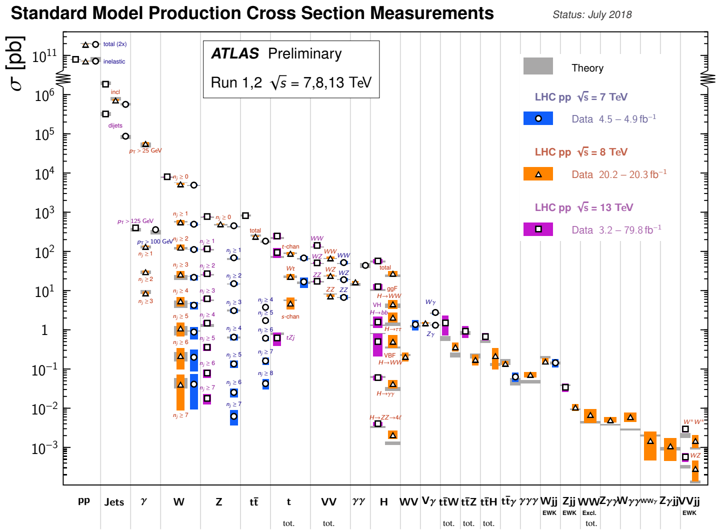
Parton showers with higher logarithmic accuracy

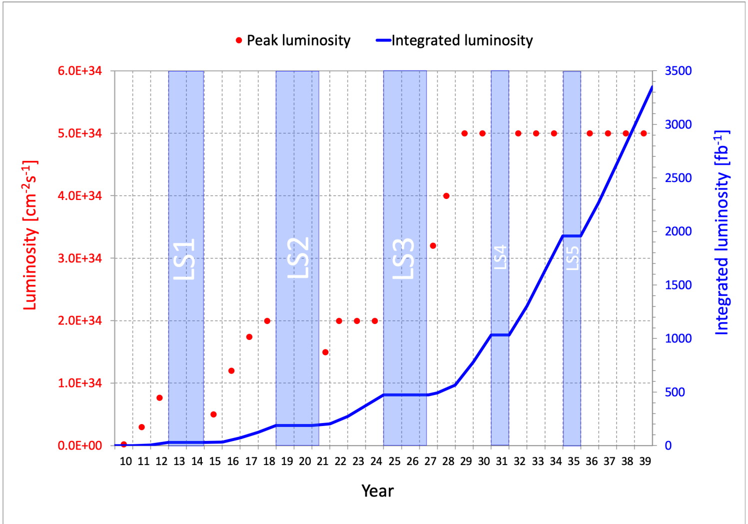
Alexander Karlberg

TUM Particle Theory Seminar

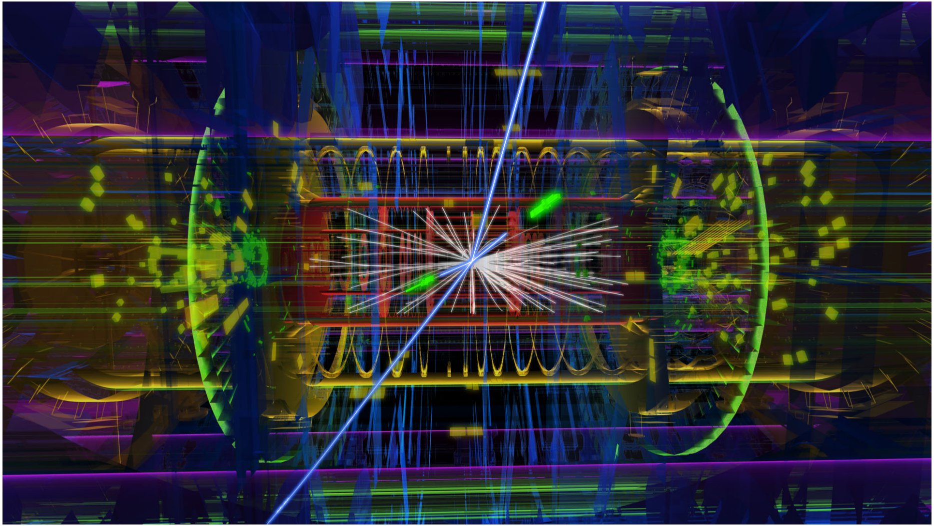
The precision era of the LHC



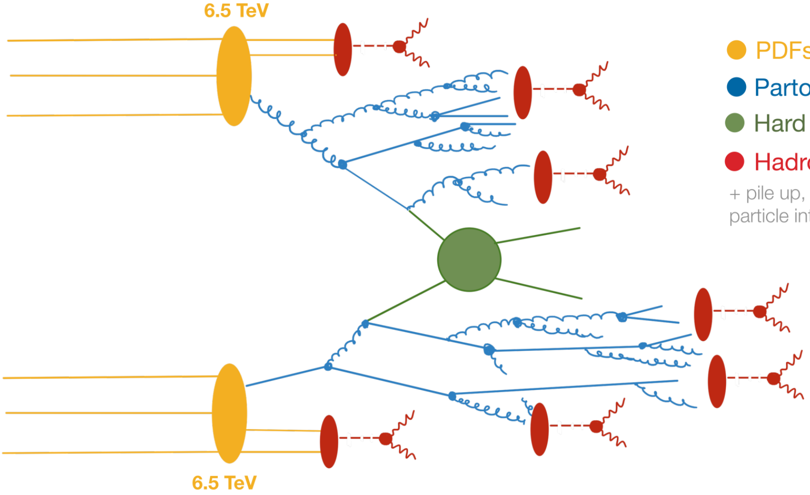
The precision era of the LHC



The LHC: A messy environment



Anatomy of an LHC collision



- PDFs / beam remnants
- Parton shower $\mathcal{O}(1-100)$ **GeV**
- Hard scattering $\mathcal{O}(0.1-1)$ **TeV**
- Hadronisation $\mathcal{O}(1)$ **GeV**

+ pile up, underlying event, multiple-particle interactions (MPI)…

courtesy M. van Beekveld

The ubiquitous Parton Shower

Pythia 8

An introduction to PYTHIA 8.2 #1

Torbjörn Sjöstrand (Lund U., Dept. Theor. Phys.), Stefan Ask (Cambridge U.), Jesper R. Christiansen (Lund U., Dept. Theor. Phys.), Richard Corke (Lund U., Dept. Theor. Phys.), Nishita Desai (U. Heidelberg, ITP) et al. (Oct 11, 2014)

Published in: *Comput.Phys.Commun.* 191 (2015) 159-177 • e-Print: 1410.3012 [hep-ph]

pdf links DOI cite 4,050 citations

Herwig 7

Herwig++ Physics and Manual #1

M. Bahr (Karlsruhe U., ITP), S. Gieseke (Karlsruhe U., ITP), M.A. Gigg (Durham U., IPPP), D. Grellscheid (Durham U., IPPP), K. Hamilton (Louvain U.) et al. (Mar, 2008)

Published in: *Eur.Phys.J.C* 58 (2008) 639-707 • e-Print: 0803.0883 [hep-ph]

pdf links DOI cite 2,644 citations

Sherpa

Event generation with SHERPA 1.1 #1

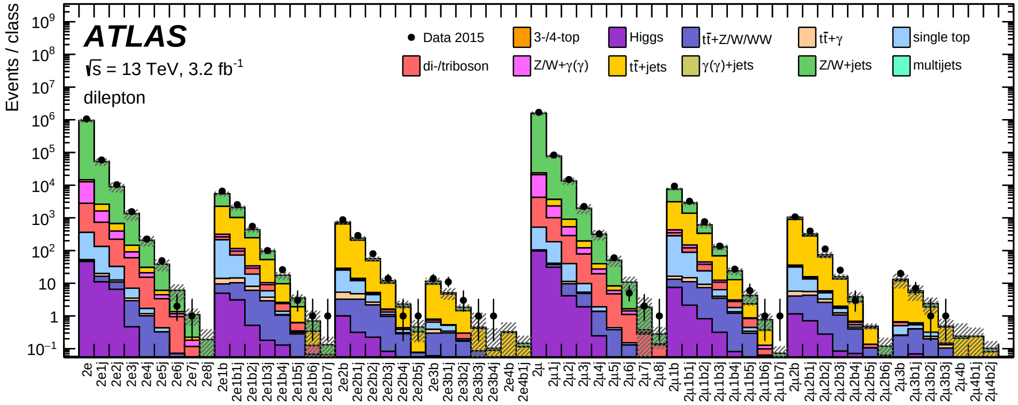
T. Gleisberg (SLAC), Stefan. Hoeche (Zurich U.), F. Krauss (Durham U., IPPP), M. Schonherr (Dresden, Tech. U.), S. Schumann (Edinburgh U.) et al. (Nov, 2008)

Published in: *JHEP* 02 (2009) 007 • e-Print: 0811.4622 [hep-ph]

pdf links DOI cite 3,386 citations

Parton Showers enter one way or another in almost 95% of all ATLAS and CMS analyses. Collider physics would not be the same without them.

