Introduction to **Audio Content Analysis**

module 4.2: regression & clustering

alexander lerch

introduction
overview

Georgia | Center for Music
Tech | Technology
College of Design

corresponding textbook section

sections 4.2 – 4.4

- **lecture content**
  - regression: non-categorical data analysis
  - clustering: unsupervised data analysis

- **learning objectives**
  - describe the basic principles of data-driven machine learning approaches
  - implement linear regression in Python
  - implement kMeans clustering in Python

introduction
overview

Georgia | Center for Music
Tech | Technology
College of Design

corresponding textbook section

sections 4.2 – 4.4

- **lecture content**
    - regression: non-categorical data analysis
    - clustering: unsupervised data analysis

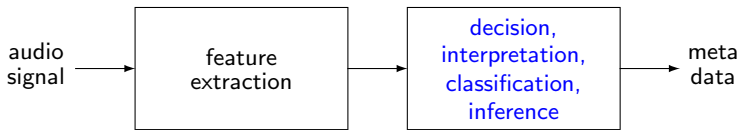- **learning objectives**
    - describe the basic principles of data-driven machine learning approaches
    - implement linear regression in Python
    - implement kMeans clustering in Python

overview ○

intro ●

regression ○○

clustering ○○

distances ○

summary ○

regression
introduction

remember the flow chart of a general ACA system:

- *classification*:
  - assign class labels to data
- *regression*:
  - estimate numerical labels for data
- *clustering*:
  - find grouping patterns in data

regression
introduction

remember the flow chart of a general ACA system:

- *classification*:
  - assign class labels to data
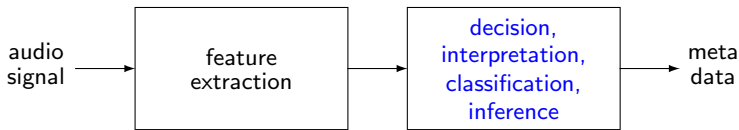- *regression*:
  - estimate numerical labels for data
- *clustering*:
  - find grouping patterns in data

regression
introduction

Georgia Tech | Center for Music Technology
College of Design

remember the flow chart of a general ACA system:

- *classification*:
  - assign class labels to data
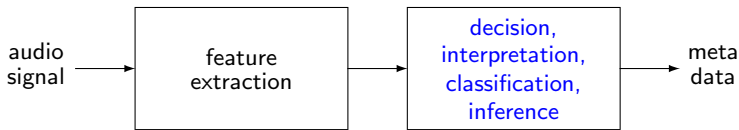- *regression*:
  - estimate numerical labels for data
- *clustering*:
  - find grouping patterns in data

regression
introduction

- given a set of pairs of data and corresponding output observations
- find model that maps input to output

- model can then be used to predict (continuous value) output for an unknown new input

regression
introduction

Georgia | Center for Music
Tech | Technology
College of Design

- given a set of pairs of data and corresponding output observations
- find model that maps input to output

- model can then be used to predict (continuous value) output for an unknown new input

regression
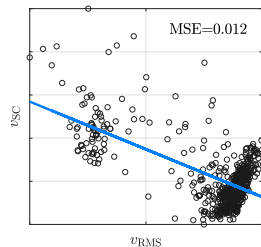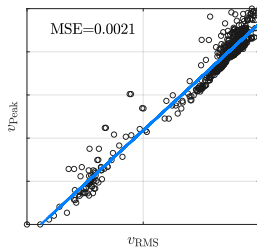linear regression

- estimate the slope m and offset b of a straight line that fits the data best:

$$\hat{y}(r) = m \cdot v(r) + b$$

- minimizing the mean squared error leads to:

$$b = \mu_y - m \cdot \mu_v$$

$$m = \frac{\sum\limits_{r=0}^{\mathcal{R}-1} (y(r) - \mu_y) \cdot (v(r) - \mu_v)}{\sum\limits_{r=0}^{\mathcal{R}-1} (v(r) - \mu_v)^2}$$

matlab source: plotLinearRegression.m

clustering
introduction

Georgia | Center for Music
Tech | Technology
College of Design

- clustering is usually unsupervised and exploratory
- group observations
  - 'similar' observations are grouped together
  - 'dissimilar' observations are in different groups
- depends on definition of 'similarity'/ distance

....

