Digital Signal Processing for Music

Part 10: Discretization 1—Sampling

alexander lerch

Georgia Tech Center for Music Technology

sampling and quantization
introduction

digital signals can only be represented with a limited number of values

$\Rightarrow$

- time discretization:
  sampling
- amplitude discretization:
  quantization

sampling and quantization
introduction

digital signals can only be represented with a limited number of values

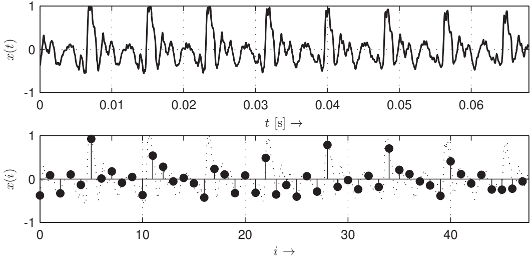
$\Rightarrow$

- time discretization:
  sampling

- amplitude discretization:
  quantization

sampling and quantization
sampling

$$T_S = \frac{1}{f_S}$$

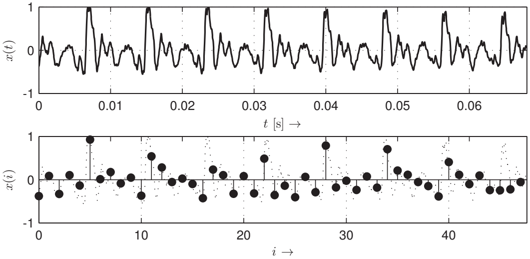


typical sample rates

- 8-16 kHz: speech (phone)
- 44.1-48 kHz: (consumer) audio/music
- higher: production audio

sampling and quantization
sampling

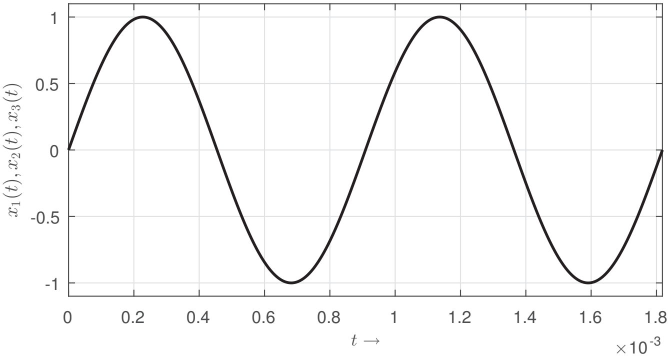
$$T_S = \frac{1}{f_S}$$



typical sample rates

- 8-16 kHz: speech (phone)
- 44.1-48 kHz: (consumer) audio/music
- higher: production audio

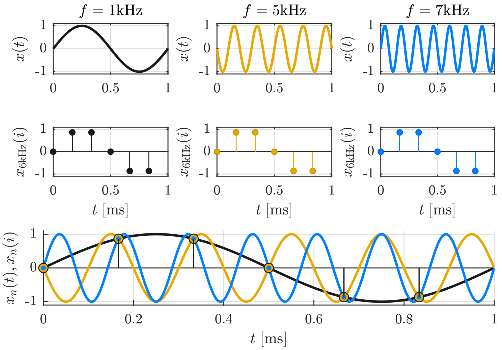
sampling and quantization
sampling ambiguity 1/4



sampling and quantization

sampling ambiguity 2/4

$$
\begin{aligned}
f_0 &= [1, 5, 7\,\mathrm{kHz}] \\
f_S &= 6\,\mathrm{kHz}
\end{aligned}
$$



matlab source: plotSampling02.m

sampling and quantization
sampling ambiguity 3/4

wagon wheel effect

sampling and quantization
sampling ambiguity 3/4

wagon wheel effect

1. $f_{wheel} < \frac{f_S}{2}$: speeding up
2. $\frac{f_S}{2} < f_{wheel} < f_S$: slowing down
3. $f_{wheel} = f_S$: standing still
4. ...

sampling and quantization

sampling ambiguity 3/4

wagon wheel effect

1. $f_{wheel} < \frac{f_S}{2}$: speeding up
2. $\frac{f_S}{2} < f_{wheel} < f_S$: slowing down
3. $f_{wheel} = f_S$: standing still
4. ...

