Introduction to **Audio Content Analysis**

module 3.7.4: feature dimensionality reduction

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introduction
overview

corresponding textbook section

section 3.7.4
appendix C

- **lecture content**
  - problems of dimensionality
  - feature selection
  - feature transformation/mapping

- **learning objectives**
  - describe potential challenges with high-dimensional feature spaces
  - discuss advantages and disadvantages of various methods for feature selection
  - summarize PCA as feature transformation method

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introduction
dimensionality reduction

- **problem**
  - many ML approaches cannot cope with large amounts of irrelevant features
  - ML algorithms might degrade in performance
- **advantages**
  - reducing storage requirements
  - reducing training complexity
  - defying the "curse of dimensionality"
- **disadvantages**
  - additional workload for reduction
  - adding an additional layer of model complexity

introduction
dimensionality reduction

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dimensionality reduction

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introduction
dimensionality issues

problems of high-dimensional data:

- increase in run-time
- overfitting
- curse of dimensionality
- required amount of training samples

introduction
dimensionality issues

problems of high-dimensional data:

- increase in run-time
- overfitting
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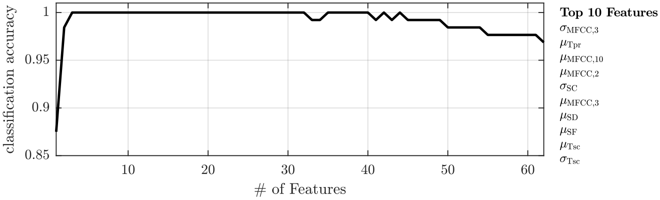
$\Rightarrow$ increasing number of input features may *decrease* classification performance



matlab source: plotSequentialForwardSelection.m

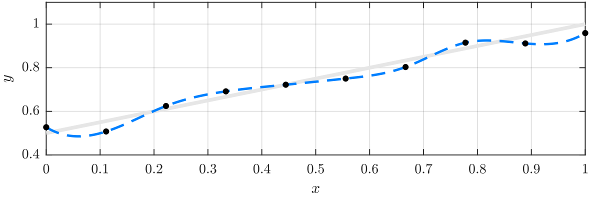
dimensionality issues
overfitting

- **overfitting**:
  - lack of training data
  - overly complex model

  ⇒ model cannot be estimated properly

- required training set size depends on
  - classifier (parametrization)
  - number of classes
  - task complexity

  ⇒ *rule of thumb*: don't bother with training sets smaller than $\mathcal{F}^2$



matlab source: plotOverfitting.m

dimensionality issues

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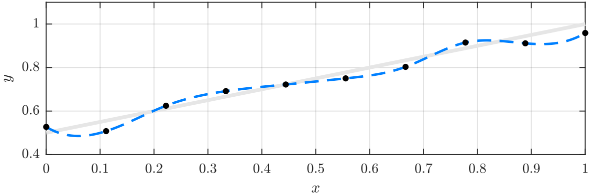

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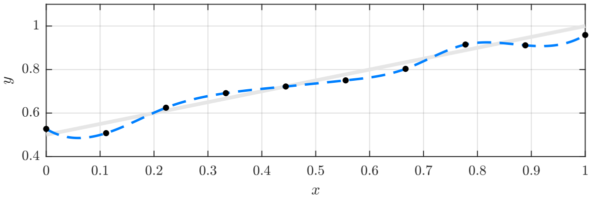


matlab source: plotOverfitting.m

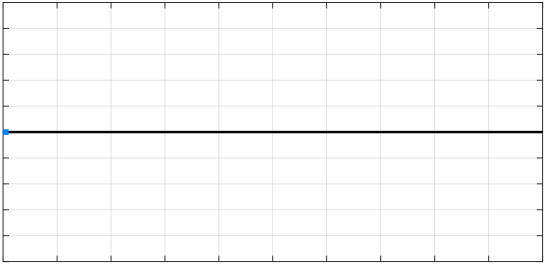
dimensionality issues
curse of dimensionality

- **curse of dimensionality**:
  - increasing dimensionality leads to sparse training data
  - neighborhoods of data points become less concentrated
  - model tends to be harder to estimate in higher-dimensional space
  - applies to distance-based algorithms

dimensionality issues
curse of dimensionality

example (uniformly distributed data): identify region on axis covering **1% of data**

- 1-D: 1% of x-axis
- 2-D: 10% of x/y-axis
- 3-D: 21.5% of x/y/z-axis
- 10-D: 63%
- 100-D: 95%



matlab source: matlab/animateCurseOfDimensionality.m

dimensionality reduction
introduction

- **feature subset selection**:
  discard least helpful features
  - high "discriminative" or descriptive power
  - non-correlation to other features
  - invariance to irrelevancies

- **feature space transformation**:
  map feature space

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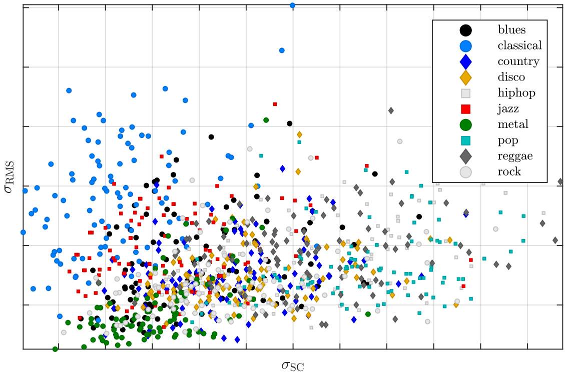
feature subset selection

manual feature selection

example scatter plots of pairs of features in a multi-class scenario



matlab source: plotFeatureScatter.m

feature subset selection
introduction

1. **wrapper methods**:
   - *description*
     - use the "classifier" itself to evaluate feature performance
   - *advantages*
     - taking into account feature dependencies
     - model dependency
   - *disadvantages*
     - complexity
     - risk of overfitting
2. **filter methods**:
   - *description*
     - use an objective function
   - *advantages*
     - easily scalable
     - independent of classification algorithm
   - *disadvantages*
     - no interaction with classifier
     - no feature dependencies

feature subset selection
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feature subset selection
wrapper methods 1/2

