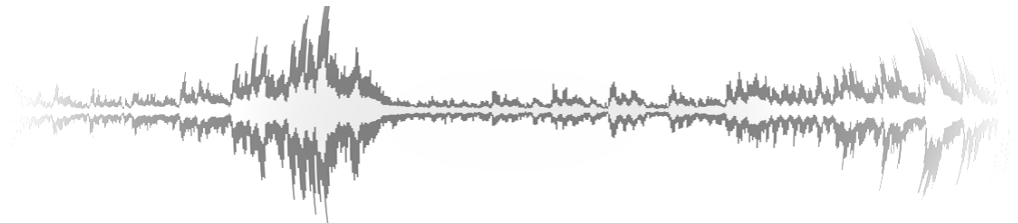


Digital Signal Processing for Music

Part 15: Digital Filters I

alexander lerch



filters

introduction 1/2

filter — broad description

system that amplifies or attenuates certain components/aspects of a signal

filter — narrow

linear time-invariant system for changing the magnitude and phase of specific frequency regions

- example for other type of filters:
 - adaptive and time-variant (e.g., denoising)
- examples for “real-world” filters
 - reverberation
 - absorption
 - echo

filters

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filters

introduction 2/2

- **audio equalization**
 - parametric EQs
 - graphic EQs
- **removal** of unwanted components
 - remove DC, rumble
 - remove hum
 - remove hiss
- **pre-emphasis/de-emphasis**
 - Vinyl
 - old Dolby noise reduction systems
- **weighting** function
 - dBA, dBC, ...
- (parameter) **smoothing**
 - smooth sudden changes



filters

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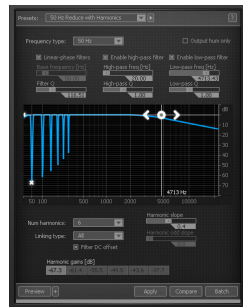
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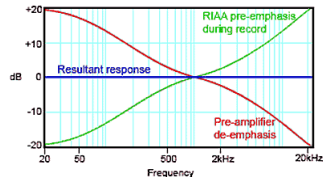
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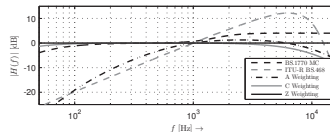
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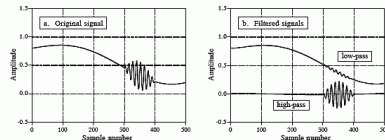
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filters

reminder: system theory

- output of a system (filter) y computed by **convolution** of input x and impulse response h

$$y(t) = x(t) * h(t)$$

- this is equivalent to a frequency domain multiplication

$$Y(j\omega) = X(j\omega) \cdot H(j\omega)$$

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)}$$

- **transfer function** $H(j\omega)$ is complex, often represented as
 - magnitude $|H(j\omega)|$ and
 - phase $\Phi_H(j\omega)$

filters

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filters

common transfer function shapes

what are typical filters/spectral filter shapes

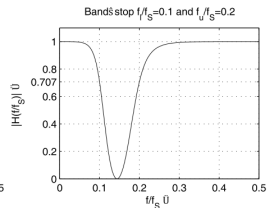
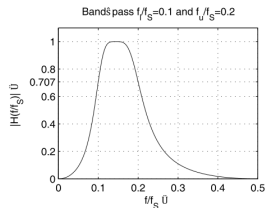
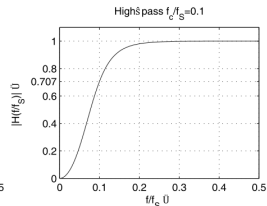
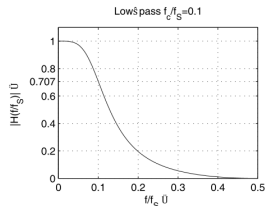


