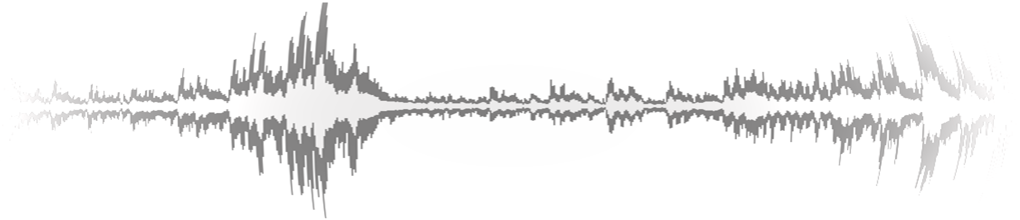
Digital Signal Processing for Music

Part 18: Sample Rate Conversion (SRC)

alexander lerch



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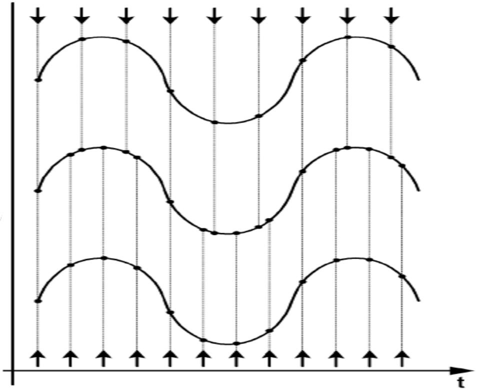
sample rate conversion
introduction

sample rate conversion

WP:"changing the sampling rate of a discrete signal to obtain a new discrete representation of the underlying continuous signal."



sample rate conversion
introduction

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typical applications

- audio file/media conversion
- word clock synchronization (multiple devices)
- DJing/scratching

sample rate conversion
introduction

- **terminology**
  - *synchronous*
    - clock rates are coupled
    - resampling factor stays constant
  - *asynchronous*
    - clock rates are independent
    - resampling factor may change
- **ideal result**
  - spectrum in the used band unchanged
  - spectral periodicity (determined by sample rate) changed

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sample rate conversion

upsampling by inserting zeros

- task: **upsample by integer factor L**
  1. insert $L-1$ zeros between all samples
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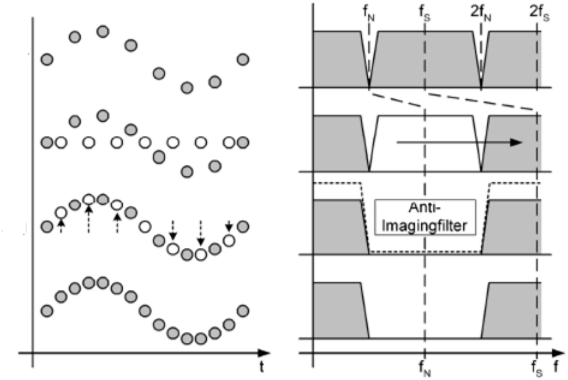

sample rate conversion
upsampling by inserting zeros

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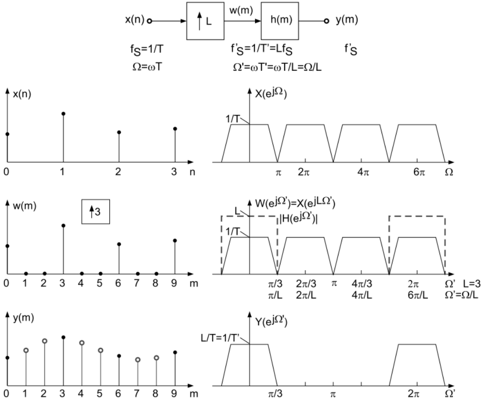

sample rate conversion

downsampling by removing samples

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- task: **downsample by integer factor M**
  1. apply anti-aliasing filter
  2. take every Mth sample

