# 2017 PME Conference Schedule

## 9:00–10:00: Registration & Breakfast - Lincoln 510

## 10:00–10:15: Welcome & Introduction - Lincoln 510

Dr. Wim Steelant, Dean, College of STEM
Gabbie Van Scoy, YSU PME Chapter President

| | Lincoln 112 | Lincoln 114 | Lincoln 115 | Lincoln 103 |
|---|---|---|---|---|
| 10:20-10:35 | Jason Khavari | Leah Bayer and Elise Eckman | Trevor Arrigoni | COMAP-MCM B Drew Asher, Nick Scoumis, and Jon Koutsourais |
| 10:40-10:55 | Gabbie Van Scoy | Elise Eckman and Leah Bayer | Robert Garrett | COMAP-MCM B Daniel Belinky |
| 11:00-11:15 | Jacob Coleman | Donald Fincher | Peter Stafford | COMAP MCM B Sky Semone and Victoria Jakicic |
| 11:20-11:35 | Mark Leadingham | Jesús Sambrine | Bradley Wolfe | COMAP MCM B Alexander Michels and Sam Hockenberry |
| 11:40-11:55 | Anthony Sulak | Lauren MacMullen | Rachael Troutman | COMAP MCM |

## 11:55–12:50: Lunch - St. John's Episcopal Church Hall

| | Lincoln 112 | Lincoln 114 | Lincoln 115 | Lincoln 103 |
|---|---|---|---|---|
| 12:50-1:05 | William Rek | Greg Knapp | Victoria Jakicic | COMAP MCM C Natalie Halavick |
| 1:10-1:25 | Sumer Kassim | Bob Krueger | Aaron Seidel | COMAP MCM |
| 1:30-1:45 | Rebekah Bright, Kallie Simpson, and Melinda Whelan | Brandon Eschborn | | COMAP MCM |

## 1:50: Closing Remarks - Lincoln 510

Youngstown State has partnered with AT&T to deliver visitor wireless Internet access on campus through the "attwifi" wireless network. This is an unsecure wireless network and is not protected by any of the University's cyber-related safeguards. The code for access is: WPSM-F19P-AH. The AT&T wireless network is supported directly by AT&T.

# Morning Session 10:20-10:35

**10:20-10:35** **Jason Khavari** **Lincoln 112**

**Hedge, or Index? An Analysis of the Costs and Benefits of Active Versus Passive Investments**

Howland High School

Advised by: Dr. Thomas Wakefield

In the world of investing, there are countless options for an investor to choose from to fit all different kinds of situations. All these options fall under two broad umbrellas, namely active and passive investing. This presentation will take an in-depth look at active and passive investing and into hedge and index funds in an attempt to determine the best possible investment choice between the two.

**10:20-10:35** **Leah Bayer and Elise Eckman** **Lincoln 114**

**Evaluating Instances of Kostant's Multiplicity Formula**

Youngstown State University

Advised by: Dr. Alicia Prieto Langarica and Dr. Pam Harris

Central to the study of the representation theory of Lie algebras is the computation of weight multiplicities, which are the dimensions of vector subspaces called weight spaces. The multiplicity of a weight can be computed using a well-known formula of Kostant that consists of an alternating sum over a finite group and involves a partition function called Kostant's partition function. There are two major obstacles in the use of this formula. First, the number of terms arising in the sum grows factorially as the rank of the Lie algebra increases and, second, the value of the partition function is often unknown. In this presentation, we specialize to the Lie algebra $\mathfrak{sl}_{r+1}(\mathbb{C})$ and focus on questions regarding the number of terms contributing nontrivially to Kostant's weight multiplicity formula. Through this study, we show that these contributing sets, called Weyl alternation sets, show interesting combinatorial and geometric properties. We dedicate a section to detailed examples that illustrate accessible techniques students may use to begin investigating the open problems we present in this area.