filters

common transfer function shapes

what are typical filters/spectral filter shapes

- very common:
 - low/high pass



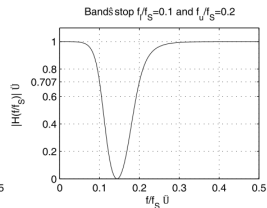
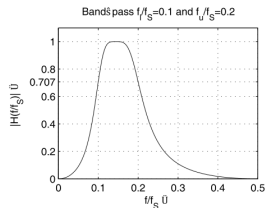
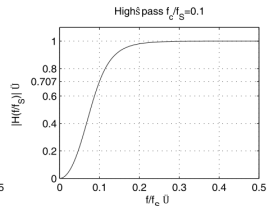
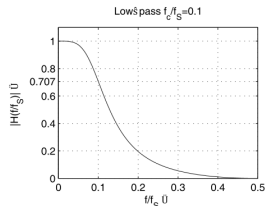
filters

common transfer function shapes

what are typical filters/spectral filter shapes



- very common:
 - low/high pass
- common for non-audio:
 - band pass/band stop

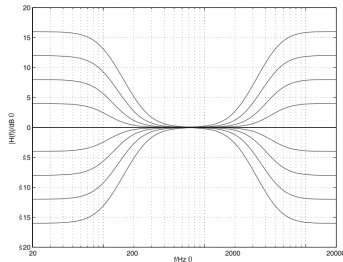


filters

common transfer function shapes

what are typical filters/spectral filter shapes

- very common:
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- common for non-audio:
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- also common in audio apps:
 - low/high shelving

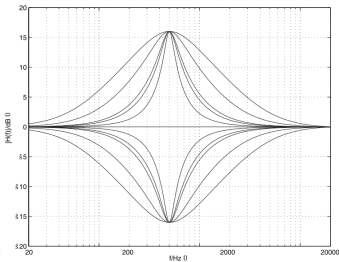
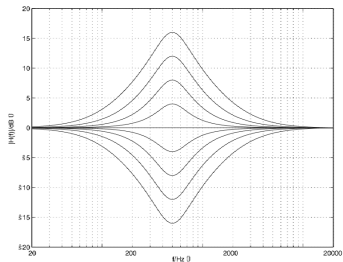


filters

common transfer function shapes

what are typical filters/spectral filter shapes

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- also common in audio apps:
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 - peak filter



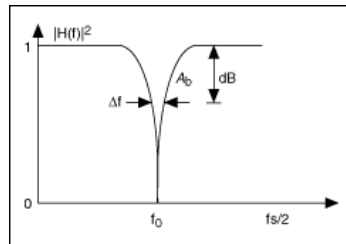
filters

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what are typical filters/spectral filter shapes

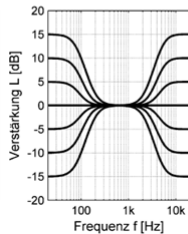
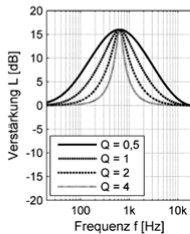
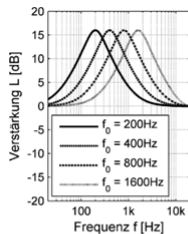
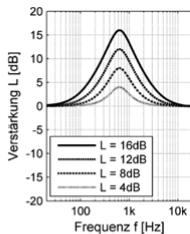


- very common:
 - low/high pass
- common for non-audio:
 - band pass/band stop
- also common in audio apps:
 - low/high shelving
 - peak filter
 - resonance/notch



