

Name: _____

Pid: _____

1. (10 points) Let $m_1, n_1, m_2, n_2 \in \mathbb{N}$, we say that $(m_1, n_1) < (m_2, n_2)$ iff either $m_1 < m_2$ or $m_1 = m_2$ and $n_1 < n_2$.

Let $P(m, n)$ be some property of pairs of integers. Assume that we can prove the following statement for all $m, n \in \mathbb{N}$:

if $P(x, y)$ is true for all $x, y \in \mathbb{N}$ such that $(x, y) < (m, n)$, then $P(m, n)$ is true.

Show that we can prove that $P(m, n)$ is true for all $m, n \in \mathbb{N}$.

2. (10 points) In the subtraction game where players may subtract 1, 2 or 5 chips on their turn, identify the N- and P-positions. (Please do not forget to prove correctness of your answer.)