Digital Signal Processing for Music

Part 15: Digital Filters I

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Georgia Center for Music Tech Technology

filters introduction 1/2

intro



filter — broad description

system that amplifies or attenuates certain components/aspects of a signal

filter — narrow description

linear time-invariant system for changing the magnitude and phase of specific frequency regions

- example for other type of filters:
 - adaptive and time-variant (e.g., denoising)
- examples for "real-world" filters
 - reverberation
 - absorption
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filters introduction 2/2



■ audio equalization

- parametric and graphic EQs
- removal of unwanted components
 - remove DC, rumble, hum, hiss
- pre-emphasis/de-emphasis
 - Vinyl
 - old noise reduction systems
- **weighting** function
 - dBA, dBC, ...
- (parameter) smoothing
 - smooth sudden changes



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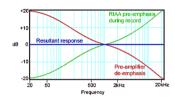
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audio equalization

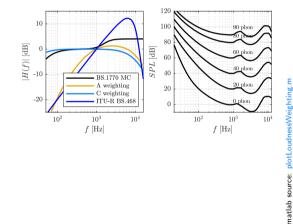
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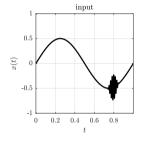
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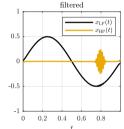


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$$y(t) = x(t) * h(t)$$

■ this is equivalent to a frequency domain multiplication

$$Y(j\omega) = X(j\omega) \cdot H(j\omega)$$

 $H(j\omega) = \frac{Y(j\omega)}{X(j\omega)}$

- transfer function $H(j\omega)$ is complex, often represented as
 - magnitude $|H(j\omega)|$ and
 - phase $\Phi_{\rm H}({\rm j}\omega)$

reminder: system theory



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filters common transfer function shapes

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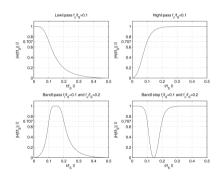
what are typical filters/spectral filter shapes



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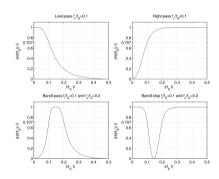
- very common:
 - low/high pass



 $^{^{1}}$ U. Zölzer, Digital Audio Signal Processing, 2nd Edition. Stuttgart: John Wiley & Sons Ltd, 2008, ISBN: 978-0-470-99785-7.



- very common:
 - low/high pass
- common for non-audio/non-parametric:
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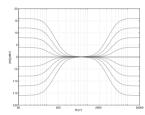
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- also common in audio apps:
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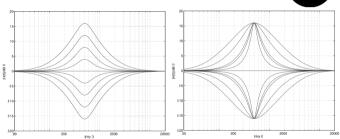
filters common transfer function shapes



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 - peak filter



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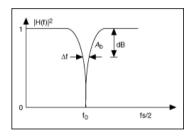
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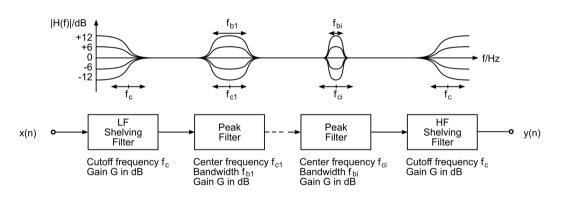






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- common for non-audio/non-parametric:
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- also common in audio apps:
 - low/high shelving
 - peak filter
 - resonance/notch

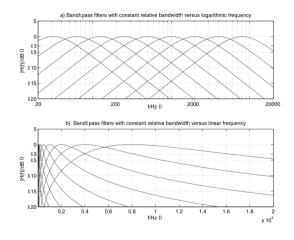




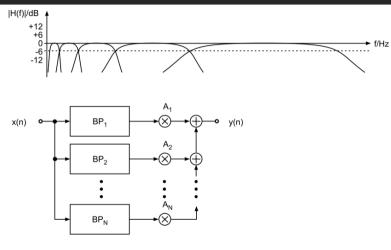
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filters filter banks — parallel connections





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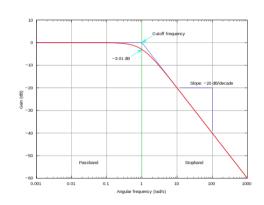


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filter parameters — lowpass/highpass



- **cut-off** frequency f_c
 - frequency marking the transition of pass to stop band
 - -3 dB of pass band level
- **slope**/steepness
 - measured in dB/Octave or dB/Decade
 - typically directly related to filter order
- sometimes: resonance
 - level increase in narrow band around cut-off frequency

