Name: _____

Pid: _____

1. (10 points) Let $S = \{n \in \mathbb{N} \ : \ x^n + y^n = z^n \text{ has an integer solution}\}$. Prove that the set S is enumeratable.

2. (10 points) Let $A, B \subseteq \mathbb{N}$ be enumeratable. Show that $A \cup B$ is also enumeratable.

3. (10 points) We say that a real number α be computable iff there is a computable function $a : \mathbb{Q} \to \mathbb{Q}$ such that $|\alpha - a(\epsilon)| \leq \epsilon$ for any rational $\epsilon > 0$.

   Show that a number $\alpha < 1$ is computable iff the function $f : \mathbb{N} \to \{0, 1, \ldots, 9\}$ such that $f(i)$ is the ith digit of the base-10 representation of α is computable.

4. (10 points) Let $X, A, B \subseteq \mathbb{N}$ such that $X = A \Delta B$ (X is the symmetric difference of A and B) and A and B are enumeratable. Prove that there are $A', B' \subseteq \mathbb{N}$ such that $X = A' \setminus B'$ and A' and B' are enumeratable.