

Digital Signal Processing for Music

Part 3: Signals

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introduction

sound

- sound is a vibration propagating through a medium
- vibrating source excites medium and vibration is received by microphone/ear
- microphone converts sound pressure (velocity) into electrical voltage
- the vibration/oscillation at each of these steps is a **signal**
- here, we are mostly interested in the electrical signal
- audio signal
 - representation of sound (speech, music, etc.)
 - main frequency content is below 12 kHz

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audio signals

categorization

- **deterministic signals:**

predictable: future shape of the signal can be known (example: sinusoidal)

- **random signals:**

unpredictable: no knowledge can help to predict what is coming next (example: white noise)

Every “real-world” audio signal can be modeled as a time-varying combination of

- (quasi-)periodic parts
- (quasi-)random parts

audio signals

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signals

properties of real-world signals

■ real-valued:

- real-world signals are usually real-valued.

■ finite:

- amplitude: $\max|x(t)| < \infty$
- energy or power:

$$E = \int_{-\infty}^{\infty} x^2(t) dt$$

$$P = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T x^2(t) dt$$

■ smooth:

- no “abrupt” changes \rightarrow finite bandwidth

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audio signals

periodic signals 1/3

periodic signals most prominent examples of deterministic signals:

$$x(t) = x(t + T_0)$$

$$f_0 = \frac{1}{T_0}$$

$$\omega_0 = \frac{2\pi}{T_0}$$

audio signals

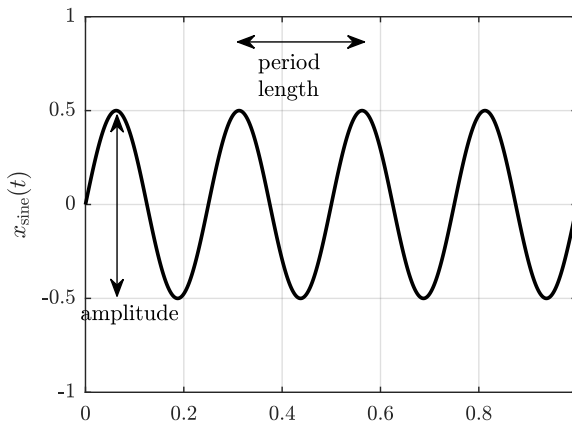
periodic signals 1/3

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audio signals

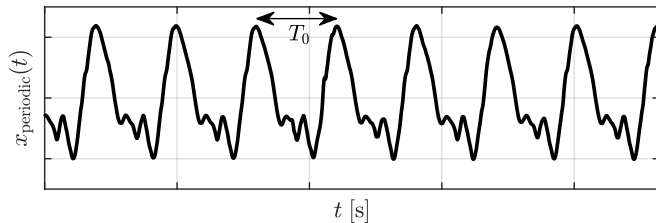
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audio signals

periodic signals 2/3

reconstruction

periodic signals can be reconstructed through a sum of sinusoids at frequencies $k \cdot \omega_0$

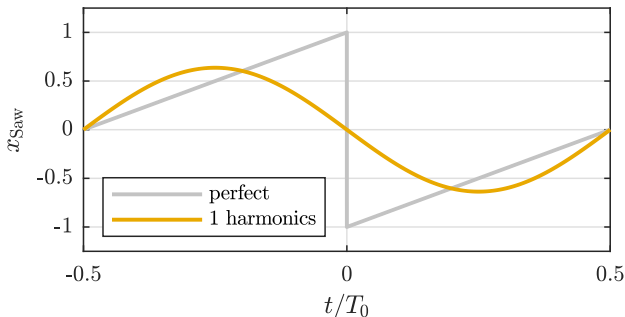
audio signals

periodic signals 2/3

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periodic signals can be reconstructed through a sum of sinusoidals at frequencies $k \cdot \omega_0$

$$\hat{x}(t) = a_1 \cdot \sin(\omega_0 t)$$



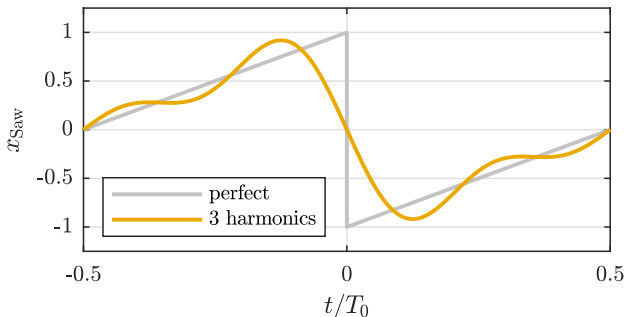
audio signals

periodic signals 2/3

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$$\hat{x}(t) = a_1 \cdot \sin(\omega_0 t) + a_2 \cdot \sin(2 \cdot \omega_0 t) + \dots + a_3 \cdot \sin(3 \cdot \omega_0 t)$$



