Digital Signal Processing for Music

Part 9: Fast Convolution

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fast convolution

introduction

convolution: measure impulse response h(i) and apply FIR filter to signal

$$\begin{aligned} y(i) &= x(i) * h(i) \\ &= \sum_{j=-\infty}^{\infty} h(j) \cdot x(i-j) \\ Y(z) &= X(z) \cdot H(z) \end{aligned}$$

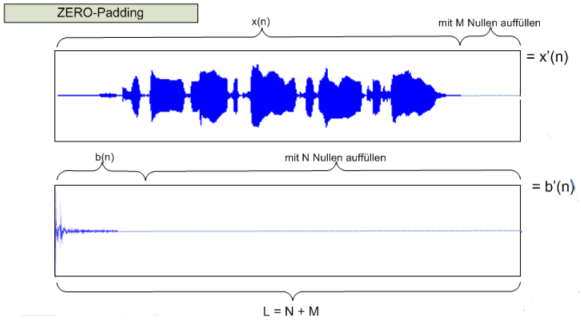
DFT convolution

signal and impulse response 1/2

- multiplication: two signals cannot be multiplied if of unequal length ($M = length(H)$, $N = length(X)$)

$\Rightarrow$ zeropad both signals

- minimum DFT length: $L \geq M + N - 1$



DFT convolution

signal and impulse response 1/2

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- minimum DFT length: $L \geq M + N - 1$

1. $X = DFT(x_{\mathrm{pad}}(i))$
2. $H = DFT(h_{\mathrm{pad}}(i))$
3. $Y = X \cdot H$
4. $y = DFT^{-1}(Y)$
5. throw away zeros if DFT was longer than $M + N$

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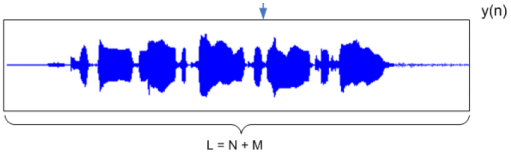


DFT convolution

signal and impulse response 2/2

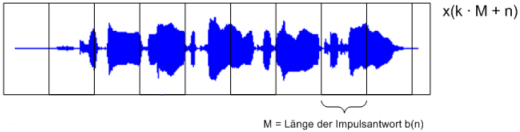
properties:

- no real-time:
  signal has to be known completely
- high memory requirements:
  - signal length N + impulse response length M
  - when FFT: next larger power of two

blocked convolution

blocked signal and impulse response 1/2

1. split input signal into blocks of length M
2. DFT convolution with each block (zeropadding)
3. overlap and save



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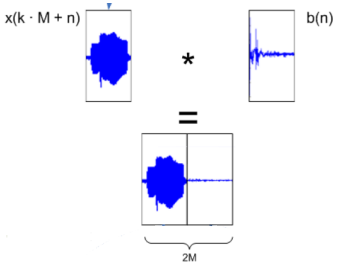

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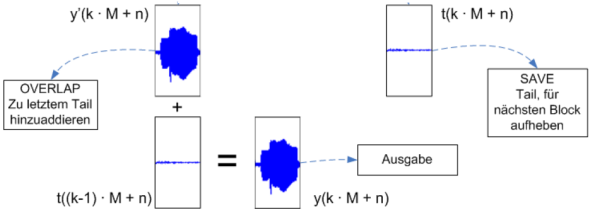

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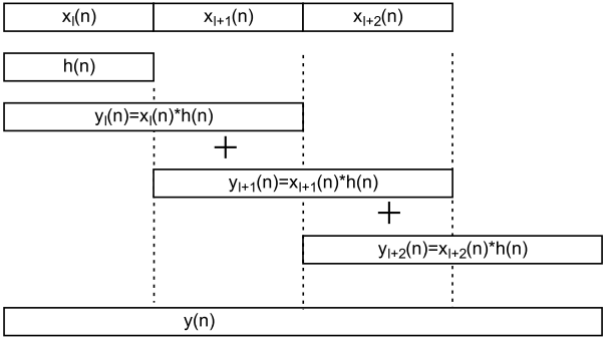


blocked convolution

blocked signal and impulse response 2/2

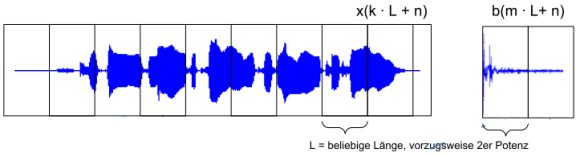
properties:

