Digital Signal Processing for Music

Part 3: Signals

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introduction
sound

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- sound is a vibration propagating through a medium
- vibrating source excites medium and vibration is received by microphone/ear
- microphone converts sound pressure (velocity) into electrical voltage

- the vibration/oscillation at each of these steps is a **signal**
- here, we are mostly interested in the electrical signal

- audio signal
  - representation of sound (speech, music, etc.)
  - main frequency content is below 12 kHz

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audio signals
categorization

- **deterministic signals**:
  predictable: future shape of the signal can be known (example: sinusoidal)
- **random signals**:
  unpredictable: no knowledge can help to predict what is coming next (example: white noise)

Every "real-world" audio signal can be modeled as a time-varying combination of

- (quasi-)periodic parts
- (quasi-)random parts

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signals

properties of real-world signals

- **real-valued**:
  - real-world signals are usually real-valued.
- **finite**:
  - amplitude: $max|x(t)| < \infty$
  - energy or power:

$$E = \int_{-\infty}^{\infty} x^2(t)dt$$

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x^2(t)dt$$

- **smooth**:
  - no "abrupt" changes → finite bandwidth

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audio signals
periodic signals 1/3

periodic signals most prominent examples of deterministic signals:

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\begin{aligned}
x(t) &= x(t + T_0) \\
f_0 &= \frac{1}{T_0} \\
\omega_0 &= \frac{2\pi}{T_0}
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audio signals
periodic signals 1/3

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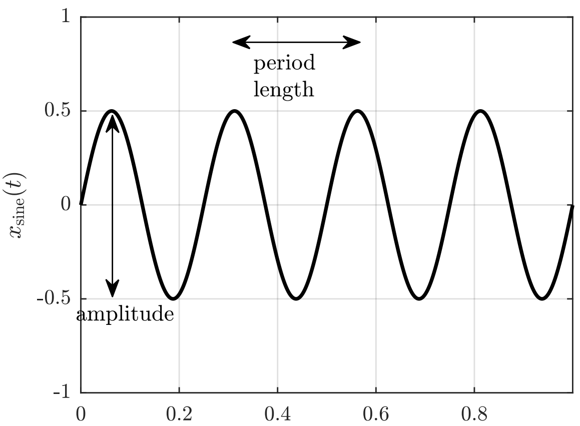

matlab source: plotSine.m

audio signals
periodic signals 1/3

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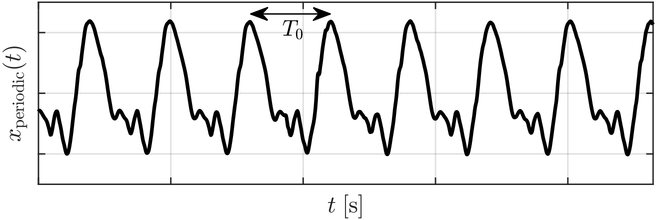


matlab source: plotPeriodic.m

audio signals
periodic signals 2/3

reconstruction

periodic signals can be reconstructed through a sum of sinusoidals at frequencies $k \cdot \omega_0$

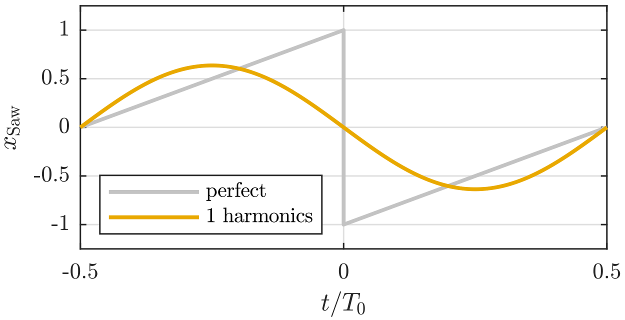
audio signals

periodic signals 2/3

reconstruction

periodic signals can be reconstructed through a sum of sinusoidals at frequencies $k \cdot \omega_0$