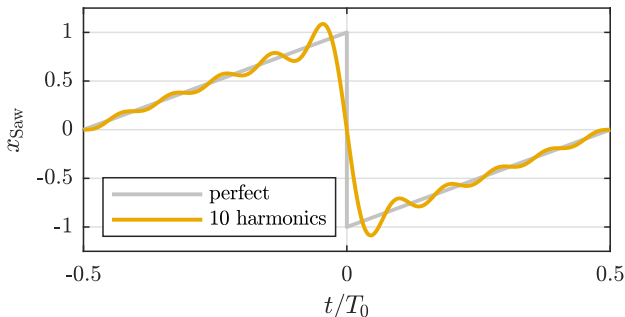
audio signals

periodic signals 2/3

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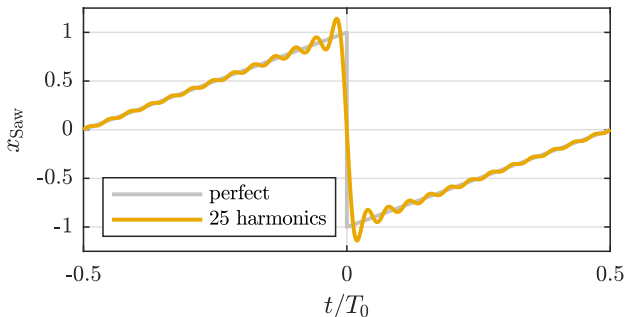
audio signals

periodic signals 2/3

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$$\hat{x}(t) = a_1 \cdot \sin(\omega_0 t) + a_2 \cdot \sin(2 \cdot \omega_0 t) + \dots + a_{25} \cdot \sin(25 \cdot \omega_0 t)$$



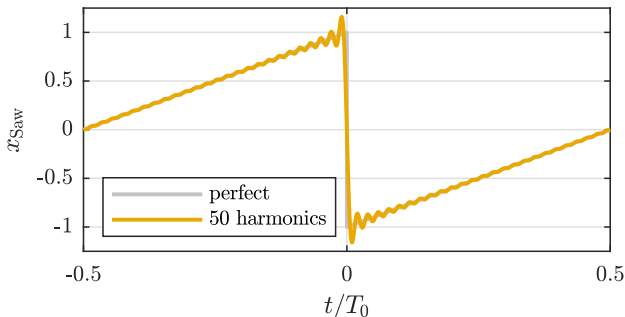
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periodic signals 2/3

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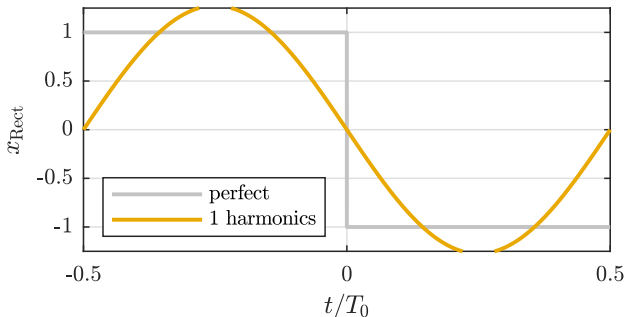
audio signals

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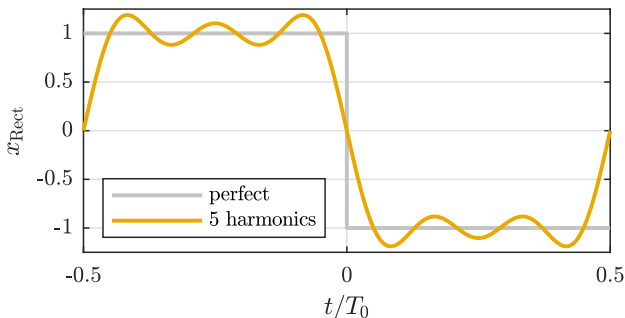
audio signals

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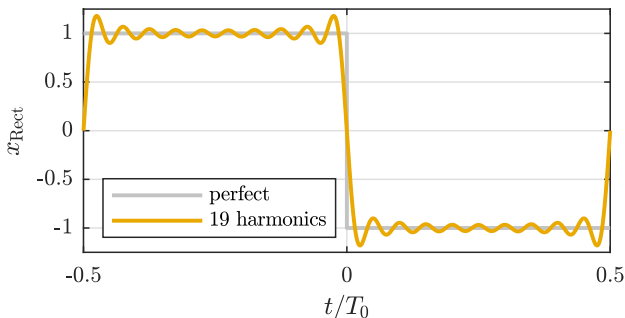
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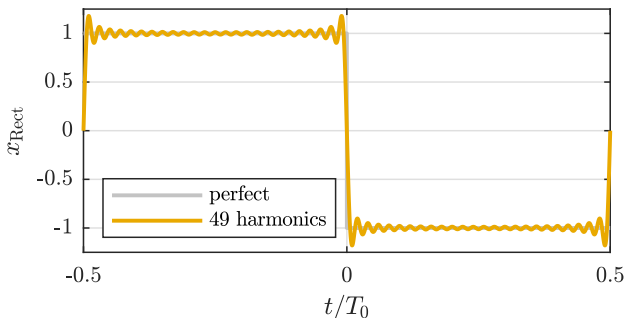
audio signals