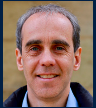
The ubiquitous Parton Shower



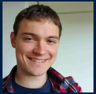
[1807.07447]

The PanScales collaboration

Oxford



Gavin Salam



Rok Medves

Frederic Dreyer

Jack Helliwell

NIKHEF

Melissa van Beekveld

Manchester

Mrinal Dasgupta

CERN

AK

Pier Monni

Alba Soto-Ontoso

Silvia Ferrario Ravasio

Monash

Ludo Scyboz

Basem El-Menoufi

UCL

Keith Hamilton

Rob Verheyen

IPhT

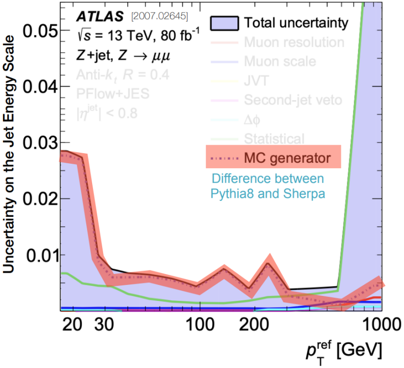
Gregory Soyez

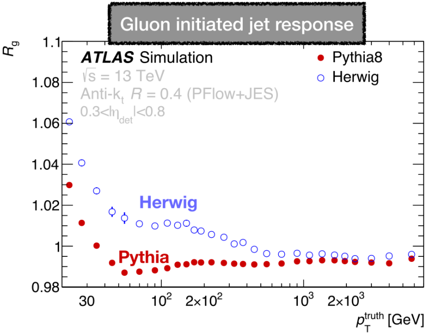
PanScales current & recent members

A project to bring logarithmic understanding and accuracy to parton showers

Differences matter!

Jet energy calibration uncertainties feed in to all jet analyses at the LHC





Differences amongst MC generators is the dominant uncertainty

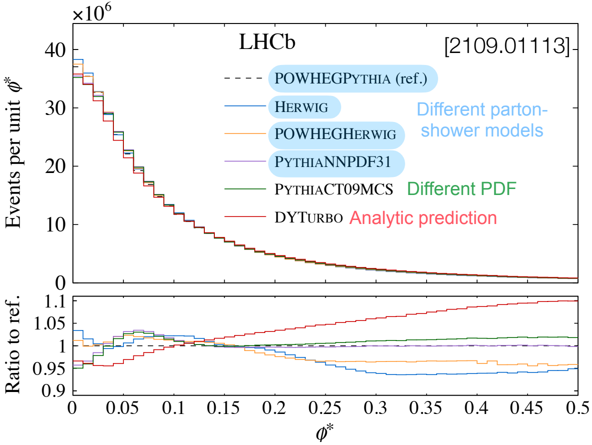
But differences matter…

Consider measurement of W boson mass

Measurements of p_T^Z in $Z/\gamma^* \to l^+l^-$ decays used to validate the MC predictions for p_T^W

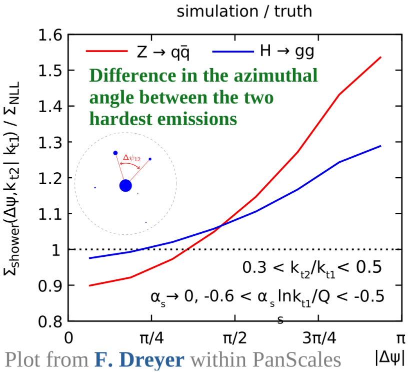
The envelope of shifts in m_W originating from differences in these shower predictions is the dominant theory uncertainty (11 MeV)

$$m_W = 80354 \pm 23_{\text{stat}} \pm 10_{\text{exp}} \pm 17_{\text{theory}} \pm 9_{\text{PDF}} \;\; \text{MeV}$$

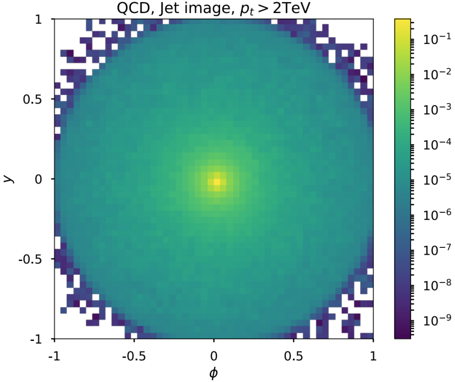


$$\phi^* = \frac{\tan((\pi - \Delta\phi)/2)}{\cosh(\Delta\eta/2)} \sim \frac{p_T^Z}{m_{ll}}$$ [1009.1580]

Machine learning and jet sub-structure



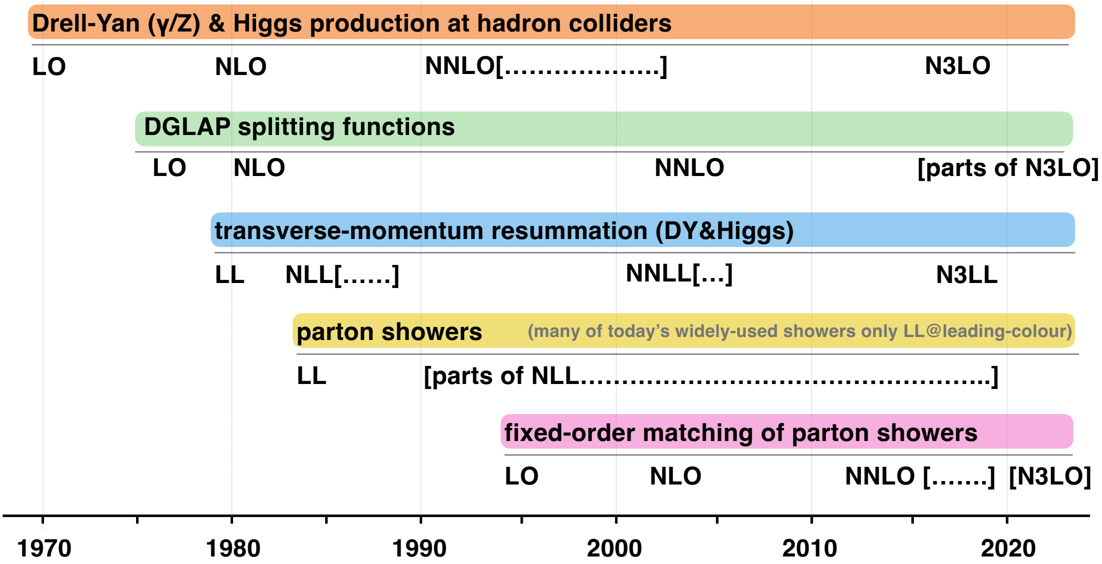
Plot from **F. Dreyer** within PanScales



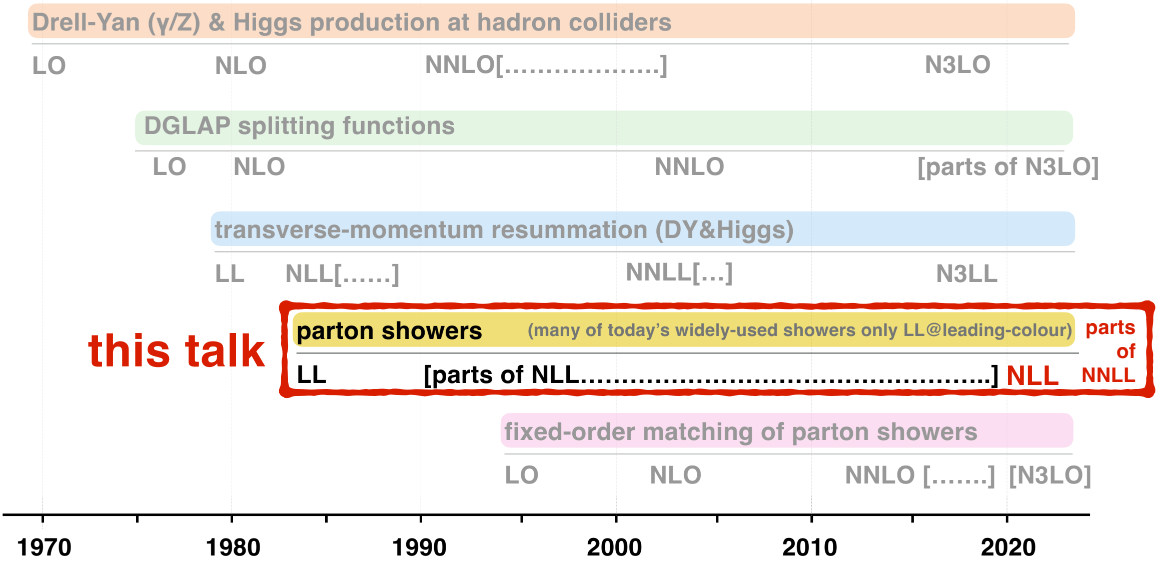
[1511.05190]

Machine learning might learn un-physical "features" from MC $\rightarrow$ can significantly impact the potential of new physics searches.

selected collider-QCD accuracy milestones



selected collider-QCD accuracy milestones



A Parton Shower in a nutshell

In one line: A Parton Shower is an iterative stochastic algorithm that takes n particles and maps them to to $n+1$ particles.