sampling and quantization
sampling ambiguity 3/4

wagon wheel effect

1. $f_{wheel} < \frac{f_S}{2}$: speeding up
2. $\frac{f_S}{2} < f_{wheel} < f_S$: slowing down
3. $f_{wheel} = f_S$: standing still
4. $\ldots$

sampling and quantization

sampling ambiguity 4/4

http://youtu.be/uENITui5_jU

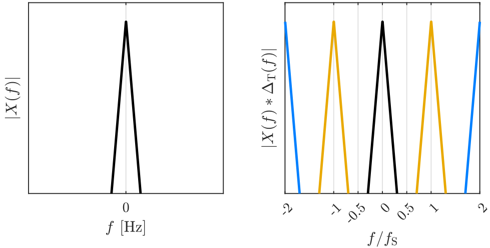
sampling and quantization
sampling

$$x(t) \mapsto X(\mathrm{j}\omega)$$

matlab source: plotSpecSampling.m

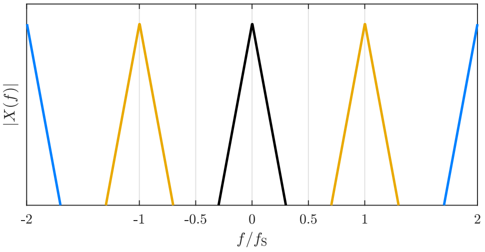
sampling and quantization
sampling

$$x(t) \mapsto X(\mathrm{j}\omega)$$
$$x(t) \cdot \delta_T \mapsto X(\mathrm{j}\omega) * \delta_{\omega_T}$$



matlab source: plotSpecSampling.m

sampling and quantization
sampling



matlab source: matlab/animateAliasing.m

sampling and quantization
sampling theorem

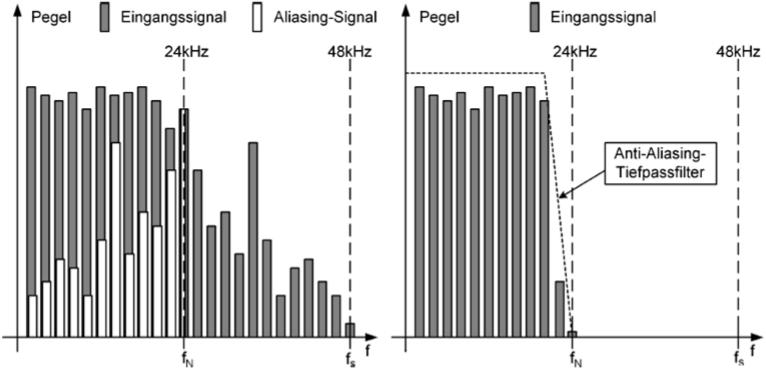
sampling theorem

A sampled audio signal can be reconstructed **without loss of information** if the sample rate f_S is higher than twice the bandwidth f_max of the signal.

$$f_\mathrm{S} > 2 \cdot f_\mathrm{max}$$

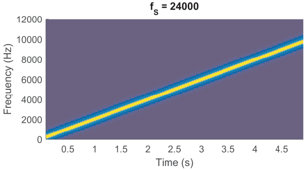
sampling and quantization

sampling: aliasing



sampling and quantization
sampling: aliasing examples 1/2

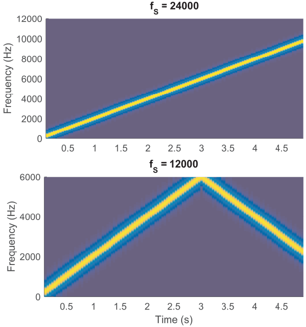
audio example: sinesweep 100–10k at 24, 12, 6k



sampling and quantization

sampling: aliasing examples 1/2

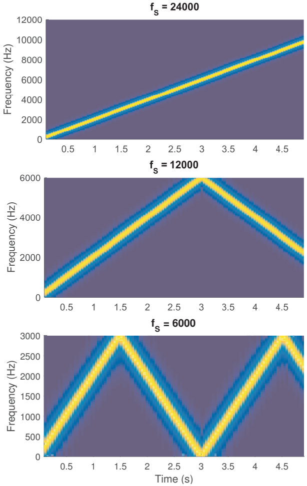
audio example: sinesweep 100–10k at 24, 12, 6k



