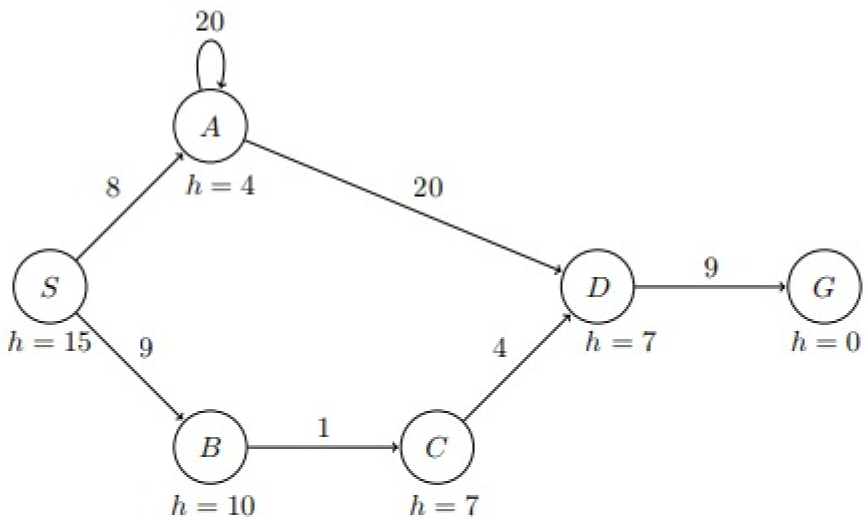
B351/Q351 WORKSHEET 6: SEARCH II (Solutions)

(1) Prove that whenever A* chooses to expand a goal node, the path to this node is optimal.
Sol:
Lets assume that $s_1(g)$ is a suboptimal goal state in the fringe and $s_2(n)$ is another state in the fringe where node 'n' lies on the optimal path to goal (g). Let us assume that the optimal cost to the goal (g) is C*, and f=g+h
for a consistent heuristic $f(s_2(n))$ will always be less than C* (as it never overestimates the optimal cost)
And since $s_1(g)$ is a suboptimal state, $f(s_1(g)) > C^*$.
So,
$f(s_2(n)) \leq C^*$
$f(s_1(g)) > C^*$
hence $f(s_1(g)) > f(s_2(n))$
f-value of suboptimal state is > f-value of any node n that lies on the optimal path, hence A* will never expand a suboptimal goal state from the fringe as it expands states with lower f-value.

(2) Consider the following graph.



(a) Is the heuristic admissible at nodes B and C? Yes/No and why?
Sol: Yes, the heuristic admissible at nodes B and C because it never overestimates the actual cost to the goal node from B and C. $\{h_B(10) < C_B(1+4+9)$ and $h_c(10) < C_c(4+9)\}$

(b) Is the heuristic consistent at nodes B and C? Yes/No and why?
Sol: heuristic is NOT consistent at nodes B and C because:
for a consistent heuristic at B,C: $h_B \leq C_{BC}+h_c$ {This equation doesn't hold true for h_b=10, C_{BC}=1 & h_c=7}

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(3) Consider a hypothetical problem in which the state space is S = {0, 1, 2, ...100}, i.e. the set of integers between 0 and 100, inclusive, and the actual cost to the goal from any state $s \in S$ is given by f(s) = s 4 − s 2 + s. Suppose you could choose between three heuristic functions for $A*$ search:

$$h1(s) = s,\ h2(s) = s^2,\ h3(s) = s^4$$

Which would you choose and why?

Sol:

h3(s) is not an admissible heuristic ($s^4 \leq s^4 - s^2 + s$ doesn't hold true for s>1)

Both h2(s) and h1(s) are admissible heuristics, we would choose h2(s) as it gives a better estimate of the actual cost without overestimating it {h2(s) >h1(s)}.

(4) Suppose we have a state graph with f = g + h and h is a consistent heuristic. Suppose that N' is a child of N for some vertices N, N' in the graph. Prove that f(N') ≥ f(N)

sol:

Let's assume the cost from $N \rightarrow N'$ is $C_{NN'}$

$f(N) = g + \mathbf{h_N}$

$f(N') = g + \mathbf{C_{NN'} + h_{N'}}$

h is a consistent heuristic function, so $h_N \leq C_{NN'} + h_{N'}$

Hence, f(N') ≥ f(N)