Name: ______________________________

Pid: ________________________________

1. (10 points) We say that L is a B-decision list

   (base case) if either L is a number $y \in \mathbb{Z}$, or

   (recursion step) L is equal to (f, v, L') where $f : \mathbb{Z} \to \{0, 1\}$, $v \in \mathbb{Z}$, and L is a B-decision list.

   We can also define the value $\mathrm{val}(L, x)$ of a B-decision list L at $x \in \mathbb{Z}$.

   (base case) If L is a number y, then $\mathrm{val}(L, x) = y$, and

   (recursion step) if $L = (f, v, L')$, then

$$\mathrm{val}(L, x) = \begin{cases} v & \text{if } f(x) = 1 \\ \mathrm{val}(L', x) & \text{otherwise} \end{cases}.$$

   Similarly one may define the length $\ell(L)$ of a B-decition list L.

   (base case) If L is a number y, then $\ell(L) = 1$, and

   (recursion step) if $L = (f, v, L')$, then $\ell(L) = \ell(L') + 1$.

   Assume that $\mathrm{val}(L, x) = x$ for any $x \in [1000]$ show that $\ell(L) \geq 1000$.

2. (10 points) Let S be the minimal set such that $3 \in S$ and $(x + y) \in S$ for any $x, y \in S$. (In other words, S is generated by $\{f\}$ from $\{3\}$, where $f(x, y) = x + y$.) Show that $S = \{3k \ : \ k \in \mathbb{N}\}$.