....

clustering
introduction

Georgia | Center for Music
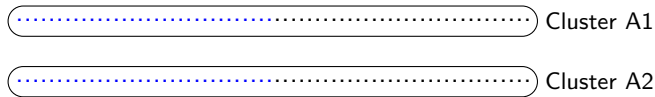Tech | Technology
College of Design

- clustering is usually unsupervised and exploratory
- group observations
  - 'similar' observations are grouped together
  - 'dissimilar' observations are in different groups
- depends on definition of 'similarity'/ distance

$$\left(\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\right)$$ Cluster A1

$$\left(\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\right)$$ Cluster A2

clustering
introduction

- clustering is usually unsupervised and exploratory
- group observations
  - 'similar' observations are grouped together
  - 'dissimilar' observations are in different groups
- depends on definition of 'similarity'/ distance

Cluster B1   Cluster B2

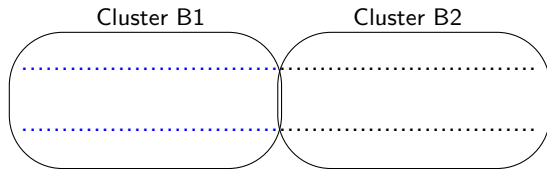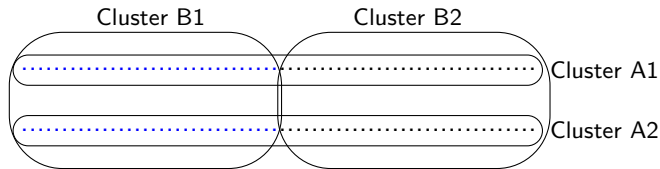## clustering
introduction

- clustering is usually unsupervised and exploratory
- group observations
  - 'similar' observations are grouped together
  - 'dissimilar' observations are in different groups
- depends on definition of 'similarity'/ distance

clustering
kMeans clustering

1. *Initialization*: randomly select K observations from the data set as initialization.

2. *Update*: compute the mean for each cluster.

3. *Assignment*: assign each observation to the cluster with the mean of the closest cluster.

4. *Iteration*: go to step 2 until the clusters converge.
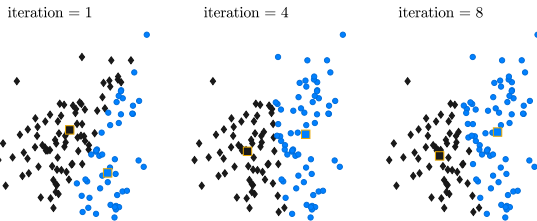
matlab source: matlab/animateKmeans.m

overview

intro

regression

clustering

distances

summary

clustering
kMeans clustering

iteration = 1　　iteration = 4　　iteration = 8

matlab source: plotKmeans.m

distances
overview

Georgia | Center for Music
Tech | Technology
College of Design

- *Euclidean Distance* (L2 Distance)

- *Manhattan Distance* (L1 Distance)

- *Cosine Similarity/Distance*
  - range is from $[-1; 1]$ ($[0; 1]$ for non-negative input),
  - not distance but similarity measure
  - independent of vector length, only on angle

- *Kullback-Leibler Divergence*
  - not symmetric: $d_{\mathrm{KL}}(\boldsymbol{v}_{\mathrm{a}}, \boldsymbol{v}_{\mathrm{b}}) \neq d_{\mathrm{KL}}(\boldsymbol{v}_{\mathrm{b}}, \boldsymbol{v}_{\mathrm{a}})$,
  - designed to measure distance between probability distributions

$$d_{\mathrm{EU}}(\boldsymbol{v}_{\mathrm{a}}, \boldsymbol{v}_{\mathrm{b}}) = \|\boldsymbol{v}_{\mathrm{a}} - \boldsymbol{v}_{\mathrm{b}}\|_2 = \sqrt{\sum_{j=0}^{\mathcal{J}-1} \left(v_{\mathrm{a}}(j) - v_{\mathrm{b}}(j) \right)^2}.$$

distances
overview

Georgia | Center for Music
Tech | Technology
College of Design

- *Euclidean Distance* (L2 Distance)

- *Manhattan Distance* (L1 Distance)

- *Cosine Similarity/Distance*
  - range is from $[-1; 1]$ ($[0; 1]$ for non-negative input),
  - not distance but similarity measure
  - independent of vector length, only on angle