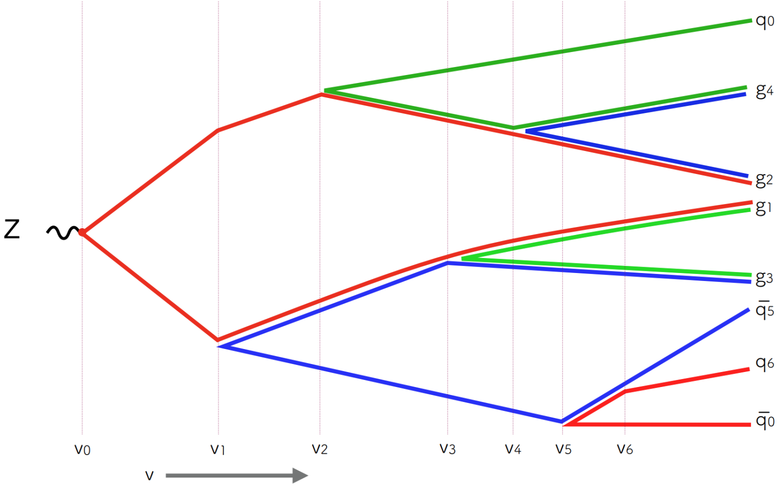
In order to do so one needs:

- A kinematic ordering variable, v, so that every phase space point is only reached once (and a cut-off $v_{cut} \sim \Lambda_{\mathrm{QCD}}$)
  - $\rightarrow$ Standard dipole showers take $v \sim k_T$ but many sensible choices exists
- A recoil map $\{p_n\} \rightarrow \{p_{n+1}\}$ to ensure momentum conservation and on-shellness of final-state particles
  - $\rightarrow$ Typically either local (only splitting dipole takes recoil) or global (all partons take recoil)
- An evolution equation governing the probability for a splitting $\widetilde{\widetilde{ij}} \rightarrow ijk$ to take place

$$\mathrm{d}\mathcal{P}_{\widetilde{\widetilde{ij}}\rightarrow ijk} \sim \frac{\alpha_S}{\pi} \mathrm{d}\ln v \, \mathrm{d}\bar{\eta} \frac{\mathrm{d}\phi}{2\pi} \left[g(\bar{\eta}) z_i P_{ik}(z_i) + g(-\bar{\eta}) z_j P_{jk}(z_j) \right] \quad (1)$$

! Governed by LO collinear splitting kernels.

A Parton Shower in a nutshell



courtesy G. Salam

Accuracy of Parton Showers

How do you even define the accuracy of an algorithm as described above?

When applying perturbation theory to total cross sections, it is easy to talk about the accuracy (LO, NLO, NNLO, ...)

$$\sigma = \sum_n c_n \alpha_S^n \tag{2}$$

Similarly for logarithmically enhanced observables we may talk about their logarithmical accuracy (LL, NLL, NNLL, ...)

$$\sigma(\mathcal{O} < e^L) = \sigma_{tot} \exp\left[\frac{1}{\alpha_S} g_1(\alpha_S L) + g_2(\alpha_S L) + \alpha_S g_3(\alpha_S L) + \cdots\right] \tag{3}$$

when $\alpha_S << 1, \quad \alpha_S L \sim -1.$

But both of these equations are *observable* dependent.

Accuracy of Parton Showers

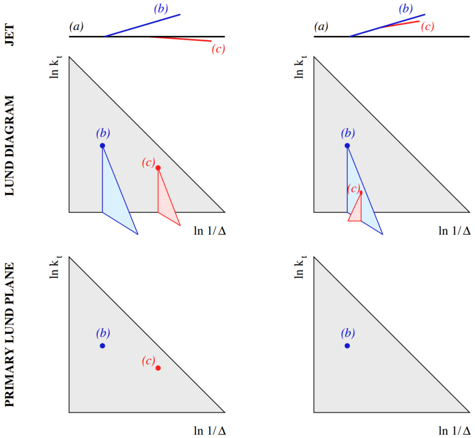
At colliders we can ask arbitrary questions about an event. The same is true for parton showers (+ hadronisation), e.g.

- Number (multiplicity) of particles in event (or jet)
- Energy in detector slice
- Angular distributions inside jets
- Even if we don't ask, machine learning might...

We therefore need to establish how to determine the logarithmic accuracy with which a parton shower can make predictions.

To do so we need to introduce the *Lund Plane* (B. Andersson et al (1989) & F. Dreyer et al. [1807.04758])

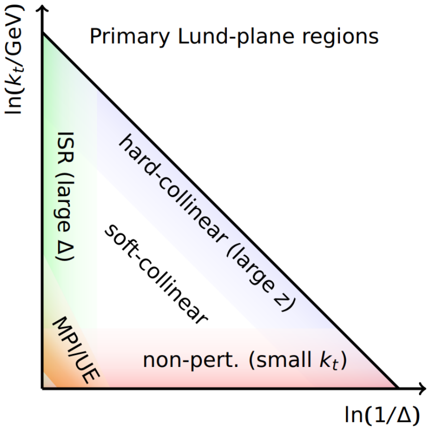
The Lund Plane



[1807.04758]

- Cluster the event with the Cambridge/Aachen algorithm, producing an angular ordered clustering sequence.
- Decluster the last clustering and record the transverse momentum and the opening angle of the declustering (plus other kinematics).
- Iterate along the hardest branch after each declustering to produce the *primary* Lund Plane.
- Following the softer branch produces the secondary, tertiary, etc Lund Plane.
- One can impose cuts easily on the declusterings (e.g. that they satisfy $z > z_{\text{cut}}$)

Logarithms in the Lund Plane



[1807.04758]

- The emission probability in the Lund Plane is then
$$\mathrm{d}\rho \sim \alpha_S \,\mathrm{d}\ln k_T \,\mathrm{d}\ln\theta$$
- Hence emissions that are well-separated in *both* directions are associated with *double logarithms* of the form $\alpha_S^n L^{2n}$
- Emissions separated along one direction are associated with *single logarithms* of the form $\alpha_S^n L^n$
- Emissions that are close in the Lund Plane are associated with a factor α_S^n
- We are now ready to state the PanScales NLL criteria for Parton Showers

NLL accurate Parton Showers

Fixed Order Matrix Element Condition

- Shower must reproduce fixed order n-particle matrix elements when emissions are well-separated in the Lund Plane, ie when the cross section is logarithmically enhanced.
- Supplement this with unitarity, 2-loop running and correct cusp anomalous dimension

Resummation Condition

- Shower must reproduce known NLL analytical resummations
- Global event shapes
- Multiplicity
- Non-global observables (slice observables), technically at leading single log (SL).

[1805.09327] & [2002.11114]

NLL accurate Parton Showers

Fixed Order Matrix Element Condition

- Fairly straightforward. Generate n emissions with your shower and compare to either factorised matrix elements (numerically very stable) or a full matrix element in some kinematic limit.
- Be careful to cover the collinear/soft phase space.

Resummation Condition

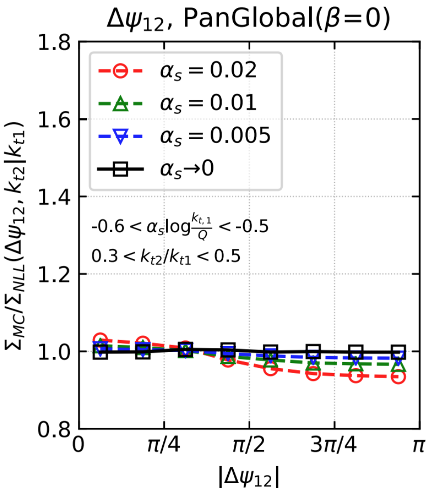
- This in general is trickier for 2 reasons:
- Requires the existence of NLL analytical results.
- Can't just compare

$$\frac{\Sigma^{\mathrm{PS}}(\alpha_S L)}{\Sigma^{\mathrm{NLL}}(\alpha_S L)} = \frac{\Sigma^{\mathrm{PS}}(\alpha_S L)}{\sigma_{tot}\exp\left[\frac{1}{\alpha_S} g_1(\alpha_S L) + g_2(\alpha_S L)\right]}$$

as the shower in general induces spurious higher order terms.

- How do we disentangle spurious "NNLL" terms from genuine NLL violations?

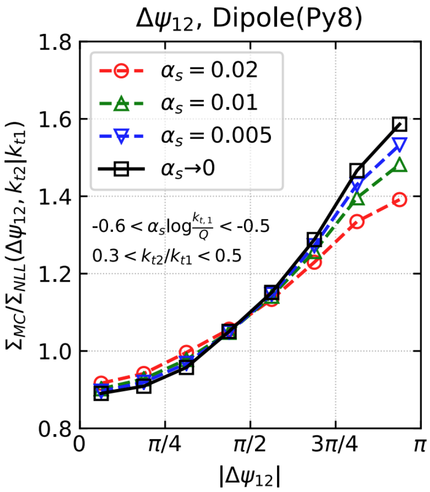
NLL tests



[2002.11114]

- Run the full shower with a specific (finite) value of $\alpha_S = \alpha_S(Q)$ and measure your favourite observable (that you can resum to NLL)
- Take the ratio to NLL and see that it is not flat.
- To see if there is an NLL mistake reduce α_S while keeping $\alpha_S L$ fixed, ie include more collinear and soft emissions.
- Genuine NLL effects are $(\alpha_S L)^n$ and are therefore unchanged. NNLL on the other hand goes as $\alpha_S(\alpha_S L)^n$ and should therefore vanish.
- Go as small in α_S as possible and extract $\alpha_S \to 0$.
- Now is it flat?

