Introduction to **Audio Content Analysis**

module 7.3.4: fundamental frequency detection in polyphonic signals

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introduction
overview

corresponding textbook section

section 7.3.4

- **lecture content**
  - overview of “historic” methods for polyphonic pitch detection
  - introduction to Non-negative Matrix Factorization (NMF)

- **learning objectives**
  - describe the task and challenges of polyphonic pitch detection
  - list the processing steps of iterative subtraction and relate them to the introduced approaches
  - describe the process of NMF and discuss advantages and disadvantages of using NMF for pitch detection

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polyphonic pitch tracking
problem statement

- **monophonic** fundamental frequency detection:
  - exactly one fundamental frequency with sinusoidals at multiples of f_0 (harmonics)
- **polyphonic** fundamental frequency detection:
  - multiple/unknown number of fundamental frequencies with harmonics
  - number of voices might change over time
  - complex mixture with overlapping frequency content

polyphonic pitch tracking
iterative subtraction: introduction

- **principle**
  1. find most salient fundamental frequency
     - e.g., with monophonic pitch tracking
  2. remove this frequency and related frequency components
     - e.g., mask or subtraction
  3. repeat until termination criterion
     - e.g., number of voices

- **challenges**
  - reliably *identify fundamental frequency* in a mixture
  - *identify/group components* and amount to subtract
    - overlapping components
    - spectral leakage
  - define *termination criterion*
    - e.g., unknown number of voices or overall energy

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polyphonic pitch tracking

iterative subtraction: Cheveigné

1. compute squared AMDF

$$\mathrm{ASMDF}_{xx}(\eta, n) = \frac{1}{i_\mathrm{e}(n) - i_\mathrm{s}(n) + 1} \sum_{i=i_\mathrm{s}(n)}^{i_\mathrm{e}(n)} \left(x(i) - x(i+\eta)\right)^2$$

2. find fundamental frequency

$$\eta_\mathrm{min} = \mathrm{argmin}\left(\mathrm{ASMDF}_{xx}(\eta, n)\right)$$

3. apply comb cancellation filter, IR:

$$h(i) = \delta(i) - \delta(i - \eta_\mathrm{min})$$

4. repeat process

polyphonic pitch tracking

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polyphonic pitch tracking

iterative subtraction: Meddis

1. auditory pitch tracking:

$$r_{zz}(c, n, \eta) = \sum_{i=0}^{\mathcal{K}-1} z_c(i) \cdot z_c(i + \eta)$$

2. detect most likely frequency for all bands
3. remove all bands with a max at detected frequency
4. reiterate until most bands have eliminated

polyphonic pitch tracking
iterative subtraction: Meddis

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iterative subtraction: spectral

1. find salient fundamental frequency (e.g. auditory approach, HPS)
2. estimate current model for harmonic magnitudes
3. subtract the model spectrum
4. repeat process

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polyphonic pitch tracking
exhaustive search

1. define set of all possible fundamental frequencies
2. compute all possible pairs of fundamental frequency
3. repeatedly filter the signal with two comb cancellation filters (all combinations)
4. find combination with minimal output energy

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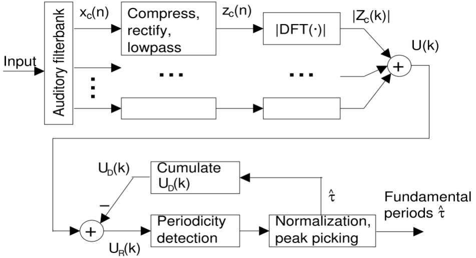
polyphonic pitch tracking
klapuri

1. gammatone **filterbank** (100 bands)
2. **normalization**, HWR, smoothing, ...
3. **STFT** per filter channel (magnitude)
4. use **delta pulse templates** to detect frequency patterns
5. **pick most salient frequencies**, remove them



graph from[1]

[1] A. P. Klapuri, "A Perceptually Motivated Multiple-F0 Estimation Method," in *Proceedings of the IEEE Workshop on Applications of Signal Processing to Audio and Acoustics (WASPAA)*, New Paltz, 2005.

