Introduction to Audio Content Analysis

Module 2.6: Fundamentals — Non-Fourier Time-Frequency Transforms

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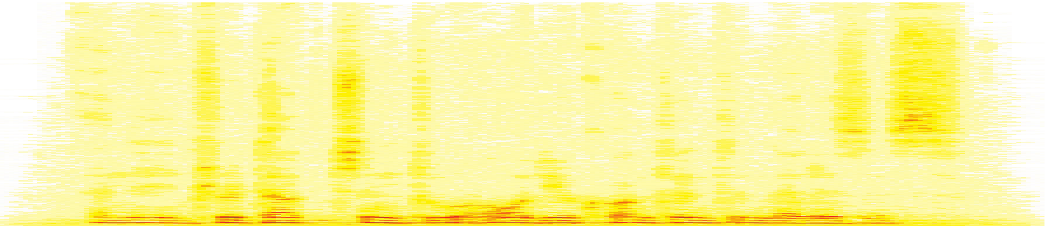

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Chapter 2 — Fundamentals: pp. 24–26

- **lecture content**
  - constant-Q transform (CQT)
  - Gammatone filterbank
- **learning objectives**
  - discussing the advantages and disadvantages of different time-frequency transforms
  - explaining the principles of the CQT and auditory filterbanks

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other time frequency transforms
introduction

- Fourier transform continues to be much-used tool in audio signal processing and MIR
- but there are disadvantages, e.g.
  - frequency axis does not directly map to (perceptual) pitch axis
  - frequency and time resolution inversely related

$\Rightarrow$ **alternative transforms** can be used

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constant-Q transform
introduction

- DFT has a *linear* frequency axis:
  - not perceptually meaningful: *logarithmic* is better match
  - low frequency resolution at low frequencies

⇒ compute DFT-like transform at specific frequencies
  - space frequencies logarithmically (constant Q)
  - resulting abscissa resolution is pitch-related

constant-Q transform

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$$\mathcal{Q} = \frac{f}{\Delta f} = \frac{1}{2^{1/c} - 1}$$

constant Q transform

implementation 1/2

$$X_{\mathrm{CQ}}(k,n) = \frac{1}{\mathcal{K}(k)} \sum_{i=i_\mathrm{s}(n)}^{i_\mathrm{e}(n)} w_k(i-i_\mathrm{s}) \cdot x(i) e^{\mathrm{j}2\pi \frac{\mathcal{Q}\cdot(i-i_\mathrm{s})}{\mathcal{K}(k)}}$$

$$\mathcal{K}(k) = \frac{f_\mathrm{S}}{f(k)}\mathcal{Q}$$

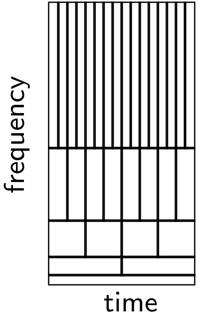
- $f(k)$: frequency of bin index k
- $\mathcal{K}(k)$: blocklength for bin index k
- $\mathcal{Q}$: measure of pitch res.
- w_k: window function
- $i_\mathrm{s}, i_\mathrm{e}$: start and stop time indices of block
- f_S: sample rate

- long window for low frequencies (high freq res, low time res)
- short window for high frequencies (low freq res, high time res)

constant Q transform

implementation 2/2

non-overlapping



overlapping

- define transformation matrix with maximum window length
- zeropad higher frequencies (left & right)

$\Rightarrow$ independent definition of block and hop length

constant Q transform
CQT vs. DFT

CQT:

- \+ perceptually/musically adapted frequency resolution
- – time resolution depends on frequency
- – not invertible
- – no optimized implementation (compare FFT)

constant Q transform

CQT vs. DFT

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auditory filterbanks
introduction

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FT and related transforms bad models of physiological properties of the human ear:

- frequency resolution (critical bands)
- frequency scale (pitch resolution)
- loudness & masking
- event perception & time integration

$\Rightarrow$ **auditory filterbanks**

not as widely used as one might think because

- computationally inefficient
- analysis only: no invertibility (mostly)
- not proven to be superior

auditory filterbanks

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auditory filterbanks

gammatone filterbank

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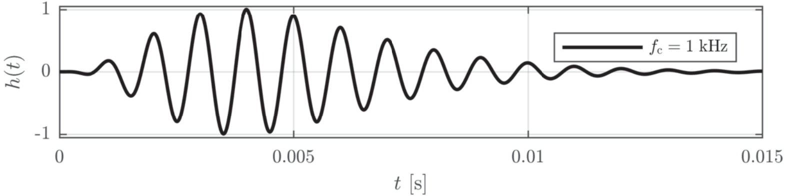
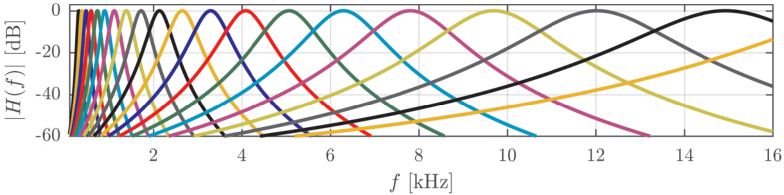
$$h(i) = \frac{a \cdot (i/f_\mathrm{S})^{\mathcal{O}-1} \cdot \cos\left(2\pi \cdot f_\mathrm{c}\frac{i}{f_\mathrm{S}}\right)}{e^{2\pi i \Delta f / f_\mathrm{S}}}$$





matlab source: matlab/displayGammatone.m

summary
lecture content

- **DFT has disadvantages**
  - low frequency resolution for low pitches
  - non-logarithmic/perceptually relevant pitch resolution
- **CQT**
  - similar to Fourier Transform but logarithmically spaced frequency bins
  - not invertible and inefficient
- **Filterbanks**
  - good model of human physiology
  - not invertible and inefficient
  - not proven to be superior