multijet NLO computation (where hierarchy among scales can spoil accuracy since resummation of logs is missing)
- how: correct weights of different NLO terms with CKKW-inspired approach:

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  - for all PS points, build the "more-likely" shower history that would have produced it (can be done by clustering kinematics with $k_T$-algo)
  - correct original NLO including $\alpha_S$ couplings evaluated at nodal scales and Sudakov FFs
  - make sure that NLO accuracy is not spoiled !

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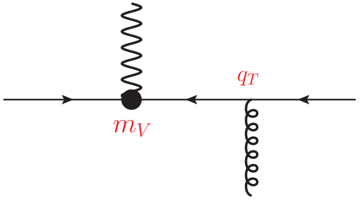

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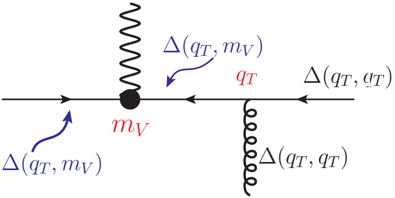
$$\bar{B}_{\rm NLO} = \alpha_{\rm S}(\mu_R)\Big[B + \alpha_{\rm S}^{\rm (NLO)} V(\mu_R) + \alpha_{\rm S}^{\rm (NLO)} \int d\Phi_{\rm r} R\Big]$$

$$\bar{B}_{\rm MiNLO} = \alpha_{\rm S}(q_T)\Delta_q^2(q_T, m_V)\Big[B\left(1 - 2\Delta_q^{(1)}(q_T, m_V)\right) + \alpha_{\rm S}^{\rm (NLO)} V(\bar{\mu}_R) + \alpha_{\rm S}^{\rm (NLO)} \int d\Phi_{\rm r} R\Big]$$




- $\bar{\mu}_R = q_T$
- $\log \Delta_{\rm f}(q_T, m_V) = -\int_{q_T^2}^{m_V^2} \frac{dq^2}{q^2} \frac{\alpha_{\rm S}(q^2)}{2\pi}\left[A_f \log \frac{m_V^2}{q^2} + B_f\right]$
- $\Delta_{\rm f}^{(1)}(q_T, m_V) = -\frac{\alpha_{\rm S}^{\rm (NLO)}}{2\pi}\left[\frac{1}{2} A_{1,{\rm f}} \log^2 \frac{m_V^2}{q_T^2} + B_{1,{\rm f}} \log \frac{m_V^2}{q_T^2}\right]$
- $\mu_F = q_T$

# MiNLO

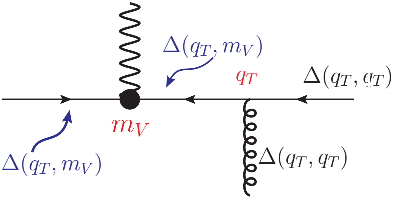
2. integrate over phase space regions where $V$ is produced with arbitrarily soft/collinear jet (i.e. finite results when integrating over all $q_T$ spectrum)

MiNLO: Multiscale Improved NLO [Hamilton,Nason,Zanderighi, 1206.3572]

- original goal: method to a-priori choose scales in multijet NLO computation (where hierarchy among scales can spoil accuracy since resummation of logs is missing)
- how: correct weights of different NLO terms with CKKW-inspired approach:

$$\bar{B}_{\mathrm{NLO}} = \alpha_{\mathrm{S}}(\mu_R)\Big[B + \alpha_{\mathrm{S}}^{(\mathrm{NLO})} V(\mu_R) + \alpha_{\mathrm{S}}^{(\mathrm{NLO})} \int d\Phi_{\mathrm{r}} R\Big]$$

$$\bar{B}_{\mathrm{MiNLO}} = \alpha_{\mathrm{S}}(q_T)\Delta_q^2(q_T, m_V)\Big[B\left(1 - 2\Delta_q^{(1)}(q_T, m_V)\right) + \alpha_{\mathrm{S}}^{(\mathrm{NLO})} V(\bar{\mu}_R) + \alpha_{\mathrm{S}}^{(\mathrm{NLO})} \int d\Phi_{\mathrm{r}} R\Big]$$




☞ Sudakov FF included on $V$+$j$ Born kinematics

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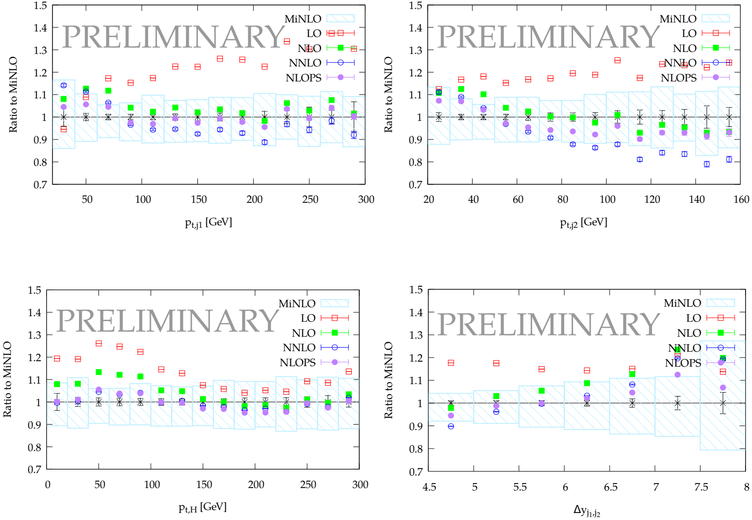
- VJ-MiNLO yields finite results also when 1st jet is unresolved ($q_T \to 0$)
- $\bar{B}_{\mathrm{MiNLO}}$ ideal to extend validity of $V$+$j$ POWHEG

# MiNLO VBF Hjjj

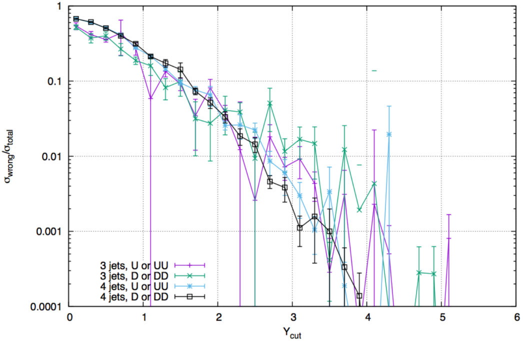


# Conclusions

- inclusive N$^3$LO are tiny and well within scale uncertainty band at NNLO
- scale uncertainty band reduced significantly
- differential NNLO corrections are sizeable, $\mathcal{O}(10\%)$, and necessary for precision phenomenology
- only moderate shrinkage of NNLO bands compared to NLO bands
- NLO-EW corrections are comparable to NNLO-QCD corrections and should be included
- "projection-to-Born" method can be extended to compute fully differential VBF Higgs at N$^3$LO

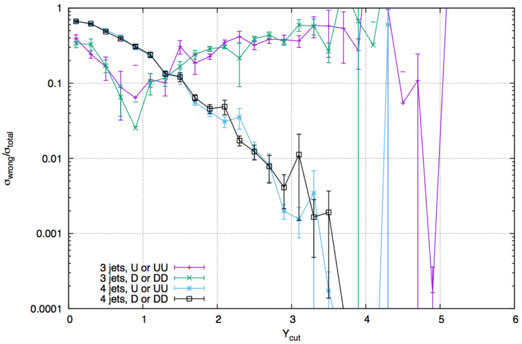
A public code, `proVBFH`, will be released in the near future. Until then total cross sections and distributions with specific cuts can be provided.

# Tagging



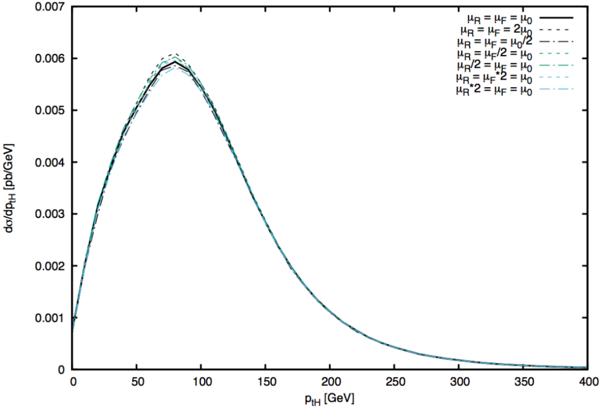

With no bug in the code, the probability of a tagged parton having wrong rapidity decreases with increasing rapidity separation between the two hardest jets.

# Tagging



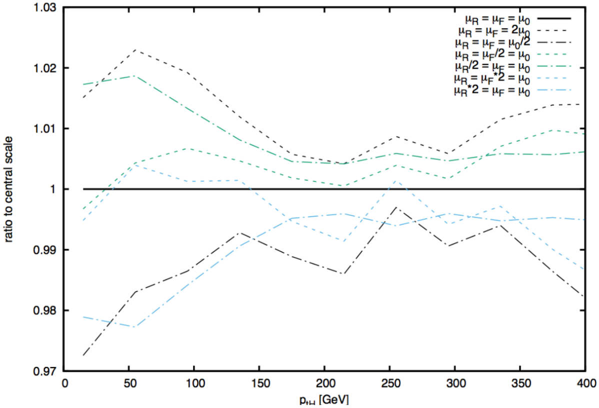

With an $\mathcal{O}(1)$ bug in the code, this is clearly not the case any more.

