

Name: \_\_\_\_\_

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1. (10 points) Let  $m_1, n_1, m_2, n_2 \in \mathbb{N}$ , we say that  $(m_1, n_1) < (m_2, n_2)$  iff either  $m_1 < m_2$  or  $m_1 = m_2$  and  $n_1 < n_2$ .

Let  $P(m, n)$  be some property of pairs of integers. Assume that we can prove the following statement for all  $m, n \in \mathbb{N}$ :

if  $P(x, y)$  is true for all  $x, y \in \mathbb{N}$  such that  $(x, y) < (m, n)$ , then  $P(m, n)$  is true.

Show that we can prove that  $P(m, n)$  is true for all  $m, n \in \mathbb{N}$ .

2. (10 points) In the subtraction game where players may subtract 1, 2 or 5 chips on their turn, identify the N- and P-positions. (Please do not forget to prove correctness of your answer.)