Principal Component Analysis

an introduction

alexander lerch

github.com/alexanderlerch/2025-PCA-Lectu

about
about me

- **education**
  - Electrical Engineering (Technical University, Berlin)
  - Tonmeister (music production, University of Arts, Berlin)

- **professional**
  - Professor, School of Music, Georgia Tech
  - Associate Dean for Research & Creative Practice, College of Design, Georgia Tech
  - 2000-2013: CEO at zplane.development

- **experience**
  - audio algorithm design (20+ years)
  - machine learning for music (15+ years)
  - professional music software engineering & development (10+ years)
  - entrepreneurship (10+ years)
  - research administration (2+ years)

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introduction

principal component analysis

goal: data reduction while maximum information content

introduction

pca use cases

- dimensionality reduction
  - overfitting, curse of dimensionality
  - visualization of high-dimensional spaces
  - performance/runtime
- remove feature correlation, redundancy
- feature analysis

principal component analysis
introduction

- **objective**
  - map features to new coordinate system

$$\boldsymbol{u}(n) = \boldsymbol{T}^{\mathrm{T}} \cdot \boldsymbol{v}(n)$$

    - $\boldsymbol{u}(n)$: transformed features (same dimension as input $\boldsymbol{v}(n)$)
    - $\boldsymbol{T}$: transformation matrix $(\mathcal{F}_v \times \mathcal{F}_v)$

$$\boldsymbol{T} = \begin{bmatrix} \boldsymbol{c}_0 & \boldsymbol{c}_1 & \dots & \boldsymbol{c}_{\mathcal{F}-1} \end{bmatrix}$$

- **properties**
  - $\boldsymbol{c}_0$ points in the direction of highest *variance* (sorted by eigenvalue)
  - variance concentrated in as few output components as possible
  - $\boldsymbol{c}_i$ orthogonal

$$\boldsymbol{c}_i^{\mathrm{T}} \cdot \boldsymbol{c}_j = 0 \quad \forall\ i \neq j$$

  - transformation is invertible

$$\boldsymbol{v}(n) = \boldsymbol{T} \cdot \boldsymbol{u}(n)$$

principal component analysis

introduction

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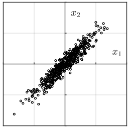
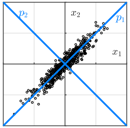
feature space transformation

PCA visualization





calculation of the transformation matrix

1. normalize input data
2. compute covariance matrix $\boldsymbol{R}$

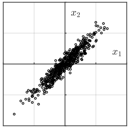
$$\boldsymbol{R} = \mathcal{E}\{(V - \mathcal{E}\{V\})(V - \mathcal{E}\{V\})\}$$

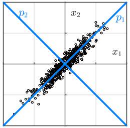
3. compute eigenvectors and eigenvalues
4. sort eigenvectors by eigenvalue and use as axes for the new coordinate system
5. apply transformation matrix

matlab source: plotPca.m

feature space transformation

PCA visualization





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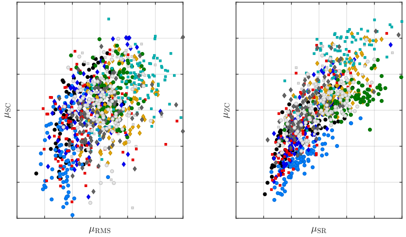
feature space transformation

PCA visualization

PCA examples

example 1

- 4-dimensional feature space: Spectral Centroid, RMS, Zero Crossing, Spectral Rolloff
- 10 classes of music signals (classical, jazz, blues, metal, ...)



matlab source: plotFeatureScatter.m

PCA examples

example 1

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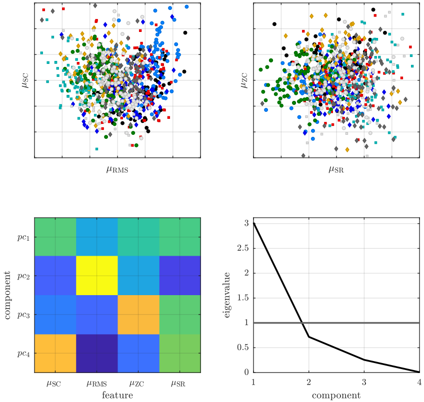
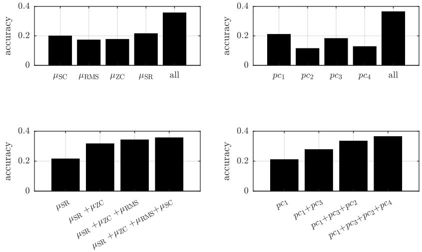


matlab source: plotFeatureScatterPca.m

PCA examples

example 1

- **observations**
  - combined feature performance is not sum of individual performance
  - variance/eigenvalue ranking does not necessarily correlate to task performance
  - simple feature selection is not necessarily inferior to PCA



matlab source: plotPcaClassification.m

PCA examples

example 2

PCA transformation matrix $\mathbf{T}^{\mathrm{T}}$

$$\begin{bmatrix} -0.5638 & -0.3596 & -0.5024 & -0.5481 \\ 0.1738 & -0.9139 & 0.3539 & 0.0965 \\ 0.2408 & -0.1882 & -0.7606 & 0.5728 \\ -0.7707 & -0.0018 & 0.2096 & 0.6017 \end{bmatrix}$$

PCA examples

example 2

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conclusion

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thank you!

links

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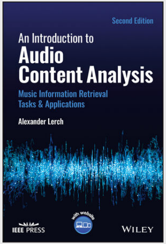
mail: alexander.lerch@gatech.edu

book: www.AudioContentAnalysis.org

music informatics group: musicinformatics.gatech.edu



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