Digital Signal Processing for Music

Part 10: Discretization 1—Sampling

alexander lerch

Georgia Tech Center for Music Technology

sampling and quantization
introduction

digital signals can only be represented with a limited number of values

⇒

- time discretization:
  sampling

- amplitude discretization:
  quantization

sampling and quantization
introduction

digital signals can only be represented with a limited number of values

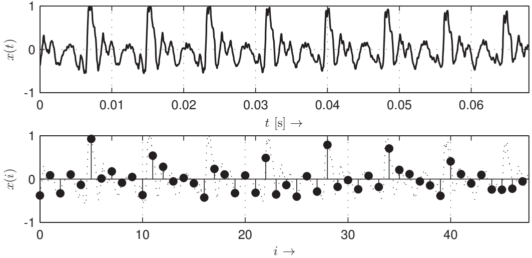
$\Rightarrow$

- time discretization:
  sampling

- amplitude discretization:
  quantization

sampling and quantization
sampling

$$T_S = \frac{1}{f_S}$$

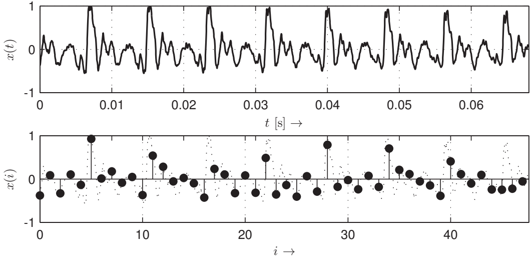


typical sample rates

- 8-16 kHz: speech (phone)
- 44.1-48 kHz: (consumer) audio/music
- higher: production audio

sampling and quantization
sampling

$$T_S = \frac{1}{f_S}$$

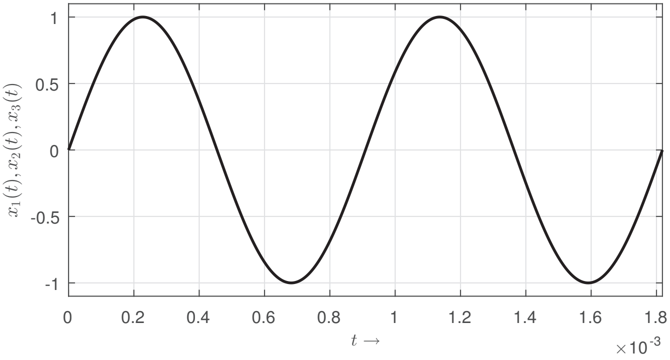


typical sample rates

- 8-16 kHz: speech (phone)
- 44.1-48 kHz: (consumer) audio/music
- higher: production audio

sampling and quantization

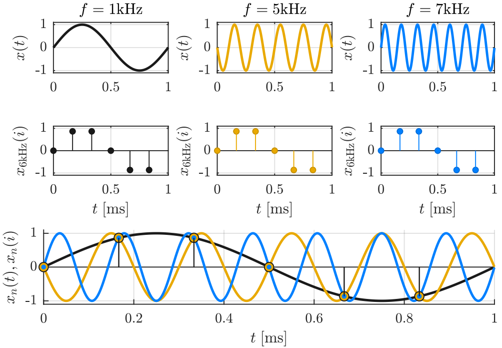
sampling ambiguity 1/4



sampling and quantization

sampling ambiguity 2/4

$$
\begin{aligned}
f_0 &= [1, 5, 7\,\mathrm{kHz}] \\
f_S &= 6\,\mathrm{kHz}
\end{aligned}
$$



matlab source: plotSampling02.m

sampling and quantization

sampling ambiguity 3/4

wagon wheel effect

sampling and quantization

sampling ambiguity 3/4

Georgia Tech | Center for Music Technology
College of Design

wagon wheel effect

1. $f_{wheel} < \frac{f_S}{2}$: speeding up
2. $\frac{f_S}{2} < f_{wheel} < f_S$: slowing down
3. $f_{wheel} = f_S$: standing still
4. ...

sampling and quantization
sampling ambiguity 3/4

wagon wheel effect

1. $f_{wheel} < \frac{f_S}{2}$: speeding up
2. $\frac{f_S}{2} < f_{wheel} < f_S$: slowing down
3. $f_{wheel} = f_S$: standing still
4. ...