**10:20-10:35** **Trevor Arrigoni** **Lincoln 115**

**Computational Analysis of Dyck Path Characteristics**

Westminster College

Advised by: Dr. Natacha Fontes-Merz

Named after 19th century German Mathematician Walther von Dyck, Dyck Paths are a special lattice path which is famous for its relationship with the Catalan Numbers. Even today, Dyck Paths are being researched in the field of combinatorics. In 2003, Dr. James Haglund of the University of Pennsylvania conjectured three statistics on Dyck Paths: Area, Bounce, and Diagonal Inversion (Dinv). When fixing the Area and Length of a Dyck Path, Haglund proved that Bounce and Dinv created equal multisets. In this presentation we will look at the following open problem that stems directly from Haglund's result:

Let $n, a \in \mathbb{N}$ and $\mathbb{P}_{n,a}$ be the set of Dyck Paths of length $n$, and area $a$ and $P_k \in \mathbb{P}_{n,a}$. Then, $\exists \Phi : \mathbb{P}_{n,a} \to \mathbb{P}_{n,a}$ such that $\Phi$ is a bijection with $B(P_k) = D(\Phi(P_k))$.

**10:20-10:35** **Lincoln 103**


**Drew Asher, Nick Scoumis, and Jon Koutsourais**


**Directed Merging**


Youngstown State University
Advised by: Dr. Thomas Madsen


Time and time again, driving is considered to be one of the most dangerous forms of transportation. The chances of getting in an accident increase dramatically when many other drivers are trying to merge into the same lane. This instance of merging is seen explicitly when leaving a barrier toll booth. To mitigate the confusion and dangers of driving in egress lanes a less autonomous form of merging will increase safety. This design of merging will be denoted as directed merging.

We illustrate why directed egress lanes will help make merging more efficient and safer for all parties involved. This model is based off of a three lane highway. There are two toll booths for each lane on the highway, and we assume the cars will evenly distribute when coming into the booths. When exiting the booths, barriers are set up to group two booths together, the far left two, the middle two, and the right two. This lets drivers focus on only one lane to merge into with one other car, rather than focusing on six lanes merging together. By predetermining the egress lane that a driver will be taking, it removes the factor of a driver crossing over multiple egress lanes, resulting in a less dangerous situation. This design is more beneficial than the standard toll booth design because it dramatically increases safety by negating the driver's ability to choose which lanes they would like to merge into. One key assumption that was made was if a driver has more choices and that ability to merge across multiple lanes, it creates a more dangerous driving situation for those around the vehicle. In our model we accounted for the different types of tolls, electronic, coin-operated, and human-operated, and how they affect that rate at which traffic accelerates while exiting the toll.

# Morning Session 10:40-10:55

**10:40-10:55** **Lincoln 112**

**Gabbie Van Scoy**
**Making Muscles: Math Models of Muscle Formation**
Youngstown State University
Advised by: Dr. Alicia Prieto Langarica and Dr. Marnie Saunders

Muscle formation is an important and complex process. At the beginning, myoblast cells, which are small round embryonic cells, have to transition into myocytes, which are elongated muscle cells. Myocytes then fuse into myotubes, which are the building blocks for muscles. Principles learned from the study of myoblast fusion not only enhance our understanding of myogenesis, but also contribute to our perspectives on membrane fusion and cell-cell fusion in a wide array of model organisms and experimental systems. A computer simulation and a mathematical model is created to help better understand this complex process.

**10:40-10:55** **Lincoln 114**

**Elise Eckman and Leah Bayer**
**Evaluating Instances of Kostant's Multiplicity Formula**
Youngstown State University
Advised by: Dr. Alicia Prieto Langarica and Dr. Pam Harris

Central to the study of the representation theory of Lie algebras is the computation of weight multiplicities, which are the dimensions of vector subspaces called weight spaces. The multiplicity of a weight can be computed using a well-known formula of Kostant that consists of an alternating sum over a finite group and involves a partition function called Kostant's partition function. There are two major obstacles in the use of this formula. First, the number of terms arising in the sum grows factorially as the rank of the Lie algebra increases and, second, the value of the partition function is often unknown. In this presentation, we specialize to the Lie algebra $\mathfrak{sl}_{r+1}(\mathbb{C})$ and focus on questions regarding the number of terms contributing nontrivially to Kostant's weight multiplicity formula. Through this study, we show that these contributing sets, called Weyl alternation sets, show interesting combinatorial and geometric properties. We dedicate a section to detailed examples that illustrate accessible techniques students may use to begin investigating the open problems we present in this area.

