Assignment 2 Comprehension Solutions

B351 / Q351

October 2, 2022

Please submit to canvas as ONE PDF file. You may type your work using LaTeX or TeX or you may handwrite them neatly. Please make sure you check your file after you submit it. If you can't read it, neither can we. The material covered in this HW can be found in Sections 2.1-2.4 and Sections 3.1-3.5 of our textbook.

1 Problems

1. Convert the following formulas to CNF (25 points)

   (a) $\neg(A \Rightarrow B) \vee (C \Rightarrow A)$

   (b) $(A \Leftrightarrow B) \Rightarrow (\neg A \wedge C)$

   (c) $(A \Rightarrow (D \Rightarrow C)) \Rightarrow (A \Rightarrow (C \Rightarrow D))$

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   Solution for problem 1
   Out of 25 points:
   1 Point - Attempting each formula
   8 Points per formula
   -2 if all Or's, And's, and Negations but not in CNF
   -4 if nowhere close to CNF, for example contains implications
   -2 if not equivalent to original formula
   2 points remain per problem for grader's discretion

   (a) $(A \vee \neg C)$

   (b) $(\neg A \vee \neg B) \wedge (C \vee A \vee B))$

   (c) $(D \vee \neg A \vee \neg C)$

2. Using resolution, prove that $\neg A \vee D, \neg C \Rightarrow \neg D \models A \Rightarrow C$ (25 points)

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   Solution for problem 2

   Step 1: setup the problem with KB $land \neg Q$ (5pts)

   $(\neg A \vee D) \wedge (\neg C \Rightarrow \neg D) \wedge \neg(A \Rightarrow C)$

   Step 2: Convert the setup to CNF (5pts)

   $(\neg A \vee D)_1 \wedge (C \vee \neg D)_2 \wedge (A)_3 \wedge (\neg C)_4$

Step 3: Preform proof by resolution (15pts total: give 13pts for small mistake, 10pts for mostly correct, 5pts or less for significant mistakes)

(a) $res(2,4) = (\neg D)_5$

(b) $res(1,5) = (\neg A)_6$

(c) $res(3,6) = ()_7\square$

3. Consider the following assumptions: (25 points)

   (a) If Zeus were able and willing to prevent evil, then he would so.

   (b) If Zeus were unable to prevent evil, then he would be impotent.

   (c) If he were unwilling to prevent evil, then he would be malevolent.

   (d) Zeus does not prevent evil.

   (e) If Zeus exists, he is neither impotent nor malevolent.

   Let the variables:

   - P: "Zeus is able to prevent evil,"
   - Q: "Zeus is willing to prevent evil,"
   - R: "Zeus prevents evil,"
   - S: "Zeus is impotent,"
   - T: "Zeus is malevolent," and
   - U: "Zeus exists."

   First translate each of the sentences into logic notation, and then prove that Zeus does not exist using resolution.

Solution for problem 3 (25 pts):
Step 1: Write statements (10 pts total)

(a) $(P \wedge Q) \Rightarrow R$ (2 pts)

(b) $\neg P \Rightarrow S$ (2 pts)

(c) $\neg Q \Rightarrow T$ (2 pts)

(d) $\neg R$ (2 pts)

(e) $U \Rightarrow (\neg S \wedge \neg T)$ (2 pts)

Key Concept: showing that U is unsatisfiable, whether as part of the proof or said at the beginning (5 pts)

Step 2: Convert statements to CNF (4 pts total)

- (a) becomes $(\neg P \vee \neg Q) \vee R$ (1 pt)

- (b) becomes $P \vee S$ (1 pt)
- (c) becomes $Q \vee T$ (1 pt)
- (d) remains $\neg R$
- (e) becomes $\neg U \vee \neg (S \vee T)$ a suitable equivalent statement with extra work in the resolution will also work here (1 pt)

Step 3: Resolution (6 pts total: award 5 for small mistake, 4 for mostly correct, 3 or less for significant conceptual errors)

- $(\neg P \vee \neg Q) \vee R$, $\neg R$ yields $\neg P \vee \neg Q$
- $\neg P \vee \neg Q$, $P \vee S$ yields $S \vee \neg Q$
- $S \vee \neg Q$, $Q \vee T$ yields $S \vee T$
- $S \vee T$, $\neg U \vee \neg (S \vee T)$ yields $\neg U$
- $\neg U$, U yields ()

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4. Using the following predicates, write each of the sentences below utilizing first order logic: $Human(x)$: x is a human, $Likes(x, y)$: x likes y, $Dancer(x)$: x is a dancer. Assume the domain is the set of all beings.(25 points)

   (a) Some humans are not dancers.
   (b) All dancers like something that is not a human.
   (c) All dancers like some humans.
   (d) Some humans do not like dancers.

