Implementing a Horseshoe Merge

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Contents

1 Abstract

When cars attempt to merge back onto the highway after exiting a toll plaza, they run into a few problems. The first is that the lanes are not clearly marked. Motorists are required to guess at where the lines should be as they begin accelerating toward the highway. Furthermore, this guessing aspect leaves a lot of room for collision since it is crowded and people aren't sure exactly where they are supposed to be. Our solution is to divide the toll plaza into two sections, right and left. From there, each half of the plaza will enter one of two branches. Those entering the right branch will travel in a semi-circle, counter-clockwise and those entering the left branch will travel in a semi-circle clockwise until both branches meet where the highway begins. This model requires less merging, and minimizes the cost of construction, while decreasing the likelihood of collisions.

2 Background

Vehicular accidents are very common. When cars are exiting a toll plaza, they are accelerating to highway speeds, about 60mph [8], and attempting to merge without much direction with all the other accelerating vehicles. Accidents that occur at higher speeds are more likely to result in fatalities. Our model ameliorates this issue by having cars enter a horseshoe road structure as soon as they exit the toll booth, meaning their speed will be at a minimum. If there is an accident, it will likely not be life threatening. Since the structure is roughly based on a roundabout, it should also decrease the overall number of accident. [7]. Once cars are about to exit this structure, they will reach highway speeds, at which point they will merge as if it were they were exiting an on-ramp. This is more safe because the cars will be done accelerating and merging with other vehicles that are going a similar speed is much easier [4]. Additionally, studies have shown that, overall, roundabouts reduce accidents by 37% [3]. Thus, the shape and travel pattern of our model should not only reduce fatalities but also reduce accidents in general.

The rules of a roundabout will still apply to this model even though it isn't technically a roundabout. Cars entering a branch will have to yield to those that have already entered the branch. This could potentially increase the wait time for cars to reach the highway. Optimization of this problem will be discussed later on.

The design for our horseshoe structure is presented in **Figure 1**. It is obvious to see that each half of the curve will only come out as far as the outermost toll booth. Since the roads already accommodate the outermost lanes, there will be no need to widen the roads. According to Google Maps, the length of the road required for a merge after a toll plaza is roughly the same length as the width of all the tolls together. Meaning, the more individual toll booths, the longer the road needs to be to merge. The calculations that we will discuss later on are based on that assumption. Since the road lengths will not have to change for our model, no additional land will need to be purchased and there will be no need to reconstruct any of the roads.

The expenses will consist of repainting the road to incorporate the new design. Water barriers could be added to help guide people to their appropriate sections. Alternately, we could add a large ghost island in the center of our model and avoid barriers all together.