sampling and quantization

sampling: aliasing examples 1/2

audio example: sinesweep 100–10k at 24, 12, 6k



sampling and quantization
sampling: aliasing examples 2/2

- **bigband**
  - original (48 kHz):
  - samples discarded (6 kHz):
  - downsampling w/ anti-aliasing filter (6 kHz):

sampling and quantization

sampling: aliasing examples 2/2

- **bigband**
  - original (48 kHz):
  - samples discarded (6 kHz):
  - downsampling w/ anti-aliasing filter (6 kHz):
- **sax**
  - original (48 kHz):
  - samples discarded (6 kHz):
  - downsampling w/ anti-aliasing filter (6 kHz):

sampling and quantization
sampling: summary 1/2

continuous input signal

↓

1. **anti-aliasing filter**

filtered continuous input signal

2. **sampling**

sampled input signal

3. **reconstruction filter**

↓

continuous output signal

sampling and quantization

sampling: summary 1/2

continuous input signal

↓

1. **anti-aliasing filter**

filtered continuous input signal

2. **sampling**

sampled input signal

3. **reconstruction filter**

↓

continuous output signal

sampling and quantization

sampling: summary 1/2

continuous input signal

↓

1. **anti-aliasing filter**

   filtered continuous input signal

2. **sampling**

   sampled input signal

3. **reconstruction filter**

   ↓

   continuous output signal

sampling and quantization

sampling: summary 1/2

continuous input signal

↓

1. **anti-aliasing filter**

filtered continuous input signal

2. **sampling**

sampled input signal

3. **reconstruction filter**

↓

continuous output signal

sampling and quantization
sampling: summary 1/2

continuous input signal

$\downarrow$

1. **anti-aliasing filter**

   filtered continuous input signal

2. **sampling**

   sampled input signal

3. **reconstruction filter**

$\downarrow$

continuous output signal

sampling and quantization
sampling: summary 1/2

continuous input signal

↓

1. **anti-aliasing filter**

filtered continuous input signal

2. **sampling**

sampled input signal

3. **reconstruction filter**

↓

continuous output signal

sampling and quantization
sampling: summary 1/2

continuous input signal

↓

1. **anti-aliasing filter**

   filtered continuous input signal

2. **sampling**

   sampled input signal

3. **reconstruction filter**

↓

continuous output signal

sampling and quantization
sampling: summary 2/2

- **sampling theorem**

  A sampled audio signal can be reconstructed without loss of information if the sample rate f_S is higher than twice the bandwidth $f\,max$ of the signal.

  - perfect reconstruction!
  - ensure accordance through filtering, otherwise aliasing (mirror frequencies)

- band of interest does not have to be base band $(0 \ldots f_S/2)$, but any band $(k \cdot f_S/2 \ldots (k+1) \cdot f_S/2)$ as long as the bandwidth is not wider

sampling and quantization
sampling: summary 2/2

- **sampling theorem**

  A sampled audio signal can be reconstructed without loss of information if the sample rate f_S is higher than twice the bandwidth f max of the signal.

  - perfect reconstruction!
  - ensure accordance through filtering, otherwise aliasing (mirror frequencies)

- band of interest does not have to be base band $(0 \ldots f_\mathrm{S}/2)$, but any band $(k \cdot f_\mathrm{S}/2 \ldots (k+1) \cdot f_\mathrm{S}/2)$ as long as the bandwidth is not wider

sampling and quantization
sampling: summary 2/2

- **sampling theorem**

  A sampled audio signal can be reconstructed without loss of information if the sample rate f_S is higher than twice the bandwidth f max of the signal.

  - perfect reconstruction!
  - ensure accordance through filtering, otherwise aliasing (mirror frequencies)

- band of interest does not have to be base band $(0 \ldots f_\mathrm{S}/2)$, but any band $(k \cdot f_\mathrm{S}/2 \ldots (k+1) \cdot f_\mathrm{S}/2)$ as long as the bandwidth is not wider