Digital Signal Processing for Music

Part 12: Non-linear Quantization

alexander lerch

Georgia Tech Center for Music Technology

non-linear quantization
introduction

we have previously looked at quantization steps that are uniformly distributed over amplitude range:

$\Rightarrow$ all neighboring steps have the same distance from each other on both input and output scale

- this is sometimes referred to as uniform (as opposed to non-linear) quantization (which is confusing as any quantization represents a **nonlinear** system)

but: what if the quantization steps are not uniformly distributed?

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non-linear quantization

introduction: uniform quantization—SNR & PDF

$$SNR = 6.02 \cdot w + c_\mathrm{S} \quad [dB]$$

$\Rightarrow$ c_S depends on signal's PDF (and scaling)

PDF	SNR
square wave	$c_S = 4.8$
sine wave	$c_S = 1.8$
rect	$c_S = 0$
tri	$c_S \approx -3$
Gauss	$c_S \approx -7$
Laplace	$c_S \approx -9$
speech	$c_S \approx -10 \ldots -15$

non-linear quantization

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non-linear quantization
introduction

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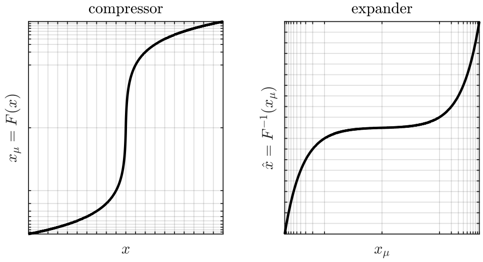
idea: quantize frequent signal values at higher resolution

- approach 1
  1. flatten PDF (companding)
  2. linear quantization
  3. extract signal (expanding)
- approach 2
  1. adapt quantization step size to PDF

$\Rightarrow$ both approaches are equivalent in their result



matlab source: plotmulaw.m

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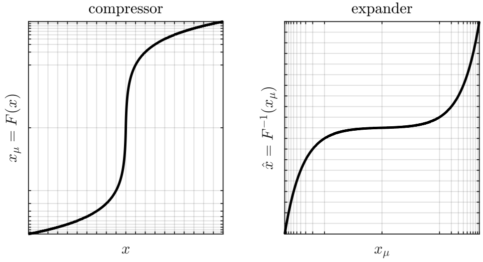

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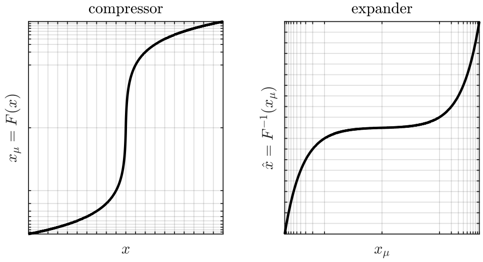

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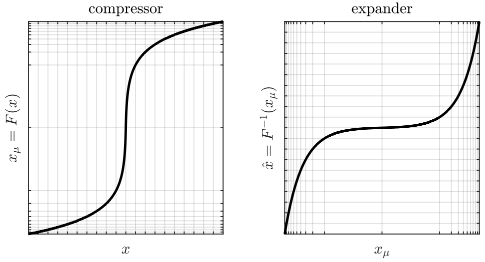

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non-linear quantization

A-Law quantization (ITU-T G.711) 1/3

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$$F(x) = sign(x)\begin{cases} \frac{A|x|}{1+\log(A)}, & |x| \leq \frac{1}{A} \\ \frac{1+\log(A|x|)}{1+\log(A)}, & \frac{1}{A} \leq |x| \leq 1 \end{cases}$$

$$F^{-1}(y) = sign(y)\begin{cases} \frac{|y|(1+\log(A))}{A}, & |y| \leq \frac{1}{1+\log(A)} \\ \frac{\exp\left(|y|(1+\log(A))-1\right)}{A}, & \frac{1}{1+\log(A)} \leq |y| \leq 1 \end{cases}$$

with $A = 87.7$

- linear and high resolution for small amplitudes
- log and increasingly low resolution for high amplitudes

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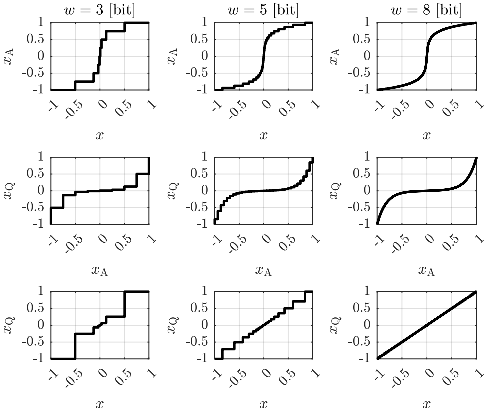
A-Law quantization 2/3



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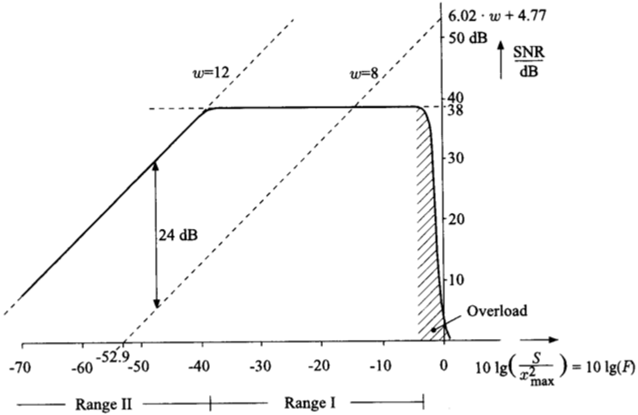
non-linear quantization

A-Law quantization 3/3



- range I: SNR is linear regardless of input level
- range II: SNR increases with input level

non-linear quantization
μ-Law quantization (ITU-T G.711)

$$F(x) = sign(x)\frac{\log(1+\mu|x|)}{\log(1+\mu)}$$

$$F^{-1}(y) = sign(y)\frac{1}{\mu}\left((1+\mu)^{|y|}-1\right)$$

with $\mu = 255$

compared to A-Law:

- higher dynamic range
- higher error at small amplitudes

non-linear quantization

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non-linear quantization
summary

- **advantages** of non-linear quantization
  - takes advantage of non-uniform distribution of input
  - in line with non-linear loudness perception of the ear
  - $\Rightarrow$ similar perceptual quality as higher resolution linear quantization

- **disadvantages**
  - processing not easily implemented in non-linear amplitude space
  - $\Rightarrow$ only used for transmission