Name: ______________________

Pid: ______________________

1. Show that there does not exist the largest integer.

Solution:

2. Show that there are irrational numbers a and b such that a^b is rational.

Solution:

3. Show that $\sum_{i=1}^{n}(2i-1) = n^2$.

Solution:

4. Prove that $\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}$.

Solution:

5. Let $a_1, \ldots, a_n$ be a sequence of real numbers. We define inductively $\prod_{i=k}^{n} a_i$ as follows:

   - $\prod_{i=1}^{1} a_i = a_1$ and
   - $\prod_{i=1}^{k+1} a_i = \left(\prod_{i=1}^{k} a_i\right) \cdot a_{k+1}$.

   Prove that $\prod_{i=1}^{n-1} \left(1 - \frac{1}{(i+1)^2}\right) = \frac{n+1}{2n}$ for all integers $n > 1$.

   Solution: