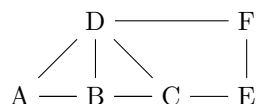


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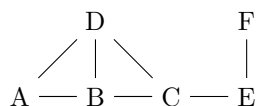
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1. (10 points) Check all the correct statements.

- ☐ The inverse of the permutation $(1, 2)(2, 4, 5)(2, 3)$ is 51234.
- ☐ There are 27 different strings of length 3 over the alphabet with 3 letters.
- ☐ Product of the permutations 13254 and 12354 is 23154.
- ☐ There are 60 different surjective functions from $[5]$ to $[4]$.
- ☐ There are 3 ways to put 4 identical balls into 3 different boxes such that all the boxes are not empty.
- ☐ A graph on 4 vertices has at most 6 edges.
- ☐ A connected graph on 5 vertices has at most 5 edges.
- ☐ If a graph on 5 vertices has 3 edges it should be disconnected.
- ☐ The following graph has an Eulerian path.



- ☐ The following graph has a Hamiltonian cycle.



2. (10 points) What is the maximal number of edges of a simple graph G on $[n]$ if it is not connected?

3. (10 points) Show that for any $n \in \mathbb{N}$, there are two permutations $p, q \in S_n$ such that any permutation from S_n can be expressed as their product (we can use each permutation multiple times).

4. (10 points) Let $a_n = 2a_{n-1} - a_{n-2}$ for $n \geq 2$, $a_1 = 2$, and $a_0 = 1$. Find a closed formula (no summation signs) for the recurrent sequence a_n .

5. (10 points) Show that among any $n + 1$ positive integers not exceeding $2n$ there must be an integer that divides one of the other integers.

6. (10 points) Give a simple closed form expression for the sum

$$\sum_{\substack{a+b+c=7 \\ a,b,c \geq 0}} \binom{7}{a,b,c}.$$