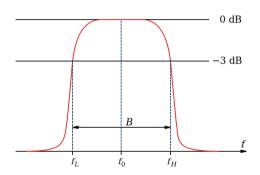


en.wikipedia.org/wiki/File:Butterworth_response.svg

filter parameters — bandpass/bandstop



- \blacksquare center frequency $f_{\rm c}$
 - frequency marking the center of the pass or stop band
- **bandwidth** ΔB
 - width of the pass band
 - at -3 dB of max pass band level
- possibly: slope
 - typically directly related to filter order



en.wikipedia.org/wiki/File:Bandwidth_2.svg

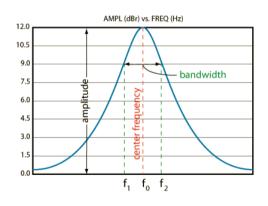
filters filter parameters — peak

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- **center** frequency f_c
 - frequency marking the center of the peak
- \blacksquare **Q** factor or bandwidth $\triangle B$
 - width of the bell
 - at -3 dB of max gain

$$Q = rac{f_{
m c}}{\Delta B}$$

- gain
 - amplification/attenuation in dB



filters filter parameters — overview



parameter	lowpass	low shelving	band pass	peak	resonance
frequency	cut-off	cut-off	center	center	center
bandwidth/Q	res. gain	_	ΔB	Q	_
gain	_	yes	_	yes	_

$$H(\mathrm{j}\omega)=\mathfrak{F}\{h(t)\}$$

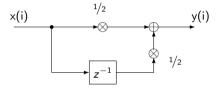
filter is defined by its

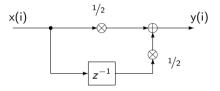
- lacktriangle complex transfer function $H(j\omega)$, or its
- impulse response h(t), or its
- list of pole and zero positions in the Z-plane

$$H(j\omega) = \mathfrak{F}\{h(t)\}$$

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$$y(i) = 0.5 \cdot x(i) + 0.5 \cdot x(i-1)$$

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example 1: transfer function 1/2

$$y(i) = 0.5 \cdot x(i) + 0.5 \cdot x(i - 1)$$

$$H(z) = 0.5 + 0.5 \cdot z^{-1}$$

$$H(j\omega) = 0.5 + 0.5 \cdot e^{-j\omega}$$

$$|H(j\omega)| = 0.5 \cdot \left| e^{-j\frac{\omega}{2}} \cdot \left(e^{j\frac{\omega}{2}} + e^{-j\frac{\omega}{2}} \right) \right|$$

$$= 0.5 \cdot \left| e^{-j\frac{\omega}{2}} \right| \cdot \left| \left(e^{j\frac{\omega}{2}} + e^{-j\frac{\omega}{2}} \right) \right|$$

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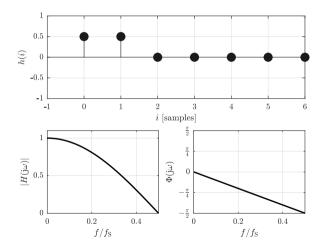
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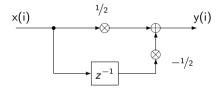
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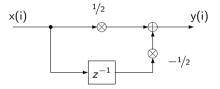
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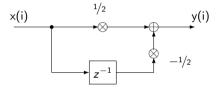




$$y(i) = 0.5 \cdot x(i) - 0.5 \cdot x(i-1)$$

 $H(z) = 0.5 - 0.5 \cdot z^{-1}$

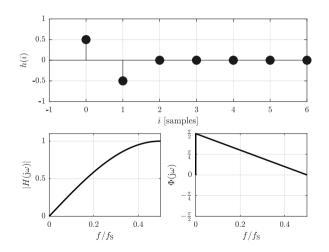




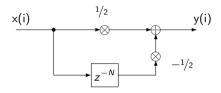
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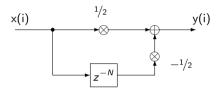
$$|H(j\omega)| = \left| \sin\left(\frac{\omega}{2}\right) \right|$$



matlab source: plotSimpleFilter.m

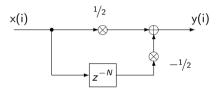






$$y(i) = 0.5 \cdot x(i) - 0.5 \cdot x(i - N)$$

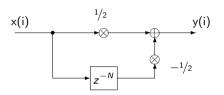
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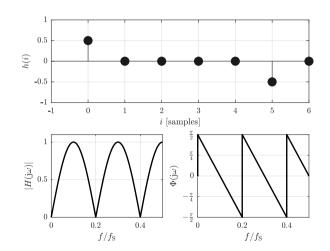


