Name: _____

Pid: _____

1. Let $p_0 = 1$, $p_1 = \cos\theta$ (for θ some fixed constant) and $p_{n+1} = 2p_1p_n - p_{n-1}$ for $n \geq 1$. Prove that $p_n = \cos(n\theta)$ for integers $n \geq 0$.

Solution:

2. Assume that there are 10 points within the sqare with side 1. Show that there are two points among these 10 such that the distance between them is at most $\sqrt{2}/3$.

Solution:

3. Show that $\binom{n}{m}\binom{m}{k} = \binom{n}{k}\binom{n-k}{m-k}$.

Solution:

4. Let $m \geq n$ be positive integers. Show that

$$S(m,n) = \sum_{i=1}^{m} S(m-i, n-1) n^{i-1}.$$

Solution: