MATH5805: Advanced Time Series Analysis
Session 1, 2014
Assignment 2
Due Wednesday April 16 at 5pm in lectures.

Question 1.(D&K Ex. 3.11.1)
Consider the autoregressive moving average, with constant, plus noise model

$$y_t = y_t^* + \epsilon_t, \quad y_t^* = \mu + \phi_1 y_{t-1}^* + \phi_2 y_{t-2}^* + \zeta_t + \theta_1 \zeta_{t-1}$$

for $t = 1, \ldots, n$ where μ is an unknown constant and the disturbances $\epsilon_t \overset{\text{i.i.d}}{\sim} \mathrm{N}(0, \sigma_\epsilon^2)$ and $\zeta_t \overset{\text{i.i.d}}{\sim} \mathrm{N}(0, \sigma_\zeta^2)$ are mutually independent sequences. The autoregressive coefficients ϕ_1, ϕ_2 are restricted such that y_t^* is a stationary process and we assume that $0 < \theta_1 < 1$. Represent this model in the state space form (3.1) and (3.2) of slide 113 of the notes (equation (3.1) of DK).

Question 2. (D&K Ex. 3.11.4)
Consider the state space model defined in equations (3.1) and (3.2) of slide 113 of the notes (or equation (3.1) of D&K) extended with regression effects

$$y_t = X_t \beta + Z_t \alpha_t + \epsilon_t, \quad \alpha_{t+1} = W_t \beta + T_t \alpha_t + R_t \eta_t,$$

for $t = 1, \ldots, n$ where X_t and W_t are fixed matrices that (partly) consist of exogenous variables and β is a vector of regression coefficients and matrices and vectors have conformable dimensions.

a) Show that this state space model can be expressed as

$$y_t = X_t^* \beta + Z_t \alpha_t^* + \epsilon_t, \quad \alpha_{t+1}^* = T_t \alpha_t^* + R_t \eta_t,$$

for $t = 1, \ldots, n$.

b) Give and expression for X_t^* in terms of X_t, W_t and the other system matrices.

Question 3. (D&K Ex. 4.14.1)
Refer to Lemma 4.1 on slide 130. Using the notation there let

$$\Sigma_* = \begin{bmatrix} \Sigma_{xx} & \Sigma_{xy} \\ \Sigma_{xy}' & \Sigma_{yy} \end{bmatrix}.$$

a) Verify

$$\Sigma_* = \begin{bmatrix} I & \Sigma_{xy}\Sigma_{yy}^{-1} \\ 0 & I \end{bmatrix} \begin{bmatrix} \Sigma_{xx} - \Sigma_{xy}\Sigma_{yy}^{-1}\Sigma_{xy}' & 0 \\ 0 & \Sigma_{yy} \end{bmatrix} \begin{bmatrix} I & 0 \\ \Sigma_{yy}^{-1}\Sigma_{xy}' & I \end{bmatrix}$$

.

b) Hence verify

$$\Sigma_*^{-1} = \begin{bmatrix} I & 0 \\ -\Sigma_{yy}^{-1}\Sigma_{xy}' & I \end{bmatrix} \begin{bmatrix} (\Sigma_{xx} - \Sigma_{xy}\Sigma_{yy}^{-1}\Sigma_{xy}')^{-1} & 0 \\ 0 & \Sigma_{yy}^{-1} \end{bmatrix} \begin{bmatrix} I & -\Sigma_{xy}\Sigma_{yy}^{-1} \\ 0 & I \end{bmatrix}$$

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c) Using the joint density

$$p(x,y) = \text{constant} \times \exp\left[-\frac{1}{2}\begin{pmatrix} x-\mu_x \\ y-\mu_y \end{pmatrix}' \Sigma_*^{-1} \begin{pmatrix} x-\mu_x \\ y-\mu_y \end{pmatrix}\right]$$

obtain $p(x|y)$ and hence prove Lemma 4.1.

Question 4.

a) (D&K Ex. 4.14.3) Suppose that the fixed vector λ is regarded as an estimate of a random vector x whose mean vector is μ. Show that the minimum mean square error matrix $E[(\lambda - x)(\lambda - x)']$ is obtained when $\lambda = \mu$.

b) (D&K Ex. 4.14.4) In the joint distribution, not necessarily normal, of x and y suppose that $\bar{x} = \beta + \gamma y$ is an estimate of x given y with mean squared error $\text{MSE}(\bar{x}) = E[(\bar{x} - x)(\bar{x} - x)']$. Use the details of the proof of Lemma 4.2 and part a) of this question show that the minimum mean square error matrix is obtained when $\bar{x} = \hat{x}$ where $\hat{x}$ and $\text{MSE}(\hat{x}) = \text{Var}(x - \hat{x})$ are as given in Lemma 4.2.

Question 5.

This exercise requires you to use the **KFAS** R-package. Locate the file `SeatBelts.R` and using the **KFAS** manual (Package 'KFAS' December 7, 2013) for reference as needed, perform the following tasks.

a) Explain what each step of the code in `SeatBelts.R` is doing. You can annotate the file with brief comments and submit as part of your answer to this question.

b) Write out using the state space formulation of Week 4 notes the model being fit. Briefly describe the key model elements and the features of the Seat Belt series being modelled by them.

c) Report the estimated values of all unknown parameters in the model and their standard errors. You may need to use the delta-method to do this.

d) Provide an estimate of the impact of the intervention (seat belt law enactment).

e) Create additional R-code to:

- Perform residual analysis as described in Week 3 Notes.
- Forecast 24 months into the future from the end of the available data with 68% prediction intervals.
- Simulate 4 samples of the signal conditional on the observed series (provide these in a four panel plot.

Include the code in an expanded '`SeatBelts.R`' and included a printout of this with any output and graphs required with your answers.