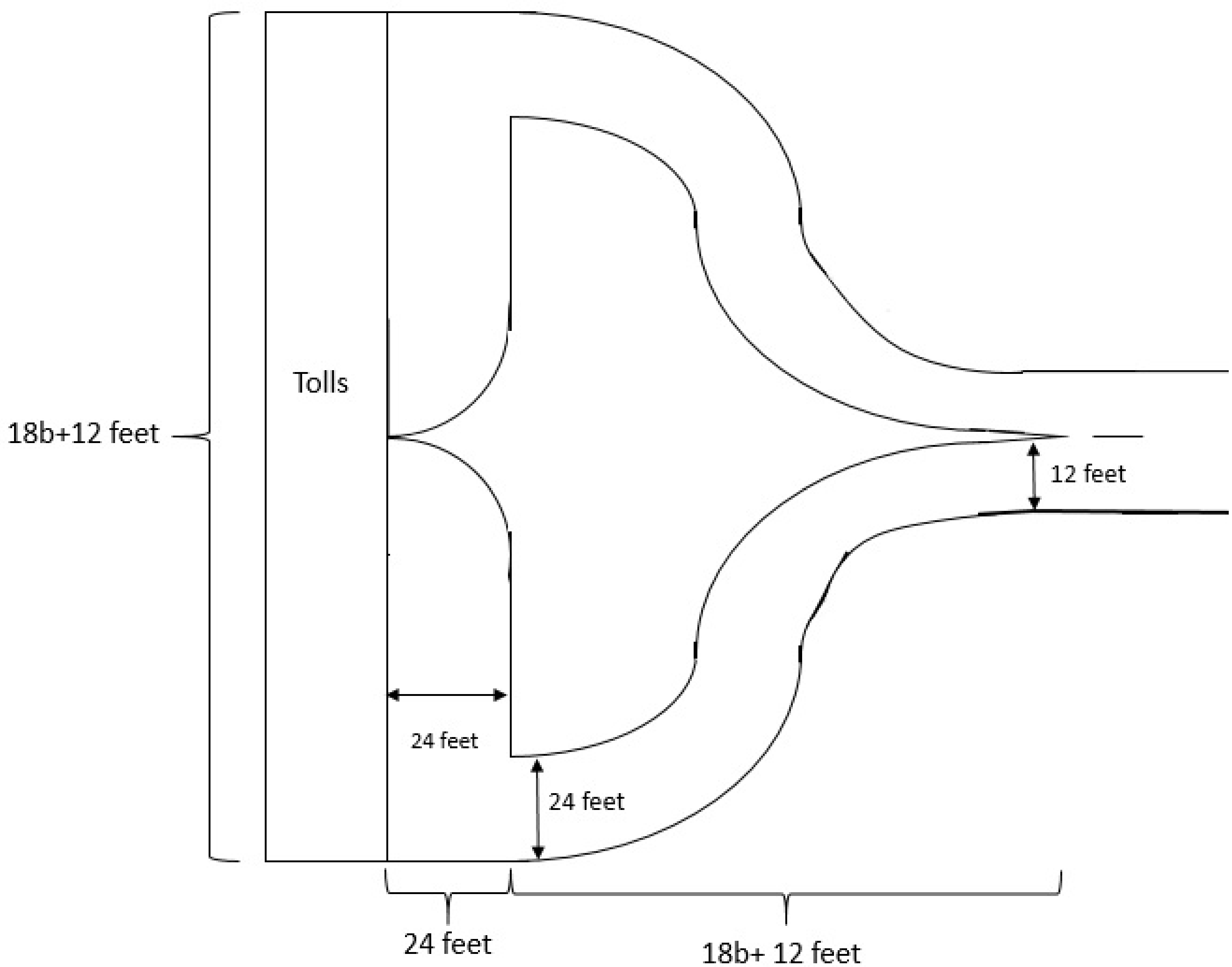

Figure 1

If we were to create this model from scratch without the preexisting road, the table in **Figure 2** shows the cost of concrete per square foot. This cost accounts for labor and other necessary materials.The concrete per square inch required to construct our design is based on the number of toll booths in the plaza.

Cost to Pave of Our Model as a Function of b and l (in USD)								
Toll booths	1 lane	2 lanes	3	4	5	6	7	8
1	\$ 6,389.77	\$ 6,389.77	\$ 8,173.84	\$ 8,173.84	\$ 8,173.84	\$ 8,173.84	\$ 8,173.84	\$ 8,173.84
2	\$ 9,874.34	\$ 9,874.34	\$ 11,989.81	\$ 11,989.81	\$ 11,989.81	\$ 11,989.81	\$ 11,989.81	\$ 11,989.81
3	\$ 13,476.93	\$ 13,476.93	\$ 15,913.84	\$ 15,913.84	\$ 15,913.84	\$ 15,913.84	\$ 15,913.84	\$ 15,913.84
4	\$ 17,127.68	\$ 17,127.68	\$ 19,891.47	\$ 19,891.47	\$ 19,891.47	\$ 19,891.47	\$ 19,891.47	\$ 19,891.47
5	\$ 20,802.61	\$ 20,802.61	\$ 23,898.99	\$ 23,898.99	\$ 23,898.99	\$ 23,898.99	\$ 23,898.99	\$ 23,898.99
6	\$ 24,491.38	\$ 24,491.38	\$ 27,924.74	\$ 27,924.74	\$ 27,924.74	\$ 27,924.74	\$ 27,924.74	\$ 27,924.74
7	\$ 28,188.86	\$ 28,188.86	\$ 31,962.30	\$ 31,962.30	\$ 31,962.30	\$ 31,962.30	\$ 31,962.30	\$ 31,962.30
8	\$ 31,892.14	\$ 31,892.14	\$ 36,008.01	\$ 36,008.01	\$ 36,008.01	\$ 36,008.01	\$ 36,008.01	\$ 36,008.01
9	\$ 35,599.48	\$ 35,599.48	\$ 48,183.96	\$ 57,343.05	\$ 57,343.05	\$ 57,343.05	\$ 57,343.05	\$ 57,343.05
10	\$ 39,309.78	\$ 39,309.78	\$ 53,157.86	\$ 63,077.33	\$ 63,077.33	\$ 63,077.33	\$ 63,077.33	\$ 63,077.33
11	\$ 43,022.30	\$ 43,022.30	\$ 58,134.73	\$ 68,815.27	\$ 68,815.27	\$ 68,815.27	\$ 68,815.27	\$ 68,815.27
12	\$ 46,736.52	\$ 46,736.52	\$ 63,113.90	\$ 79,975.15	\$ 97,350.19	\$ 97,350.19	\$ 97,350.19	\$ 97,350.19
13	\$ 50,452.10	\$ 50,452.10	\$ 68,094.89	\$ 86,261.39	\$ 104,817.25	\$ 104,817.25	\$ 104,817.25	\$ 104,817.25
14	\$ 54,168.74	\$ 54,168.74	\$ 73,077.29	\$ 92,549.43	\$ 112,286.39	\$ 112,286.39	\$ 112,286.39	\$ 112,286.39
15	\$ 57,886.24	\$ 57,886.24	\$ 78,060.87	\$ 98,838.98	\$ 119,852.90	\$ 147,925.56	\$ 147,925.56	\$ 147,925.56
16	\$ 61,604.47	\$ 61,604.47	\$ 83,045.43	\$ 105,129.71	\$ 127,465.40	\$ 157,137.44	\$ 157,137.44	\$ 157,137.44

