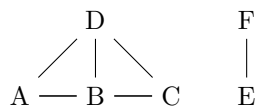


Name: _____

Pid: _____

1. (10 points) Check all the correct statements.

- ☐ The number of different strings you can get by reordering letters in the word aabbc is 30.
- ☐ The following graph is connected.



- ☐ There are 32 different strings of length 5 over the alphabet with 2 letters.
- ☐ There are 4 different surjective functions from $[4]$ to $[4]$.
- ☐ There are 3 ways to put 4 identical balls into 3 different boxes such that all the boxes are not empty.
- ☐ A graph on 4 vertices has at most 6 edges.
- ☐ A disconnected graph on 5 vertices has at most 6 edges.
- ☐ If a graph on 5 vertices has 3 edges it should be disconnected.
- ☐ K_5 has an Eulerian cycle.
- ☐ K_6 has a Hamiltonian cycle.

2. Let us consider graph $G = ([5] \times [5], E)$ such that $((x_1, y_1), (x_2, y_2)) \in E$ iff

- either $x_1 = x_2$ and $|y_1 - y_2| = 1$
- or $|x_1 - x_2| = 1$ and $y_1 = y_2$.

(a) (5 points) Is it possible to cover the graph G with 8 paths of length 5 (these paths should not visit any vertex twice)?

Note that by covering we mean that every edge of G is an edge of one of the paths.

(b) (5 points) Is it possible to cover the graph G with 5 paths of length 8 (these paths should not visit any vertex twice)?

Note that by covering we mean that every edge of G is an edge of one of the paths.

3. (10 points) In the Durmstrang Institute every semester consists of two parts with two midterms in each of them. Find a closed formula for number of ways to organize semester if there are n days in this semester.

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4. (10 points) Let p be a permutation, we say that $i < j$ form an inversion in p iff $p_i > p_j$. Let us denote by $I(n, k)$ the number of permutations on $[n]$ with k inversions. Show that $I(n, k) = I(n, \binom{n}{2} - k)$.

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5. (10 points) Elements of \mathbb{Z}^2 are colored in black and white, show that there are $x_1, x_2, y_1, y_2 \in \mathbb{Z}$ such that $(x_1, y_1), (x_1, y_2), (x_2, y_1), (x_2, y_2)$ are colored in the same color.

6. (10 points) A certain academic term contains 40 days of class. A teacher needs to give 5 exams on these days, but any two of these exams must be ≥ 3 class days apart (for example, if there is an exam on Day 12, then there can be no exam on Day 13 or Day 14, but an exam on Day 15 is alright). How many ways can the teacher do this?