

Introduction to Audio Content Analysis

Module A.2: Fundamentals — Convolution

alexander lerch

introduction

overview

corresponding textbook section

Section A.2

■ lecture content

- LTI systems
- convolution
- filter examples

■ learning objectives

- basic understanding of linearity and time-invariance
- basic understanding of the convolution operation
- ability to implement simple filters



introduction

overview

corresponding textbook section

Section A.2

■ lecture content

- LTI systems
- convolution
- filter examples

■ learning objectives

- basic understanding of linearity and time-invariance
- basic understanding of the convolution operation
- ability to implement simple filters

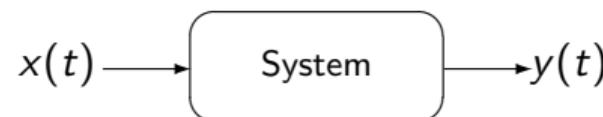


systems

introduction

a system:

- any process producing an output signal in response to an input signal



name examples for systems in signal processing

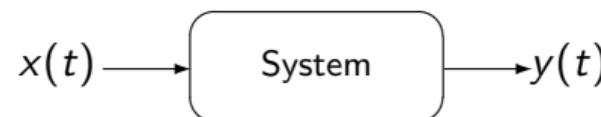


systems

introduction

a system:

- any process producing an output signal in response to an input signal



name examples for systems in signal processing



- filters, effects
- vocal tract
- room
- (audio) cable
- ...

systems

LTI systems

LTI: Linear Time-Invariant Systems

are a great model for many real-world systems

■ **linearity**

- 1 *homogeneity*: $f(ax) = af(x)$
- 2 *superposition* (additivity): $f(x + y) = f(x) + f(y)$

■ **time invariance**: $f(x(t - \tau)) = f(x)(t - \tau)$

convolution

introduction

convolution

convolution operation describes the **output of an LTI system**:

$$y(t) = (x * h)(t) := \int_{-\infty}^{\infty} x(\tau)h(t - \tau)d\tau$$

$$y(i) = (x * h)(i) := \sum_{j=-\infty}^{\infty} x(j)h(i - j)$$

overview



convolution animation

systems



convolution



filter examples



summary



matlab source: [matlab/animateConvolution.m](#)

convolution properties

■ identity:

$$x(i) = \delta(i) * x(i)$$

■ commutativity:

$$h(i) * x(i) = x(i) * h(i)$$

■ associativity:

$$(g(i) * h(i)) * x(i) = g(i) * (h(i) * x(i))$$

■ distributivity:

$$g(i) * (h(i) + x(i)) = (g(i) * h(i)) + (g(i) * x(i))$$

■ circularity:

$h(i)$ periodic $\Rightarrow y(i) = h(i) * x(i)$ periodic

convolution properties

■ identity:

$$x(i) = \delta(i) * x(i)$$

■ commutativity:

$$h(i) * x(i) = x(i) * h(i)$$

■ associativity:

$$(g(i) * h(i)) * x(i) = g(i) * (h(i) * x(i))$$

■ distributivity:

$$g(i) * (h(i) + x(i)) = (g(i) * h(i)) + (g(i) * x(i))$$

■ circularity:

$$h(i) \text{ periodic} \Rightarrow y(i) = h(i) * x(i) \text{ periodic}$$

convolution properties

■ identity:

$$x(i) = \delta(i) * x(i)$$

■ commutativity:

$$h(i) * x(i) = x(i) * h(i)$$

■ associativity:

$$(g(i) * h(i)) * x(i) = g(i) * (h(i) * x(i))$$

■ distributivity:

$$g(i) * (h(i) + x(i)) = (g(i) * h(i)) + (g(i) * x(i))$$

■ circularity:

$h(i)$ periodic $\Rightarrow y(i) = h(i) * x(i)$ periodic

convolution properties

■ identity:

$$x(i) = \delta(i) * x(i)$$

■ commutativity:

$$h(i) * x(i) = x(i) * h(i)$$

■ associativity:

$$(g(i) * h(i)) * x(i) = g(i) * (h(i) * x(i))$$

■ distributivity:

$$g(i) * (h(i) + x(i)) = (g(i) * h(i)) + (g(i) * x(i))$$

■ circularity:

$$h(i) \text{ periodic} \Rightarrow y(i) = h(i) * x(i) \text{ periodic}$$

convolution properties

■ identity:

$$x(i) = \delta(i) * x(i)$$

■ commutativity:

$$h(i) * x(i) = x(i) * h(i)$$

■ associativity:

$$(g(i) * h(i)) * x(i) = g(i) * (h(i) * x(i))$$

■ distributivity:

$$g(i) * (h(i) + x(i)) = (g(i) * h(i)) + (g(i) * x(i))$$

■ circularity:

$$h(i) \text{ periodic} \Rightarrow y(i) = h(i) * x(i) \text{ periodic}$$

filter

example 1: Moving Average

$$y(i) = \sum_{j=0}^{\mathcal{J}-1} b(j) \cdot x(i-j)$$

- replaces current sample with average of \mathcal{J} samples
- smooths a signal (low pass)
- IR: rectangular
- linear phase, but inefficient for many coefficients
- Finite Impulse Response (FIR)

filter

example 2: Single-Pole

$$y(i) = (1 - \alpha) \cdot x(i) + \alpha \cdot y(i - 1)$$

- **recursive system:** output depends on previous *output*
- the larger alpha, the less the current input is taken into account (low pass)
- alpha from 0...1
- efficient, but non-linear phase
- Infinite Impulse Response (IIR)

summary

lecture content

■ LTI system

- good model for many real-world system
- linear (homogeneity, superposition) and time-invariant
- impulse response reflects all system properties

■ convolution

- operation that computes the output of an LTI system from the input

■ low pass filters

- often used to smooth a signal
- typical examples are moving average (FIR) and single pole (IIR)