$$\hat{x}(t) = a_1 \cdot sin(\omega_0 t)$$



matlab source: plotAdditiveSynthesis.m

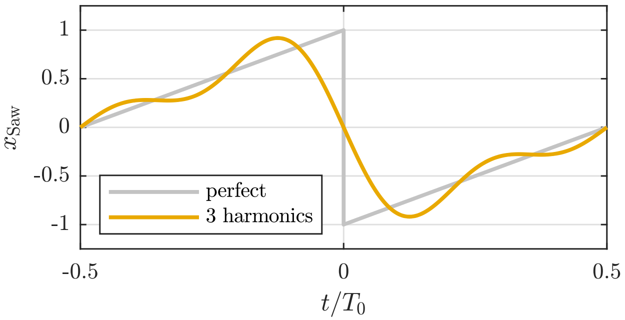
audio signals

periodic signals 2/3

reconstruction

periodic signals can be reconstructed through a sum of sinusoidals at frequencies $k \cdot \omega_0$

$$\hat{x}(t) = a_1 \cdot sin(\omega_0 t) + a_2 \cdot sin(2 \cdot \omega_0 t) + \ldots + a_3 \cdot sin(3 \cdot \omega_0 t)$$



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audio signals
periodic signals 2/3

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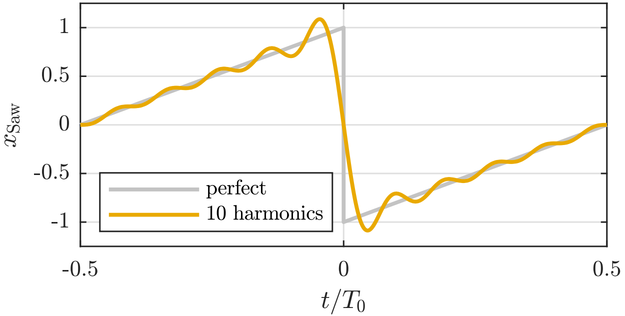

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audio signals

periodic signals 2/3

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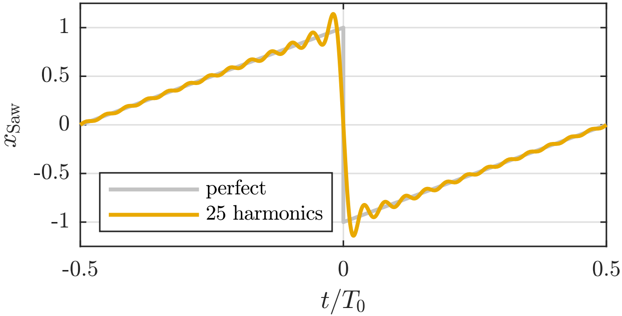
$$\hat{x}(t) = a_1 \cdot sin(\omega_0 t) + a_2 \cdot sin(2 \cdot \omega_0 t) + \ldots + a_{25} \cdot sin(25 \cdot \omega_0 t)$$



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audio signals

periodic signals 2/3

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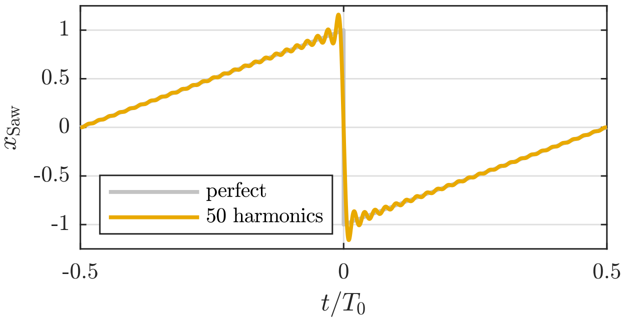
$$\hat{x}(t) = a_1 \cdot sin(\omega_0 t) + a_2 \cdot sin(2 \cdot \omega_0 t) + \ldots + a_{50} \cdot sin(50 \cdot \omega_0 t)$$



