

Name: _____

Pid: _____

1. (a) (10 points) Let ϕ , ψ , and χ be propositional formulas on Ω . Show that $(\phi \vee (\psi \wedge \chi))|_{\rho} = ((\phi \vee \psi) \wedge (\phi \vee \chi))|_{\rho}$ for any assignment ρ to the variables Ω .

- (b) (10 points) Let $\psi_{1,1}, \dots, \psi_{1,n}, \psi_{2,1}, \dots, \psi_{2,m}$ be propositional formulas on Ω . Let $\phi_1 = \bigwedge_{i=1}^n \psi_{1,i}$ and $\phi_2 = \bigwedge_{j=1}^m \psi_{2,j}$. Show that $(\phi_1 \vee \phi_2)|_{\rho} = (\bigwedge_{i=1}^n \bigwedge_{j=1}^m (\psi_{1,i} \vee \psi_{2,j}))|_{\rho}$ for any assignment ρ to the variables Ω .

- (c) (10 points) Let Ω be a set of variables. We say that a propositional formula is a literal if the formula is equal to x or $\neg x$ for $x \in \Omega$.

We say that a propositional formula on Ω is in conjunctive normal form if it is equal to $\bigwedge_{i=1}^n \bigvee_{j=1}^{m_i} \psi_{i,j}$, where $\psi_{i,j}$ is a literal.

Let ϕ be a propositional formula on Ω . Show using structural induction that there is a propositional formula ψ on Ω in conjunctive normal form such that $\psi|_{\rho} = \phi|_{\rho}$ for any assignment ρ to Ω .