# 3-point vs 7-point scale variations



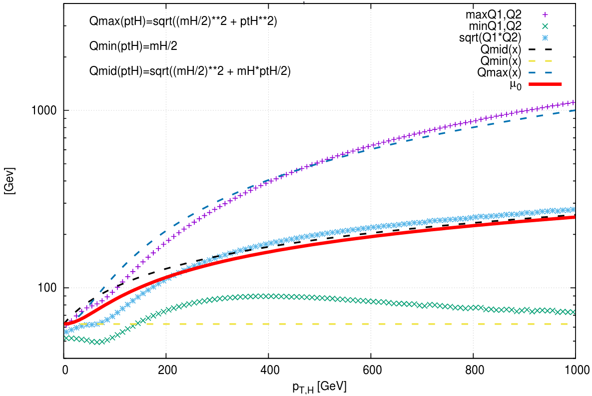

3- and 7-point scale variations are very close to eachother.

# 3-point vs 7-point scale variations



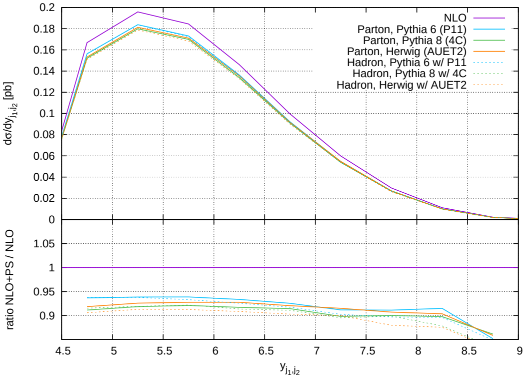

3- and 7-point scale variations are very close to eachother.

# Choice of scale



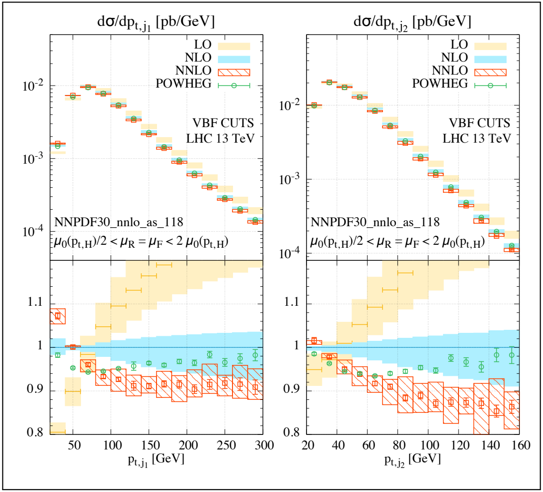

Our choice of $\mu_0^2(p_{t,H}) = \frac{M_H}{2}\sqrt{\left(\frac{M_H}{2}\right)^2 + p_{t,H}^2}$ is very close to a choice of $\mu = \sqrt{Q_1 Q_2}$.

# VBF and Parton Shower



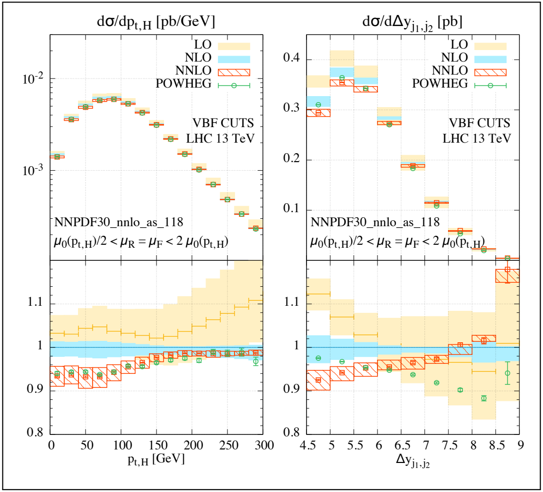

- different parton showers give relatively similar results
- hadronisation effects are consistently small

# NNLO/NLO/LO PDFs




- LO results with LO PDFs
- NLO results with NLO PDFs
- NNLO results with NNLO PDFs

# NNLO/NLO/LO PDFs




- LO results with LO PDFs
- NLO results with NLO PDFs
- NNLO results with NNLO PDFs