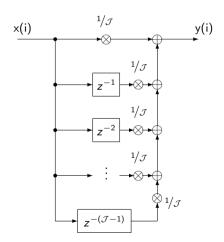
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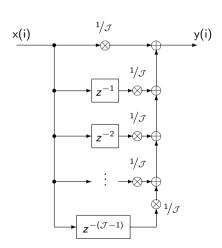
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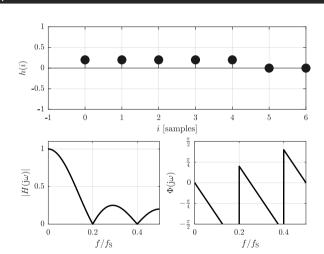
$$= \left| \sin \left(\frac{N\omega}{2} \right) \right|$$







$$y(i) = \frac{1}{\mathcal{J}} \sum_{j=0}^{\mathcal{J}-1} x(i-j)$$



$$H(j\omega) = \mathrm{e}^{-\mathrm{j}\mathcal{J}rac{\omega}{2}}rac{\sin\left(\mathcal{J}\cdotrac{\omega}{2}
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example 4: recursive implementation

$$y(i) = \sum_{j=0}^{\mathcal{J}-1} \frac{1}{\mathcal{J}} \cdot x(i-j)$$

$$= \frac{1}{\mathcal{J}} \cdot (x(i) - x(i-\mathcal{J})) + \sum_{j=1}^{\mathcal{J}} \frac{1}{\mathcal{J}} \cdot x(i-j)$$

$$= \frac{1}{\mathcal{J}} \cdot (x(i) - x(i-\mathcal{J})) + y(i-1)$$

y(i-1)

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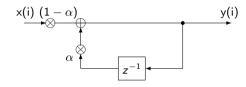
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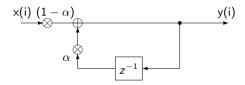
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not applicable with windowed coefficients!





examples

000000000000000000

$$y(i) = (1-\alpha) \cdot x(i) + \alpha \cdot y(i-1)$$

= $x(i) + \alpha \cdot (y(i-1) - x(i))$

$$y(i) = (1 - \alpha) \cdot x(i) + \alpha \cdot y(i - 1)$$

$$H(z) = \frac{1 - \alpha}{1 - \alpha z^{-1}}$$

$$H(j\omega) = \frac{1 - \alpha}{1 - \alpha e^{-j\omega}}$$

$$|H(j\omega)| = \left| \frac{1 - \alpha}{1 - \alpha e^{-j\omega}} \right|$$

$$= \frac{1 - \alpha}{\sqrt{(1 + \alpha^2 - 2\alpha \cos(\omega))}}$$

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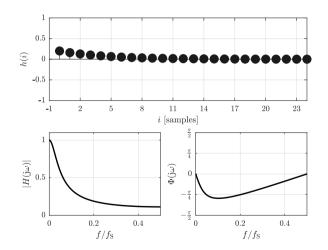
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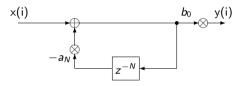
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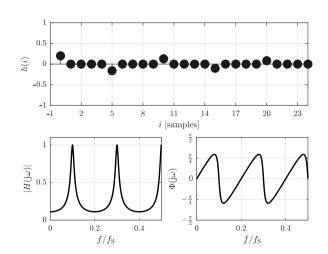
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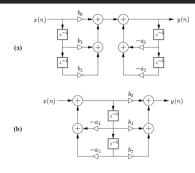
$$= \frac{|1 - \alpha e^{-J\omega}|}{\sqrt{(1 + \alpha^2 - 2\alpha \cos(\omega))}}$$





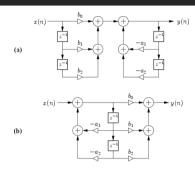
$$y(i) = b_0 \cdot x(i) - a_N \cdot y(i - N)$$





diff eq :
$$y(i) = \sum_{k=0}^{K_1} b_k \cdot x(i-k) + \sum_{k=1}^{K_2} -a_k \cdot y(i-k)$$

rans. fct : $H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{K_1} b_k \cdot z^{-k}}{1 + \sum_{k=0}^{K_2} a_k \cdot z^{-k}}$



$$\begin{array}{lll} \text{diff eq} : y(i) & = & \displaystyle \sum_{k=0}^{K_1} b_k \cdot x(i-k) + \sum_{k=1}^{K_2} -a_k \cdot y(i-k) \\ \\ \text{trans. fct} : H(z) & = & \displaystyle \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{K_1} b_k \cdot z^{-k}}{1 + \sum_{k=1}^{K_2} a_k \cdot z^{-k}} \end{array}$$

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 - changing the sound quality of a signal
 - hiding unwanted frequency components
 - smoothing
 - processing for measurement and transmission
- most common audio filter types are
 - low/high pass
 - peak
 - shelving
- filter parameters include
 - frequency (mid, cutoff)
 - bandwidth or Q
 - gain
- filter orders
 - typical orders are 1st, 2nd, maybe 4th
 - higher order give more flexibility wrt transfer function
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