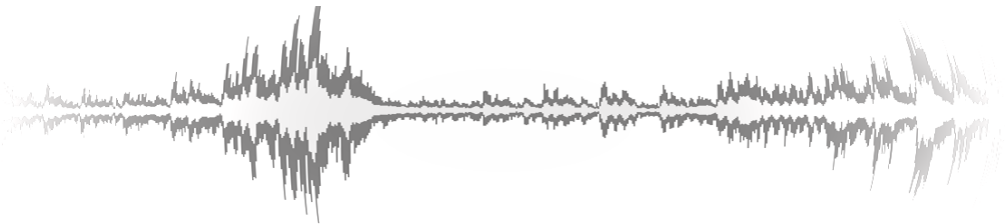
# Digital Signal Processing for Music

## Part 11: Discretization 2—Quantization

alexander lerch



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# sampling and quantization

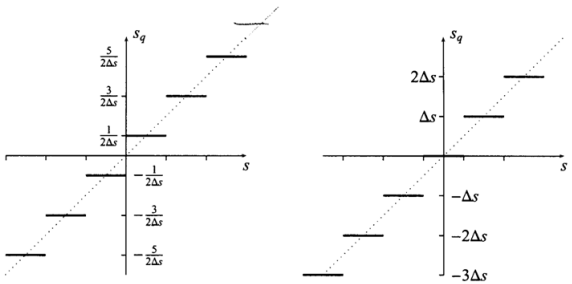
## quantization introduction 1/2



**quantizer**:

continuous $\mapsto$ discrete (pre-defined set of allowed values)

- quantization is **non-linear**
- quantization is **irreversible**




mid-rise mid-tread

# sampling and quantization

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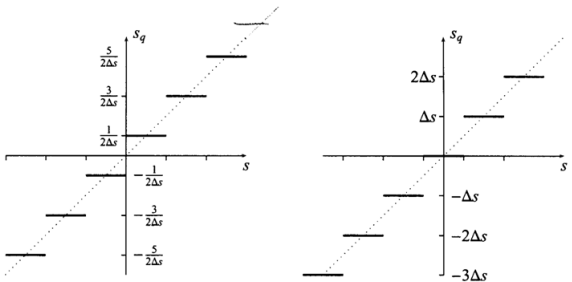


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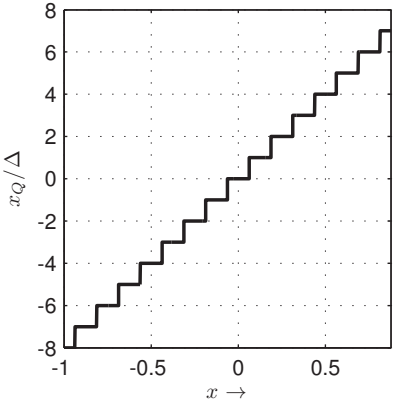
# sampling and quantization

## quantization introduction 2/2



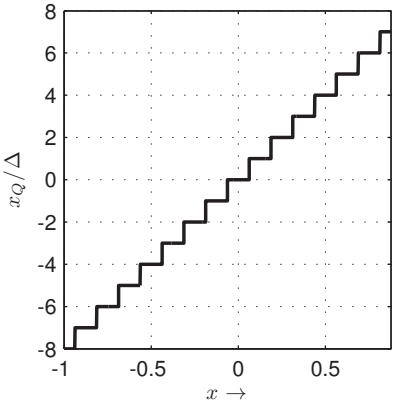



- **number of quantization steps**: $\mathcal{M} = 16$
- **word length (bits)**:

# sampling and quantization

## quantization introduction 2/2



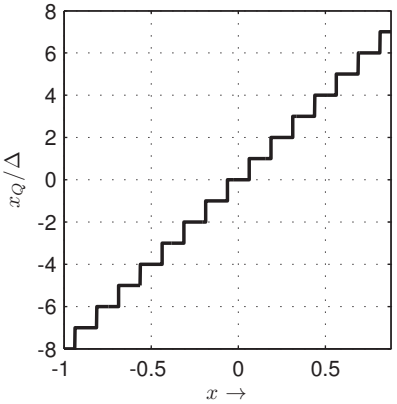



- **number of quantization steps**: $\mathcal{M} = 16$
- **word length (bits)**: ?