sampling and quantization
sampling ambiguity 3/4

wagon wheel effect

1. $f_{wheel} < \frac{f_S}{2}$: speeding up
2. $\frac{f_S}{2} < f_{wheel} < f_S$: slowing down
3. $f_{wheel} = f_S$: standing still
4. $\ldots$

sampling and quantization

sampling ambiguity 4/4

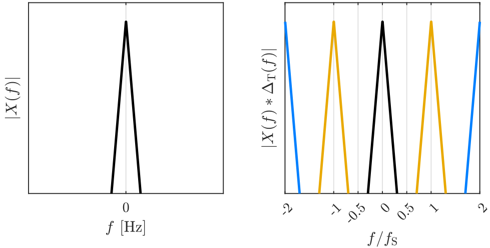
http://youtu.be/uENITui5_jU

sampling and quantization
sampling

$$x(t) \mapsto X(\mathrm{j}\omega)$$

matlab source: plotSpecSampling.m

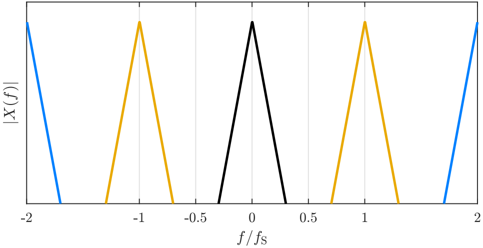
$$x(t) \mapsto X(\mathrm{j}\omega)$$
$$x(t) \cdot \delta_T \mapsto X(\mathrm{j}\omega) * \delta_{\omega_T}$$



matlab source: plotSpecSampling.m

sampling and quantization

sampling



matlab source: matlab/animateAliasing.m

sampling and quantization

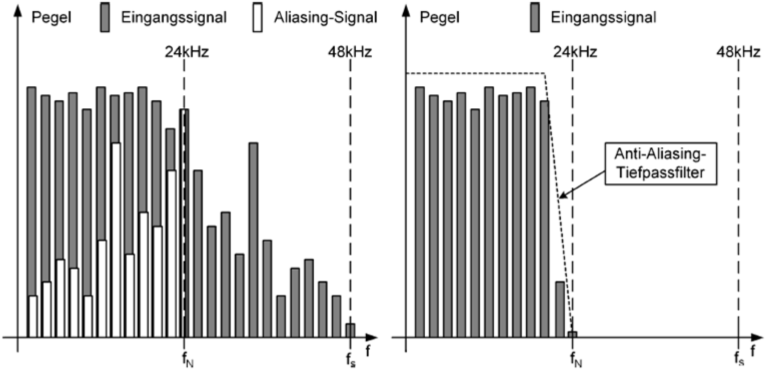
sampling theorem

sampling theorem

A sampled audio signal can be reconstructed **without loss of information** if the sample rate f_S is higher than twice the bandwidth f_max of the signal.

$$f_\mathrm{S} > 2 \cdot f_\mathrm{max}$$

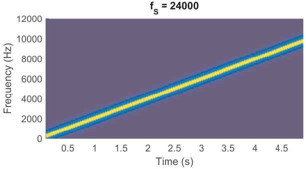
sampling and quantization
sampling: aliasing



sampling and quantization

sampling: aliasing examples 1/2

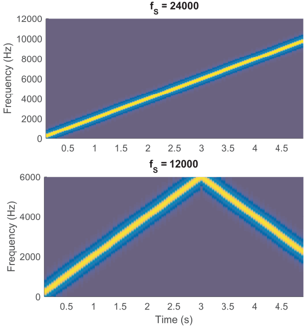
audio example: sinesweep 100–10k at 24, 12, 6k



sampling and quantization

sampling: aliasing examples 1/2

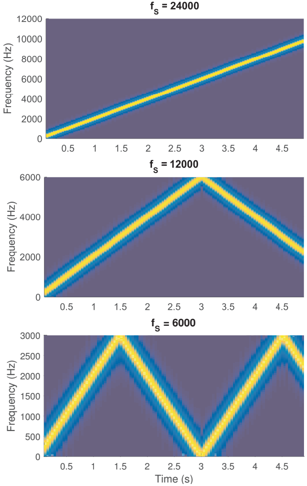
audio example: sinesweep 100–10k at 24, 12, 6k



