

Name: _____

Pid: _____

1. (10 points) We say that L is a B -decision list

(base case) if either L is a number $y \in \mathbb{Z}$, or

(recursion step) L is equal to (f, v, L') where $f : \mathbb{Z} \rightarrow \{0, 1\}$, $v \in \mathbb{Z}$, and L' is a B -decision list.

We can also define the value $\text{val}(L, x)$ of a B -decision list L at $x \in \mathbb{Z}$.

(base case) If L is a number y , then $\text{val}(L, x) = y$, and

(recursion step) if $L = (f, v, L')$, then

$$\text{val}(L, x) = \begin{cases} v & \text{if } f(x) = 1 \\ \text{val}(L', x) & \text{otherwise} \end{cases}.$$

Similarly one may define the length $\ell(L)$ of a B -decision list L .

(base case) If L is a number y , then $\ell(L) = 1$, and

(recursion step) if $L = (f, v, L')$, then $\ell(L) = \ell(L') + 1$.

Assume that $\text{val}(L, x) = x$ for any $x \in [1000]$ show that $\ell(L) \geq 1000$.

2. (10 points) Let S be the minimal set such that $3 \in S$ and $(x + y) \in S$ for any $x, y \in S$. (In other words, S is generated by $\{f\}$ from 3, where $f(x, y) = x + y$.) Show that $S = \{3k : k \in \mathbb{N}\}$.