

Introduction to Audio Content Analysis

Module 7.3.4: Fundamental Frequency Detection in Polyphonic Signals

alexander lerch

introduction

overview

corresponding textbook section

Section 7.3.4

■ lecture content

- overview of “historic” methods for polyphonic pitch detection
- introduction to Non-negative Matrix Factorization (NMF)

■ learning objectives

- describe the task and challenges of polyphonic pitch detection
- list the processing steps of iterative subtraction and relate them to the introduced approaches
- describe the process of NMF and discuss advantages and disadvantages of using NMF for pitch detection



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polyphonic pitch tracking

problem statement

- **monophonic** fundamental frequency detection:
 - exactly one fundamental frequency with sinusoidals at multiples of f_0 (harmonics)
- **polyphonic** fundamental frequency detection:
 - multiple/unknown number of fundamental frequencies with harmonics
 - number of voices might change over time
 - complex mixture with overlapping frequency content

polyphonic pitch tracking

iterative subtraction: introduction

■ principle

- 1 find most salient fundamental frequency
 - ▶ e.g., with monophonic pitch tracking
- 2 remove this frequency and related frequency components
 - ▶ e.g., mask or subtraction
- 3 repeat until termination criterion
 - ▶ e.g., number of voices

■ challenges

- *reliably identify fundamental frequency in a mixture*
- *identify/group components and amount to subtract*
 - ▶ overlapping components
 - ▶ spectral leakage
- *define termination criterion*
 - ▶ e.g., unknown number of voices or overall energy

polyphonic pitch tracking

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polyphonic pitch tracking

iterative subtraction: Cheveigné

1 compute squared AMDF

$$\text{ASMDF}_{xx}(\eta, n) = \frac{1}{i_e(n) - i_s(n) + 1} \sum_{i=i_s(n)}^{i_e(n)} (x(i) - x(i + \eta))^2$$

2 find fundamental frequency

$$\eta_{\min} = \operatorname{argmin} (\text{ASMDF}_{xx}(\eta, n))$$

3 apply comb cancellation filter, IR:

$$h(i) = \delta(i) - \delta(i - \eta_{\min})$$

4 repeat process

polyphonic pitch tracking

iterative subtraction: Cheveigné

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polyphonic pitch tracking

iterative subtraction: Meddis

1 auditory pitch tracking:

$$r_{zz}(c, n, \eta) = \sum_{i=0}^{\mathcal{K}-1} z_c(i) \cdot z_c(i + \eta)$$

- 2 detect most likely frequency for all bands
- 3 remove all bands with a max at detected frequency
- 4 reiterate until most bands have eliminated

polyphonic pitch tracking

iterative subtraction: Meddis

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polyphonic pitch tracking

iterative subtraction: spectral

- 1 find salient fundamental frequency (e.g. auditory approach, HPS)
- 2 estimate current model for harmonic magnitudes
- 3 subtract the model spectrum
- 4 repeat process

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polyphonic pitch tracking

exhaustive search

- 1 define set of all possible fundamental frequencies
- 2 compute all possible pairs of fundamental frequency
- 3 repeatedly filter the signal with two comb cancellation filters (all combinations)
- 4 find combination with minimal output energy

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polyphonic pitch tracking

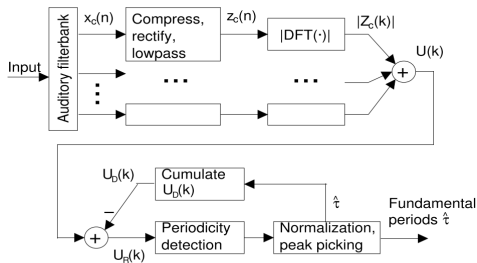
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polyphonic pitch tracking

klapuri

- 1 gammatone **filterbank** (100 bands)
- 2 **normalization**, HWR, smoothing, ...
- 3 **STFT** per filter channel (magnitude)
- 4 use **delta pulse templates** to detect frequency patterns
- 5 **pick most salient frequencies**, remove them



