


Lecture 2 Sets, Functions, and Relations

Definition

A set is a well-defined collection of objects

Exercise

- Give four collections that are sets
- Give two collections that aren't sets

Definition Let S be a set and x be some object. We denote the statement " x belongs to S " by $x \in S$ 

Exercise Which statements are true?

- $-1 \in \mathbb{N}$
- $\sqrt{2} \in \mathbb{Q}$
- $\sqrt{2} \in \mathbb{R}$
- $-1 \in \mathbb{Z}$

Definition

We use several ways to define a set

Listing elements: $\{1, 2, 3\}$ $\{1, 2, 3, \dots\}$

Conditional definitions: $[n] = \{x \in \mathbb{Z} : 1 \leq x \leq n\}$

Constructive definitions:

$$\{x^2 : x \in \mathbb{Z}\} = \{x \in \mathbb{R} : x^2 \in \mathbb{Z}\}$$

Exercise Show that in any vector space V ,
if a set of vectors $\{x_1, \dots, x_n\}$ spans V ,
then there is a basis $B \subseteq \{x_1, \dots, x_n\}$ of V .
 \downarrow they are essentially the same

We prove using induction by n that
for any set $S \subseteq V$ of size n , if S spans V ,
then there is a basis $B \subseteq S$ of V

Now let's prove for $n=1$. In this case
 $S = \{x\}$ and is L.I.. Therefore S is a basis.

Assume that the statement is true for k .
Consider S consisting of $k+1$ vectors
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 $\{x_1, \dots, x_{k+1}\}$

- If S is L.I., then $B = S$ is a basis.

- otherwise, WLOG $x_{k+1} = \alpha_1 x_1 + \dots + \alpha_k x_k$

Therefore $\{x_1, \dots, x_k\}$ spans V . By the I.H.,

there is $B \subseteq \{x_1, \dots, x_k\} \subseteq S$ s.t. B is
a basis.