



Introduction to **Audio Content Analysis**

module A.3: fundamentals — correlation

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introduction

overview

corresponding textbook section

appendix A.3

■ lecture content

- cross correlation function (CCF)
- auto-correlation function (ACF)

■ learning objectives

- describe use cases of correlation
- implement cross- and auto-correlation



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correlation function

definition

correlation function: compute similarity between two *stationary* signals x, y

$$r_{xy}(\tau) = \mathcal{E}\{x(t)y(t + \tau)\}$$

■ **continuous:**

$$r_{xy}(\tau) = \int_{-\infty}^{\infty} x(t) \cdot y(t + \tau) dt$$

■ **discrete:**

$$r_{xy}(\eta) = \sum_{i=-\infty}^{\infty} x(i) \cdot y(i + \eta)$$

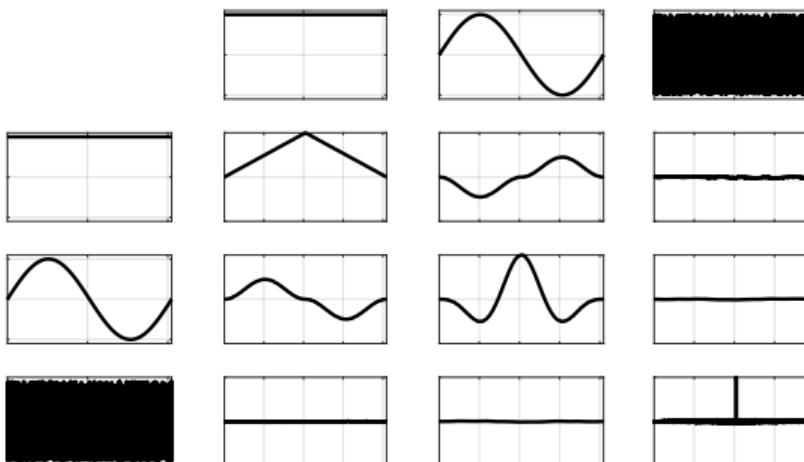
correlation function animation

$$r_{xy}(\eta) = \sum_{i=-\infty}^{\infty} x(i) \cdot y(i + \eta)$$



correlation function examples

- rectangular window vs.
- sine vs.
- noise



overview
o

CCF
oo

blocked correlation
oo

normalization
o

ACF
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apps
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summary
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correlation function

blocked correlation: animation

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matlab source: [matlab/animateBlockedCorrelation.m](#)

correlation function

normalization

$$\lambda_c = \frac{1}{\sqrt{\left(\sum_{i=i_s(n)}^{i_e(n)} x^2(i) \right) \cdot \left(\sum_{i=i_s(n)}^{i_e(n)} y^2(i) \right)}}$$

methods of dealing with the triangular weighting/shape for blocked correlation:

- 1 different block lengths ($\mathcal{K}, 3\mathcal{K}$)
- 2 circular application
- 3 modified normalization

$$\lambda_c(\eta) = \frac{\mathcal{K}}{(\mathcal{K} - |\eta|) \cdot \sqrt{\left(\sum_{i=i_s(n)}^{i_e(n)} x^2(i) \right) \cdot \left(\sum_{i=i_s(n)}^{i_e(n)} y^2(i) \right)}}.$$

correlation function

normalization

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autocorrelation function

definition & properties

autocorrelation function properties

- ACF is correlation function with the signal itself $r_{xx}(\eta)$
- ACF at lag 0:
 $r_{xx}(0, n) = 1$ if normalized, energy otherwise
- maximum:
 $|r_{xx}(\eta, n)| \leq r_{xx}(0, n)$
- symmetry:
 $r_{xx}(\eta, n) = r_{xx}(-\eta, n)$
- periodicity:
The ACF of a periodic signal is periodic (period length of input signal)



(auto-)correlation function

applications and use cases

what are the use cases of correlation



(auto-)correlation function

applications and use cases



what are the use cases of correlation

- *cross-correlation:*

- compute similarity between different signals (correlation meter)
- detect shift between two similar but shifted signals (radar)

- *autocorrelation:*

- detect self-similarity of (shifted) signal (lpc coefficients, noisiness)
- detect periodicity of signal

summary

lecture content

■ correlation function

- measure of similarity between two signals
- use case example: find time lag between signals

■ normalized correlation

- results in value between $-1 \dots 1$
- correlation coefficient: normalized correlation at lag $\eta = 0$

■ autocorrelation

- measure of self-similarity
- use case example: find periodicity