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audio signals

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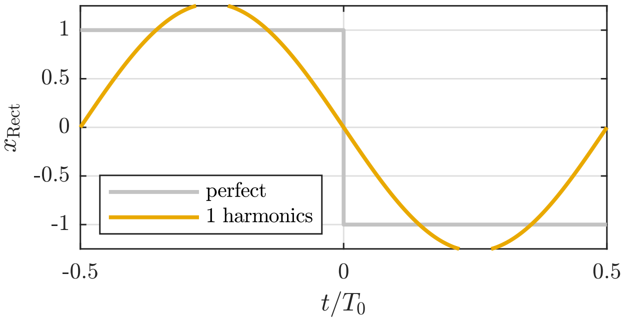

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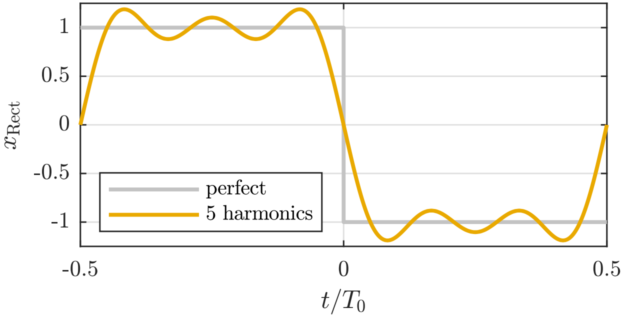

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audio signals

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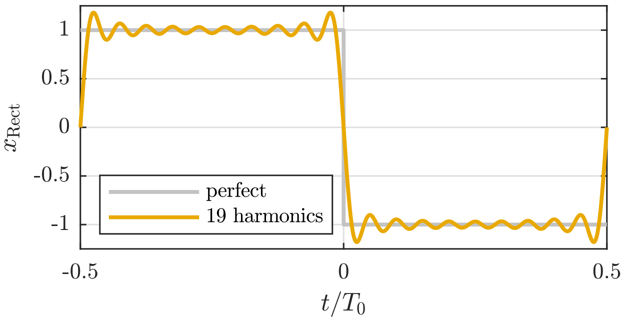
$$\hat{x}(t) = a_1 \cdot sin(\omega_0 t) + a_3 \cdot sin(3 \cdot \omega_0 t) + \ldots + a_{19} \cdot sin(19 \cdot \omega_0 t)$$



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audio signals

periodic signals 2/3

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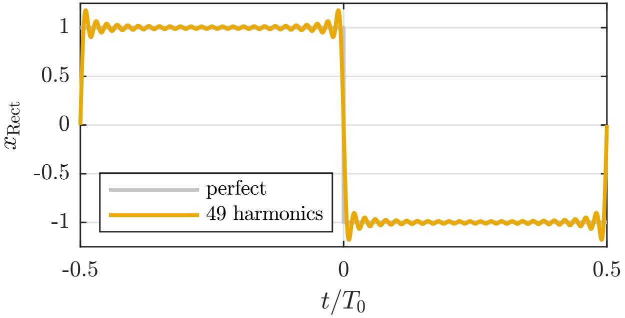
$$\hat{x}(t) = a_1 \cdot sin(\omega_0 t) + a_3 \cdot sin(3 \cdot \omega_0 t) + \ldots + a_{49} \cdot sin(49 \cdot \omega_0 t)$$



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audio signals

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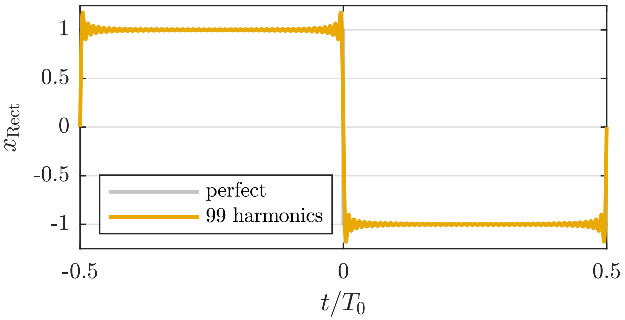