1. **single variable classification**:
   - *procedure*
     - evaluate each feature individually
     - choose the top N
   - *complexity*
     - subsets to test: $\mathcal{F}$
   - *challenges*
     - inter-feature correlation is not considered
     - feature combinations are not considered
2. **brute force subset selection**
   - *procedure*
     - evaluate all possible feature combinations
     - choose the optimal combination
   - *complexity*
     - subsets to test: $2^{\mathcal{F}}$

feature subset selection

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feature subset selection

wrapper methods 2/2

4. **sequential forward selection**
   - *procedure*
     1. init: empty feature subset $\mathcal{V}_\mathrm{s} = \emptyset$
     2. find feature v_j maximizing objective function

$$v_j = \underset{\forall j | v_j \notin \mathcal{V}_\mathrm{s}}{\operatorname{argmax}} J(\mathcal{V}_\mathrm{s} \bigcup v_j)$$

     3. add feature v_j to $\mathcal{V}_\mathrm{s}$
     4. go to step 2
5. **sequential backward elimination**
   - *procedure*
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feature space transformation
PCA introduction

- **objective**
  - map features to new coordinate system

$$\boldsymbol{u}(n) = \boldsymbol{T}^{\mathrm{T}} \cdot \boldsymbol{v}(n)$$

    - $\boldsymbol{u}(n)$: transformed features (same dimension as $\boldsymbol{v}(n)$)
    - $\boldsymbol{T}$: transformation matrix ($\mathcal{F} \times \mathcal{F}$)

$$\boldsymbol{T} = \begin{bmatrix} \boldsymbol{c}_0 & \boldsymbol{c}_1 & \dots & \boldsymbol{c}_{\mathcal{F}-1} \end{bmatrix}$$

- **properties**
  - $\boldsymbol{c}_0$ points in the direction of highest *variance*
  - variance concentrated in as few output components as possible
  - $\boldsymbol{c}_i$ orthogonal

$$\boldsymbol{c}_i^{\mathrm{T}} \cdot \boldsymbol{c}_j = 0 \quad \forall \ i \neq j$$

  - transformation is invertible

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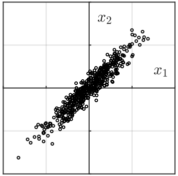
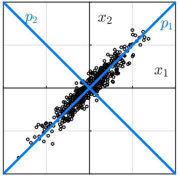
feature space transformation
PCA visualization





calculation of the transformation matrix

1. compute covariance matrix $\boldsymbol{R}$

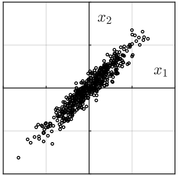
$$\boldsymbol{R} = \mathcal{E}\{(V - \mathcal{E}\{V\})(V - \mathcal{E}\{V\})\}$$

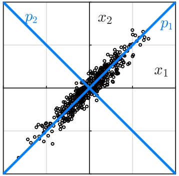
2. choose eigenvectors as axes for the new coordinate system

matlab source: plotPca.m

feature space transformation

PCA visualization





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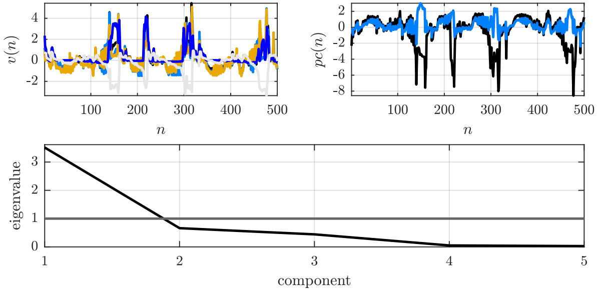
introduction

PCA example



matlab source: plotPcaExample.m

introduction
PCA example

pca transformation matrix

$$\begin{bmatrix} -0.4187 & 0.3467 & -0.4569 & 0.4143 & -0.1271 & -0.5549 \\ -0.3908 & 0.1815 & 0.8136 & -0.0289 & 0.2060 & -0.3304 \\ -0.4516 & 0.3384 & 0.0859 & 0.2413 & -0.2919 & 0.7285 \\ -0.4337 & 0.1699 & -0.3337 & -0.7243 & 0.3747 & 0.0816 \\ 0.3802 & 0.5599 & -0.0381 & 0.2808 & 0.6622 & 0.1524 \\ 0.3679 & 0.6245 & 0.0956 & -0.4071 & -0.5267 & -0.1495 \end{bmatrix}$$

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summary

lecture content

- **dimensionality problems**
  - overfitting
  - insufficient training data $\Rightarrow$ sparse feature space
- **feature selection**
  - select subset of features performing "best"
  - wrapper methods use classifier itself as objective function
  - filter methods use different objective function
- **feature transformation**
  - map feature space into new space minimizing irrelevant information
  - still requires computation of all features
  - new dimensions commonly lack interpretability