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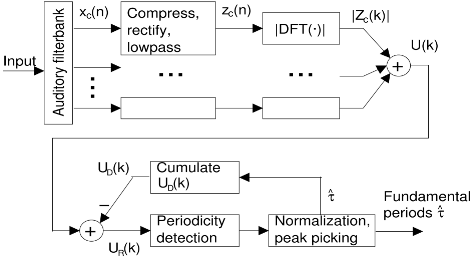

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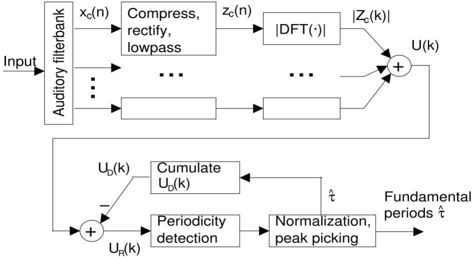

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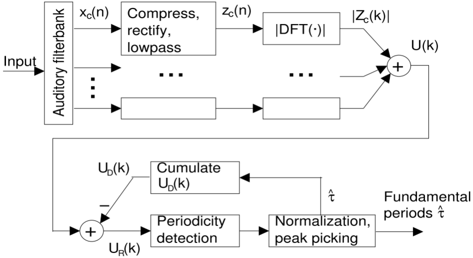

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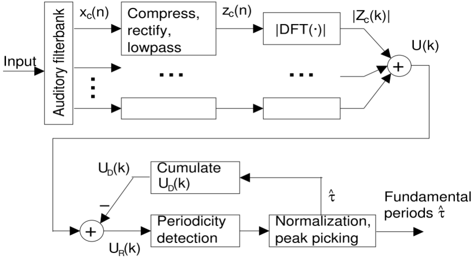

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non-negative matrix factorization
introduction

- **Non-negative Matrix Factorization (NMF)**
  Given a $m \times n$ matrix V, find a $m \times r$ matrix W and a $r \times n$ matrix H such that

$$V \approx WH$$

  - all matrices must be non-negative
  - rank r is usually smaller than m and n

- **advantage of non-negativity?**
  - additive model
  - relates to probability distributions
  - efficiency?

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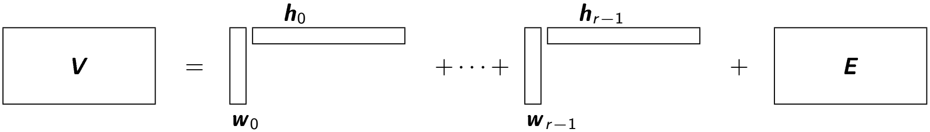
alternative formulation[2] to $V \approx WH$

$$V = \sum_{i=1}^{r} w_i \cdot h_i + E$$

- $V \in \mathbb{R}^{m \times n}$
- $W = [w_1, w_2, \ldots, w_r] \in \mathbb{R}^{m \times r}$
- $H = [h_1, h_2, \ldots, h_r]^T \in \mathbb{R}^{r \times n}$
- E is the error matrix



[2] A Cichocki, R Zdunek, A. Phan, *et al.*, *Nonnegative matrix and tensor factorizations: applications to exploratory multi-way data analysis and blind source separation*. John Wiley & Sons, 2009.

objective function
distance and divergence

- task: **iteratively minimize objective function** $D(V||WH)$
- typical distance measures ($B = WH$):
  - squared Euclidean distance:

$$D_{\mathrm{EU}}(V \parallel B) = \parallel V - B \parallel^2 = \sum_{ij}(V_{ij} - B_{ij})^2$$

  - generalized K-L divergence:

$$D_{\mathrm{KL}}(V \parallel B) = \sum_{ij}\left(V_{ij} \log\left(\frac{V_{ij}}{B_{ij}}\right) - V_{ij} + B_{ij}\right)$$

  - others[3]: Bregman Divergence, Alpha-Divergence, Beta-Divergence, ...

