

Name: \_\_\_\_\_

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1. Let  $U$  be a Gödel Universal function, and let  $S = \{n : U_n \text{ is a surjective function}\}$ . Is  $S$  enumerable?  
Is  $\mathbb{N} \setminus S$  enumerable?

2. Let  $U$  be a Universal Gödel function. Let  $H = \{(n, x) \in \mathbb{N}^2 : U(n, x) \text{ terminates}\}$ . Show that  $H \in \Sigma_1$  and that  $A \leq_m H$  for any  $A \in \Sigma_1$ .

3. Show that there is a number  $r \in \mathbb{R}$  such that  $\{q \in \mathbb{Q} : q < r\}$  is not enumerable.

4. Let  $A$  be an algorithm computing Gödel universal function. We say that the running time of  $A$  at  $n$  is polynomial iff there is an integer  $k$  such that  $A(n, x)$  terminates after  $k + \log^k(x)$  steps.

Show that the set

$$\{n : \text{the running time of } A \text{ at } n \text{ is polynomial}\}$$

is undecidable.