NLL tests



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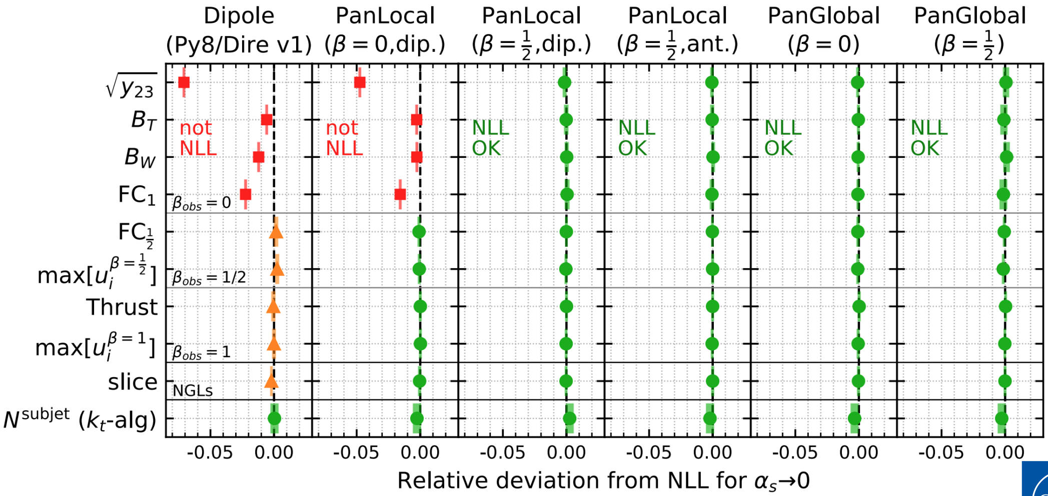
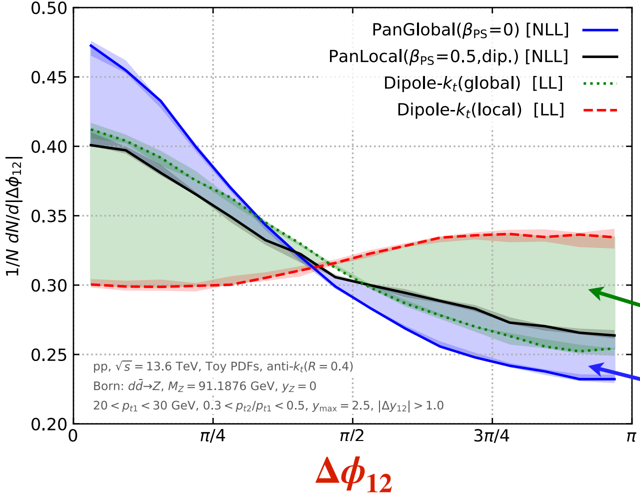
NLL tests summary



COMMISSIONING

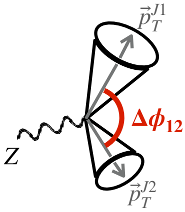
$m_{\ell\ell} = m_Z$

Azimuthal angle between leading jets (DY)



LL showers

NLL showers



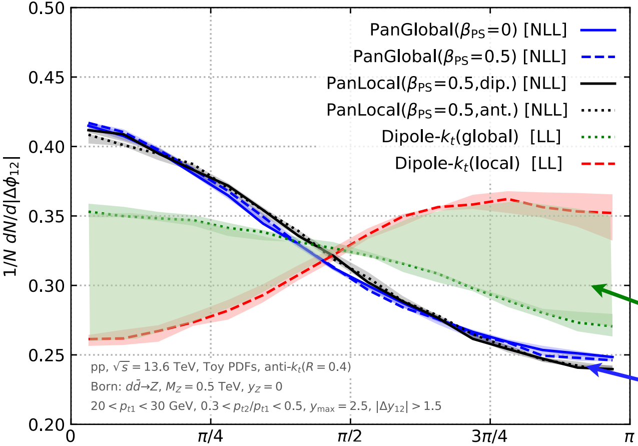
for more exclusive quantities, also see clear shape differences in going to NLL

van Beekveld, Ferrario Ravasio, GPS, Soto Ontoso, Soyez, Verheyen, Hamilton: 2207.09467

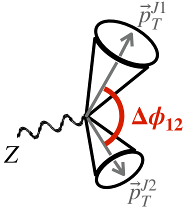
$m_{\ell\ell} = 500\,\mathrm{GeV}$

COMMISSIONING

Azimuthal angle between leading jets (DY)



$\Delta\phi_{12}$



for more exclusive quantities, also see clear shape differences in going to NLL

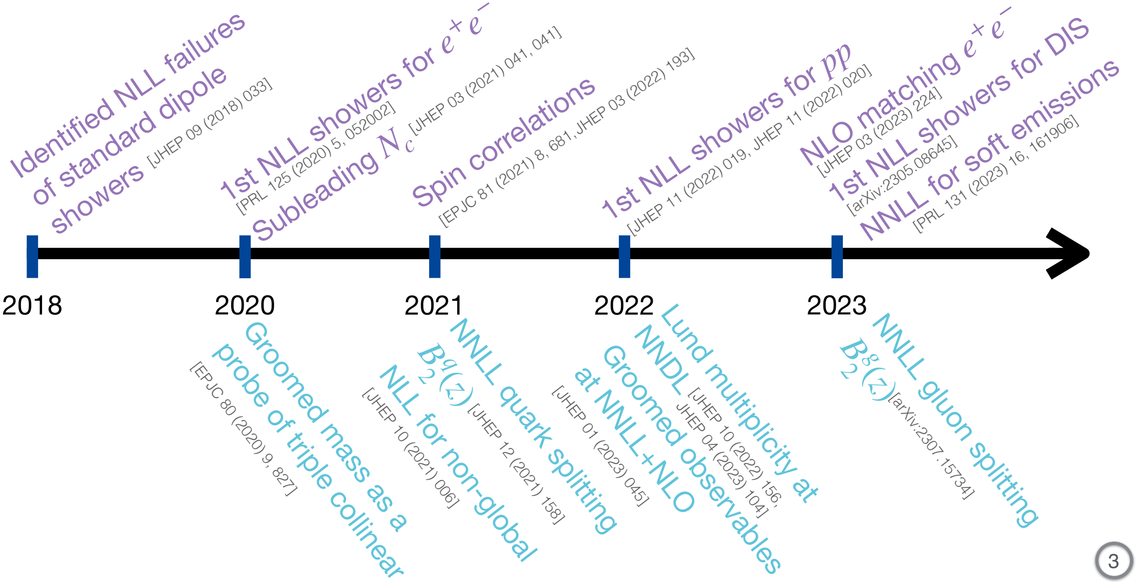
especially at larger scales

LL showers

NLL showers

van Beekveld, Ferrario Ravasio, GPS, Soto Ontoso, Soyez, Verheyen, Hamilton: 2207.09467

Time-evolution of the PanScales project



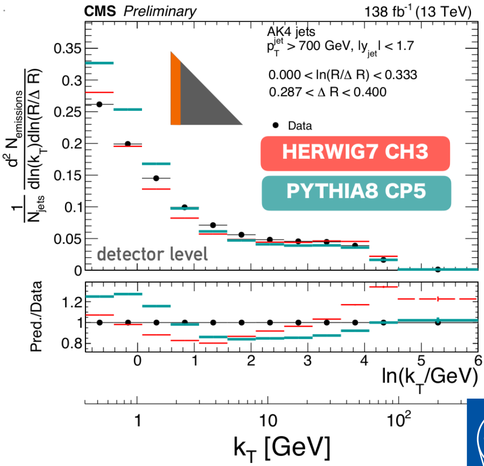
Going beyond NLL

- In order to go *beyond NLL* we have to be able to describe configurations in the Lund Plane, where at most two emissions are close to each other.
- This in particular includes when an emission is close to the top of the Lund Plane (where the initial “hard” parton sits), but it also includes configurations with for instance two commensurate energy wide-angle emissions (double-soft configurations).
- NLO matching has been understood for more than two decades, but the interplay with NLL showers could be first be investigated now
- Double-soft corrections have also been studied within Vincia and Sherpa, but since neither are NLL it is unclear what formal improvement that brings
- For NLL showers it is clear - these corrections bring NNDL to event-shapes and jet multiplicites, along with NSL for non-global observables.

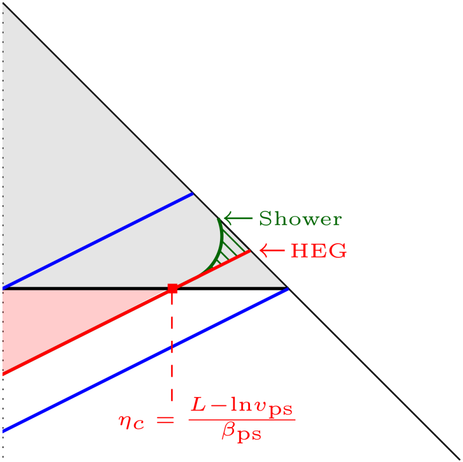
An experimental aside

- The region of interest, ie the central region in the Lund Plane, has recently been measured by CMS
- Neither Pythia nor Herwig manage to describe the data better than $10-30\%$
- Matching already (presumably) included here, but not double-soft corrections
- Will be interesting to see how PanScales compares when we have tuning and matching in pp

Lund Plane
(Recent CMS results; also ATLAS & ALICE)