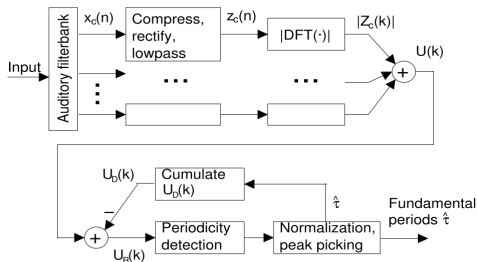
graph from¹

¹A. P. Klapuri, "A Perceptually Motivated Multiple-F0 Estimation Method," in *Proceedings of the IEEE Workshop on Applications of Signal Processing to Audio and Acoustics (WASPAA)*, New Paltz, 2005.

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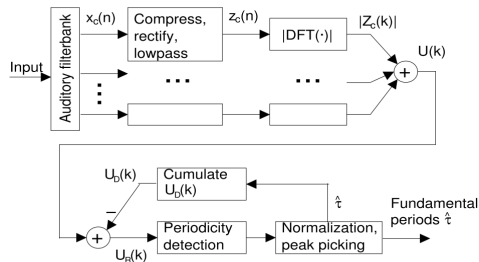
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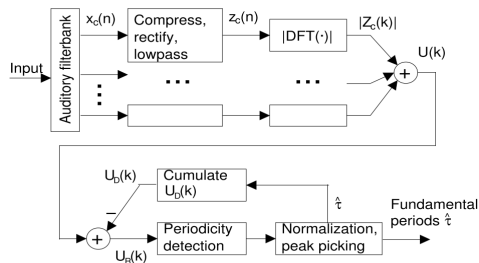
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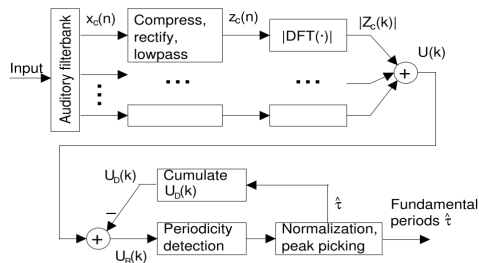
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non-negative matrix factorization

introduction

■ Non-negative Matrix Factorization (NMF)

Given a $m \times n$ matrix V , find a $m \times r$ matrix W and a $r \times n$ matrix H such that

$$V \approx WH$$

- all matrices must be non-negative
- rank r is usually smaller than m and n

■ advantage of non-negativity?

- additive model
- relates to probability distributions
- efficiency?

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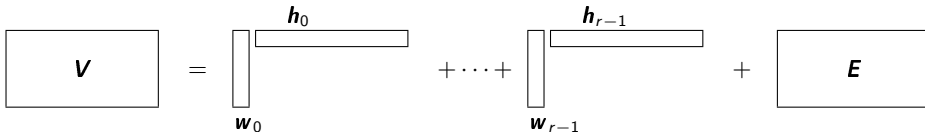
non-negative matrix factorization

overview

alternative formulation² to $V \approx WH$

$$V = \sum_{i=1}^r w_i \cdot h_i + E$$

- $V \in \mathbb{R}^{m \times n}$
- $W = [w_1, w_2, \dots, w_r] \in \mathbb{R}^{m \times r}$
- $H = [h_1, h_2, \dots, h_r]^T \in \mathbb{R}^{r \times n}$
- E is the error matrix



²A Cichocki, R Zdunek, A. Phan, et al., *Nonnegative matrix and tensor factorizations: applications to exploratory multi-way data analysis and blind source separation*. John Wiley & Sons, 2009.

objective function

distance and divergence

- task: **iteratively minimize objective function** $D(V||WH)$
- typical distance measures ($B = WH$):
 - squared Euclidean distance:

$$D_{\text{EU}}(V \parallel B) = \|V - B\|^2 = \sum_{ij} (V_{ij} - B_{ij})^2$$

- generalized K-L divergence:

$$D_{\text{KL}}(V \parallel B) = \sum_{ij} \left(V_{ij} \log \left(\frac{V_{ij}}{B_{ij}} \right) - V_{ij} + B_{ij} \right)$$

- others³: Bregman Divergence, Alpha-Divergence, Beta-Divergence, ...