**10:40-10:55** **Lincoln 115**

**Robert Garrett**
**Crash-Safety Ratings and the True Assessment of Injuries by Vehicle**
Miami University
Advised by: Dr. Thomas Fisher

Every year the National Highway Traffic Safety Association (NHTSA) and Insurance Institute for Highway Safety (IIHS) release safety ratings for popular makes and models of vehicles produced. We link these safety ratings with the accident data provided in the National Automotive Sampling System (NASS). In this project we create two different views of the data. We achieve this by using Shiny to graphically outline these views. We create a metric by looking at actual injuries that have occurred in each type of vehicle and the variant factors that could have caused a crash. Secondly, we outline the difference in the distributions of each type of vehicle when accounting for user-defined conditions. Thirdly we outline the statistical difference between the NHTSA and IIHS ratings systems and how they correspond to the data in NASS.

**10:40-10:55** **Lincoln 103**

**Daniel Belinky**
**For Whom the Lane Tolls**
Youngstown State University
Advised by: Dr. George Yates

Toll plazas on highways are often of designs which expand the roadway to include more booths than there are lanes. While this is excellent for preventing bottlenecks, the open space beyond the booths has proven to be quite dangerous. As the expanded plaza contracts back to the original amount of lanes, drivers must merge in accordance with many other vehicles at the same time. Resultantly, toll plazas are considered some of the least safe portions of a highway.

Through rigorous research and unique design, we aim to reduce collisions and increase safety around the toll plaza without hampering throughput efficiency. We attempt to accomplish this primary goal while minimizing construction costs involved with the implementation of the design.

We propose a plaza arrangement never before attempted. The plaza is split down the middle, with express lanes for electronic toll collection on the left and traditional booths on the right. The toll lanes on the right side of the plaza are separated by traffic cone barriers so that each lane may only access a select few booths. This reduces safety hazards caused by drivers making drastic lane changes at the last second. All booths accept both cash and electronic payment so that all vehicles may proceed.

The most innovative part of our design comes in our toll booth alignment. Inspired by runners lining up on a track, our booths are cascaded so that the driver furthest to the left may be processed first. This way, drivers to the left accelerate before the other lanes and merging becomes a much more refined process–akin to merging onto the highway from an entrance ramp.

We tested our design against that of a more typical plaza in an advanced work-flow simulation software program called Arena. In Arena, we used basic kinematic equations and empirical data from toll plazas to create realistic traffic flows through the two designs. Our model was markedly more efficient in tests of light, normal, and heavy traffic. According to sensitivity analysis, our model was able to maintain effectiveness under a variety of changing circumstances.

Through our simulation results, we conclude that our proposed design could revolutionize highways across the nation. We discuss these results and further considerations within the contents of this report.

# Morning Session 11:00-11:15

**11:00-11:15** | **Lincoln 112**

**Jacob Coleman**
**Distributive Embeddings of Groups into Monoids of Binary Operations**
West Virginia Wesleyan College
Advised by: Dr. Scott Zinzer

Gregory Mezera describes a method for embedding groups into a monoid of binary operations on a set. He also states in that article that finding minimal embeddings would be an interesting problem in itself. We explore this question by first doing small case-wise analysis. We then prove several structural properties that distributive binary operations must possess and use that information to narrow down the set of operations enough so that a computer may be used to find the sets suitable for the distributive embeddings of groups. We then describe explicit embeddings of groups into the set of binary operations of sets with various size and conclude by describing future work with larger sets as well as the possibility of working with infinite sets.

**11:00-11:15** | **Lincoln 114**

**Donald Fincher**
**Solving Differential Equations with a Decomposition Method**
Kent State University
Advised by: Dr. Mahmoud Najafi

In this work, we will show how the decomposition method pioneered by George Adomian can be used to solve differential equations. This method is important because of the fast convergence of the series solution. Additionally, using the so-called aftertreatment technique, a closed form analytical solution may be found. We will apply the decomposition method and the aftertreatment technique to a wave equation that models conductor galloping of overhead power lines, which is an oscillation of the power lines caused by strong winds. Improved transmission tower designs are needed which are capable of combating excessive stresses exerted on the tower by the galloping power lines. Our model of this wave equation shows that the oscillations can be reduced over time by adding a dampener at the transmission tower. Through this research, power transmission systems that are more reliable and resistant to galloping can be engineered.