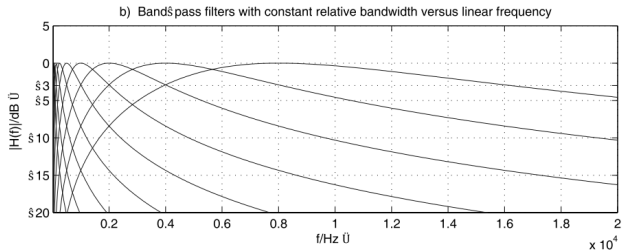
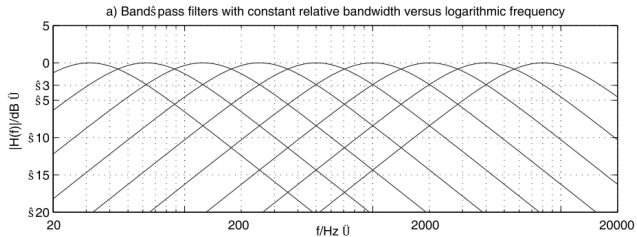
filters

common transfer function shapes



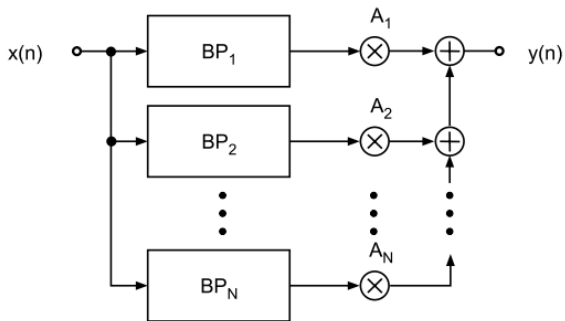
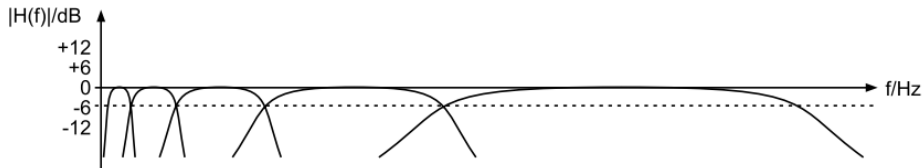
filters

filter banks



filters

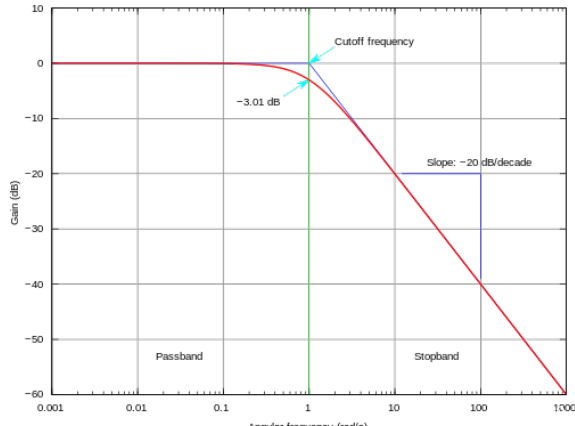
filter banks



filters

filter parameters — lowpass/highpass

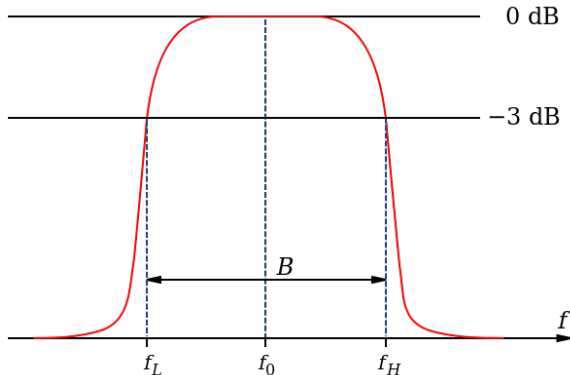
- **cut-off** frequency f_c
 - frequency marking the transition of pass to stop band
 - -3 dB of pass band level
- **slope**/steepness
 - measured in dB/Octave or dB/Decade
 - typically directly related to filter order
- sometimes: **resonance**
 - level increase in narrow band around cut-off frequency



filters

filter parameters — bandpass/bandstop

- **center** frequency f_c
 - frequency marking the center of the pass or stop band
- **bandwidth** ΔB
 - width of the pass band
 - at -3 dB of max pass band level
- possibly: **slope**
 - typically directly related to filter order