# sampling and quantization

## quantization introduction 2/2






- **number of quantization steps**: $\mathcal{M} = 16$
- **word length (bits)**: $w = \log_2(\mathcal{M}) = 4\,\mathrm{bit}$

# sampling and quantization

## quantization: word length & number of steps

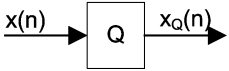


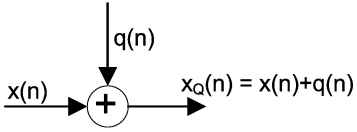
| $w$ | $\mathcal{M} = 2^w$ |
|---|---|
| 1 | 2 |
| 2 | 4 |
| 4 | 16 |
| 8 | 256 |
| 12 | 4096 |
| 16 | 65536 |
| 20 | 1048576 |
| 24 | 16777216 |

# sampling and quantization

## quantization error: definition






$$q(i) = x_{\mathrm{Q}}(i) - x(i)$$

# sampling and quantization

## quantization error: max. amplitude



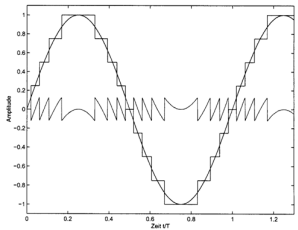
**What is the maximum amplitude of the quantization error**

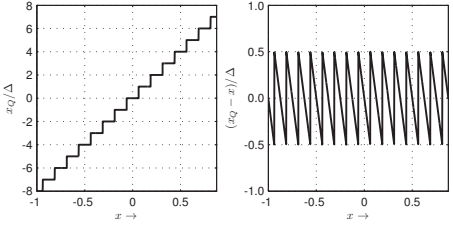
# sampling and quantization

## quantization error: max. amplitude



**What is the maximum amplitude of the quantization error**



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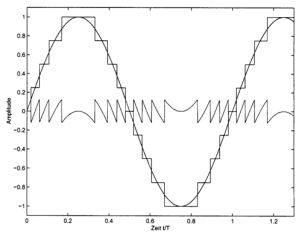
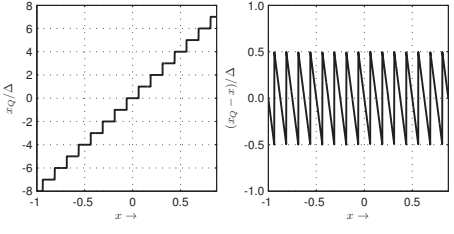



$$|q(i)| \leq \frac{\Delta}{2}$$

# sampling and quantization

## quantization error: pdf 1/2



**What is the pdf of the quantization error**

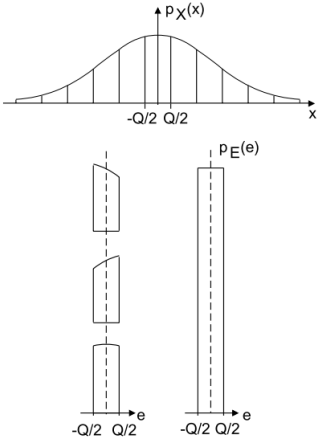
# sampling and quantization

## quantization error: pdf 1/2



**What is the pdf of the quantization error**

assuming $\Delta \ll max(|x(i)|)$



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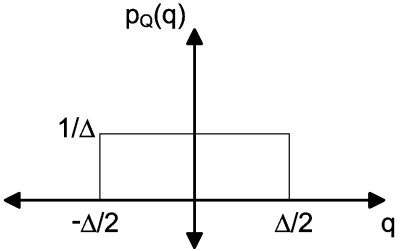

# sampling and quantization

## quantization error: pdf 2/2



it can be shown that the pdf of the quantization error depends (without derivation)

- on the **variance of the input** signal in relation to the step size
- on the **pdf of the input** signal

→ will be **uniform for large values of** $\frac{\sigma_X}{\Delta}$

# sampling and quantization

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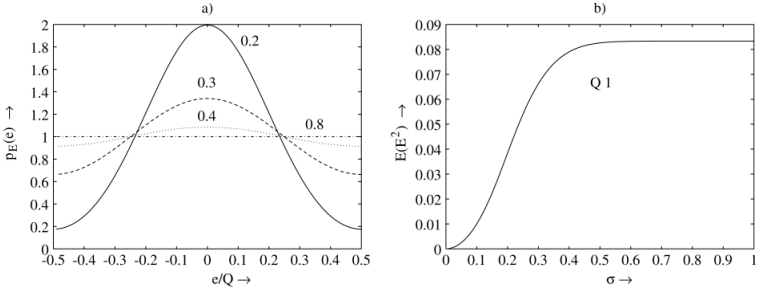


**Figure 2.16** (a) PDF of quantization error for different standard deviations of a Gaussian PDF input. (b) Variance of quantization error for different standard deviations of a Gaussian PDF input.