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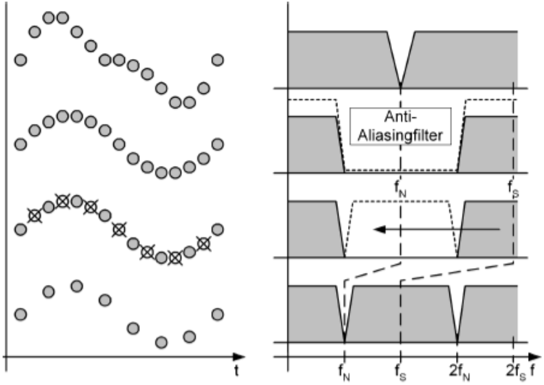

sample rate conversion

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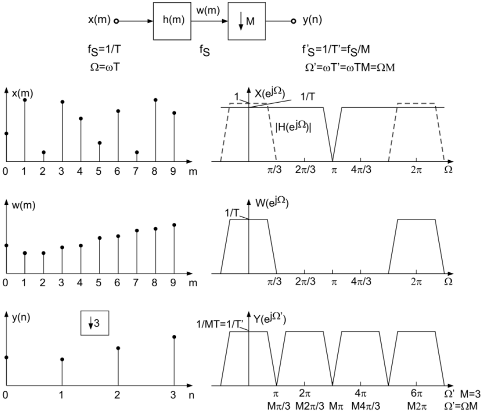

sample rate conversion

resampling by rational factor

- task: **convert sample rate to any** other (coupled) sample rate
  1. convert sample rate ratio to integer factors
     e.g.: $^{48}/_{44.1} \Rightarrow L = 160, M = 147$
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sample rate conversion

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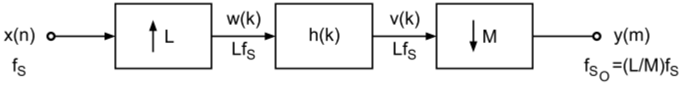
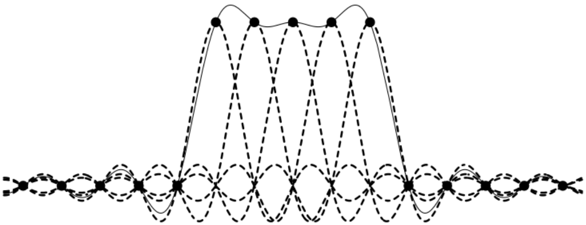


sample rate conversion
sinc interpolation 1/2

- perfect reconstruction of the sampled spectrum is possible with ideal filter

⇒ resampling should be possible by time domain convolution with sinc



$$x(i-\alpha) = \sum_{m=-\infty}^{\infty} x(m) \frac{\Omega_C}{\pi} \frac{sin\left(\Omega_C(i-\alpha-m)\right)}{\Omega_C(i-\alpha-m)}$$

Ω_C is the cutoff frequency of the ideal lowpass

sample rate conversion

sinc interpolation 1/2

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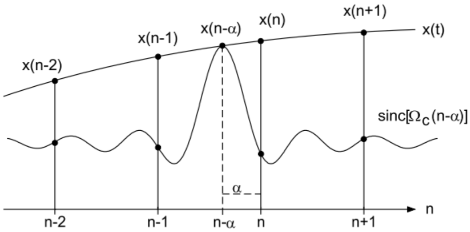

sample rate conversion
sinc interpolation 2/2

- practical implementation: **windowed sinc**

sample rate conversion
polynomial interpolation

- **interpolation methods**
  - can be interpreted as filters with time-variant filter coefficients
  - not based on traditional filter desing methods
- polynomial interpolation