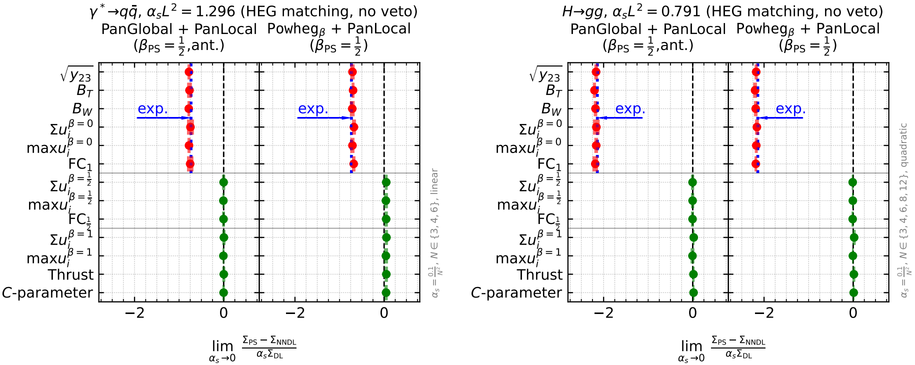


POWHEG$_\beta$ and NNDL accuracy

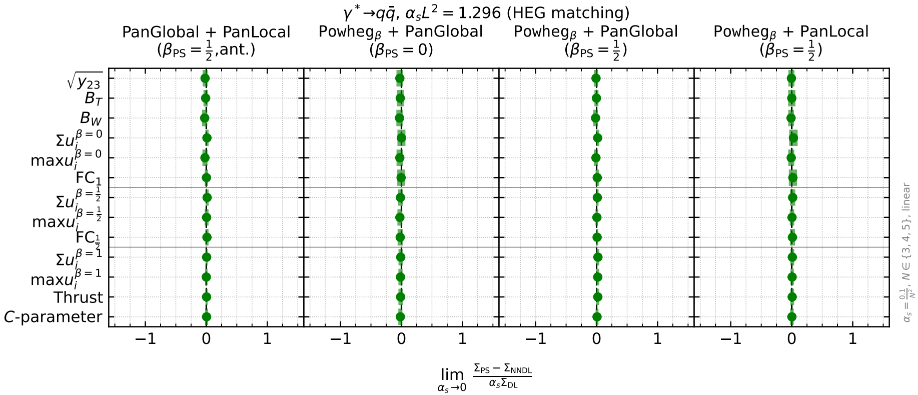


- We have so far explored $e^+e^- \to q\bar{q}$ @ NLO
- For matching schemes that leave the shower first emission untouched (such as MC@NLO, KrkNLO, and MAcNLOPS) the matching works more or less out of the box.
- For POWHEG style matchings (including MiNNLO and GENEVA) log accuracy is more subtle to maintain.
- Main concern related to kinematic mismatch between shower and hardest emission generator, but there are also issues related to how one partitions the $g \to gg$ splitting function.

HEG-matching without a veto is not NNDL accurate



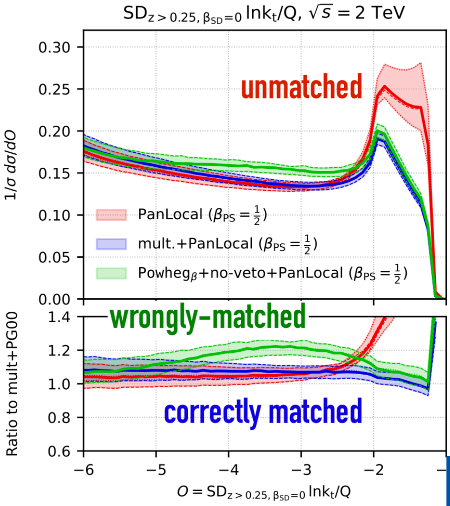
Proper HEG-matching achieves NNDL accuracy



Phenomenological impact

- Incorrect matching leads to breaking of NLL and exponentiation
- Correct matching on the other hand augments the shower from NLL to NLL+NNDL for event shapes.
- Impact of NLL breaking terms vary - for SoftDrop they have a big impact due to the single-logarithmic nature of the observable

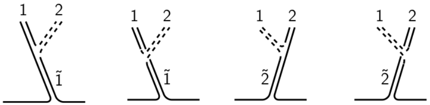
$$\partial_L \Sigma_{\mathrm{SD}}(L) = \bar{\alpha} c e^{\bar{\alpha} c L - \bar{\alpha}\Delta} - 2\bar{\alpha} L e^{-\bar{\alpha} L^2}(1 - e^{-\bar{\alpha}\Delta})$$



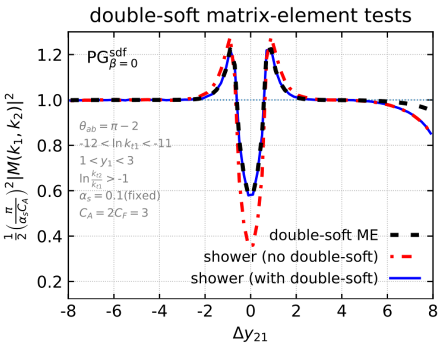
First steps towards NNLL accuracy for PanGlobal

- NLO matching is a necessary ingredient for going beyond NLL
- However, there are in general two other contributions that are needed to reach NNLL
- For now we have focused on the so-called double-soft emissions (and their associated virtual corrections)
- To get the right we must ensure that any pair of soft emissions with commensurate energy and angles should be produced with the correct ME
- Any additional soft radiation off that pair must also come with the correct ME
- Must still get the single-soft emission rate right at NLO (CMW-scheme)
- This should achieve NNDL accuracy for multiplicities, ie terms $\alpha_S^n L^{2n}$, $\alpha_S^n L^{2n-1}$, $\alpha_S^n L^{2n-2}$
- and next-to-single-log (NSL) accuracy for non-global logarithms, for instance the energy in a rapidity slice, $\alpha_S^n L^n$ and $\alpha_S^n L^{n-1}$ (albeit only at leading-N_C)

The double-soft ME

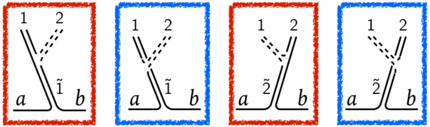


- Any two-emission configuration in a dipole-shower comes with four histories
- We accept any such configuration with the true ME divided by the shower's *effective double-soft ME* divided by the sum over all histories that could have lead to that configuration.

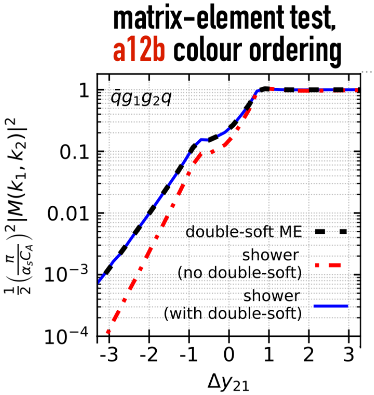


$$P_{\text{accept}} = \frac{|M^2_{\text{DS}}|}{\sum_h |M^2_{\text{shower},h}|}$$

Correcting the colour-ordering



- We have two distinct colour orderings $a12b$ and $a21b$
- We need to get the relative fractions $F^{(12)}$ and $F^{(21)}$ right in order to ensure that any further emissions are also correct.
- In practice we accept a colour ordering if the shower generates too little of it, and swap them if the shower generates too much (and similarly for $q\bar{q}$ vs gg branchings).



$$P_{\text{swap}} = \frac{F^{(12)}_{\text{shower}} - F^{(12)}_{\text{DS}}}{F^{(12)}_{\text{shower}}}$$

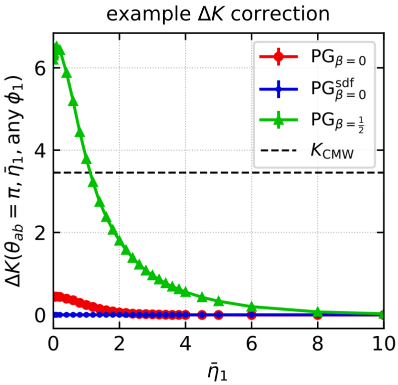
Single-soft emissions

- The PanScales showers are already correct at NLO in the soft-collinear region due to the use of the CMW-coupling

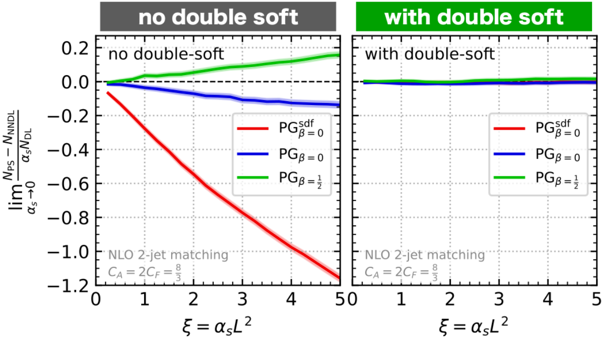
$$\alpha_S \to \alpha_S + \alpha_S^2 K_{\text{CMW}}/2\pi$$

- This in general is not enough the get to soft wide-angle region right and we need to add a ΔK_{CMW} which depends on the rapidity of the single soft emission

$$\Delta K = \int_r d\Phi^{(\text{PS})}_{12/\tilde{1}} |M^{(\text{PS})}_{12/\tilde{1}}|^2 - \int_{r_{\text{sc}}} d\Phi^{(\text{PS})}_{12/\tilde{1}_{\text{sc}}} |M^{(\text{PS})}_{12/\tilde{1}_{\text{sc}}}|^2.$$



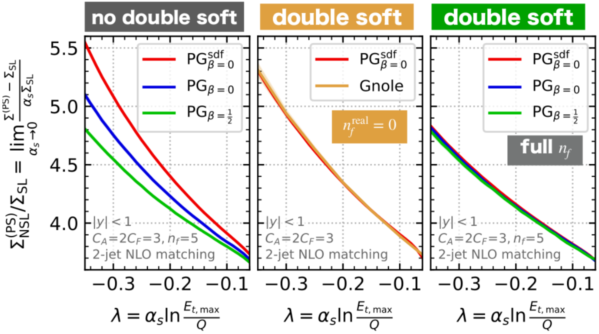
Lund Multiplicities



$$\lim_{\alpha_S\to 0} \frac{N_{(PS)} - N_{NNDL}}{\alpha_S N_{(PS)}}\Big|_{\text{fixed } \alpha_S L^2}$$

- Reference NNDL ($\alpha_S^n L^{2n-2}$) analytic result in Medves, Soto Ontoso, Soyez [2205.02861]
- We take $\alpha_S \to 0$ limit to isolate NNDL terms. This is significantly more challenging that at NDL due to presence of $1/\alpha_S$ in denominator.
- Showers without double-soft corrections show clear differences from reference (and each other).
- Adding the double-soft corrections brings NNDL agreement.

