

Name: _____

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1. Let U be a Gödel Universal function, and let $S = \{n : U_n \text{ is a surjective function}\}$. Is S enumerable?
Is $\mathbb{N} \setminus S$ enumerable?

2. Let U be a Universal Gödel function. Let $H = \{(n, x) \in \mathbb{N}^2 : U(n, x) \text{ terminates}\}$. Show that $H \in \Sigma_1$ and that $A \leq_m H$ for any $A \in \Sigma_1$.

3. Show that there is a number $r \in \mathbb{R}$ such that $\{q \in \mathbb{Q} : q < r\}$ is not enumerable.

4. Let A be an algorithm computing Gödel universal function. We say that the running time of A at n is polynomial iff there is an integer k such that $A(n, x)$ terminates after $k + \log^k(x)$ steps.

Show that the set

$$\{n : \text{the running time of } A \text{ at } n \text{ is polynomial}\}$$

is undecidable.