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objective function

gradient descent

■ minimization of objective function

■ **gradient descent**: minimum can be found as zero of derivative

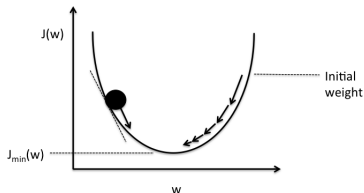
- 2D example: given a function $f(x_1, x_2)$, find the minimum $x_1 = a$ and $x_2 = b$

1 initialize $x_i(0)$ with random numbers

2 update points iteratively:

$$x_i(n+1) = x_i(n) - \alpha \cdot \frac{\partial f}{\partial x_i}, \quad i = [1, 2]$$

⇒ as iteration number n increases, x_1, x_2 will be closer to a, b .



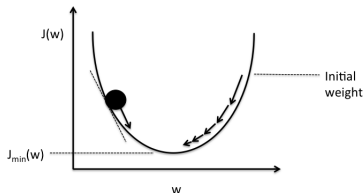
objective function

gradient descent

- minimization of objective function
- **gradient descent**: minimum can be found as zero of derivative
 - 2D example: given a function $f(x_1, x_2)$, find the minimum $x_1 = a$ and $x_2 = b$
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objective function

additive vs. multiplicative update rules

optimization of objective function⁴ $D_{\text{EU}}(V \parallel WH) = \|V - WH\|^2$

■ **additive** update rules:

$$H \leftarrow H + \alpha \frac{\partial J}{\partial H} = H + \alpha[(W^T V) - (W^T WH)]$$

$$W \leftarrow W + \alpha \frac{\partial J}{\partial W} = W + \alpha[(VH^T) - (WHH^T)]$$

■ **multiplicative** update rules:

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objective function

additional cost function constraints

- additional penalty terms (regularization terms) may be added to objective function
- example: sparsity in W or H

$$D = \|V - WH\|^2 + \alpha J_W(W) + \beta J_H(H)$$





- α, β : coefficients for controlling degree of sparsity
- J_W and J_H : typically L_1, L_2 norm

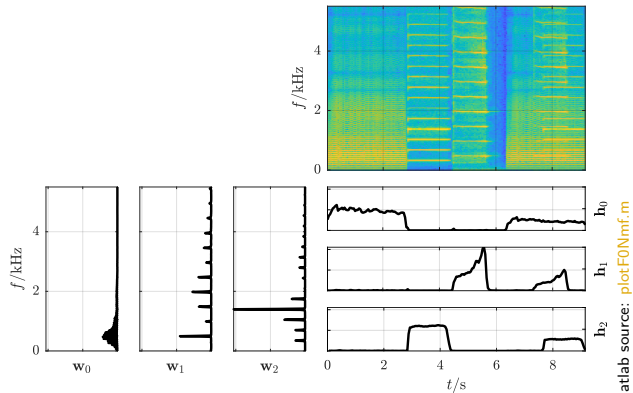
nmf example

template extraction

- unsupervised extraction of templates and activations

- input audio:

-  horn
-  oboe
-  violin
-  mix



nmf use cases

piano transcription

■ separate template adaptation from activation matrix adaptation:

- 1 train/set template matrix
- 2 order template matrix to have fixed pitch mapping
- 3 keep template matrix fixed and only update activation matrix
- 4 pick activation magnitude to determine active pitches

■ potential problems

- detuned piano
- template differs significantly from sound analyzed

summary

lecture content

■ polyphonic pitch detection

- highly challenging task with
 - ▶ unknown number of sources
 - ▶ unknown harmonic structure
 - ▶ spectral overlap of sources
 - ▶ time-varying mixture

■ traditional approaches

- iterative subtraction (detect one pitch, remove it, repeat analysis)
- multi-band processing

■ non-negative matrix factorization

- iterative process minimizing an objective function
- split a matrix into a template matrix and an activation matrix