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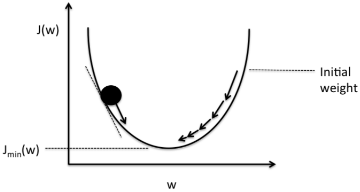
objective function
gradient descent

- minimization of objective function
- **gradient descent**: minimum can be found as zero of derivative
    - 2D example: given a function $f(x_1, x_2)$, find the minimum $x_1 = a$ and $x_2 = b$
        1. initialize $x_i(0)$ with random numbers
        2. update points iteratively:

$$x_i(n+1) = x_i(n) - \alpha \cdot \frac{\partial f}{\partial x_i}, \quad i = [1, 2]$$

$\Rightarrow$ as iteration number n increases, x_1, x_2 will be closer to a, b.



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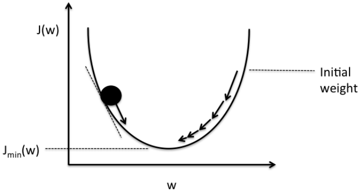

objective function
additive vs. multiplicative update rules

optimization of objective function[4] $D_{\mathrm{EU}}(V \parallel WH) = \parallel V - WH \parallel^2$

- **additive** update rules:

$$H \leftarrow H + \alpha \frac{\partial J}{\partial H} = H + \alpha[(W^T V) - (W^T WH)]$$

$$W \leftarrow W + \alpha \frac{\partial J}{\partial W} = W + \alpha[(VH^T) - (WHH^T)]$$

- **multiplicative** update rules:

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objective function

additional cost function constraints

- additional penalty terms (regularization terms) may be added to objective function
- example: sparsity in W or H

$$D = \| V - WH \|^2 + \alpha J_{\mathrm{W}}(W) + \beta J_{\mathrm{H}}(H)$$

  - α, β: coefficients for controlling degree of sparsity
  - J_{W} and J_{H}: typically L_1, L_2 norm

nmf example
template extraction

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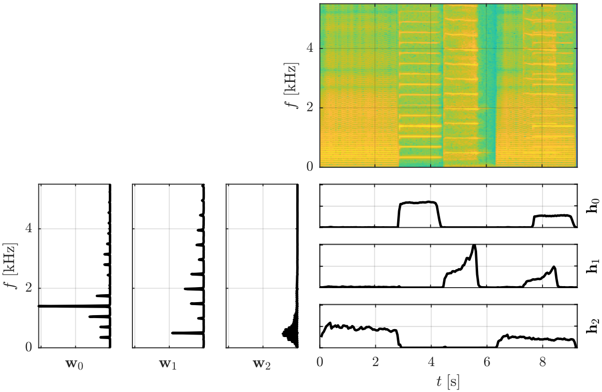
- unsupervised extraction of templates and activations
- input audio:
  - horn
  - oboe
  - violin
  - mix



matlab source: plotF0Nmf.m

nmf use cases

piano transcription

- separate template adaptation from activation matrix adaptation:
  1. train/set template matrix
  2. order template matrix to have fixed pitch mapping
  3. keep template matrix fixed and only update activation matrix
  4. pick activation magnitude to determine active pitches

- potential problems
  - detuned piano
  - template differs significantly from sound analyzed

summary
lecture content

- **polyphonic pitch detection**
  - highly challenging task with
    - unknown number of sources
    - unknown harmonic structure
    - spectral overlap of sources
    - time-varying mixture
- **traditional approaches**
  - iterative subtraction (detect one pitch, remove it, repeat analysis)
  - multi-band processing
- **non-negative matrix factorization**
  - iterative process minimizing an objective function
  - split a matrix into a template matrix and an activation matrix