

Name: \_\_\_\_\_

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1. (a) (10 points) Let  $\phi$ ,  $\psi$ , and  $\chi$  be propositional formulas on  $\Omega$ . Show that  $(\phi \vee (\psi \wedge \chi))|_{\rho} = ((\phi \vee \psi) \wedge (\phi \vee \chi))|_{\rho}$  for any assignment  $\rho$  to the variables  $\Omega$ .

- (b) (10 points) Let  $\psi_{1,1}, \dots, \psi_{1,n}, \psi_{2,1}, \dots, \psi_{2,m}$  be propositional formulas on  $\Omega$ . Let  $\phi_1 = \bigwedge_{i=1}^n \psi_{1,i}$  and  $\phi_2 = \bigwedge_{j=1}^m \psi_{2,j}$ . Show that  $(\phi_1 \vee \phi_2)|_{\rho} = (\bigwedge_{i=1}^n \bigwedge_{j=1}^m (\psi_{1,i} \vee \psi_{2,j}))|_{\rho}$  for any assignment  $\rho$  to the variables  $\Omega$ .

- (c) (10 points) Let  $\Omega$  be a set of variables. We say that a propositional formula is a literal if the formula is equal to  $x$  or  $\neg x$  for  $x \in \Omega$ .

We say that a propositional formula on  $\Omega$  is in conjunctive normal form if it is equal to  $\bigvee_{i=1}^n \bigwedge_{j=1}^{m_i} \psi_{i,j}$ , where  $\psi_{i,j}$  is a literal.

Let  $\phi$  be a propositional formula on  $\Omega$ . Show using structural induction that there is a propositional formula  $\psi$  on  $\Omega$  in conjunctive normal form such that  $\psi|_{\rho} = \phi|_{\rho}$  for any assignment  $\rho$  to  $\Omega$ .