matlab source: plotAdditiveSynthesis.m

audio signals
periodic signals 3/3

youtube — mechanical additive synthesis:
http://youtu.be/8KmVDxkia_w

audio signals

superposition of sinusoidals 1/2

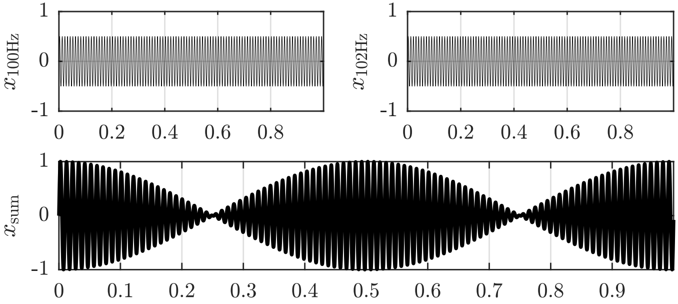
- **partials**:
  a set of frequencies comprising a (pitched) sound
- **overtones**:
  as partials but without the fundamental frequency
- **harmonics**:
  integer multiples of the fundamental frequency, including the fundamental frequency

audio signals

superposition of sinusoidals 2/2

$$y(t) = \underbrace{\sin\left(2\pi(f + \frac{\Delta f}{2})t\right)}_{\sin(2\pi f)\cos\left(2\pi t\frac{\Delta f}{2}\right)+\cos(2\pi f)\sin\left(2\pi t\frac{\Delta f}{2}\right)} + \underbrace{\sin\left(2\pi(f - \frac{\Delta f}{2})t\right)}_{\sin(2\pi f)\cos\left(-2\pi t\frac{\Delta f}{2}\right)+\cos(2\pi f)\sin\left(-2\pi t\frac{\Delta f}{2}\right)}$$

$$= 2\sin(2\pi f)\cdot\cos\left(2\pi\frac{\Delta f}{2}t\right)$$



matlab source: plotBeating.m

audio signals

superposition of sinusoidals 2/2

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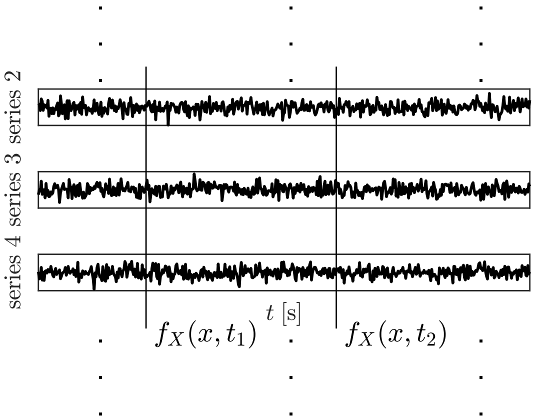
&= 2\sin(2\pi f)\cdot\cos\left(2\pi\frac{\Delta f}{2}t\right)
\end{aligned}
$$

audio examples: addition of sines

500Hz	+1000	+750	+667	+625	+600	+530	+502	+501

random process: ensemble of random series



special cases:

- **stationarity**:
  all parameters (such as the mean) are time invariant
- **ergodicity**:
  process with equal time and ensemble mean (implies stationarity)

matlab source: plotRandomProcess.m

deterministic prototype signals

periodic signals

- **sinusoidal**

$$x(t) = \sin(\underbrace{2\pi f}_{\omega} t + \Phi)$$

- **sawtooth**

$$x(t) = 2\left(\frac{t}{T_0} - \text{floor}\left(\frac{1}{2} + \frac{t}{T_0}\right)\right)$$

- **square wave**

$$x(t) = \text{sign}\left(\sin(\omega t)\right)$$

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non-periodic deterministic signals

- **DC**

$$x(t) = 1$$

- **impulse**

$$\delta(t) = \begin{cases} \infty & t = 0 \\ 0 & t \neq 0 \end{cases}$$

$$\delta(t) = \int\limits_{-\infty}^{\infty} \delta(t)dt = 1$$

- **exponential**

$$x(t) = e^{-\alpha t}$$

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- two basic signal classes, **deterministic** and **random**
- *deterministic* signals can be described by a function and are predictable
  - special case: periodic signals — sum of sinusoidals with freq. integer ratio
- *random signals* are not predictable
  - special case: ergodic signals can be described statistically