sampling and quantization

sampling: aliasing examples 1/2

audio example: sinesweep 100–10k at 24, 12, 6k



sampling and quantization

sampling: aliasing examples 2/2

- **bigband**
  - original (48 kHz):
  - samples discarded (6 kHz):
  - downsampling w/ anti-aliasing filter (6 kHz):

sampling and quantization

sampling: aliasing examples 2/2

- **bigband**
  - original (48 kHz):
  - samples discarded (6 kHz):
  - downsampling w/ anti-aliasing filter (6 kHz):
- **sax**
  - original (48 kHz):
  - samples discarded (6 kHz):
  - downsampling w/ anti-aliasing filter (6 kHz):

sampling and quantization
sampling: summary 1/2

continuous input signal

↓

1. **anti-aliasing filter**

   filtered continuous input signal

2. **sampling**

   sampled input signal

3. **reconstruction filter**

↓

continuous output signal

sampling and quantization
sampling: summary 1/2

continuous input signal

↓

1. **anti-aliasing filter**

filtered continuous input signal

2. **sampling**

sampled input signal

3. **reconstruction filter**

↓

continuous output signal

sampling and quantization
sampling: summary 1/2

continuous input signal

↓

1. **anti-aliasing filter**

   filtered continuous input signal

2. **sampling**

   sampled input signal

3. **reconstruction filter**

   ↓

   continuous output signal

sampling and quantization
sampling: summary 1/2

continuous input signal

↓

1. **anti-aliasing filter**

filtered continuous input signal

2. **sampling**

sampled input signal

3. **reconstruction filter**

↓

continuous output signal

sampling and quantization
sampling: summary 1/2

continuous input signal

↓

1. **anti-aliasing filter**

   filtered continuous input signal

2. **sampling**

   sampled input signal

3. **reconstruction filter**

   ↓

   continuous output signal

sampling and quantization
sampling: summary 1/2

continuous input signal

↓

1. **anti-aliasing filter**

   filtered continuous input signal

2. **sampling**

   sampled input signal

3. **reconstruction filter**

↓

continuous output signal

sampling and quantization

sampling: summary 1/2

continuous input signal

↓

1. **anti-aliasing filter**

   filtered continuous input signal

2. **sampling**

   sampled input signal

3. **reconstruction filter**

↓

continuous output signal

sampling and quantization

sampling: summary 2/2

- **sampling theorem**

  A sampled audio signal can be reconstructed without loss of information if the sample rate f_S is higher than twice the bandwidth f max of the signal.

  - perfect reconstruction!
  - ensure accordance through filtering, otherwise aliasing (mirror frequencies)

- band of interest does not have to be base band $(0 \ldots f_S/2)$, but any band $(k \cdot f_S/2 \ldots (k+1) \cdot f_S/2)$ as long as the bandwidth is not wider

sampling and quantization
sampling: summary 2/2

- **sampling theorem**

  A sampled audio signal can be reconstructed without loss of information if the sample rate f_S is higher than twice the bandwidth $f\,max$ of the signal.

  - perfect reconstruction!
  - ensure accordance through filtering, otherwise aliasing (mirror frequencies)

- band of interest does not have to be base band $(0 \ldots f_\mathrm{S}/2)$, but any band $(k \cdot f_\mathrm{S}/2 \ldots (k+1) \cdot f_\mathrm{S}/2)$ as long as the bandwidth is not wider

sampling and quantization

sampling: summary 2/2

- **sampling theorem**

  A sampled audio signal can be reconstructed without loss of information if the sample rate f_S is higher than twice the bandwidth f max of the signal.

  - perfect reconstruction!
  - ensure accordance through filtering, otherwise aliasing (mirror frequencies)

- band of interest does not have to be base band $(0 \ldots f_\mathrm{S}/2)$, but any band $(k \cdot f_\mathrm{S}/2 \ldots (k+1) \cdot f_\mathrm{S}/2)$ as long as the bandwidth is not wider

Temporary page!

LaTeX was unable to guess the total number of pages correctly. As there was some unprocessed data that should have been added to the final page this extra page has been added to receive it.

If you rerun the document (without altering it) this surplus page will go away, becau

LaTeX now knows how many pages to expect for this document.