

# Introduction to Audio Content Analysis

Module A.3: Fundamentals — Correlation

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# introduction

## overview

corresponding textbook section

## Appendix A.3

### ■ lecture content

- cross correlation function (CCF)
- auto-correlation function (ACF)

### ■ learning objectives

- describe use cases of correlation
- implement cross- and auto-correlation



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# correlation function

## definition

**correlation function:** compute similarity between two *stationary* signals  $x, y$

$$r_{xy}(\tau) = \mathcal{E}\{x(t)y(t + \tau)\}$$

■ **continuous:**

$$r_{xy}(\tau) = \int_{-\infty}^{\infty} x(t) \cdot y(t + \tau) dt$$

■ **discrete:**

$$r_{xy}(\eta) = \sum_{i=-\infty}^{\infty} x(i) \cdot y(i + \eta)$$

# correlation function animation

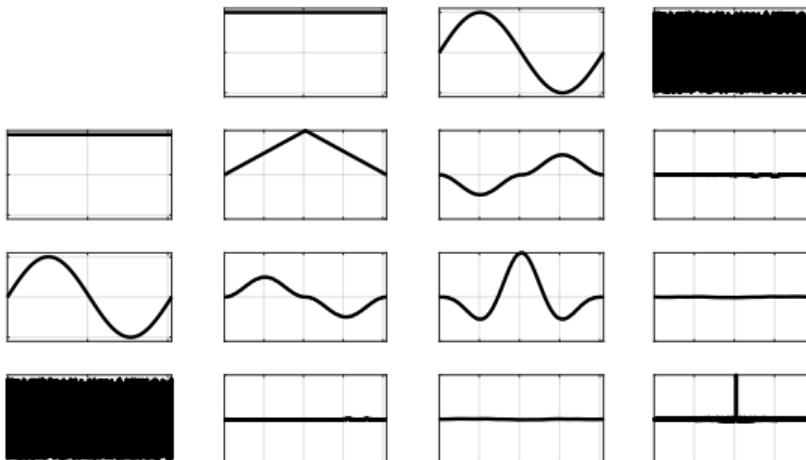
$$r_{xy}(\eta) = \sum_{i=-\infty}^{\infty} x(i) \cdot y(i + \eta)$$



matlab source: [matlab/animateCorrelation.m](#)

# correlation function examples

- rectangular window vs.
- sine vs.
- noise



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[CCF](#)

oo

[blocked correlation](#)

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# correlation function

blocked correlation: animation



matlab source: [matlab/animateBlockedCorrelation.m](#)

# correlation function

## normalization

$$\lambda_c = \frac{1}{\sqrt{\left( \sum_{i=i_s(n)}^{i_e(n)} x^2(i) \right) \cdot \left( \sum_{i=i_s(n)}^{i_e(n)} y^2(i) \right)}}$$

methods of dealing with the triangular weighting/shape for blocked correlation:

- 1 different block lengths ( $\mathcal{K}, 3\mathcal{K}$ )
- 2 circular application
- 3 modified normalization

$$\lambda_c(\eta) = \frac{\mathcal{K}}{(\mathcal{K} - |\eta|) \cdot \sqrt{\left( \sum_{i=i_s(n)}^{i_e(n)} x^2(i) \right) \cdot \left( \sum_{i=i_s(n)}^{i_e(n)} y^2(i) \right)}}.$$

# correlation function

## normalization

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methods of dealing with the triangular weighting/shape for blocked correlation:

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# autocorrelation function

## definition & properties

### autocorrelation function properties

- ACF is correlation function with the signal itself  $r_{xx}(\eta)$
- ACF at lag 0:  
 $r_{xx}(0, n) = 1$  if normalized, energy otherwise
- maximum:  
 $|r_{xx}(\eta, n)| \leq r_{xx}(0, n)$
- symmetry:  
 $r_{xx}(\eta, n) = r_{xx}(-\eta, n)$
- periodicity:  
The ACF of a periodic signal is periodic (period length of input signal)



# (auto-)correlation function

## applications and use cases

**what are the use cases of correlation**



# (auto-)correlation function

## applications and use cases



### what are the use cases of correlation

- *cross-correlation:*

- compute similarity between different signals (correlation meter)
- detect shift between two similar but shifted signals (radar)

- *autocorrelation:*

- detect self-similarity of (shifted) signal (lpc coefficients, noisiness)
- detect periodicity of signal

### ■ correlation function

- measure of similarity between two signals
- use case example: find time lag between signals

### ■ normalized correlation

- results in value between  $-1 \dots 1$
- correlation coefficient: normalized correlation at lag  $\eta = 0$

### ■ autocorrelation

- measure of self-similarity
- use case example: find periodicity