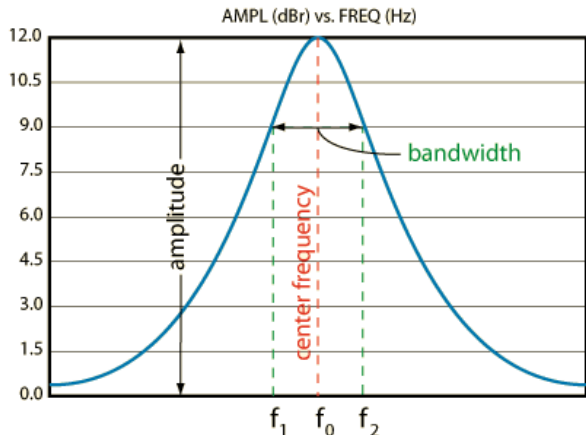
filters

filter parameters — peak

- **center** frequency f_c
 - frequency marking the center of the peak
- **Q factor** or **bandwidth** ΔB
 - width of the bell
 - at -3 dB of max gain

$$Q = \frac{f_c}{\Delta B}$$

- **gain**
 - amplification/attenuation in dB



filters

filter parameters — overview

<i>parameter</i>	lowpass	low shelving	band pass	peak	resonance
<i>frequency</i>	cut-off	cut-off	center	center	center
<i>bandwidth/Q</i>	res. gain	—	ΔB	Q	—
<i>gain</i>	—	yes	—	yes	—

filters

digital filter description

filter is defined by its

- complex transfer function $H(j\omega)$, or its
- impulse response $h(t)$, or its
- *list of pole and zero positions in the Z-plane*

$$H(j\omega) = \mathfrak{F}\{h(t)\}$$

filters

digital filter description

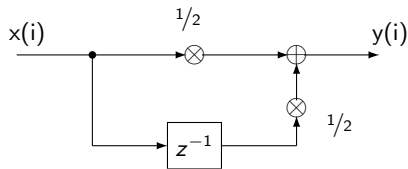
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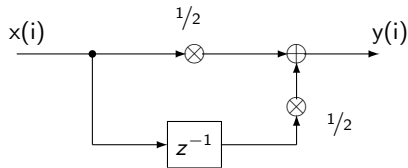
filters

example 1



filters

example 1



$$y(i) = 0.5 \cdot x(i) + 0.5 \cdot x(i - 1)$$

filters

example 1: transfer function 1/2

$$y(i) = 0.5 \cdot x(i) + 0.5 \cdot x(i-1)$$

$$H(z) = 0.5 + 0.5 \cdot z^{-1}$$

$$H(j\omega) = 0.5 + 0.5 \cdot e^{-j\omega}$$

$$\begin{aligned} |H(j\omega)| &= 0.5 \cdot \left| e^{-j\frac{\omega}{2}} \cdot \left(e^{j\frac{\omega}{2}} + e^{-j\frac{\omega}{2}} \right) \right| \\ &= 0.5 \cdot \underbrace{\left| e^{-j\frac{\omega}{2}} \right|}_1 \cdot \underbrace{\left| \left(e^{j\frac{\omega}{2}} + e^{-j\frac{\omega}{2}} \right) \right|}_{|2 \cos(\frac{\omega}{2})|} \end{aligned}$$

$$= \left| \cos\left(\frac{\omega}{2}\right) \right|$$

filters

example 1: transfer function 1/2

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$$H(z) = 0.5 + 0.5 \cdot z^{-1}$$

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filters

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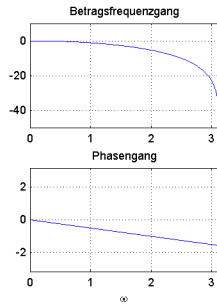
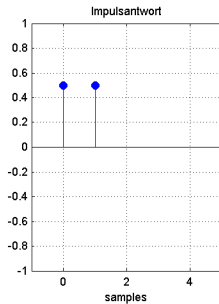
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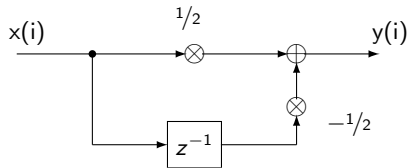
filters

example 1: transfer function 2/2



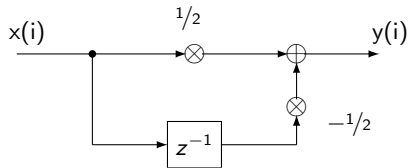
filters

example 2



filters

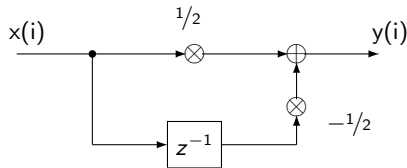
example 2



$$y(i) = 0.5 \cdot x(i) - 0.5 \cdot x(i-1)$$
$$H(z) = 0.5 - 0.5 \cdot z^{-1}$$

filters

example 2



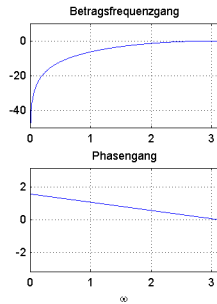
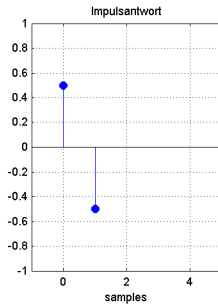
$$y(i) = 0.5 \cdot x(i) - 0.5 \cdot x(i-1)$$

$$H(z) = 0.5 - 0.5 \cdot z^{-1}$$

$$|H(j\omega)| = \left| \sin\left(\frac{\omega}{2}\right) \right|$$

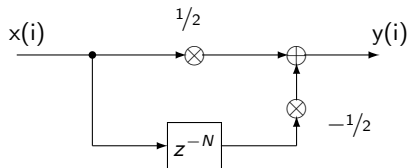
filters

example 2: transfer function



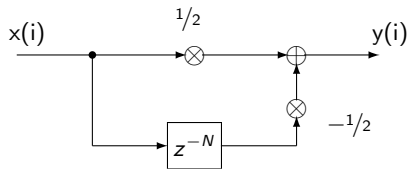
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example 3



filters

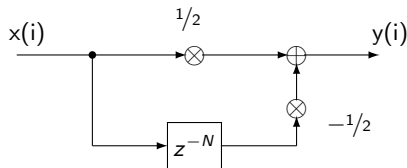
example 3



$$y(i) = 0.5 \cdot x(i) - 0.5 \cdot x(i - N)$$
$$H(z) = 0.5 - 0.5 \cdot z^{-N}$$

filters

example 3



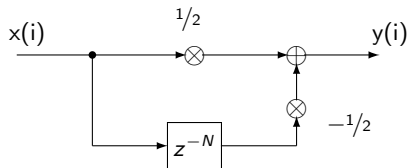
$$y(i) = 0.5 \cdot x(i) - 0.5 \cdot x(i - N)$$

$$H(z) = 0.5 - 0.5 \cdot z^{-N}$$

$$|H(j\omega)| = 0.5 \cdot \left| e^{-j\frac{N\omega}{2}} \cdot \left(e^{j\frac{N\omega}{2}} - e^{-j\frac{N\omega}{2}} \right) \right|$$

filters

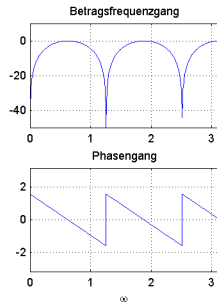
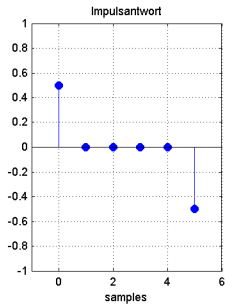
example 3



$$\begin{aligned}y(i) &= 0.5 \cdot x(i) - 0.5 \cdot x(i - N) \\H(z) &= 0.5 - 0.5 \cdot z^{-N} \\|H(j\omega)| &= 0.5 \cdot \left| e^{-j\frac{N\omega}{2}} \cdot \left(e^{j\frac{N\omega}{2}} - e^{-j\frac{N\omega}{2}} \right) \right| \\&= \left| \sin \left(\frac{N\omega}{2} \right) \right|\end{aligned}$$