Figure 2

3 The Math

3.1 Length and Amplitude Calculations

The three main variables that we need to take into consideration in our model are b, the number of toll booths in the plaza, l, the number of lanes on the highway, and r, the number of branches.The ratio of branches to booths is significant because a decrease in branches yields an increase in traffic through the reduced number of branches. Adding branches on the other hand increases the area required for construction which would increase the cost. Another noteworthy relationship is the ratio of lanes to branches. There are three scenarios to be taken into consideration:

- Same number of branches as there are lanes
- More branches than lanes
- Less branches than lanes

Since there are going to be two lanes per branch, if r and l are the same, say 2 and 2, then there will need to be two merges. The two lanes of cars in each branch will merge within the branches and proceed onto the highway without having to merge with another branch. Meaning there will be as many merges as there are branches in this scenario. If r is greater than l, (i.e. 3 branches and 2 lanes), the two lanes in the branches will have to merge and the now merged branches will also have to merge together. This yields 4 merges which is not ideal. If r is less than l, a maximum of one branch would have to merge it's two lanes. This is the optimal case. Taking the tradeoffs into account we get the following formula relating our three variables:

$$r \geq 2$$

$$r = min(\lfloor b/3 \rfloor, l)$$

The minimum number of branches that we will have is two, hence the restriction that r must be greater than two. From there, if we have more than 8 booths, we would need to incorporate another branch in order to avoid traffic jams. With this equation, if we have up to 8 lanes the minimum of the floor function and lanes will be less than or equal to 2. That means we will have only two branches. Once we get to 9 booths, the floor function will start yielding 3. If we have nine booths, l will only be slightly less than 9, so 3 would be the minimum which gives us the additional branch that we need. If the number of toll booths were to continue increasing, this model would account for that.

The width of our configuration is dependent on the width of the plaza. As the number of booths increases, we will need to widen our model. In every plaza, there is one more lane than there are booths. Since each lane is 12 feet wide and each booth is 6 feet wide [6], we can express the relationship with the following equation:

$$6b + 12(b + 1) = 18b + 12 \tag{1}$$

As discussed in the background, we are assuming that the length of the merge area, prior to reaching the highway, is the same as the width of the plaza. Therefore the equation for the length will be the same as the previous equation.

To find the arc length of our configuration we used Fermats formula for arc lengths. It is expressed as:

$$\int_a^b 1 + (f'(x))^2$$

[9]

The function that we chose to represent our configuration is a sine curve. It is similar to the shape of our model over the interval $[\frac{\pi}{2}$ - $\frac{3\pi}{2}]$. We want the sign curve to lie in the center of the branch, thus making it the average of the arc length of one branch segment. To do this, we made the amplitude equal to our function times the width of the plaza. Since we will never have less than two branches, using the width alone would be assuming we have too much space. To account for this, we will divide our width by the number of branches that we have, and since a branch is two dimensional we will account for that by multiplying by two. Finally, in order to ensure that the sine curve lies in the center of our branch, we will subtract a lane width (12 feet) because each branch has two lanes. The amplitude is modeled by the following expression.