**11:00-11:15** | **Lincoln 115**

**Peter Stafford**
**Retention and Graduation Rates at Colleges and Universities in Ohio**
Youngstown State University
Advised by: Dr. Lucy Kerns

Retention and graduation rates are major concerns for colleges and universities in Ohio and across the country. Low retention and graduation rates can result in decreased funding, a bad reputation, and decreased enrollment. Therefore, many colleges and universities make it a priority to find out what causes students to decide not to return, and fail to graduate. In this study, we examine various factors that may be related to low retention and graduation rates at higher education institutions across Ohio. We look at the statistics for 50 colleges and universities, and analyze the factors that correlate with low retention and graduation rates.

**11:00-11:15** | **Lincoln 103**

**Sky Semone and Victoria Jakicic**
**Modelling Traffic Flow through Barrier Tolls**
Indiana University of Pennsylvania
Advised by: Dr. John Chrispell

All people are familiar with traffic congestion and the safety problems and wasted time it causes. Barrier tolls found on multi-lane toll highways present a unique challenge for drivers. This talk will propose a method for constructing better suited barrier toll exit ramps to increase traffic flow and decrease accidents. We will use both macroscopic and microscopic models to determine the short term and long term behavior at toll booth merge zones. We then developed a model that exhibited improved capacity and flow.

## Morning Session 11:20-11:35

**11:20-11:35** **Lincoln 112**

**Mark Leadingham**
**Chaos: Pulsars and their Quirks**
West Virginia Wesleyan College
Advised by: Dr. Scott Zinzer

Chaos Theory is the study of systems that are highly dependent on their initial conditions. Any seemingly insignificant change will create completely different results than those expected. Deterministic Chaos focuses on strange results occurring from seemingly predictable systems. The applications are apparent in computer science, physics, and astronomy, which is the interest of this talk. Astronomical objects called pulsars exhibit chaotic behavior in the emission of their radio pulses. We will discuss famous attractors (Lorenz & Rossler) as well as the methods of describing these phenomena, such as building models, the fourth-order Runge-Kutta Method, and the concept of embedding dimensions.

**11:20-11:35** **Lincoln 114**

**Jesús Sambrine**
**Persistence Homology**
Kent State University
Advised by: Kent Professors

We'll quickly run down the background necessary to formulate the persistence homology technology. Persistence homology will be explained and how it pertains to analyzing high dimensional data. Potentially, an application might be shown.

**11:20-11:35** **Lincoln 115**

**Bradley Wolfe**
**The Zariski Topology**
Edinboro University of Pennsylvania
Advised by: Dr. Richard White

Let $P$ be a polynomial in $n$ real variables, and denote its solution set as $Z(P)$. Then the complements of solution sets form a basis for a topology on $\mathbb{R}^n$. This topology is called the Zariski Topology. First, it will be shown that the solution sets form a basis for a Topology. Second, it will be shown that the Zariski Topology is not Hausdorff. Finally, it will be shown that every finite set is closed.

**11:20-11:35** **Lincoln 103**

**COMAP Problem B**
**Alexander Michels and Sam Hockenberry**
Westminster College
Advised by: Dr. Carolyn Cuff

When cars attempt to merge back onto the highway after exiting a toll plaza, they run into a few problems. The first is that the lanes are not clearly marked. Motorists are required to guess at where the lines should be as they begin accelerating toward the highway. Furthermore, this guessing aspect leaves a lot of room for collision since it is crowded and people aren't sure exactly where they are supposed to be. Our solution is to divide the toll plaza into two sections, right and left. From there, each half of the plaza will enter one of two branches. Those entering the right branch will travel in a semi-circle, counter-clockwise and those entering the left branch will travel in a semi-circle clockwise until both branches meet where the highway begins. This model requires less merging, and minimizes the cost of construction, while decreasing the likelihood of collisions.

# Morning Session 11:40-11:55

**11:40-11:55** | **Lincoln 112**

**Anthony Sulak**
**3-Colorable Graphs and Gröbner Bases**
Cleveland State University
Advised by: Dr. Leah Gold

Coloring started when mathematicians in the nineteenth century tried coloring maps with the least amount of colors. Ever since then coloring has evolved from pure mathematical joy to real world applications. To make coloring the most efficient, algorithms had to be developed. Gröbner bases is a computational tool that has been growing in popularity since Buchberger introduced his algorithm to compute Gröbner bases. We will use Macaulay2 to compute Gröbner bases. We will use those Gröbner bases to find out if a graph $\mathcal{G}$ is 3-colorable or not and how to color $\mathcal{G}$ if it is.