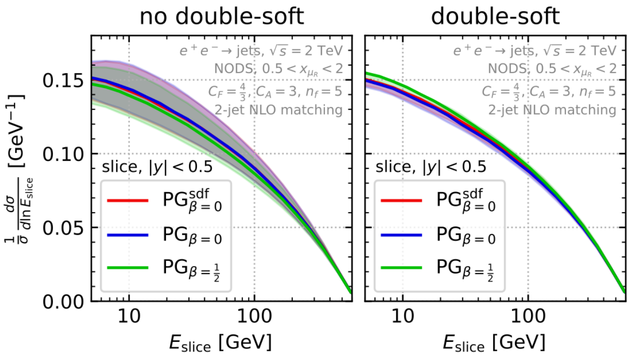
Energy in a slice



$$\lim_{\alpha_S \to 0} \frac{\Sigma^{(\mathrm{PS})} - \Sigma_{\mathrm{SL}}}{\alpha_S}\Bigg|_{\text{fixed } \alpha_S L}$$

- Reference NSL ($\alpha_S^n L^{n-1}$) from "Gnole" Banfi, Dreyer, Monni [2111.02413].
- We did this test semi-blind: only compared to Gnole after we had agreement between the three PanGlobal variants.
- We have NSL agreement with Gnole (using $n_f^{\mathrm{real}} = 0$) and agreement between all showers with full-n_f dependence (first calculation of this kind as a by-product!)

What about pheno?



- We studied energy flow between two hard (1 TeV) jets as a preliminary pheno case
- The three PanGlobal variants are remarkably close without double-soft corrections, but have large uncertainties
- With double-soft corrections we see a small shift in central values but a dramatic reduction in uncertainties.

Conclusions

- Parton showers with controlled logarithmic accuracy are emerging. Main benefit in going from LL $\rightarrow$ NLL accuracy are reduced (and reliable) uncertainties
- We have NLO matching working for simple processes and are working towards more phenomenologically interesting hadron-collider processes
- Also work in progress on massive quarks and tuning
- We have taken the first steps towards NNLL accurate showers
- So far the program has brought NNDL to event shapes and multiplicities, and NSL for non-global observables.
- The code will become public soon, although no definite date set
- Parton showers with full NNLL accuracy are coming too...

BACKUP

Let's match!

- The first matching procedure we consider is multiplicative matching (also often called Matrix Element Corrections). The hardest emission cross section can be written as

$$\mathrm{d}\sigma_{\text{mult}} = \bar{B}(\Phi_{\text{B}}) \left[S_{\text{PS}}(v_{\Phi}^{\text{PS}}, \Phi_{\text{B}}) \times \frac{R_{\text{PS}}(\Phi)}{B_0(\Phi_{\text{B}})} \mathrm{d}\Phi \otimes \frac{R(\Phi)}{R_{\text{PS}}(\Phi)} \right] \times I_{\text{PS}}(v_{\Phi}^{\text{PS}}, \Phi)\,.$$

- With the parton shower Sudakov given by

$$S_{\text{PS}}(v, \Phi_{\text{B}}) = \exp\left[-\int_{v_{\Phi}^{\text{PS}} > v} \frac{R_{\text{PS}}(\Phi)}{B_0(\Phi_{\text{B}})} \mathrm{d}\Phi_{\text{rad}} \right],$$

- and the NLO normalisation factor written as

$$\bar{B}(\Phi_{\text{B}}) = B_0(\Phi_{\text{B}}) + V(\Phi_{\text{B}}) + \int R(\Phi)\mathrm{d}\Phi_{\text{rad}}\,,$$

Multiplicative matching

- In practice the multiplicative matching can only work if $R(\Phi) \leqslant R_{\mathrm{PS}}(\Phi)$ in order for the first emission probability to be bounded by 1.
- Since $R(\Phi)$ and $R_{\mathrm{PS}}(\Phi)$ agree in the soft/collinear limits, the matching has no impact in these limits, and from a logarithmic point of view we therefore expect NLL accuracy to be retained.
- This type of matching has to be implemented directly inside the relevant shower code, and cannot be achieved with external tools.

MC@NLO matching

- In the MC@NLO scheme the hardest emission cross section takes the form

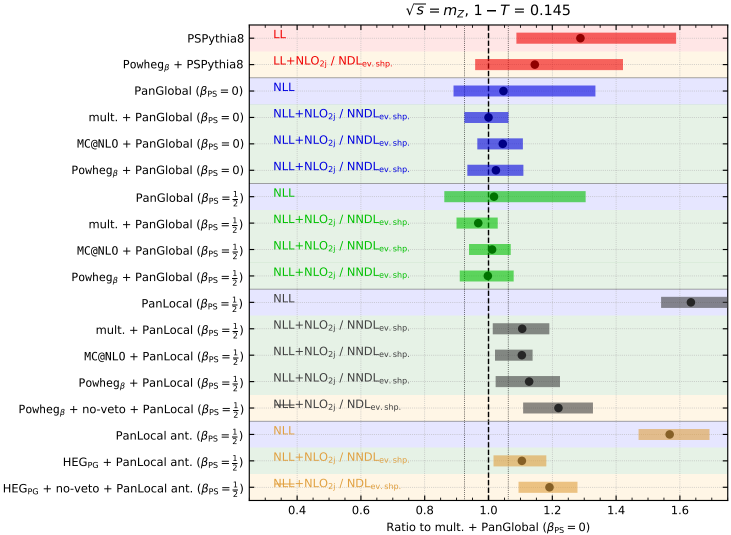
$$d\sigma_{\text{MC@NLO}} = \bar{B}_{\text{PS}}(\Phi_{\text{B}})\, S_{\text{PS}}(v_{\Phi}^{\text{PS}}, \Phi_{\text{B}}) \times \frac{R_{\text{PS}}(\Phi)}{B_0(\Phi_{\text{B}})}\, d\Phi \times I_{\text{PS}}(v_{\Phi}^{\text{PS}}, \Phi) + \\ + [R(\Phi) - R_{\text{PS}}(\Phi)]\, d\Phi \times I_{\text{PS}}(v^{\max}, \Phi)\,,$$

- with

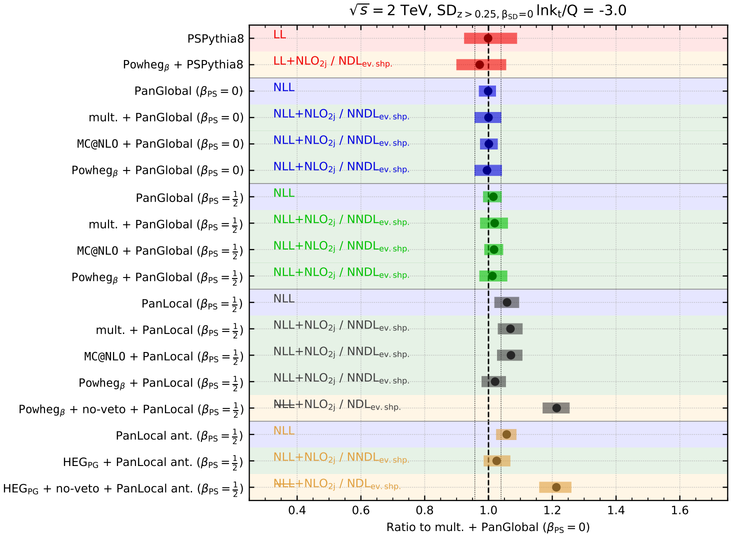
$$\bar{B}_{\text{PS}}(\Phi_{\text{B}}) = B_0(\Phi_{\text{B}}) + V(\Phi_{\text{B}}) + \int R_{\text{PS}}(\Phi)\, d\Phi_{\text{rad}}\,.$$

- Interpretation: Generate events with the shower (modifying the normalisation) and supplement these with a set of finite hard events.
- Specifically, this ensures that the shower is preserved in the infrared and collinear regions.

Summary for thrust



Summary for SD



NLO Matching - a solved problem?

- Event generators with NLO accuracy have become the *de facto* tool for particle collision simulations.
- There are a number of solutions available, going back more than 20 years, but by far the two most widely used are MC@NLO [Frixione, Webber '02] and POWHEG [Nason '04, Frixione, Nason, Oleari '07].
- Both were formulated at a time when parton showers had limited (i.e. leading) logarithmic accuracy.
- For this reason the concern was mainly to improve the fixed order side of things, without breaking the shower.
- With the advent of NLL showers (of which a number have emerged in recent years in addition to the PanScales showers) it has become relevant to return to the question of formal shower accuracy in the context of NLO matching.
- Will discuss the two-body decay processes $\gamma^* \to q\bar{q}$ and $h \to gg$ in the following.