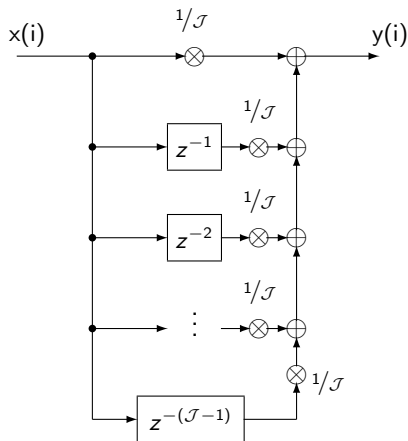
filters

example 3: transfer function



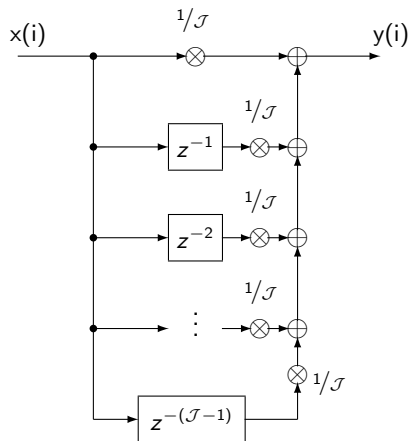
filters

example 4



filters

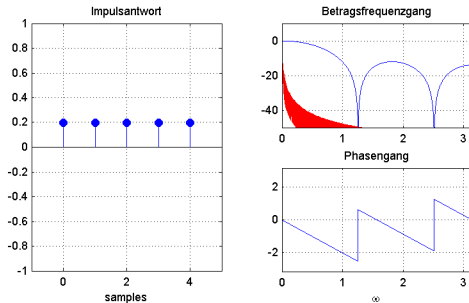
example 4



$$y(i) = \frac{1}{\mathcal{J}} \sum_{j=0}^{\mathcal{J}-1} x(i-j)$$

filters

example 4: transfer function



$$H(j\omega) = e^{-j\mathcal{J}\frac{\omega}{2}} \frac{\sin\left(\mathcal{J} \cdot \frac{\omega}{2}\right)}{\mathcal{J} \cdot \sin\left(\frac{\omega}{2}\right)}$$

filters

example 4: recursive implementation

$$\begin{aligned}y(i) &= \sum_{j=0}^{\mathcal{J}-1} \frac{1}{\mathcal{J}} \cdot x(i-j) \\&= \frac{1}{\mathcal{J}} \cdot (x(i) - x(i - \mathcal{J})) + \underbrace{\sum_{j=1}^{\mathcal{J}} \frac{1}{\mathcal{J}} \cdot x(i-j)}_{y(i-1)} \\&= \frac{1}{\mathcal{J}} \cdot (x(i) - x(i - \mathcal{J})) + y(i-1)\end{aligned}$$

not applicable with windowed coefficients!

filters

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filters

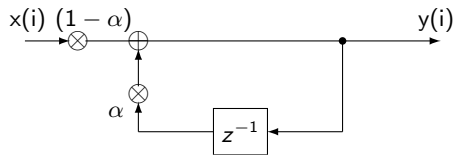
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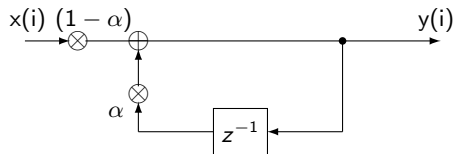
filters

example 5



filters

example 5



$$\begin{aligned} y(i) &= (1 - \alpha) \cdot x(i) + \alpha \cdot y(i - 1) \\ &= x(i) + \alpha \cdot (y(i - 1) - x(i)) \end{aligned}$$