**11:40-11:55** | **Lincoln 114**

**Lauren MacMullen**
**Fractals and Linear Algebra**
Indiana University of Pennsylvania
Advised by: Dr. Rachelle Bouchat

In this talk, we will look at fractals, which are objects that are self-similar on an infinite number of scales. This means that you can continually zoom in and see a similar picture. We will show how matrices and linear algebra can be used to create fractals. Specifically, we will look at affine transformations.

**11:40-11:55** | **Lincoln 115**

**Rachael Troutman**
**Nine-Point Circle Theorem**
Edinboro University of Pennsylvania
Advised by: Dr. Rick White

Using Euclidean geometry, this talk will demonstrate a proof of the Nine-Point Circle Theorem. This theorem states the following: If $\triangle ABC$ is any triangle, then the midpoints of the sides of $\triangle ABC$, the feet of the altitudes of $\triangle ABC$, and the midpoints of the segments joining the orthocenter of $\triangle ABC$ to the three vertices all lie on a single circle.

**11:40-11:55** | **COMAP Modeling Discussion** | **Lincoln 103**

All are welcome to discuss this year's COMAP problems and potential solutions to both the discrete and continuous problems. This informal session is meant to share ideas and strategies for the approach to the problems, which are provided on Page 14 of this abstract book.

# Afternoon Session 12:50-1:05

**12:50-1:05** **Lincoln 112**

**William Rek**
**Napoleon Triangles**
Youngstown State University
Advised by: Dr. Thomas Smotzer

In this project we prove both inner and outer Napoleon Triangles are equilateral. We also prove that the difference of both the inner and outer Napoleon Triangle's areas is equivalent in area to the original triangle. Napoleon Triangles are named after Napoleon Bonaparte, though it is uncertain if he actually proved Napoleon Triangles are equilateral.

**12:50-1:05** **Lincoln 114**

**Greg Knapp**
**Measures of Non-Uniqueness in the Ring of Integer-Valued Polynomials**
Case Western Reserve University
Advised by: Dr. Paul Baginski

We examine the ring of integer-valued polynomials, i.e. the set of all $f \in \mathbb{Q}[x]$ such that $f(\mathbb{Z}) \subseteq \mathbb{Z}$. This ring, denoted $\mathrm{Int}(\mathbb{Z})$, has elements which factor non-uniquely into irreducible polynomials of $\mathrm{Int}(\mathbb{Z})$. We quantify the non-uniqueness of the factorizations of $f \in \mathrm{Int}(\mathbb{Z})$ using two concepts: the elasticity, $\rho(f)$, and the catenary degree, $\mathrm{cat}(f)$. Given a factorization, $z$ of $f \in \mathrm{Int}(\mathbb{Z})$, if the length of $z$ is defined to be the number of irreducible elements appearing in $z$, then the elasticity of $f$ is defined to be the ratio between the maximal and minimal elements of the set of lengths of $f$. The catenary degree gives a more fine-grained measure of how dissimilar the factorizations of $f$ are.

Given a polynomial of degree $n$, we prove general upper bounds on the elasticity and catenary degree in terms of $n$ and prove that they are sharp. In addition, given $n \in \mathbb{N}$, we characterize all attainable catenary degrees and elasticities of polynomials with degree $n$. While these are general bounds, when $f$ has a particular form, far smaller bounds can be established. As an example, we will discuss the catenary degree of polynomials $f \in \mathbb{Z}[x]$ which are a product of linear terms. Lastly, we will discuss open problems and conjectures.

This project was supported by NSF grant DMS-1358454.

**12:50-1:05** **Lincoln 115**

**Victoria Jakicic**
**Magic Polygons and Their Properties**
Indiana University of Pennsylvania
Advised by: Dr. Rachelle Bouchat

Magic squares are square arrays, where the sum of each row, each column, and both main diagonals is the same. The concept of a magic square with 3 rows and 3 columns is generalized to encompass regular polygons. These magic polygons have the same sum on each edge, as well as each of the diagonals. Construction of these magic polygons, as well as their existence will be discussed.

**12:50-1:05** **Lincoln 103**


**Natalie Halavick**


**Analyzing the Effects of Autonomous Cars in Seattle, Washington**


Youngstown State University
Advised by: Dr. George Yates


The recent interest in autonomous cars has left many political leaders wondering whether their areas of living should invest now or wait. In their infancy, these cars seem like the bright future of transportation, however many worry about how worthwhile it will be. Seattle, Washington experiences multiple traffic delays in the Interstate system, and their governor has tasked us with evaluating possible traffic scenarios involving the inclusion of autonomous vehicles.