# sampling and quantization

## quantization error: power



**how to compute the power $W_Q$ of the quantization error**

# sampling and quantization

## quantization error: power



**how to compute the power $W_\mathrm{Q}$ of the quantization error**

from PDF:

$$W_\mathrm{Q} = \int_{-\Delta/2}^{\Delta/2} q^2 \cdot \underbrace{p_\mathrm{Q}(q)}_{1/\Delta}\, dq$$

# sampling and quantization

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$$= \frac{1}{\Delta} \int_{-\Delta/2}^{\Delta/2} q^2\, dq$$

# sampling and quantization

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\begin{aligned}
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&= \frac{1}{\Delta} \int_{-\Delta/2}^{\Delta/2} q^2\, dq \\
&= \frac{1}{\Delta} \left[ \frac{1}{3} q^3 \right]_{-\Delta/2}^{\Delta/2}
\end{aligned}
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&= \frac{\Delta^2}{12}
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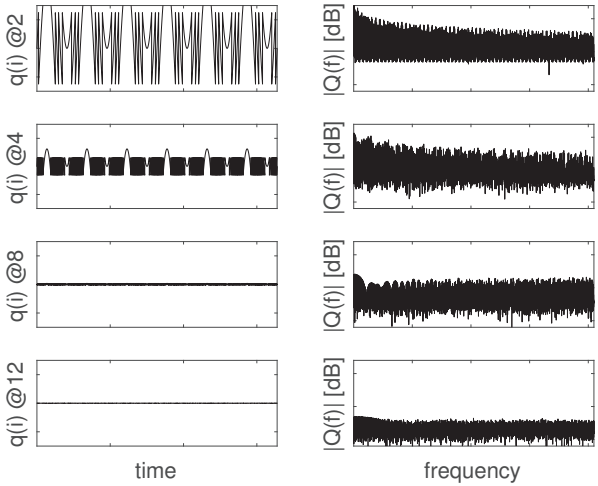
# sampling and quantization

## quantization error: spectrum



quantization error of a full-scale sinusoidal



# sampling and quantization

## quantization: audio examples

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| $w$ | $x_{\mathrm{Q,sine}}(i)$ | $q_{\mathrm{sine}}(i)$ | $x_{\mathrm{Q,speech}}(i)$ | $q_{\mathrm{speech}}(i)$ | $x_{\mathrm{Q,music}}(i)$ | $q_{\mathrm{music}}(i)$ |
|---|---|---|---|---|---|---|
| 16 | | | | | | |
| 12 | | | | | | |
| 8 | | | | | | |
| 6 | | | | | | |
| 4 | | | | | | |
| 2 | | | | | | |

# sampling and quantization

## quality assessment of a quantizer: SNR



Signal-to-Noise Ratio (SNR):

- power of the signal in relation to power of the (quantization) noise

$$SNR' = \frac{\text{signal energy}}{\text{noise energy}} = \frac{W_S}{W_Q}$$

- often in decibel

$$SNR = 10 \cdot \log_{10}\left(\frac{W_S}{W_Q}\right) \; [dB]$$

- SNR grows by
  - reducing the noise power
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# sampling and quantization

## quantization: SNR 1/3

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**What is the SNR of a quantized full-scale sinusoidal**

# sampling and quantization

## quantization: SNR 1/3



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$$SNR = 10 \cdot \log_{10}\left(\frac{W_S}{W_Q}\right) \; [dB]$$

use: $sin^2(t) = \frac{1-cos(2t)}{2}$

# sampling and quantization

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$$W_S = \frac{A^2}{2} \xrightarrow[\text{full-scale}]{} W_S = \frac{(\Delta \cdot 2^{w-1})^2}{2}$$

$$W_Q = \frac{\Delta^2}{12}$$

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# sampling and quantization

## quantization: SNR 1/3



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$$SNR = w \cdot 20\log_{10}(2) + 10 \cdot \log_{10}\left(\frac{3}{2}\right) \; [dB]$$

# sampling and quantization

## quantization: SNR 2/3



**derive the SNR for a full-scale square wave**

# sampling and quantization

## quantization: SNR 2/3



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$$SNR = 10 \cdot \log_{10}\left(\frac{W_S}{W_Q}\right) \; [dB]$$

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## quantization: SNR 2/3



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$$SNR = w \cdot 20 \log_{10}(2) + 10 \cdot \log_{10}(3) \; [dB]$$