NLO Matching - revisited

- To understand the interplay between matching and logarithmic accuracy, it is instructive to discuss the example of event shapes, for which the probability of some observable $\mathcal{O}$ to have a value below e^L is given by

$$\Sigma(O < e^L) = (1 + C_1\alpha_s + \ldots)\, e^{\alpha_s^{-1} g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \ldots}, \qquad L \ll -1\,.$$

- Here g_1 is responsible for LL terms ($\alpha_s^n L^{n+1}$), g_2 for NLL terms ($\alpha_s^n L^n$) and C_1 and g_3 for NNLL terms ($\alpha_s^n L^{n-1}$).
- Σ can also be written in terms of a double-logarithmic expansion

$$\Sigma(O < e^L) = h_1(\alpha_S L^2) + \sqrt{\alpha_S}\, h_2(\alpha_S L^2) + \alpha_S h_3(\alpha_S L^2) + \ldots, \qquad |L| \gg 1\,,$$

- with h_1 responsible for DL terms ($\alpha_S^n L^{2n}$), h_2 for NDL ($\alpha_S^n L^{2n-1}$), and h_3 for NNDL terms ($\alpha_S^n L^{2n-2}$).
- In analytic resummation C_1 is typically obtained through NLO matching, and its inclusion is enough to achieve NNDL for event shapes.

NLO Matching - revisited

- Hence, for event shapes there is an obvious logarithmic correspondence with NLO matching: A good NLO matching scheme should augment an NLL shower to NNDL.
- However, this is not the case in general.
- As is know from analytic resummation NLO matching is a necessary ingredients to achieve NNLL accuracy in general, since a term α_s contributes to the $\alpha_s^n L^{n-1}$ logarithmic tower.
- So instead of thinking of NLO matching as a way of achieving better fixed order accuracy we can think of it as a step towards having NNLL accurate event generators.

Matching in a nut-shell

- **Multiplicative**: Modify the shower's first emission through a veto on $P_{\text{exact}}/P_{\text{shower}}$, which itself is expected to go to 1 in the infrared/collinear limit.
- **MC@NLO**: Supplement the shower events with a set of hard events, $P_{\text{exact}} - P_{\text{shower}}$, which vanish in the infrared/collinear limit.
- **POWHEG**: Handle the hardest emission generation with a special Hardest Emission Generator (HEG) that acheives NLO acuracy for the hardest emission.
- There is also **KrkNLO** which is similar in spirit to multiplicative matching and **MAcNLOPS** which is multiplicative when $P_{\text{exact}} < P_{\text{shower}}$ and MCNLO otherwise.
- Here I will mainly discuss POWHEG, as both Multiplicative and MC@NLO matching achieves NNDL without any furhter considerations.

POWHEG$_\beta$

- Let us consider a simple version of POWHEG matching given by

$$\mathrm{d}\sigma_{\text{POWHEG-simple}} = \bar{B}(\Phi_{\mathrm{B}})\, S_{\mathrm{HEG}}(v_{\Phi}^{\mathrm{HEG}}, \Phi_{\mathrm{B}}) \times \frac{R_{\mathrm{HEG}}(\Phi)}{B_0(\Phi_{\mathrm{B}})}\, \mathrm{d}\Phi \times I_{\mathrm{PS}}(v_{\Phi}^{\mathrm{HEG}}, \Phi)\,.$$

- In this variant of POWHEG the HEG generates an event at a scale v_{Φ}^{HEG} that is then handed over to the shower, which continues showering starting at the same scale.
- In order to preserve leading logarithmic accuracy, the ordering variable of the HEG and the shower need to coincide in the simulatneously soft and collinear limit.
- This is for instance the case in standard transverse-momentum ordered `POWHEG-BOX`+Pythia8 usage.
- It would however not be the case if one were to use a $\beta = 1/2$ variant of one of the PanScales showers.

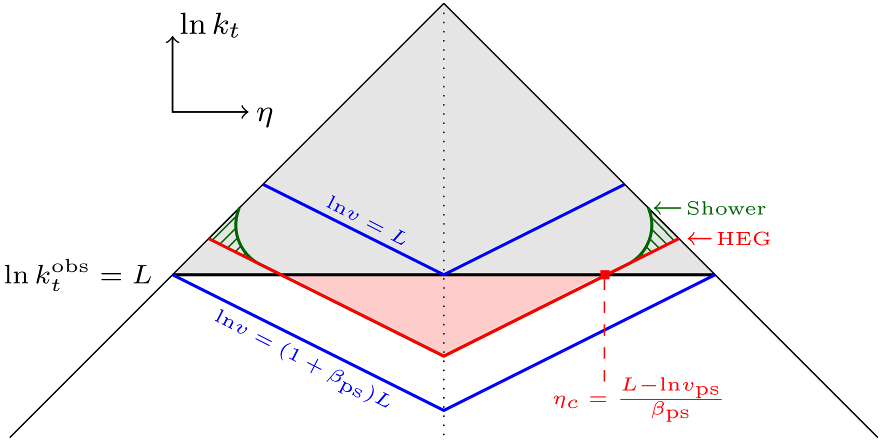
POWHEG$_\beta$

- One can however fairly easily modify the POWHEG ordering variable to have the necessary β dependence such that it coincides with the PanScales showers in the simultaneously soft and collinear limit

$$\bar{\eta} = -\ln\tan\left(\frac{\arccos y}{2}\right), \quad \ln v = \ln\frac{\sqrt{s}}{2} + \ln\sin\left[2\arctan e^{-\bar{\eta}}\right] + \ln\xi - \beta|\bar{\eta}|.$$

- Inside the PanScales framework we call this POWHEG$_\beta$.
- Even so there can still be mismatches in both the hard-collinear and soft wide-angle regions of the Lund Plane.
- This is something that has been known for some time [Corke, Sjöstrand '10], and is connected to the question of under-/double-counting in matching. It is mostly solved by the usual veto
- To address the logarithmic impact we again return to the Lund Plane...

POWHEG$_\beta$ and NNDL accuracy



~~NLL~~ - so what?

- Okay, we broke NLL, but in a very technical way. Maybe this breaking will not be very relevant for phenomenology, since the NLL breaking starts at $\mathcal{O}(\alpha_S^4)$ and the NNDL breaking a relative $\mathcal{O}(\alpha_S)$ in Σ?
- Hard to say without running the code, but one needs to keep in mind that there are other observables than event shapes, and that some of these could potentially be more sensitive to the problem.
- One such is the mass of the first SoftDrop ($\beta = 0$) splitting, which is sensitive to the hard-collinear region by construction, and does not have double-logarithmic terms. It has the following single-logarithmic structure

$$\partial_L \Sigma_{\mathrm{SD}}(L) = \bar{\alpha} c\, e^{\bar{\alpha} c L}$$

- Taking the shower/HEG mismatch into account, one instead finds

$$\partial_L \Sigma_{\mathrm{SD}}(L) = \bar{\alpha} c\, e^{\bar{\alpha} c L - \bar{\alpha}\Delta} - 2\bar{\alpha} L e^{-\bar{\alpha} L^2}\left(1 - e^{-\bar{\alpha}\Delta}\right),$$

- This again gives rise to terms $\alpha_S^n L^{2n-2}$ in the logarithm, but more importantly when $\alpha_S L^2 \sim 1$ the second term is only suppressed by a relative $\mathcal{O}(\sqrt{\alpha_S})$ compared to the first one, which is parametrically larger than the $\mathcal{O}(\alpha_S)$ effect for event shapes.

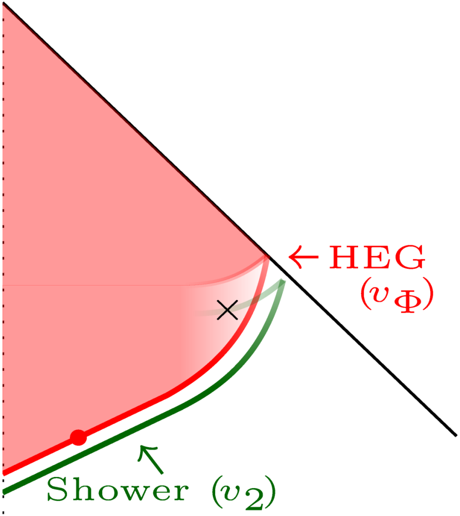
Solution to the problem

- The solution to the problem is actually well-known and already applied in typical POWHEG usage.
- After the HEG hands over the hardest emission, the shower should not start from v_{Φ}^{HEG} but rather from the maximum scale, and then veto *all* emissions with a hardness scale above v_{Φ}^{HEG}.
- We can write this procedure as

$$\mathrm{d}\sigma_{\mathtt{POWHEG}\text{-veto}} = \bar{B}(\Phi_{\mathrm{B}})\, S_{\mathrm{HEG}}(v_{\Phi}^{\mathrm{HEG}}, \Phi_{\mathrm{B}}) \times \frac{R_{\mathrm{HEG}}(\Phi)}{B_0(\Phi_{\mathrm{B}})}\, \mathrm{d}\Phi \times I_{\mathrm{PS}}(v^{\max}, \Phi | v_i^{\mathrm{HEG}} < v_{\Phi}^{\mathrm{HEG}}),$$

- As we shall see, this will be enough to restore NNDL accuracy, with a proviso having to do with gluon splittings...

