Assignment 4 Comprehension

B351 / Q351

October 5, 2022

Note: Please submit this assignment as a single pdf file on canvas. If multiple files are submitted, only one will be graded

On α-β pruning

1. Suppose you have performed α-β pruning on a game tree with horizon h where you used an integer-valued, static evaluation function $f(x)$ to estimate how good the positions at the horizon h are. Now you realized that you made a mistake when computing $f(x)$ and that a better heuristic is actually given by

$$g(x) = \begin{cases} (f(x))^2 & \text{if } f(x) \geq 0 \text{ and} \\ -(f(x))^2 & \text{if } f(x) < 0 \end{cases}.$$

   How does this change in heuristic affect the previous pruning? Carefully justify your answer.

   Solution: (=) This is somewhat of a trick question because the new heuristic is not any "better" than the old one. An answer as short as a sentence can get full credit on this question.

   (a) Recognizes that $f(x) < f(x')$ is equivalent to $g(x) < g(x')$. (5pts)

   (b) Justification (5pts) for this proceeds in a piecewise manner: if $f(x)$ and $f(x')$ are both positive, then $g(x) = f(x)^2$ and $g(x) = f(x')^2$ are both positive, and since f is integer-valued, $f(x) > f(x') \iff f(x)^2 > f(x')^2$, and so forth for other cases.

   (c) Recognizes that the new heuristic function is identical for the purposes of minimax and alpha-beta pruning. (10pts)

   (d) Justification (5pts) includes anything that explains or proves that minimax and alpha-beta pruning depend only on the ordering given by the heuristic.

2. Draw an example of a game tree (this tree does not have to be associated to a particular game) of depth 3 and branching factor 2 where α-β pruning will NOT prune anything.

Solution: (=)

(a) Correct Depth and Branching Factor:
   i. No pruning: 25 points
   ii. 1 leaf pruned: 20 points
   iii. 2 leaves pruned: 15 points
   iv. 3+ leaves pruned: 10 points

(b) Incorrect Depth (d=2)
   i. No pruning: 15 points
   ii. Pruning happens: (10 points)

Mancala

3. Mancala is a two-player, turn-based strategy game where the objective is to collect the most pieces by the end of the game. The Mancala board is made up of two rows of six holes, or pits, each filled with 4 stones. At the ends of each of these pits are two 'Mancalas' or stores which hold all your captured stones. You and your opponent take turns moving stones according to the following rules:

   (a) You can only move the stones on your side
   (b) Each time you move, you pick up all the stones in a pit and distribute them in a in a counterclockwise direction to the following pits
   (c) If the last stone of a move landed in your Mancala, you can move again
   (d) Stones can not leave your Mancala once placed inside
   (e) If the last stone of a move landed on an empty pit on your side and there are some stones in the opposite pit, then the stones in the two pits will be captured to your Mancala

   Many of you might not be familiar with the game, and if so might still be confused about how it is played. We highly suggest you check out the following website and play a couple of practice game to get a grasp on how the game is played.

   https://mancala.playdrift.com/

   Once you have a good understanding of the rules, answer the following question.

Below is an example Mancala board. If you are player 1 (bottom player), what is the best sequence of moves before your opponent's next turn you can make to get the most number of stones in your Mancala this turn?

8	2	4	7	0	6	0	10
	1	0	4	3	3	0	

Solution (Zach):
In this scenario, there are two ways to maximize the number of stones - taking extra turns (last placed stone is in the mancala) and capturing opponent's stones (placing last stone in a blank spot). If these two are independent (as is the case here), the former should be done first. Therefore, the move sequence with the highest immediate return is:
1. Move stones from the pit containing the leftmost 3 (extra turn)
2. Move stone from the pit containing the rightmost 1 (extra turn)
3. Move stones from the pit containing the 4 (extra turn)
4. Move stone from the pit containing the rightmost 1 (extra turn)
5. Move stone from the pit containing the 1 (capturing the 4 stones on the opposing side, along with the 1)
Total gain = 9 stones.

Thinking about a simple game

4. Consider the following game played on a $1 \times N$ board: Two players take turns writing on the board. Each player can place an S or an O, and the first player who produces three consecutive boxes that spell SOS wins the game. For example, if $N = 3$ a possible game is as follows:

   The first player writes an S in the first square, so the game is at position

   S _ _

   The second player places an S in the last square, so the game is at position

   S _ S

   [this is not a very smart move, by the way]

   The first player places an O in the second square, so the game is at position

   S O S

   (so the first player wins).

(a) Suppose that the first player places an S in the first square and that $N = 4$. Was this a good move? Justify your answer.

(b) Who wins the game if $N = 7$? (the possibilities are the first player, the second player, or a draw. You must assume the players are playing to win!) Justify your answer.

Solutions (=)

a) This is a very bad move, player 2 will always win by just placing an S on the far right empty slot forcing player 1 no matter what their move to lose
b) Player 1 will always win on this board as long as they are playing optimally, if they place an S in the middle slot, then they can force player 1 no matter where they go into the same scenario as in part (a)

Bonus question 10%

5. Consider the heuristic function for the 8-puzzle given by: $h(s)$ = sum of permutation inversions. For example, $h(N) = 4+6+3+1+0+2+0+0 = 16$ (there are 4 numbers smaller than 5 that come after 5, 6 numbers smaller than 8 that come after 8, and so on) for the following board configuration N:

5		8
4	2	1
7	3	6

STATE(N)

Is h admissible? Justify your answer with a proof or counterexample.
Solution (=): h is not admissible. For example, for the puzzle:
1 5 2
4 8 3
7 0 6
The sum of permutation inversions is
0 + 3 + 0 +
1 + 3 + 0 +
1 + 0 + 0 =
8
Whereas this can be solved in 5 moves