$$f(t) = \sum_{k=0}^{\mathcal{O}} x_k p_k(t)$$

$$p_k(t) = \prod_{j=0}^{\mathcal{O}} \frac{t - t_j}{t_k - t_j}$$

sample rate conversion
polynomial interpolation

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sample rate conversion

polynomial interpolation example

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\begin{aligned}
x(t) &= \frac{1}{t} \\
\text{nodes: } t &= [2, 4, 5] \\
p_0(t) &= \frac{(t-4)(t-5)}{(2-4)(2-5)} = \frac{(t-4)(t-5)}{6} \\
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\Rightarrow f(t) &= p_0\frac{1}{2} + p_1\frac{1}{4} + p_2\frac{1}{5} \\
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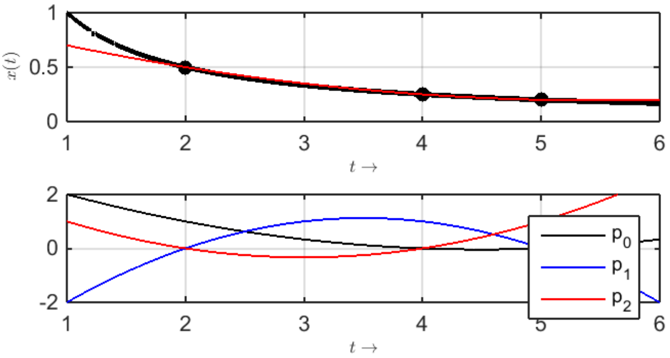

sample rate conversion

special case: linear interpolation

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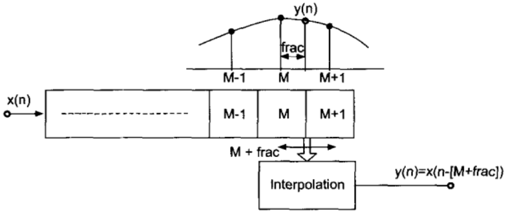
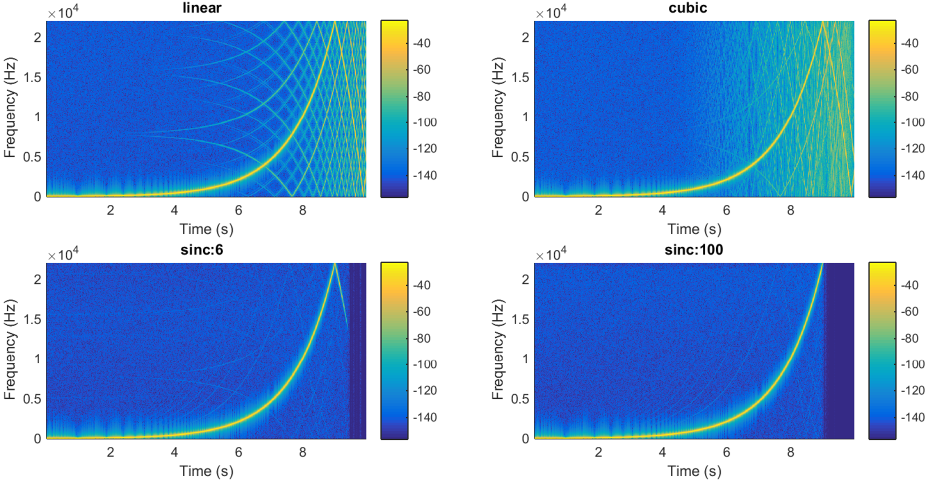


$$\hat{x} = x_l \cdot (1 - frac) + x_r \cdot frac$$

sample rate conversion

interpolation comparison



sample rate conversion

downsampling audio examples

	orig (48 kHz)	ds (6 kHz, w/o filt)	ds (6 kHz, w/ filt)
sax			
big band			

sample rate conversion

summary

- resampling: estimate different sample points of underlying continuous signal
- as with sampling, proper filtering has to take place
- some interpolation approaches have filter “built-in”
- perfect reconstruction impossible (infinite sinc), however, perceptually artifact-free resampling is possible
  - main issue: filter cut-off and steepness vs. aliasing