Introduction to **Audio Content Analysis**

module A.3: fundamentals — correlation

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corresponding textbook section

appendix A.3

- **lecture content**
  - cross correlation function (CCF)
  - auto-correlation function (ACF)
- **learning objectives**
  - describe use cases of correlation
  - implement cross- and auto-correlation

corresponding textbook section

appendix A.3

- **lecture content**
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- **learning objectives**
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correlation function
definition

correlation function: compute similarity between two *stationary* signals x,y

$$r_{\mathrm{xy}}(\tau) = \mathcal{E}\{x(t)y(t+\tau)\}$$

- **continuous**:

$$r_{\mathrm{xy}}(\tau) = \int\limits_{-\infty}^{\infty} x(t)\cdot y(t+\tau)dt$$

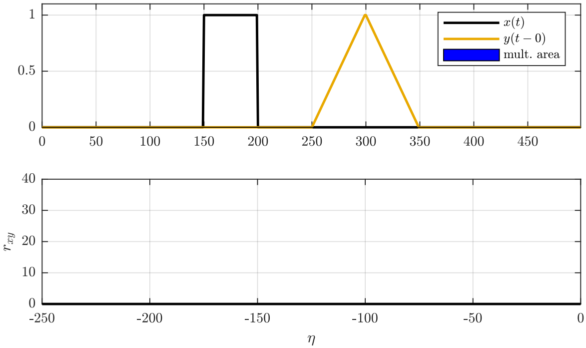
- **discrete**:

$$r_{\mathrm{xy}}(\eta) = \sum_{i=-\infty}^{\infty} x(i)\cdot y(i+\eta)$$

correlation function

animation

$$r_{\mathrm{xy}}(\eta) = \sum_{i=-\infty}^{\infty} x(i) \cdot y(i+\eta)$$

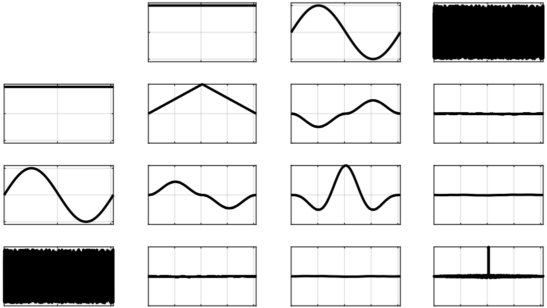


matlab source: matlab/animateCorrelation.m

correlation function

examples

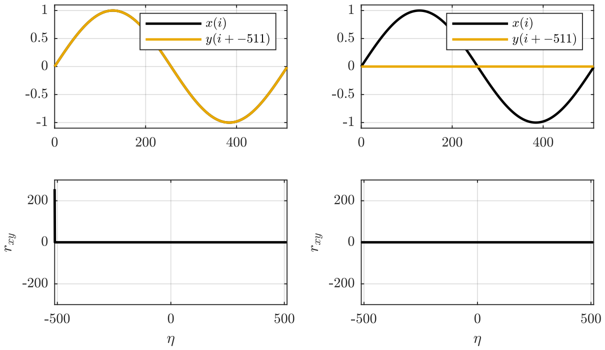
- rectangular window vs.
- sine vs.
- noise



matlab source: plotCorrelation.m

correlation function

blocked correlation: animation



matlab source: matlab/animateBlockedCorrelation.m

correlation function

normalization

$$\lambda_c = \frac{1}{\sqrt{\left(\sum\limits_{i=i_\mathrm{s}(n)}^{i_\mathrm{e}(n)} x^2(i)\right) \cdot \left(\sum\limits_{i=i_\mathrm{s}(n)}^{i_\mathrm{e}(n)} y^2(i)\right)}}$$

methods of dealing with the triangular weighting/shape for blocked correlation:

1. different block lengths $(\mathcal{K}, 3\mathcal{K})$
2. circular application
3. modified normalization

$$\lambda_c(\eta) = \frac{\mathcal{K}}{(\mathcal{K} - |\eta|) \cdot \sqrt{\left(\sum\limits_{i=i_\mathrm{s}(n)}^{i_\mathrm{e}(n)} x^2(i)\right) \cdot \left(\sum\limits_{i=i_\mathrm{s}(n)}^{i_\mathrm{e}(n)} y^2(i)\right)}}.$$

correlation function

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autocorrelation function
definition & properties

autocorrelation function properties

- ACF is correlation function with the signal itself $r_{xx}(\eta)$
- ACF at lag 0:
  $r_{xx}(0, n) = 1$ if normalized, energy otherwise
- maximum:
  $|r_{xx}(\eta, n)| \leq r_{xx}(0, n)$
- symmetry:
  $r_{xx}(\eta, n) = r_{xx}(-\eta, n)$
- periodicity:
  The ACF of a periodic signal is periodic (period length of input signal)

(auto-)correlation function

applications and use cases

what are the use cases of correlation

(auto-)correlation function

applications and use cases

what are the use cases of correlation

- *cross-correlation*:
  - compute similarity between different signals (correlation meter)
  - detect shift between two similar but shifted signals (radar)
- *autocorrelation*:
  - detect self-similarity of (shifted) signal (lpc coefficients, noisiness)
  - detect periodicity of signal

summary
lecture content

- **correlation function**
  - measure of similarity between two signals
  - use case example: find time lag between signals
- **normalized correlation**
  - results in value between $-1 \dots 1$
  - correlation coefficient: normalized correlation at lag $\eta = 0$
- **autocorrelation**
  - measure of self-similarity
  - use case example: find periodicity