$$sin(x)((18b + 12)/2r - 12) = ((9b + 6)/r) - 12sin(x) \tag{2}$$

Since the road will need to be longer than the current period of sine will allow, we elongated the period so that it is over the interval [(18b+12). 3/2(18b+12)] The equation is expressed as the following:

$$((9b+6)/r) - 12sin(pi * x/(18b+12)) \tag{3}$$

The two lanes of these branches, will in many circumstances, have to merge. When this happens, the maximum height of the amplitude will increase linearly by $(6)\frac{(18b+12)}{2}$. The first half of the expression will remain the same. However, when we merge the two lanes, the second half of the expression will be as shown below.

$$y_1 - \frac{y_2}{x_2 - x_1} \tag{4}$$

$$\frac{(6-0)}{(18b+12)} - \frac{1}{2(18b+12)}$$

$$\frac{6}{9b+6}$$

To find the point at which the two lanes will intersect, we plug in the end points ((18b+12),6) to the linear equation $y = mx + b$ and accounting for the period shift, the new peak height is $\frac{6}{(9b+6)x-12}$. Adding the two halves of the equation together, we have the following equation to model the amplitude.

$$(\frac{9b+6}{r} - 12 + ((\frac{6x}{9b+6})x - 12))sin(\frac{\pi x}{18b+12}) = (\frac{9b+6}{r} + (\frac{6x}{9b+6} - 24))sin(\frac{\pi x}{18b+12})$$

3.2 Area Calculations

To explain this briefly, each branch has a quarter circle of radius 24 feet and a portion of a rectangular region. That portion depends on the number of branches. So the rectangular region is $\frac{18b+12}{r}$, minus the width taken up by the quarter-circle (24 feet) times the width of the two lane road (24 feet).

r_m is the number of branches whose two lanes merge into one. The area is the width of the road (24 feet) times the arc length of the first half of the road. Next we add the average width of the second half of the road and multiply the arc length of the second half of the road. That function is where the amplitude changes linearly as a function of r_{r-r_m}. Simply put, it is accounting for the branches that do not merge. This can be calculated as the integral over the entire interval using the arc length function.

$$r(\tfrac{\pi}{4(24^2)}) + (\tfrac{18b+12}{r} - 24)24) + 24r_m \int_{\frac{18b+12}{2}}^{18b+12} \sqrt{1 + ((\tfrac{\pi}{2r} - \tfrac{2\pi}{3b+2})cos(\tfrac{\pi x}{18b+12}))^2} dx +$$
$$18 \int_{18b+12}^{\frac{3(18b+12)}{2}} \sqrt{1 + \tfrac{\pi}{2r} cos \tfrac{\pi x}{(18b+12)} + \tfrac{\pi x}{27(b^2)+36b+12} cos \tfrac{\pi x}{18b+12} + \tfrac{2}{3b+2} sin \tfrac{\pi x}{18b+12} - (\tfrac{4\pi}{3b+2}) cos(\tfrac{\pi x}{18b+12})^2)} dx$$
$$+r_{r-r_m}(24 \int_{\frac{1}{18b+12}}^{\frac{3}{2(18b+12)}} \sqrt{(1 + ((\tfrac{\pi}{2*r} - (\tfrac{2\pi}{3B+2})cos(\tfrac{\pi x}{18b+12})^2)} dx$$

This equation was too tedious and convoluted to attempt hand calculations. We created a computer program to do the computations for us and area values are shown below in **Figure 3**.