# sampling and quantization

## quantization: SNR 3/3



### Signal-to-Noise Ratio

$$SNR = 6.02 \cdot w + c_{\mathrm{S}} \quad [dB]$$

- every additional bit adds app. 6 dB SNR
- constant $c_{\mathrm{S}}$ depends on signal (scaling and PDF shape)

SNR for different input signal examples

- square wave (full scale): $c_{\mathrm{S}} = 4.77\,\mathrm{dB}$
- sinusoidal wave (full scale): $c_{\mathrm{S}} = 1.76\,\mathrm{dB}$
- rectangular PDF (full scale): $c_{\mathrm{S}} = 0\,\mathrm{dB}$
- Gaussian PDF (full scale $= 4\sigma_g$): $c_{\mathrm{S}} = -7.27\,\mathrm{dB}$

# sampling and quantization

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# sampling and quantization

## quantization: word length and SNR



| w | Δ | Max. Amp | theo. SNR |
|---|---|---|---|
| 8 (Int) | $\pm 1$ | $0 \ldots 255$ | $\approx 48\,\text{dB}$ |
| 16 (Int) | $\pm 1$ | $-32768 \ldots 32767$ | $\approx 96\,\text{dB}$ |
| 20 (Int) | $\pm 1$ | $-524288 \ldots 524287$ | $\approx 120\,\text{dB}$ |
| 24 (Int) | $\pm 1$ | $-16777216 \ldots 16777215$ | $\approx 144\,\text{dB}$ |
| 32 (Float) | $\pm 1.175 \cdot 10^{-38}$ | $\pm 3.403 \cdot 10^{1038}$ | 1529 dB |
| 64 (Float) | $\pm 2.225 \cdot 10^{-308}$ | $\pm 1.798 \cdot 10^{10308}$ | 12318 dB |

# sampling and quantization

## quantization: SNR and auditory sensation area



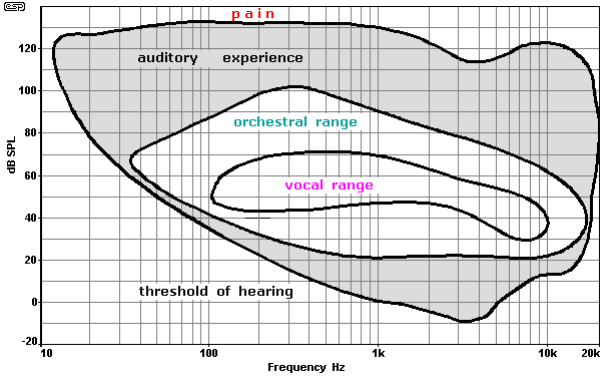
**so how many bits do we need**

# sampling and quantization

## quantization: SNR and auditory sensation area



**so how many bits do we need**



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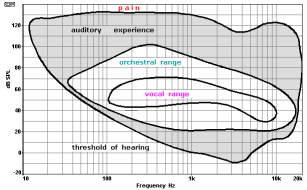
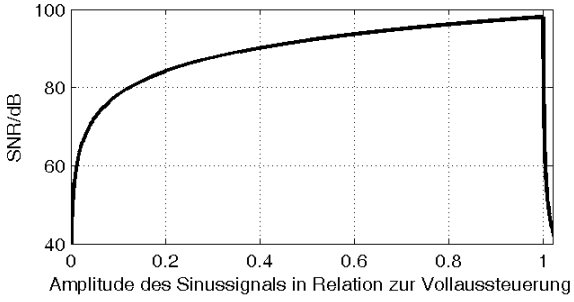



- to cover the whole range of hearing: 20–24 bit
- practically, a lower range is sufficient as the dynamic range of recordings has to be much lower
- in production with many processing and possible requantization steps, high resolution (if possible floating point) is recommended

# sampling and quantization

## quantization: SNR and signal scaling






full scale:

- absolute maximum before clipping
- usually 1 (in floating point systems)
- marks 0 dbFS

# sampling and quantization

## quantization: summary



- quantization is **non-linear** & **irreversible**
  - information is lost
  - error is introduced
- quantization **error**
  - power is determined by number of bits (wordlength)
  - is approximately white noise (flat spectrum and uncorrelated to signal) when the signal power is much higher than the quantization step size
  - special severe case: clipping
- **SNR** is used to assess quantizer quality
  - depends on both signal power and quant error power (ratio)
  - each additional bit gains 6 dB SNR
  - different signals with identical maximum amplitude yield different SNRs
- **typical word lengths** include
  - 8 bit: phone
  - 16 bit: consumer audio
  - 24 bit and higher: production audio

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