Name: ______________________

Pid: ______________________

1. Let $a_1, \ldots, a_n$ be a sequence of real numbers. We define inductively $\prod_{i=k}^{n} a_i$ as follows:

   - $\prod_{i=1}^{1} a_i = a_1$ and
   - $\prod_{i=1}^{k+1} a_i = \left(\prod_{i=1}^{k} a_i\right) \cdot a_{k+1}$.

   Prove that $\prod_{i=1}^{n-1} \left(1 - \frac{1}{(i+1)^2}\right) = \frac{n+1}{2n}$ for all integers $n > 1$.

   Solution:

2. Let $A_1, \ldots, A_n, B_1, \ldots, B_n$ be some sets. Show that $\bigcap_{i=1}^n (A_i \cap B_i) = (\bigcap_{i=1}^n A_i) \cap (\bigcap_{i=1}^n B_i)$.

Solution:

3. Let $a_1, \ldots, a_n, b_1, \ldots, b_n \in \mathbb{R}$ be some numbers such that $a_1 \leq b_1, \ldots, a_n \leq b_n$. Show that $\sum_{i=1}^{n} a_i \leq \sum_{i=1}^{n} b_i$.

Solution:

4. Show that two following sets are the same: $\{x : 0 \leq x \leq 2^{n+1} - 1\}$ and $\left\{ \sum_{i=0}^{n} a_i 2^i \ : \ a_1, \ldots, a_n \in \{0, 1\} \right\}$.

Solution:

5. Show that $\sum_{i=0}^{n-1} (i+1)x^i = \frac{1-(n+1)x^n+nx^{n+1}}{(1-x)^2}$.

Solution: