MAT1856/APM466 Assignment 1

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Fundamental Questions - 25 points

1.

   (a) Governments issue bonds instead of printing money to prevent inflation and currency devaluation.

   (b) The long-term part of a yield curve might flatten if investors are less optimistic about the economy and expect slower economic growth or a potential recession.

   (c) Quantitative easing is a monetary policy tool where central banks purchase securities, such as government bonds, to increase the money supply and lower long-term interest rates (Bank of Canada, 2020). During the COVID-19 pandemic, the U.S. Federal Reserve has used QE by buying large amounts of Treasury securities to stabilize financial markets and support economic recovery (Brookings Institution, 2024).

2. I selected these 10 bonds based on their maturity within the 0-5 year range, ensuring relatively similar coupon rates and a balanced distribution of issue and maturity dates. Most were issued in April or October and are set to mature in either September or March, with one exception. This structure simplifies my curve calculations.

   Selected Bonds

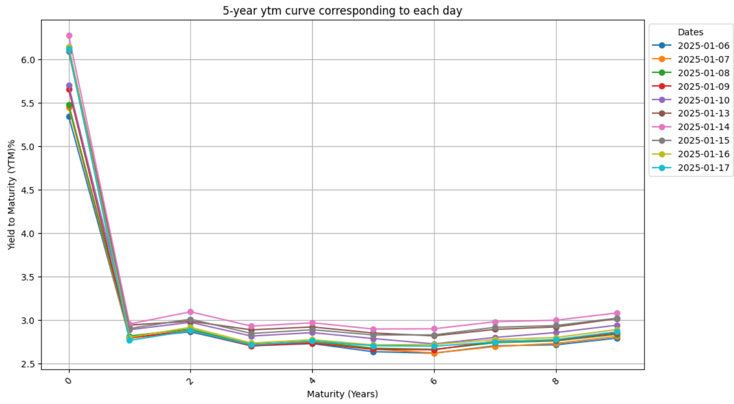
   CAN 3.25 Sep 28, CAN 3.50 Mar 28, CAN 3.50 Sep 29, CAN 4.00 Mar 29, CAN 1.25 Mar 25, CAN 0.50 Sep 25, CAN 2.75 Sep 27, CAN 0.25 Mar 26, CAN 1.00 Sep 26, CAN 1.25 Mar 27

3. Eigenvalues and eigenvectors of the covariance matrix allow us to find the main patterns in the stochastic processes. The eigenvalues show how much variance each pattern or principal component explains, while the eigenvectors give us the direction of the patterns. If the eigenvalue is larger this means that they capture the most variation among the stochastic processes. This is Principal Component Analysis, which helps simplify datasets by focusing on the most meaningful variations. (Jaadi, 2024)

Empirical Questions - 75 points

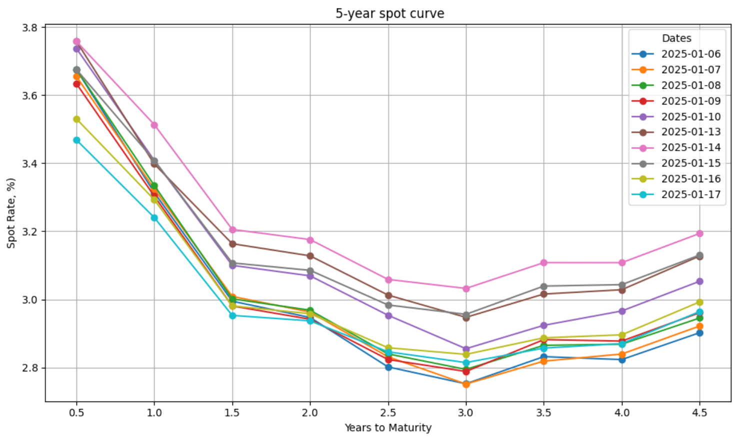
4.

   (a) The yield to maturity (YTM) for each bond is calculated using the bond pricing equation, with accrued interest added to the clean price: Dirty Price = Clean Price + $\frac{n}{365}$ × Coupon Rate For zero-coupon bonds, the yield is obtained by solving: $r = -\frac{\ln\left(\frac{P}{F}\right)}{T}$ For coupon-paying bonds, the bootstrapping method determines $P = \sum_{i=1}^{N} C \cdot e^{-r_i t_i} + F \cdot e^{-r_N T}$ I used a piecewise linear interpolation, where Yield to Maturity values are connected with straight lines to estimate the yield curve.



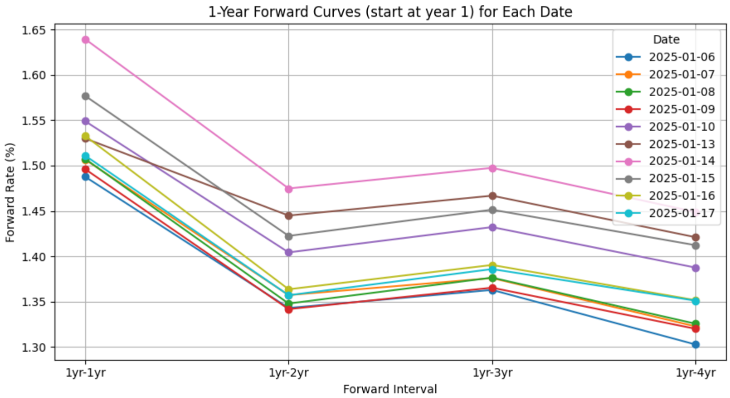
(b) **Algorithm:**

- Read the CSV file containing dirty prices of the the bonds.
- Convert Maturity Date to years until maturity.Time to Maturity $= \frac{\text{Maturity Date} - \text{Today}}{365}$
- Extract the latest price for each bond.
- Sort bonds by Time to Maturity (1 to 5 years).
- Compute Spot Rates via bootstrapping:
- For the shortest maturity bond: $P = F \cdot e^{-rT}$, $r = -\frac{\ln(P/F)}{T}$
- For longer maturity bonds (Coupon-Paying Bonds) $P = \sum_{i=1}^{N} C \cdot e^{-r_i t_i} + F \cdot e^{-r_N T}$
- Repeat bootstrapping for last 10 days.



(c) **Algorithm**

- Read the CSV file containing spot rate
- Compute forward rates using spot rates: $f_{m,n} = \left(\frac{(1+s_n/2)^{2n}}{(1+s_m/2)^{2m}}\right)^{\frac{1}{2(n-m)}} - 1$
- Computing Specific Forward Rates $f_{1,2} = \left(\frac{(1+s_2/2)^4}{(1+s_1/2)^2}\right)^{\frac{1}{2}} - 1$, $f_{1,3} = \left(\frac{(1+s_3/2)^6}{(1+s_1/2)^2}\right)^{\frac{1}{3}} - 1$ $f_{1,4} = \left(\frac{(1+s_4/2)^8}{(1+s_1/2)^2}\right)^{\frac{1}{4}} - 1$, $f_{1,5} = \left(\frac{(1+s_5/2)^{10}}{(1+s_1/2)^2}\right)^{\frac{1}{5}} - 1$
- Repeat bootstrapping for the last 10 trading days.



5. Covariance Matrix of Yield Log-Returns $\begin{bmatrix} 0.000811 & 0.000240 & 0.000221 & 0.000363 & 0.000320 \\ 0.000240 & 0.000408 & 0.000399 & 0.000484 & 0.000495 \\ 0.000221 & 0.000399 & 0.000558 & 0.000527 & 0.000543 \\ 0.000363 & 0.000484 & 0.000527 & 0.000628 & 0.000635 \\ 0.000320 & 0.000495 & 0.000543 & 0.000635 & 0.000647 \end{bmatrix}$

   Covariance Matrix of Forward Rate Log-Returns $\begin{bmatrix} 0.001108 & 0.000684 & 0.000680 & 0.000649 \\ 0.000684 & 0.000825 & 0.000796 & 0.000783 \\ 0.000680 & 0.000796 & 0.000781 & 0.000770 \\ 0.000649 & 0.000783 & 0.000770 & 0.000766 \end{bmatrix}$

6. Eigenvalues of Yield Covariance Matrix

$$\begin{bmatrix} 2.342348 \times 10^{-3} & 6.017065 \times 10^{-4} & 8.770975 \times 10^{-5} & 1.908198 \times 10^{-5} & 3.537925 \times 10^{-7} \end{bmatrix}$$

   Eigenvectors of Yield Covariance Matrix

$$\begin{bmatrix} -0.353884 & -0.926401 & -0.100857 & -0.063896 & 0.047883 \\ -0.394179 & 0.149122 & 0.470588 & -0.772259 & -0.067440 \\ -0.439573 & 0.271552 & -0.825823 & -0.223566 & -0.032714 \\ -0.511698 & 0.105371 & 0.207473 & 0.467533 & -0.682224 \\ -0.514130 & 0.186281 & 0.208199 & 0.361889 & 0.725713 \end{bmatrix}$$

   Eigenvalues of Forward Rate Covariance Matrix

$$\begin{bmatrix} 3.052 \times 10^{-3} & 4.130 \times 10^{-4} & 1.300 \times 10^{-5} & 2.000 \times 10^{-6} \end{bmatrix}$$

   Eigenvectors of Forward Rate Covariance Matrix

$$\begin{bmatrix} -0.513331 & -0.857671 & 0.028634 & 0.008541 \\ -0.505388 & 0.302837 & 0.240661 & -0.771333 \\ -0.495379 & 0.271396 & -0.805354 & 0.179857 \\ -0.485463 & 0.314698 & 0.540986 & 0.610429 \end{bmatrix}$$

The largest eigenvalue for Yields is 0.002342, which has the eigenvector: [-0.35388408 -0.39417946 -0.4395727 -0.51169751 -0.51413044]

The largest eigenvalue for Forward Rates is 0.003052, which has the eigenvector: [-0.51333053 -0.50538802 -0.49537864 -0.48546341]

The largest eigenvalue represents the most significant variation in the yield curve, with its associated eigenvector defining the primary mode of movement across maturities.

References and GitHub Link to Code

GitHub Repository: https://github.com/alexanderlee10/APM466-A1

- Bank of Canada. (2020, December). How quantitative easing works. Retrieved from https://www.bankofcanada.ca/2
quantitative-easing-works/

- Brookings Institution. (2024, January). The Fed's response to COVID-19: Addressing economic and financial challenges. Retrieved from https://www.brookings.edu/articles/fed-response-to-covid19/

- Jaadi, Z. (2024, February). A step-by-step explanation of principal component analysis (PCA). Built In. Retrieved from https://builtin.com/data-science/step-step-explanation-principal-component-analysis

- RiskLab. (n.d.). APM466/MAT1856 library. Retrieved from https://seco.risklab.ca/apm466-mat1856-library/