Area of Our Model as a Function of b and l (in square feet)								
Toll booths	1 lane	2 lanes	3	4	5	6	7	8
1	2,218.67	2,218.67	2,838.14	2,838.14	2,838.14	2,838.14	2,838.14	2,838.14
2	3,428.59	3,428.59	4,163.13	4,163.13	4,163.13	4,163.13	4,163.13	4,163.13
3	4,679.49	4,679.49	5,525.64	5,525.64	5,525.64	5,525.64	5,525.64	5,525.64
4	5,947.11	5,947.11	6,906.76	6,906.76	6,906.76	6,906.76	6,906.76	6,906.76
5	7,223.13	7,223.13	8,298.26	8,298.26	8,298.26	8,298.26	8,298.26	8,298.26
6	8,503.95	8,503.95	9,696.09	9,696.09	9,696.09	9,696.09	9,696.09	9,696.09
7	9,787.80	9,787.80	11,098.02	11,098.02	11,098.02	11,098.02	11,098.02	11,098.02
8	11,073.66	11,073.66	12,502.78	12,502.78	12,502.78	12,502.78	12,502.78	12,502.78
9	12,360.93	12,360.93	16,730.54	19,910.78	19,910.78	19,910.78	19,910.78	19,910.78
10	13,649.23	13,649.23	18,457.59	21,901.85	21,901.85	21,901.85	21,901.85	21,901.85
11	14,938.30	14,938.30	20,185.67	23,894.19	23,894.19	23,894.19	23,894.19	23,894.19
12	16,227.96	16,227.96	21,914.55	27,769.15	33,802.15	33,802.15	33,802.15	33,802.15
13	17,518.09	17,518.09	23,644.06	29,951.87	36,394.88	36,394.88	36,394.88	36,394.88
14	18,808.59	18,808.59	25,374.06	32,135.22	38,988.33	38,988.33	38,988.33	38,988.33
15	20,099.39	20,099.39	27,104.47	34,319.09	41,615.59	51,363.04	51,363.04	51,363.04
16	21,390.44	21,390.44	28,835.22	36,503.37	44,258.82	54,561.61	54,561.61	54,561.61

Figure 3

3.3 Traditional Flare Calculations

The total area from the tolls to the beginning of the highway can be roughly modeled by a trapezoid. The first base and the height would be equal to the width of the toll plaza. The second base would be the width of the total number of lanes. The equation is found by:

$$A = \frac{b_1 + b_2}{2h}$$
$$A = \frac{(18b + 12) + 12l}{2}(18b + 12)$$
$$A = (9b + 6 + 6l)(18b + 12)$$
$$A = 162(b^2) + 108b + 108b + 72 + 108bl + 72l$$
$$A = 162(b^2) + 216b + 108bl + 72l + 72$$

4 Sensitivity Analysis

4.1 Cost Analysis

In our model, we had to take into consideration that cost could be a large issue. As described, we plan on our model using the preexisting pavement rather than demolishing and rebuilding our model from scratch. But, if we did have to rebuild our model, the cost is displayed below in **Figure 4**. This shows the cost difference between our model and the current toll exit system implemented on the New Jersey Turnpike.

Cost Difference between Models (Our Cost - Traditional Cost)

Toll booths	1 lane	2 lanes	3	4	5	6	7	8
1	$ 4,575.37	$ 4,056.97	$ 5,322.64	$ 4,804.24	$ 4,285.84	$ 3,767.44	$ 3,249.04	$ 2,730.64
2	$ 5,727.14	$ 4,897.70	$ 6,183.73	$ 5,354.29	$ 4,524.85	$ 3,695.41	$ 2,865.97	$ 2,036.53
3	$ 6,063.81	$ 4,923.33	$ 6,219.76	$ 5,079.28	$ 3,938.80	$ 2,798.32	$ 1,657.84	$ 517.36
4	$ 5,515.52	$ 4,064.00	$ 5,376.27	$ 3,924.75	$ 2,473.23	$ 1,021.71	$ (429.81)	$ (1,881.33)
5	$ 4,058.29	$ 2,295.73	$ 3,629.55	$ 1,866.99	$ 104.43	$ (1,658.13)	$ (3,420.69)	$ (5,183.25)
6	$ 1,681.78	$ (391.82)	$ 967.94	$ (1,105.66)	$ (3,179.26)	$ (5,252.86)	$ (7,326.46)	$ (9,400.06)
7	$ (1,619.14)	$ (4,003.78)	$ (2,614.98)	$ (4,999.62)	$ (7,384.26)	$ (9,768.90)	$ (12,153.54)	$ (14,538.18)
8	$ (5,847.38)	$ (8,543.06)	$ (7,122.87)	$ (9,818.55)	$ (12,514.23)	$ (15,209.91)	$ (17,905.59)	$ (20,601.27)
9	$ (11,004.68)	$ (14,011.40)	$ (4,433.64)	$ 1,718.73	$ (1,287.99)	$ (4,294.71)	$ (7,301.43)	$ (10,308.15)
10	$ (17,092.14)	$ (20,409.90)	$ (9,879.58)	$ (3,277.87)	$ (6,595.63)	$ (9,913.39)	$ (13,231.15)	$ (16,548.91)
11	$ (24,110.50)	$ (27,739.30)	$ (16,255.67)	$ (9,203.93)	$ (12,832.73)	$ (16,461.53)	$ (20,090.33)	$ (23,719.13)
12	$ (32,060.28)	$ (36,000.12)	$ (23,562.58)	$ (10,641.17)	$ 2,794.03	$ (1,145.81)	$ (5,085.65)	$ (9,025.49)
13	$ (40,941.82)	$ (45,192.70)	$ (31,800.79)	$ (17,885.17)	$ (3,580.19)	$ (7,831.07)	$ (12,081.95)	$ (16,332.83)
14	$ (50,755.42)	$ (55,317.34)	$ (40,970.71)	$ (26,060.49)	$ (10,885.45)	$ (15,447.37)	$ (20,009.29)	$ (24,571.21)
15	$ (61,501.28)	$ (66,374.24)	$ (51,072.57)	$ (35,167.42)	$ (19,026.46)	$ 4,173.24	$ (699.72)	$ (5,572.68)
16	$ (73,179.53)	$ (78,363.53)	$ (62,106.57)	$ (45,206.29)	$ (28,054.60)	$ (3,566.56)	$ (8,750.56)	$ (13,934.56)