- minimum latency:
  impulse response length
- long FFT, but more efficient
- FFT of impulse response *is only computed once*

partitioned convolution

blocked signal and blocked impulse response 1/3

1. split **both** input signal and impulse response into blocks of arbitrary length
2. DFT convolution with each signal block with each impulse response block (zeropadding)
3. overlap and save



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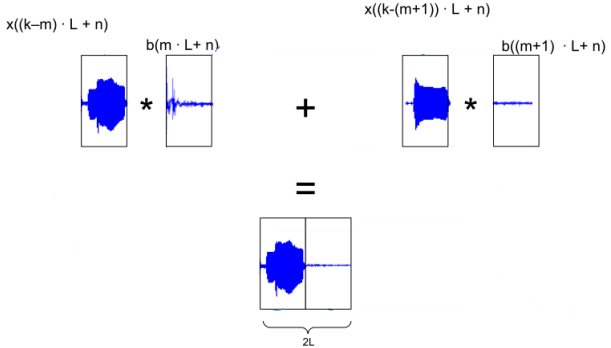

partitioned convolution

blocked signal and blocked impulse response 1/3

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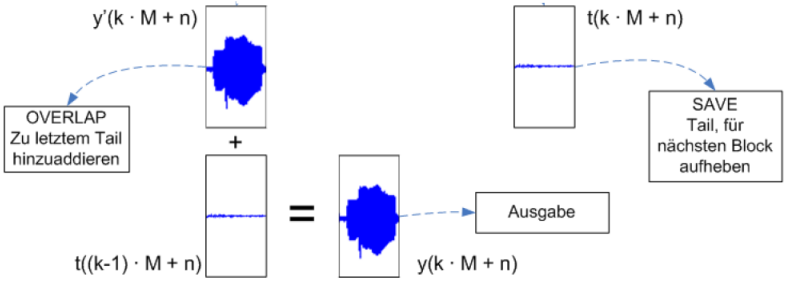
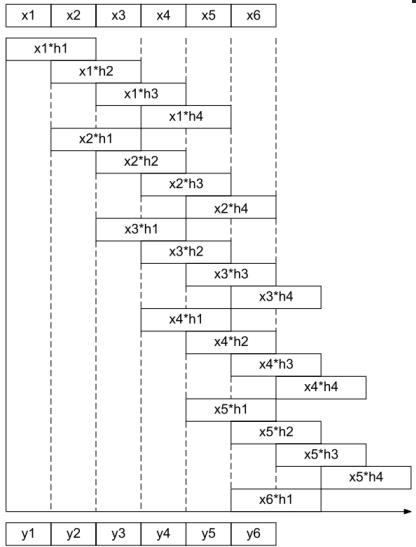


partitioned convolution

blocked signal and blocked impulse response 2/3



partitioned convolution

blocked signal and blocked impulse response 3/3

properties:

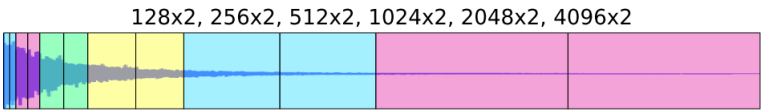
- arbitrary choice of latency/FFT length
  - long FFT: high latency, low workload
  - short FFT: short latency, high workload
- FFTs of IR computed only once

non-uniform partitioned convolution

different block lengths

- fast convolution: latency still formidable for efficient implementation

⇒ **non-uniform block lengths**



- **advantages**:
  - *any* desirable latency
- **disadvantages**:
  - less efficient due to multiple FFT lengths (but: inefficiency of short FFT partly compensated by very long FFTs)
  - complex implementation
  - comparably high memory usage (IR in many different FFT lengths)

non-uniform partitioned convolution

different block lengths

- fast convolution: latency still formidable for efficient implementation

$\Rightarrow$ **non-uniform block lengths**

128x2, 256x2, 512x2, 1024x2, 2048x2, 4096x2

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