MATH5805 Advanced Time Series Analysis

Session 1 2014

William T.M. Dunsmuir

June 5, 2014

Topic: 12: Notes on Preparing for Final Exam

Structure of the Final Exam

Stucture of Exam:

- **Question 1** A theory question on linear Gaussian state space models.
- **Question 2** A theory question on nonlinear non-Gaussian state space models.
- **Question 3** Focus on the Stochastic Volatility model – see below.
- **Question 4** A (mostly) practical question on glarma models - including interpretation of output, specifying models mathematically and in R code using $glarma$ package.

Allowable materials

You can bring to the exam:

- any material available on Moodle for this course including handouts, notes, R code examples and output.
- Manuals and vignettes available from CRAN for the KFAS and glarma packages.
- Calculators will be provided if required.
- any lecture notes you have prepared based on lectures and revision study.
- the text book Durbin and Koopman but only that book. No other reference books.

Preparation – derivations in notes and assignments

It is essential that you go through all the handout lecture notes and fill in any incomplete mathematical details that were covered in more detail in lectures. You should be able to reproduce any argument or derivation in those notes.

You should also check that you fully understand the solutions to all questions on the three assignments. Partial solutions and R code is available for each of these. Solutions not made available are for the questions that were generally done well. We also discussed solutions in detail in class at the relevant time.

Preparation – Linear Gaussian state space models

At a minimum you should revise the following concepts:

- Linear state space models of different types: local level, trend, regression, arma, seasonal components and how to combine these in an overall state space model mathematically and using KFAS software.
- Be able to explain in these models what are the unknown parameters and how they are estimated in KFAS. How to extract point estimates and their standard errors.
- What are smoothed and filtered estimates of state, signals, noise terms. What these represent in terms of conditional distribution features and how to extract the variances of these conditional on the observed data. Use of KFAS to do all of these things.
- How to deal with missing data in linear state space models and setting up forecasts into the future.
- **Basic** derivations associated with key parts of the Kalman Filter Smoother recursions.

Preparation – nonlinear non-Gaussian state space models

- Mode estimation for linear Gaussian and nonlinear non-Gaussian models as covered in the notes - particularly focussing on the conditional independent response, Gaussian linear state model.
- Laplace approximation and its relationship to the Durbin-Koopman approximate likelihood.
- Importance sampling and its use to refine approximate likelihoods.
- Application of ideas to the exponential family cases: Poisson and binary and binomial (could you write out the modifications to the theory and computing methods we developed for the binary to the binomial?).
- How to implement the above using R software (as covered in lecture examples).

Preparation on Stochastic Volatility model

.

- Review the definition of this model in notes and Durbin and Koopman.
- For the specific model:

$$y_t = \mu + exp(\frac{1}{2}\theta_t)\epsilon_t, \quad \epsilon \sim \mathrm{N}(0, \sigma_\epsilon)$$

  where $\theta_t = Z_t\alpha_t$ and α_t is modelled as the autoregression

$$\alpha_{t+1} = \phi\alpha_t + \eta_t, \quad \eta_t \sim \mathrm{N}(0, \sigma_\eta^2)$$

  with $\alpha_1 \sim \mathrm{N}(0, \sigma_\eta^2/(1-\phi^2))$ you should be able to develop the approximating linear Gaussian model, the modal approximation, the Laplace approximation and specify how importance sampling would be set up to estimate the likelihood.
- Implement these ideas in R code by adapting software that you have used for Poisson or binary data.