Figure 4

Figure 4 shows that our model is more expensive when the amount of highway lanes and toll booths are small. As the amount lanes and booths gets large, our model proves to be more cost effective than producing the exit as a section of tapering pavement.

4.2 Model Analysis

4.2.1 Design

As we designed our model, we discovered that the optimal amount of branches our model should have is two. We found that we only need two branches to keep traffic flow constant. Something we did not take into consideration is the percentage of commercial vehicles that will not be using ETC to pay. Our model does not correctly support the size of large commercial vehicles using coin/full service toll booth. Thus as more vehicles exit the toll booths, throughput may decrease. Furthermore, as the number of booths increases we found that two branches was not enough. Displayed below in **Figure 5** is the updated model where we implement three branches into out toll exiting system.

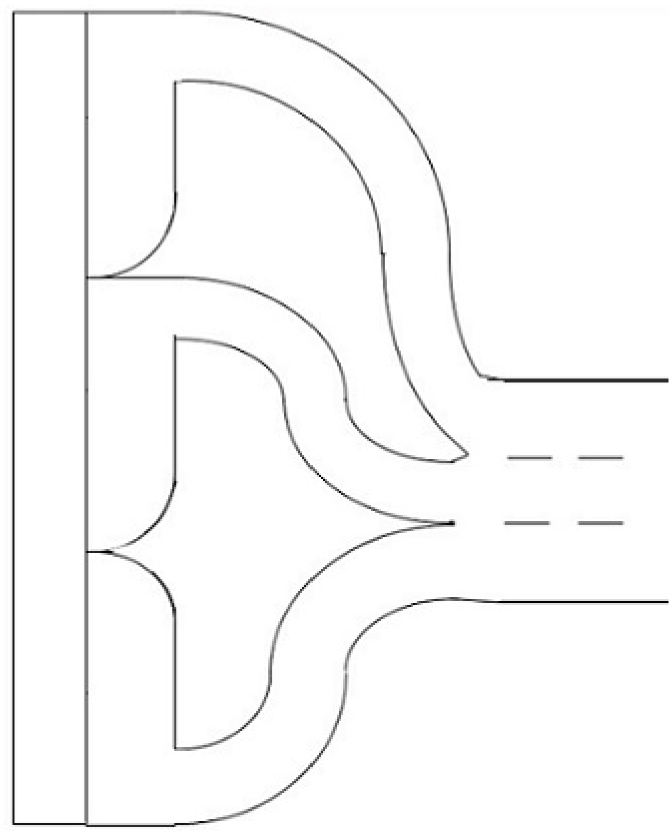

Figure 5

In this diagram, the middle branch will include all three types of payment methods due to the location of the booths. Having an extra branch will improve traffic flow. Our cost analysis includes the price of additional branches using the equation that related lanes and branches.

4.2.2 Implementation of Autonomous Cars

When completing research on autonomous cars, we found that these cars would only increase the amount of throughput our model creates. Since autonomous cars always follow the driving laws, merge faster than regular cars, and keep a safe driving distance [1]. These cars would increase the flow of traffic through the toll booth.

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