Using a matrix-based cellular automata in MatLab, we use simple matrix transformations to represent the movement of both human-driven and self-driving vehicles on one of the worst stretches of Seattle's Route 90. This 7-mile strip incurs changes from 5 lanes to 3 to 4, often causing a large amount backup and citizen distress. We focused our efforts on this piece of road, testing the effective ness of different ratios of autonomous vehicles to human vehicles, additionally trying various densities of vehicles in the area as well.

We utilize a metric analagous to throughput, counting the amount of iterations necessary for all cars to leave that piece of road. This allows us to compare the different car ratios and densities with a similar test, additionally discretizing the measure in an already-discrete model. We found no difference in including autonomous vehicles on the road, and we are confident that the simplicity of our model will be well-received by the public.

## Afternoon Session 1:10-1:25

**1:10-1:25** | **Lincoln 112**

**Sumer Kassim**
**A Variation on the Tower Of Lire**
Westminster College
Advised by: Dr. Natacha Fontes-Merz

The Leaning Tower of Lire is a common physics problem that concerns stacking blocks of equal mass over a table. The goal is to stack the blocks over the table as far as possible. In this talk we discuss the problem with a use of different masses, and find the combination of blocks that maximizes the total overhang.

**1:10-1:25** | **Lincoln 114**

**Bob Krueger**
**Degree Conditions Forcing Hamiltonian Cycles**
Miami University
Advised by: Dr. Louis DeBiasio

One major question of graph theory is which graphs have Hamiltonian Cycles, cycles that traverse every vertex of the graph exactly once. In particular, many people have investigated what conditions imply this property of Hamiltonicity. Dirac first showed that if the minimum degree (the number of edges coming from a vertex) of a graph is at least half the number of vertices, then the graph contains a Hamiltonian cycle. Similar degree conditions have been created for balanced $k$-partite graphs (where the vertices are divided into $k$ disjoint equally sized parts). We extend these results to not-necessarily balanced $k$-partite graphs. In the talk I will give an overview and history of the problem (suitable for an audience without much graph theory experience), some insight into the generalization, and if time permits, an outline of our proof.

**1:10-1:25** | **Lincoln 115**

**Aaron Seidel**
**Using Mathematics to Investigate Methane Gas Emissions by Wetlands in Western Pennsylvania**
Indiana University of Pennsylvania
Advised by: Dr. Rachelle Bouchat

Recent studies involving methane budgets of western Pennsylvania have sought to quantify the amount and rate of fugitive methane released during industrial natural gas development and from distribution systems. However, in order to fully understand the carbon cycle and budget of the area, contributions from natural systems such as wetlands are also needed. This talk will focus on the mathematics behind the collection techniques to measure the methane emissions from the wetlands overseen by the Evergreen Conservancy in Tanoma, Pennsylvania.

**1:10-1:25** | **COMAP Modeling Discussion** | **Lincoln 103**

All are welcome to discuss this year's COMAP problems and potential solutions to both the discrete and continuous problems. This informal session is meant to share ideas and strategies for the approach to the problems, which are provided on Page 14 of this abstract book.

# Afternoon Session 1:30-1:45

**1:30-1:45** **Rebekah Bright, Kallie Simpson, and Melinda Whelan** **Lincoln 112**

**Business Analytics Presentation**

Slippery Rock University

Advised by: Dr. Singhababu

Every spring, the Manhattan College in New York hosts the Business Analytics Competition & Conference. This is an opportunity for undergraduate students studying Business Analytics or related fields to test their knowledge and hone their skills. Competing teams engage in the "art & science" of decision-making as well as practice their ability to draw business insight from a comprehensive analysis of relevant data. In the spring of 2016, competitors were given a data set from the online clothing store Gilt. We used different statistical and data analysis techniques to analyze this data set, and gave a presentation with business insights advising the company. We focused on a few main aspects throughout the process, and used these topics to give advice to our client. These main aspects dealt with commonalities between items that did not sell, different factors that would effect an article of clothing from selling, how discounts affect rates of sales, and what type of implications can be drawn from different lengths of sale. In this presentation we will talk about the process of the competition, and describe the different statistical and analytical techniques that we used.

