

Name: _____

Pid: _____

1. (10 points) Derive $(A \wedge B) \Rightarrow C$ from $A \Rightarrow (B \Rightarrow C)$ using natural derivations.

Solution:

1		$A \Rightarrow (B \Rightarrow C)$	
2			
3			
4			
5			
6			
7			

2		$A \wedge B$	
3			
4			
5			
6			
7			

3		A	$\wedge E, 2$
4		$B \Rightarrow C$	$\Rightarrow E, 1, 3$
5		B	$\wedge E, 2$
6		C	$\Rightarrow E, 4, 5$
7		$(A \wedge B) \Rightarrow C$	$\Rightarrow I, 1-6$

2. (10 points) Derive $A \vee B$ from $X \vee A$ and $\neg X \vee B$ using natural derivations.

Solution:

1		$x \vee A$		
2		$\neg x \vee B$		
<hr/>				
3			A	
<hr/>				
4			$A \vee B$ $\vee I, 3$	
5			x	
<hr/>				
6				$\neg x$
<hr/>				
7				\perp $\neg E, 6$
8				$A \vee B$ $\perp E, 7$
9				B
<hr/>				
10				$A \vee B$ $\vee I, 9$
11			$A \vee B$	$\vee E, 2, 6\text{--}10, 10\text{--}9$
12		$A \vee B$		$\vee E, 1, 3\text{--}4, 5\text{--}11$