

2. (10 points) Derive $A \vee B$ from $X \vee A$ and $\neg X \vee B$ using natural derivations.

Solution:

1		$x \vee A$	
2		$\neg x \vee B$	
<hr/>			
3			A
			<hr/>
4			$A \vee B$ $\vee\text{I}, 3$
5			x
			<hr/>
6			
			$\neg x$
			<hr/>
7			
			\perp $\neg\text{E}, 6$
8			
			$A \vee B$ $\perp\text{E}, 7$
9			
			B
			<hr/>
10			
			$A \vee B$ $\vee\text{I}, 9$
11			$A \vee B$ $\vee\text{E}, 2, 6\text{--}10, 10\text{--}9$
12		$A \vee B$ $\vee\text{E}, 1, 3\text{--}4, 5\text{--}11$	