**1:30-1:45** **Brandon Eschborn** **Lincoln 114**

**The History of Number Systems**

Edinboro University of Pennsylvania

Advised by: Dr. Frank Marzano

How peoples have depicted numbers has changed numerous times throughout the millennia. This talk discusses the various number systems used over written history, from bases 2 to 60.

**1:30-1:45** **COMAP Modeling Discussion** **Lincoln 103**

All are welcome to discuss this year's COMAP problems and potential solutions to both the discrete and continuous problems. This informal session is meant to share ideas and strategies for the approach to the problems, which are provided on Page 14 of this abstract book.

# 2017 MCM / ICM - COMAP Modeling Problems

**Continuous Modeling (Problem A)**
**Managing the Zambezi River**

The Kariba Dam on the Zambezi River is one of the larger dams in Africa. Its construction was controversial, and a 2015 report by the Institute of Risk Management of South Africa included a warning that the dam is in dire need of maintenance. A number of options are available to the Zambezi River Authority (ZRA) that might address the situation. Three options in particular are of interest to ZRA:

**(Option 1)** Repairing the existing Kariba Dam,

**(Option 2)** Rebuilding the existing Kariba Dam, or

**(Option 3)** Removing the Kariba Dam and replacing it with a series of ten to twenty smaller dams along the Zambezi River.

There are two main requirements for this problem:

**Requirement 1** ZRA management requires a brief assessment of the three options listed, with sufficient detail to provide an overview of potential costs and benefits associated with each option. This requirement should not exceed two pages in length, and must be provided in addition to your main report.

**Requirement 2** Provide a detailed analysis of Option (3)–removing the Kariba Dam and replacing it with a series of ten to twenty smaller dams along the Zambezi River. This new system of dams should have the same overall water management capabilities as the existing Kariba Dam while providing the same or greater levels of protection and water management options for Lake Kariba that are in place with the existing dam. Your analysis must support a recommendation as to the number and placement of the new dams along the Zambezi River.

In your report for Requirement 2, you should include a strategy for modulating the water flow through your new multiple dam system that provides a reasonable balance between safety and costs. In addition to addressing known or predicted normal water cycles, your strategy should provide guidance to the ZRA managers that explains and justifies the actions that should be taken to properly handle emergency water flow situations (i.e. flooding and/or prolonged low water conditions). Your strategy should provide specific guidance for extreme water flows ranging from maximum expected discharges to minimum expected discharges. Finally, your recommended strategy should include information addressing any restrictions regarding the locations and lengths of time that different areas of the Zambezi River should be exposed to the most detrimental effects of the extreme conditions.

## Discrete Modeling (Problem B)
## Merge After Toll

Multi-lane divided limited-access toll highways use "ramp tolls" and "barrier tolls" to collect tolls from motorists. A ramp toll is a collection mechanism at an entrance or exit ramp to the highway and these do not concern us here. A barrier toll is a row of tollbooths placed across the highway, perpendicular to the direction of traffic flow. There are usually (always) more tollbooths than there are incoming lanes of traffic (see former 2005 MCM Problem B). So when exiting the tollbooths in a barrier toll, vehicles must "fan in" from the larger number of tollbooth egress lanes to the smaller number of regular travel lanes. A toll plaza is the area of the highway needed to facilitate the barrier toll, consisting of the fan-out area before the barrier toll, the toll barrier itself, and the fan-in area after the toll barrier. For example, a three-lane highway (one direction) may use 8 tollbooths in a barrier toll. After paying toll, the vehicles continue on their journey on a highway having the same number of lanes as had entered the toll plaza (three, in this example).

Consider a toll highway having $L$ lanes of travel in each direction and a barrier toll containing $B$ tollbooths ($B > L$) in each direction. Determine the shape, size, and merging pattern of the area following the toll barrier in which vehicles fan in from $B$ tollbooth egress lanes down to $L$ lanes of traffic. Important considerations to incorporate in your model include accident prevention, throughput (number of vehicles per hour passing the point where the end of the plaza joins the $L$ outgoing traffic lanes), and cost (land and road construction are expensive). In particular, this problem does not ask for merely a performance analysis of any particular toll plaza design that may already be implemented. The point is to determine if there are better solutions (shape, size, and merging pattern) than any in common use.

