

Digital Signal Processing for Music

Part 5: Signal Similarity — Correlation

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signal similarity

introduction

■ correlation function

- indicates (linear) dependencies between two signals
- shifts the signals to find the dependency for each shift in time

■ lecture content

- (cross) correlation function (CCF)
- auto correlation function (ACF)

signal similarity

correlation function

compute similarity between two **stationary** signals x, y

$$r_{xy}(\tau) = \mathcal{E}\{x(t)y(t + \tau)\}$$

■ continuous:

$$r_{xy}(\tau) = \int_{-\infty}^{\infty} x(t) \cdot y(t + \tau) dt$$

■ discrete:

$$r_{xy}(\eta) = \sum_{i=-\infty}^{\infty} x(i) \cdot y(i + \eta)$$

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signal similarity

correlation function: animation

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signal similarity

correlation function: use cases

- find (linear!) similarity between two signals (e.g., clean and noisy)
- find time shift between two similar signals
- example: **radar**
 - correlate sent signal with received signal
 - pick maximum location and convert to distance of object

signal similarity

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signal similarity

normalization & correlation coefficient

- normalization

$$r_{xy}(\tau) = \frac{\mathcal{E}\{(X - \mu_X)(Y - \mu_Y)\}}{\sigma_X \sigma_Y}$$

- special case: **Pearson Correlation Coefficient** $r_{xy}(0)$ after normalization

What are possible reasons for normalization



signal similarity

normalization & correlation coefficient

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What are possible reasons for normalization



- ensuring that function will always be between -1 and 1
- shifting and scaling one signal will not change the coefficient

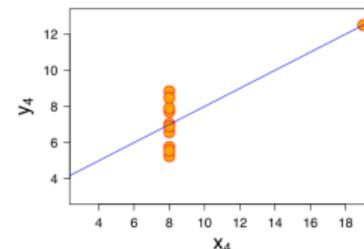
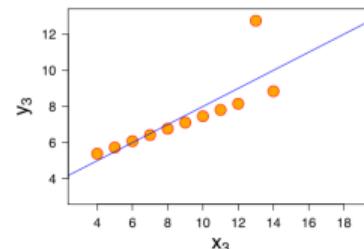
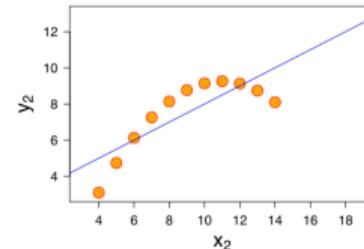
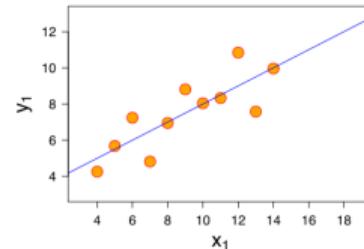
signal similarity

correlation function: problems as summary statistic

Anscombe's quartet:

■ descriptive statistics

- identical mean: 7.5
- identical variance: 4.2
- identical **Pearson correlation coefficient**: 0.816



correlation function

examples 1/2

Describe the (cross) correlation function between the following signals



correlation function

examples 1/2

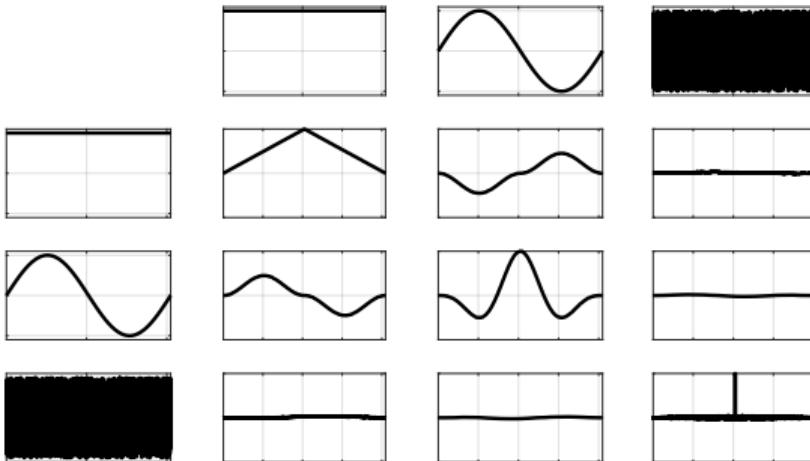
Describe the (cross) correlation function between the following signals



- rectangular window vs.
- windowed sine vs.
- noise

correlation function

examples 2/2



introduction
o

CCF
ooo

correlation coefficient
o

possible issues
o

examples
ooo●

ACF
o

summary
o

correlation function

blocked correlation: animation

Georgia Tech | Center for Music Technology
College of Design



matlab source: [matlab/animateBlockedCorrelation.m](#)

correlation function

auto correlation function

$$r_{xx}(\tau) = \mathcal{E}\{x(t)x(t + \tau)\}$$

autocorrelation function properties

- **power:** $r_{xx}(0) = \mathcal{E}\{X^2\}$
- **symmetry** $r_{xx}(\tau) = r_{xx}(-\tau)$
(substitute $t = t' + \tau$)
- **global max:** $r_{xx}(\tau) \leq r_{xx}(0)$
- **periodicity:**
The ACF of a periodic signal is periodic (period length of input signal)

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correlation function

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- correlation function is useful tool to
 - *determine the similarity* between two signals (CCF)
 - *identify a shift/latency* between two similar signals (CCF)
 - *identify periodicity* vs. noisiness in a signal (ACF)
- continues to be standard approach for all applications related to the above tasks
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