- *Kullback-Leibler Divergence*
  - not symmetric: $d_{\mathrm{KL}}(\boldsymbol{v}_{\mathrm{a}}, \boldsymbol{v}_{\mathrm{b}}) \neq d_{\mathrm{KL}}(\boldsymbol{v}_{\mathrm{b}}, \boldsymbol{v}_{\mathrm{a}})$,
  - designed to measure distance between probability distributions

$$d_{\mathrm{M}}(\boldsymbol{v}_{\mathrm{a}}, \boldsymbol{v}_{\mathrm{b}}) = \|\boldsymbol{v}_{\mathrm{a}} - \boldsymbol{v}_{\mathrm{b}}\|_1 = \sum_{j=0}^{\mathcal{J}-1} |v_{\mathrm{a}}(j) - v_{\mathrm{b}}(j)|.$$

overview   intro   regression   clustering   distances   summary

distances
overview

Georgia | Center for Music
Tech | Technology
College of Design

- *Euclidean Distance* (L2 Distance)

- *Manhattan Distance* (L1 Distance)

- *Cosine Similarity/Distance*
  - range is from $[-1; 1]$ ($[0; 1]$ for non-negative input),
  - not distance but similarity measure
  - independent of vector length, only on angle

- *Kullback-Leibler Divergence*
  - not symmetric:
    $d_{\mathrm{KL}}(\boldsymbol{v}_{\mathrm{a}}, \boldsymbol{v}_{\mathrm{b}}) \neq d_{\mathrm{KL}}(\boldsymbol{v}_{\mathrm{b}}, \boldsymbol{v}_{\mathrm{a}})$,
  - designed to measure distance between probability distributions

$$s_{\mathrm{C}}(\boldsymbol{v}_{\mathrm{a}}, \boldsymbol{v}_{\mathrm{b}}) = \frac{\sum\limits_{j=0}^{\mathcal{J}-1} v_{\mathrm{a}}(j) \cdot v_{\mathrm{b}}(j)}{\sqrt{\sum\limits_{j=0}^{\mathcal{J}-1} v_{\mathrm{a}}(j)^2} \cdot \sqrt{\sum\limits_{j=0}^{\mathcal{J}-1} v_{\mathrm{b}}(j)^2}}.$$

$$d_{\mathrm{C}}(\boldsymbol{v}_{\mathrm{a}}, \boldsymbol{v}_{\mathrm{b}}) = 1 - s_{\mathrm{C}}(\boldsymbol{v}_{\mathrm{a}}, \boldsymbol{v}_{\mathrm{b}}).$$

distances
overview

Georgia Center for Music
Tech Technology
College of Design

- *Euclidean Distance* (L2 Distance)

- *Manhattan Distance* (L1 Distance)

- *Cosine Similarity/Distance*
  - range is from $[-1; 1]$ ($[0; 1]$ for non-negative input),
  - not distance but similarity measure
  - independent of vector length, only on angle

- *Kullback-Leibler Divergence*
  - not symmetric:
    $d_{\mathrm{KL}}(\boldsymbol{v}_{\mathrm{a}}, \boldsymbol{v}_{\mathrm{b}}) \neq d_{\mathrm{KL}}(\boldsymbol{v}_{\mathrm{b}}, \boldsymbol{v}_{\mathrm{a}})$,
  - designed to measure distance between probability distributions

$$d_{\mathrm{KL}}(\boldsymbol{v}_{\mathrm{a}}, \boldsymbol{v}_{\mathrm{b}}) = \sum_{j=0}^{\mathcal{J}-1} v_{\mathrm{a}}(j) \cdot \log\left(\frac{v_{\mathrm{a}}(j)}{v_{\mathrm{b}}(j)}\right).$$

summary
lecture content

Georgia | Center for Music
Tech | Technology
College of Design

- **regression**
  - model to estimate numeric labels from features
  - linear regression assumes model is straight line

- **clustering**
  1. unsupervised grouping
  2. feature space and distance measure determine result
  3. number of clusters usually has to be known
  4. kMeans is simple way of clustering

- **distances**
  - L1 and L2 are most common distances
  - not all 'distances' are consistent. a real distance
    - ▶ cannot be negative
    - ▶ is symmetric