B351/Q351 WORKSHEET 3: FOL II (Solutions)

Q(1) Transform $\forall x(\forall y(\text{Animal}(y) \Rightarrow \text{Loves}(x, y)) \Rightarrow (\exists y\ \text{Loves}(y, x)))$ to CNF.

Sol:
1. Eliminate implications: $\forall x\ [\neg\forall y\ \neg\text{Animal}(y) \lor \text{Loves}(x, y)] \lor [\exists y\ \text{Loves}(y, x)]$
2. Move ¬ inwards
    - $\forall x\ [\exists y\ \neg(\neg\text{Animal}(y) \lor \text{Loves}(x, y))] \lor [\exists y\ \text{Loves}(y, x)]$
    - $\forall x\ [\exists y\ \neg\neg\text{Animal}(y) \land \neg\text{Loves}(x, y)] \lor [\exists y\ \text{Loves}(y, x)]$ (De Morgan)
    - $\forall x\ [\exists y\ \text{Animal}(y) \land \neg\text{Loves}(x, y)] \lor [\exists y\ \text{Loves}(y, x)]$ (double negation)
3. Standardize variables: $\forall x\ [\exists y\ \text{Animal}(y) \land \neg\text{Loves}(x, y)] \lor [\exists z\ \text{Loves}(z, x)]$
4. Skolemization: $\forall x\ [\text{Animal}(F(x)) \land \neg\text{Loves}(x, F(x))] \lor [\text{Loves}(G(x), x)]$
5. Drop universal quantifiers: $[\text{Animal}(F(x)) \land \neg\text{Loves}(x, F(x))] \lor [\text{Loves}(G(x), x)]$
6. Distribute $\lor$ over $\land$: $[\text{Animal}(F(x)) \lor \text{Loves}(G(x), x)] \land [\neg\text{Loves}(x, F(x)) \lor \text{Loves}(G(x), x)]$

Q2. For the following sets of literals find the most general unifier (mgu) or explain why it does not exist. Letters a, b, c represent constants.
(a) {P(a, x), P(a, y)}
=> {x=y}
(b) {P(x, f(y), z), P(g(a), f(w), u), P(v, f(b), c)}
=>{x=v=g(a), y=w=b, z=u=c}
(c) {P(x), R(x)}
=>Function P ≠ Function R, MGU does not exist.
(d) {Q(h(x, y), w), Q(h(g(v), a), f(v)), Q(h(g(v), a), f(b))}
=>{x=g(v),y=a,w=f(b),v=b}

Q3. Suppose that Linda, a student in this class, owns a red convertible. Everyone who owns a red convertible has gotten a speeding ticket. Prove using resolution that someone in this class has gotten a speeding ticket.
Sol:
c(x): x is in this class.
r(x): x owns a red convertible.
t(x): x has gotten a speeding ticket.

1. $\forall x(r(x) \rightarrow t(x))$ Hypothesis
2. $r(\text{Linda}) \rightarrow t(\text{Linda})$ Universal instantiation using (1)
3. r(Linda) Hypothesis
4. t(Linda) Modus ponens using (2) and (3)
5. c(Linda) Hypothesis
6. $r(\text{Linda}) \land c(\text{Linda})$ Conjunction using (4) and (5)
7. $\exists x(c(x) \land t(x))$ Existential generalization using (6)