Our preceding analysis made a simplifying assumption that contracts between consumers and banks could only be initiated in period 0. This served to streamline the analysis, as the alternative to a period 0 contract was autarky (for the consumer) and zero profits (for the bank). We now discuss how the problem changes if unbanked consumers are permitted to sign two-period contracts in period 1.

The main change relates to the formulation of reservation values and their implications for the shape and feasibility of the period 0 contract. Consider the monopolist bank facing a sophisticated hyperbolic discounter. If a contract were not signed in period 0, they would meet again in period 1. In period 1, the contract must satisfy the One-self's participation constraint, which would be determined by some unbanked consumption path $C_1^{A'}$. This can be stated formally. Given some autarky consumption path $C_1^{A'}$, the bank solves:

$$\begin{aligned} &\underset{C_1}{max}\Pi_1\left(C_1; C_1^{A'}\right) \\ &\text{s.t.}U_1\left(C_1\right) \geq U_1\left(C_1^{A'}\right) \end{aligned}$$

Let the solution be denoted C_1^{m1}. Two observations can be made. First, the bank can always offer a period 1 contract that delivers nonnegative profits. This is because any contract will satisfy One-self's optimality condition, $u'\left(c_1^{m1}\right) = \beta u'\left(c_2^{m1}\right)$. So, except in the special case where the autarky consumption path satisfies this condition, the bank can make positive profits in period 1. Second, the autarky consumption path $C_1^{A'}$ might differ from C_1^A, the consumer's autarky utility in the absence of banking. In other words, C_1^A maximizes $U_0\left(C_0^A\right)$ while $C_1^{A'}$ maximizes $U_0\left(c_0^{A'}, c_1^{m1}, c_2^{m2}\right)$. In the latter case, period 0 anticipates that consumption across periods 1 and 2 is guaranteed to satisfy period 1's optimality condition. We can denote $C_0^B \equiv \left(c_0^{A'}, c_1^{m1}, c_2^{m2}\right)$, which corresponds to a Zero-self utility of U_0^B.

In period 0, any contract must meet the Zero-self's reservation utility, U_0^B:

$$\begin{aligned} &\underset{C_0}{max}\Pi_0\left(C_0; Y_0\right) \\ &\text{s.t.}U_0\left(C_0\right) \geq U_0^B \end{aligned}$$

The maximization problem looks familiar, apart from the modified reservation utility. The Zero-self's discounted utility from such a contract is no longer monotonic in her full autarky utility, U_0^A. For example, consider two hypothetical consumers who in autarky must consume their income streams, which deliver the same autarky utility but through different consumption paths: consumer X has $c_1^A = c_2^A$ while consumer Y has $c_1^A > c_2^A$ in a way that satisfies period 1's optimality condition. Then, for consumer X, $U_0^B < U_0^A$ while for consumer Y, $U_0^B = U_0^A$. It follows that, since period 0 contracts depend on the distribution of future consumption, a consumer who fares relatively better in the absence of a bank may fare relatively worse under a banking contract.

Given this benchmark full-commitment contract, the renegotiation-proof contract can be solved for by adding a no-renegotiation constraint to the above maximization problem. The constraint is the same as used previously, and again narrows the set of contracts that can be offered in period 0. As in Proposition 2 (parts a and c), the renegotiation-proof constraint results in lower profits and greater period 0 consumption relative to full-commitment. These results are independent of the period 0 reservation utility and therefore remain unchanged.

A key difference here, however, is that a renegotiation-proof contract will be offered to all consumers (unlike before, where the bank was better off not contracting with consumers whose autarky utility left them close enough to the first-best). Intuitively, this is because the alternative to a period 0 contract is not autarky; rather, it is a period 1 contract that tilts consumption in period 1's favor. Since, in period 0, the bank can at least offer the consumer a consumption path of C_0^B, it ensures that a contract will be accepted.

By opening up the possibility of period 1 contracts, we introduce an additional consideration–the bank that offers commitment itself creates a need for it. By threatening to fully indulge the One-self's preferences, the bank is always able to induce the Zero-self to accept an offer of partial commitment, no matter how weak.

Finally, observe that the bank's decision about whether to operate as a nonprofit is subject to the same tradeoff between improved commitment and reduced enjoyment of profits. However, the attractiveness of nonprofit status drops (relative to the case where period 1 contracts are disallowed) due to the fact that even the for-profit bank finds it profitable to offer contracts to consumers at all levels of autarky utility.

Next, we turn to competition. In most cases, the possibility of period 1 contracts leaves our previous analysis unaltered. This is because competitive contracts do not depend on autarky utility. The only modification to our previous results relates to Proposition 3 (part b). Now, the consumer will accept a period 0 renegotiation-proof contract at any autarky utility since not doing so exposes her to a period 1 contract that fully satisfies period 1's taste for imbalanced consumption.