Example 5: transfer function 1/2

$$y(i) = (1 - \alpha) \cdot x(i) + \alpha \cdot y(i - 1)$$

$$H(z) = \frac{1 - \alpha}{1 - \alpha z^{-1}}$$

$$H(j\omega) = \frac{1 - \alpha}{1 - \alpha e^{-j\omega}}$$

$$|H(j\omega)| = \left| \frac{1 - \alpha}{1 - \alpha e^{-j\omega}} \right|$$

$$= \frac{1 - \alpha}{\sqrt{(1 + \alpha^2 - 2\alpha \cos(\omega))}}$$

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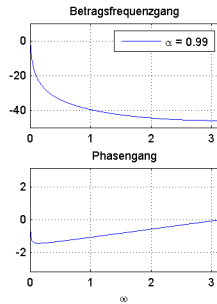
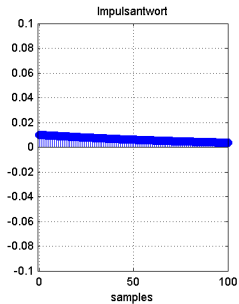
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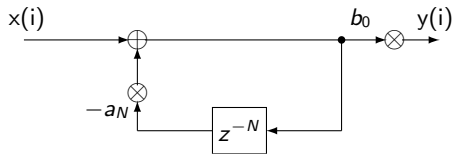
filters

example 5: transfer function 2/2



filters

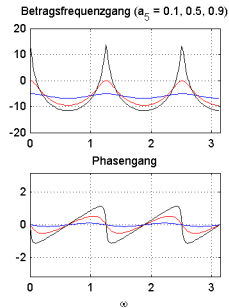
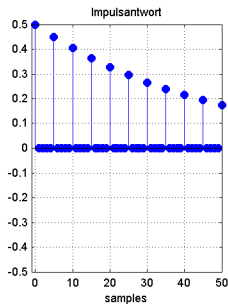
example 6



$$y(i) = b_0 \cdot x(i) - a_N \cdot y(i - N)$$

filters

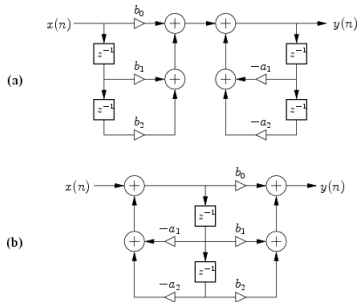
example 6: transfer function



$$H(j\omega) = \frac{b_0}{1 - a_N \cdot e^{-j\omega N}}$$

filters

biquad: structure

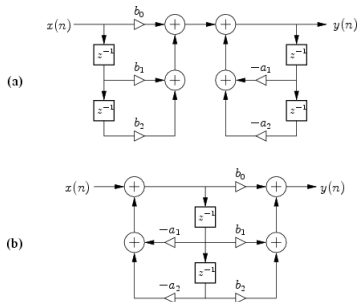


$$\text{diff eq : } y(i) = \sum_{k=0}^{K_1} b_k \cdot x(i-k) + \sum_{k=1}^{K_2} -a_k \cdot y(i-k)$$

$$\text{trans. fct : } H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{K_1} b_k \cdot z^{-k}}{1 + \sum_{k=1}^{K_2} a_k \cdot z^{-k}}$$

filters

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filters

summary

- filter (equalization) can be used for various tasks
 - changing the sound quality of a signal
 - hiding unwanted frequency components
 - smoothing
 - processing for measurement and transmission
- most common audio filter types are
 - low/high pass
 - peak
 - shelving
- filter parameters include
 - frequency (mid, cutoff)
 - bandwidth or Q
 - gain
- filter orders
 - typical orders are 1st, 2nd, maybe 4th
 - higher order give more flexibility wrt transfer function
 - higher orders are difficult to design and control

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