Further subtleties



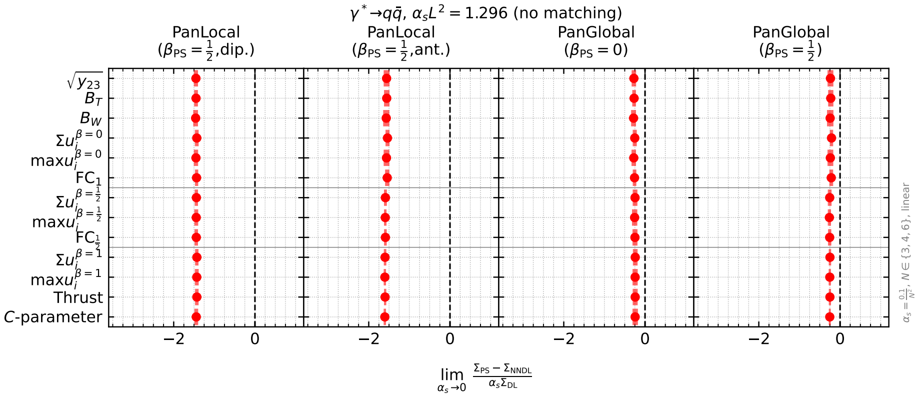
- Even when the contours are fully aligned there are issues associated with how dipole showers partition the $g \to gg(q\bar{q})$ splitting function.
- In PanScales we use

$$\frac{1}{2!}P_{gg}^{\mathrm{asym}}(\zeta) = C_A\left[\frac{1+\zeta^3}{1-\zeta} + (2\zeta-1)w_{gg}\right],$$

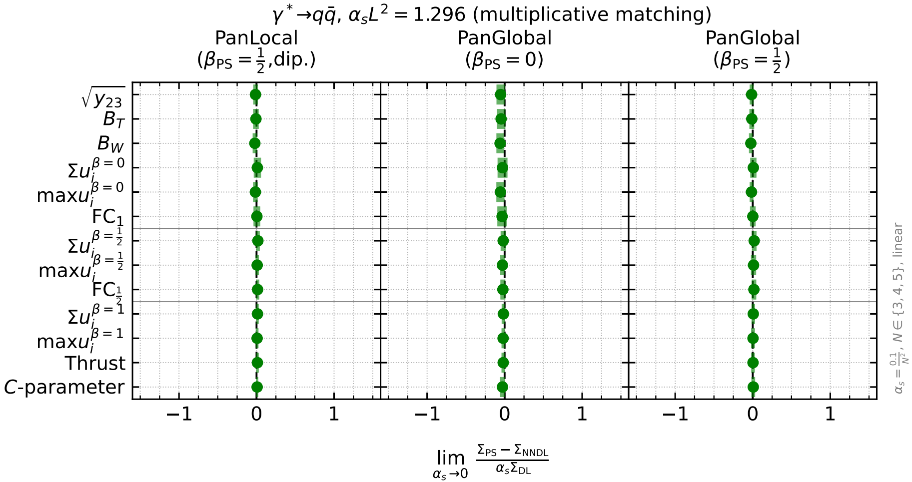
such that $P_{gg}^{\mathrm{asym}}(\zeta) + P_{gg}^{\mathrm{asym}}(1-\zeta) = 2P_{gg}(\zeta)$

- This partitioning takes place to isolate the two soft divergences in the splitting function ($\zeta \to 0$ and $\zeta \to 1$), but there is some freedom in how one handles the non-singular part.
- Similarly, in the HEG one needs to handle this issue, and in general if the shower and the HEG do not agree on this procedure, one can induce similar NNDL breaking to what was seen above.

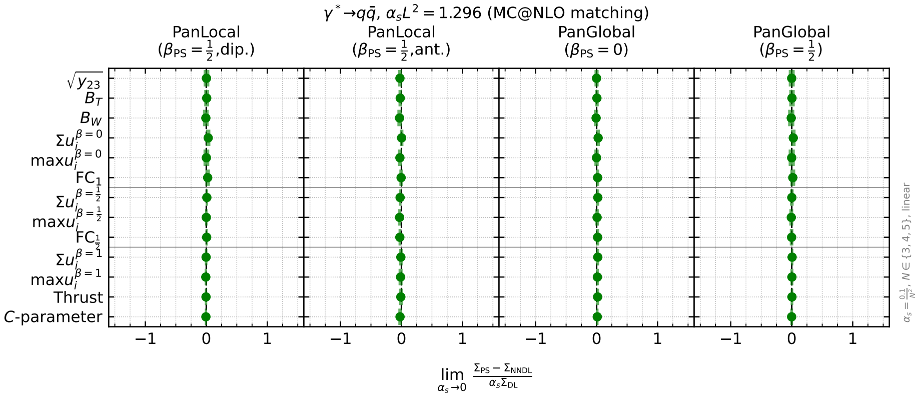
Showers without matching are not NNDL accurate



Multiplicative matching achieves NNDL accuracy



MC@NLO matching achieves NNDL accuracy



NLL

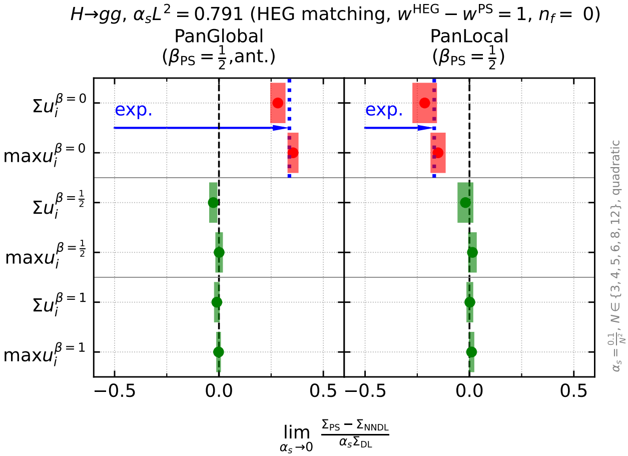
- While breaking of NNDL is not desirable, one could take the view that as long as NLL is not broken, the matching still achieved its goal.
- Eq. (**??**) gives the impression that NLL is not broken, as the term $\propto \alpha_S(\alpha_S L^2)^n$ vanishes when $\alpha_S \to 0$.
- However, if we take the logarithm of eq. (**??**) we get

$$\ln \Sigma = -\bar{\alpha} L^2 - \sum_{n=2}^{\infty} \frac{2\beta^{n-1}\Delta}{(n-1)!} \cdot \bar{\alpha}^n L^{2n-2} + \mathcal{O}(\bar{\alpha}^n L^{2n-3}) \,.$$

which fails to satisfy the exponentiation criterion, that there are no terms $\alpha_S^n L^m$ in $\ln \Sigma$ with $m > n+1$ (starting at $\mathcal{O}(\alpha_S^4)$).

- Alternatively one can view these terms as spurious super-leading logarithms induced by the matching.

HEG-matching with $w^{\mathrm{HEG}}! = w^{\mathrm{PS}}$ is not NNDL accurate



Phenomenological considerations

- Now that we have improved the logarithmic accuracy of our showers, we also want to assess the impact on phenomenology.
- However, in order to make a fair comparison, we need to understand their uncertainty.
- To this effect we include scale compensation, for an emission carrying away a momentum fraction z, given by[1]

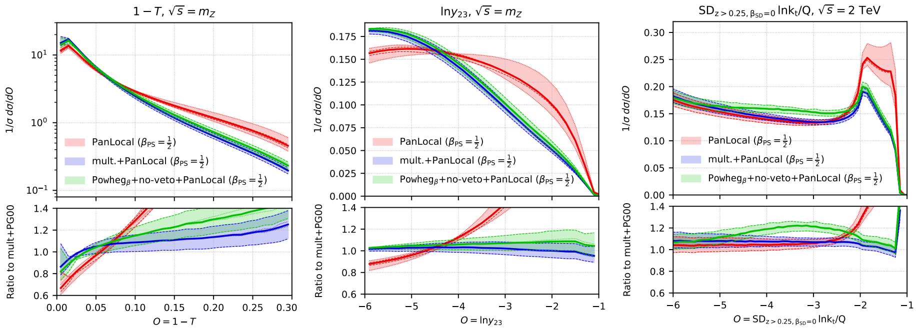
$$\alpha_S(\mu_R)\left(1+\frac{K\alpha_S(\mu_R)}{2\pi}+\frac{2(1-z)\beta_0\alpha_S(\mu_R)}{2\pi}\ln(x_R)\right), \quad \mu_R = x_R\,\mu_R^{\text{central}}.$$

  where the factor $1-z$ ensures that we only apply the scale compensation in the soft limit, and not the hard, where the shower includes all the necessary ingredients. For showers that are not NLL we include the term proportional to K (CMW scheme) but omit the $1-z$ term.
- In order to assess missing terms in the hard matching region we take the emission strengt proportional to (unless matching that emission)

$$P_{\text{splitting}}(x_{\text{hard}}) = P^{\text{(default)}}_{\text{splitting}} \times \left[1+(x_{\text{hard}}-1)\min\left(\frac{4\kappa_\perp^2}{Q^2},1\right)\right],$$

[1] Inspired by [Mrenna, Skands '16]

No matching vs multiplicative vs no veto



- Large effect of matching, with good agreement between showers after matching
- Omitting the veto in POWHEG leads to sizable effects in SD (expected), moderate effects in thrust (surprising as it is $\beta = 1$) and little effect in $\sqrt{y_{23}}$ (disappointing as it is $\beta = 0$).