   Solutions :

   (a) $\exists x \; Human(x) \; \wedge \; \neg Dancer(x)$ (5points)
   (b) $\forall x \; Dancer(x) \implies \exists y \; [\; Likes(x, y) \; \wedge \; \neg \, Human(y) \;]$ (7 points)
   (c) $\forall x \; Dancer(x) \implies \exists y \; [\; Likes(x, y) \; \wedge \; Human(y) \;]$ (6 points)
   (d) $\forall x \; \exists y \; Human(y) \; \wedge \; (Dancer(x) \implies \neg Likes(y, x))$ (7 points)

2 Bonus Problems (10%)

1. Prove using resolution that if (5 points)

   $KB = (p \Rightarrow q) \wedge (r \vee s) \wedge (\neg s) \wedge (\neg s \Rightarrow \neg t) \wedge (\neg q \vee s) \wedge (\neg p \wedge r \Rightarrow u) \wedge (w \vee t)$,

   then $KB \models (u \wedge w)$

Solutions :
First convert KB into CNF:
$KB = (\neg p \vee q) \wedge (r \vee s) \wedge (\neg s) \wedge (s \vee \neg t) \wedge (\neg q \vee s) \wedge (p \vee \neg r \vee u) \wedge (w \vee t)$ (2 points).
Then we prove by contradiction to show $KB \wedge \neg(u \wedge w)$ is unsatisfiable (1 point).
Keep using resolution rule, then we can prove it in various ways (2 points).
$(P \vee Q)_1 \wedge (\neg Q \vee R)_2 \wedge (\neg P \vee \neg S \vee T)_3 \wedge (\neg R)_4 \wedge (Q \vee U)_5 \vee (Q \vee S)_6 \wedge (\neg T)_7$

2. Consider the following predicates:

   (a) $Dad(Fa, So)$
   (b) $Alive(So)$
   (c) $\forall x \forall y (Dad(x, y) \Rightarrow Child(y, x))$
   (d) $\forall x \forall y ((Child(x, y) \wedge Alive(x)) \Rightarrow Younger(x, y))$

   Use resolution to prove that $Younger(So, Fa)$. [Hint: The process to convert first-order logic statements into CNF is on page 40 of our book and in Worksheet 3. In this case, Step 2 of that process can be ignored since we don't have any existential quantifier.]

Solutions :
Award half credit for any attempt that is not fully correct.
It is the student's choice whether to apply universal instantiation before or after CNF.
Convert each statement into CNF:

(a) $Dad(Fa, So)$ (no grade)
(b) $Alive(So)$ (no grade)
(c) $\neg Dad(x, y) \vee Child(y, x)$ (.5pt)
(d) $\neg Child(x, y) \vee \neg Alive(x) \vee Younger(x, y)$ (.5pt)

Instantiate (c) and (d) with this particular Father-Son pair: $\neg Dad(Fa, So) \vee Child(So, Fa)$ and $\neg Child(So, Fa) \vee \neg Alive(So) \vee Younger(So, Fa)$ (1pt).
End up with the following KB: $Dad(Fa, So)_1 \wedge Alive(So)_2 \wedge (\neg Dad(Fa, So) \vee Child(So, Fa))_3 \wedge (\neg Child(So, Fa) \vee \neg Alive(So) \vee Younger(So, Fa))_4$ (1pt).
If using contradiction, will also have $\neg Q$: $\neg Younger(So, Fa)_5$
Prove $Younger(So, Fa)$ using resolution (2pt). Example proof without contradiction:
$Res(2, 4) \Rightarrow (\neg Child(So, Fa) \vee Younger(So, Fa))_6$
$Res(1, 3) \Rightarrow Child(So, Fa)_7$
$Res(6, 7) \Rightarrow Younger(So, Fa)_8$
With contradiction, also need $Res(5, 8) \Rightarrow \vee(\varnothing)_9 = f$.