Determine the performance of your solution in light and heavy traffic. How does your solution change as more autonomous (self-driving) vehicles are added to the traffic mix? How is your solution affected by the proportions of conventional (human-staffed) tollbooths, exact-change (automated) tollbooths, and electronic toll collection booths (such as electronic toll collection via a transponder in the vehicle)?

**Interdisciplinary (Problem C)**
**Cooperate and Navigate**

Traffic capacity is limited in many regions of the United States due to the number of lanes of roads. For example, in the Greater Seattle area drivers experience long delays during peak traffic hours because the volume of traffic exceeds the designed capacity of the road networks. This is particularly pronounced on Interstates 5, 90, and 405, as well as State Route 520, the roads of particular interest for this problem.

Self-driving, cooperating cars have been proposed as a solution to increase capacity of highways without increasing number of lanes or roads. The behavior of these cars interacting with the existing traffic flow and each other is not well understood at this point.

The Governor of the state of Washington has asked for analysis of the effects of allowing self-driving, cooperating cars on the roads listed above in Thurston, Pierce, King, and Snohomish counties. In particular, how do the effects change as the percentage of self-driving cars increases from 10% to 50% to 90%? Do equilibria exist? Is there a tipping point where performance changes markedly? Under what conditions, if any, should lanes be dedicated to these cars? Does your analysis of your model suggest any other policy changes? Your answer should include a model of the effects on traffic flow of the number of lanes, peak and/or average traffic volume, and percentage of vehicles using self-driving, cooperating systems. Your model should address cooperation between self-driving cars as well as the interaction between self-driving and non-self-driving vehicles. Your model should then be applied to the data for the roads of interest, provided in the Excel spreadsheet.

**2017 MCM-COMAP Participants from YSU**

| | | |
|---|---|---|
| Jenna Wise | Zack While | Joseph Wilaj |
| Thomas Stoner | Natalie Halavick | Daniel Belinky |
| Luke Nappi | Jospeh Miller | Devon Rambo |
| Andrew Asher | John Gaboriault-Whitcomb | Uyen Vuong |
| Nicholas Scoumis | Kathryn Platt | Theoni Kasamias |
| Jon-Michael Koutsourais | Kenneth Diogo | Kendall Orris |

## 2017 PME National Meeting at MAA MathFest

Please join us at this year's meeting to be held July 26 through July 29, 2017, in Chicago, Illinois. Students are invited to give fifteen minute talks on any mathematical topic or application in areas such as statistics, computing, or operations research. Topics including expository research, interesting applications, problems, etc. are also welcome. Transportation reimbursement is also available to those who qualify. Visit the National Pi Mu Epsilon website at `http://www.math-pme.org` for more details.

## Ohio Section of MAA Spring Meeting at Sinclair Community College

The Ohio Section of the Mathematical Association of America will hold its annual spring meeting at Sinclair Community College on Friday, March 31 and Saturday, April 1, 2017. The meeting consists of talks by mathematics faculty, graduate students, and undergraduates from around the state. The Section especially welcomes talks and participation by undergraduate students. In addition to student talks, there is an undergraduate problem solving competition with cash prizes, and a pizza party. We encourage you to give a talk at the meeting or participate in the competition or pizza party.

If you are participating in the problem solving competition, we ask that you register at:

`http://constum.ohiomaa.org/`

If you have any questions, please do not hesitate to contact Tom Wakefield by phone `330-941-1395` or by email `tpwakefield@ysu.edu`.

## A Warm Welcome to the Participating Schools:

- Ashland University
- Boardman High School
- Case Western Reserve University
- Clarion University of Pennsylvania
- Cleveland State University
- Edinboro University of Pennsylvania
- Fairmont State University
- Howland High School
- Indiana University of Pennsylvania
- iSTEM Geauga Early College High School
- Kent State University
- Miami University
- Penn State Erie, The Behrend College
- Slippery Rock University
- The University of Akron
- West Virginia Wesleyan College
- Westminster College
- Youngstown State University

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**President:** Gabbie Van Scoy

**Vice President:** Zack While

**Secretary:** Emily Hoopes

**Treasurer:** Natalie Hoopes

**Historian:** Joseph Wilaj

**Webmaster:** Jenna Wise

**Pi Mu Epsilon Faculty Advisors**

Dr. G. Jay Kerns

Dr. Alicia Prieto Langarica

Dr. Thomas Madsen

Dr. Angela Spalsbury

Dr. Thomas Wakefield

Dr. George Yates

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