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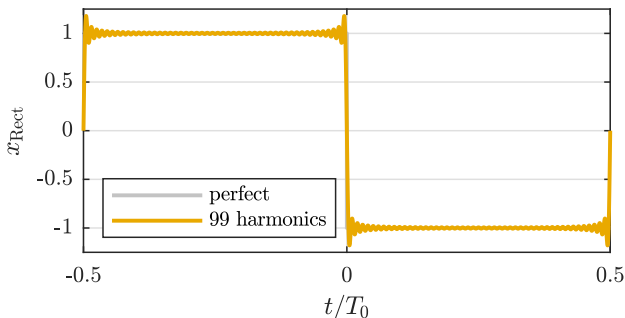
audio signals

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audio signals

periodic signals 3/3

youtube — mechanical additive synthesis:

http://youtu.be/8KmVDxkia_w

audio signals

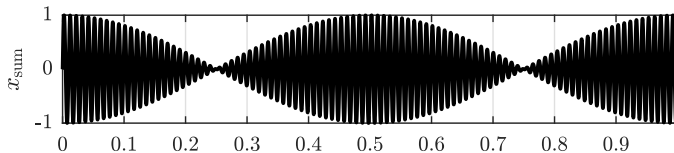
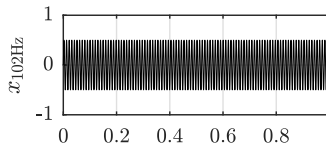
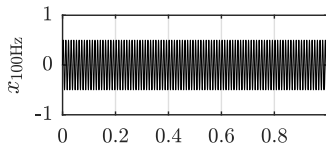
superposition of sinusoids 1/2

- **partials:**
a set of frequencies comprising a (pitched) sound
- **overtones:**
as partials but without the fundamental frequency
- **harmonics:**
integer multiples of the fundamental frequency, including the fundamental frequency

audio signals

superposition of sinusoids 2/2

$$\begin{aligned} y(t) &= \underbrace{\sin\left(2\pi\left(f + \frac{\Delta f}{2}\right)t\right)}_{\sin(2\pi f) \cos\left(2\pi t \frac{\Delta f}{2}\right) + \cos(2\pi f) \sin\left(2\pi t \frac{\Delta f}{2}\right)} + \underbrace{\sin\left(2\pi\left(f - \frac{\Delta f}{2}\right)t\right)}_{\sin(2\pi f) \cos\left(-2\pi t \frac{\Delta f}{2}\right) + \cos(2\pi f) \sin\left(-2\pi t \frac{\Delta f}{2}\right)} \\ &= 2 \sin(2\pi f) \cdot \cos\left(2\pi \frac{\Delta f}{2} t\right) \end{aligned}$$












audio signals

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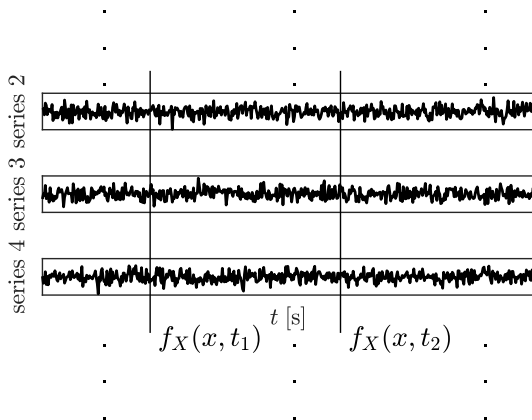
audio examples: addition of sines

500Hz	+1000	+750	+667	+625	+600	+530	+502	+501
								

audio signals

random process

random process: ensemble of random series



special cases:

- **stationarity:**
all parameters (such as the mean) are time invariant
- **ergodicity:**
process with equal time and ensemble mean (implies stationarity)

deterministic prototype signals

periodic signals

■ sinusoidal

$$x(t) = \sin(\underbrace{2\pi f t}_{\omega} + \Phi)$$

■ sawtooth

$$x(t) = 2 \left(\frac{t}{T_0} - \text{floor} \left(\frac{1}{2} + \frac{t}{T_0} \right) \right)$$

■ square wave

$$x(t) = \text{sign}(\sin(\omega t))$$

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deterministic prototype signals

non-periodic deterministic signals

■ DC

$$x(t) = 1$$

■ impulse

$$\delta(t) = \begin{cases} \infty & t = 0 \\ 0 & t \neq 0 \end{cases}$$

$$\delta(t) = \int_{-\infty}^{\infty} \delta(t) dt = 1$$

■ exponential

$$x(t) = e^{-\alpha t}$$

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audio signals

summary

- two basic signal classes, **deterministic** and **random**
- *deterministic* signals can be described by a function and are predictable
 - special case: periodic signals — sum of sinusoidals with freq. integer ratio
- *random signals* are not predictable
 - special case: ergodic signals can be described statistically