Problem Sheet 3

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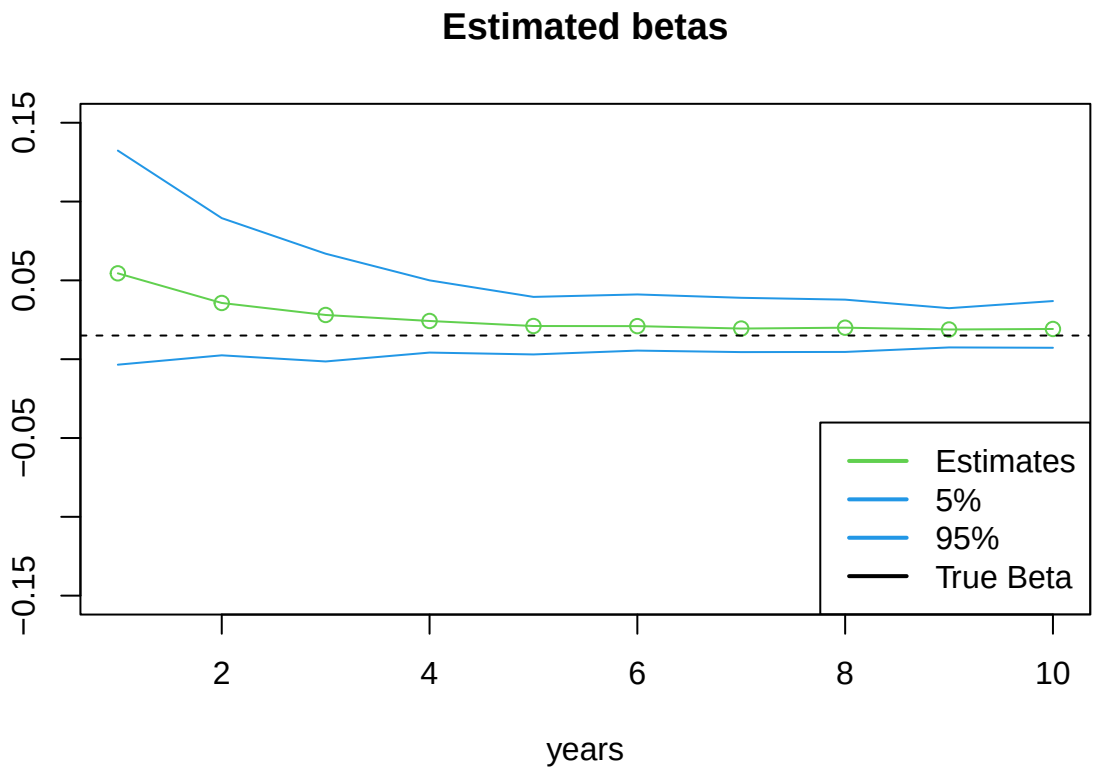
Code is posted at the following site: https://github.com/alexanderober/BUSI525/tree/main/PS3

Part 1

The bias decreases as the sample size increases. This is something Stambaugh points out; it's a small sample bias.

```
library(MASS)

## initialize parameters
beta = 0.015
sigma_u = 0.053
sigma_v = 0.044
rho = 0.98
rho_uv = -0.8
sigma_uv = rho_uv*sigma_u*sigma_v
Sigma <- matrix(c(sigma_u^2, sigma_uv, sigma_uv, sigma_v^2),2,2)

## initialize vectors to hold the estimates
mean_betas = rep(0, 10)
betas_5 = rep(0, 10)
betas_95 = rep(0, 10)

B <- 250

for (t in 120*1:10){
  beta_hat = rep(0, B)
  for (b in 1:B){
    ## simulate the data forward
    rx = matrix(rep(0, (t+1)*2), nrow = t+1)
    uv = mvrnorm(n = t, rep(0, 2), Sigma)
    for (i in 1:t){
      rx[i+1, ] = c(beta, rho)*rx[i, 2] + uv[i,]
    }

    ## perform the regression and hold the coefficient
    mod <- lm(rx[(2:(t+1)), 1]~rx[(1:t), 2])
    summ <- summary(mod)
    beta_hat[b] <- summ$coefficients[2, 1]
    summ$coefficients[2, 1]
  }

  ## form aggregate statistics
```

```
  beta_hat = sort(beta_hat)
  mean_betas[t/120] = mean(beta_hat)
  betas_5[t/120] = beta_hat[round(0.05*B)]
  betas_95[t/120] = beta_hat[round(0.95*B)]
}

## plot the data
plot(mean_betas, main = "Estimated betas",
     col = 3,
     xlab = 'years', ylab = '',
     ylim = c(-0.15, 0.15))
lines(x = 1:10, mean_betas, col = 3)
lines(x = 1:10, y = betas_5, col =20)
lines(x = 1:10, y = betas_95, col =20)
abline(h = beta, lty = 'dashed')
legend(x = 'bottomright', legend = c(NULL,'Estimates', '5%', '95%', 'True Beta'),
       col = c(3, 20, 20, 'black'), lwd = 2)
```

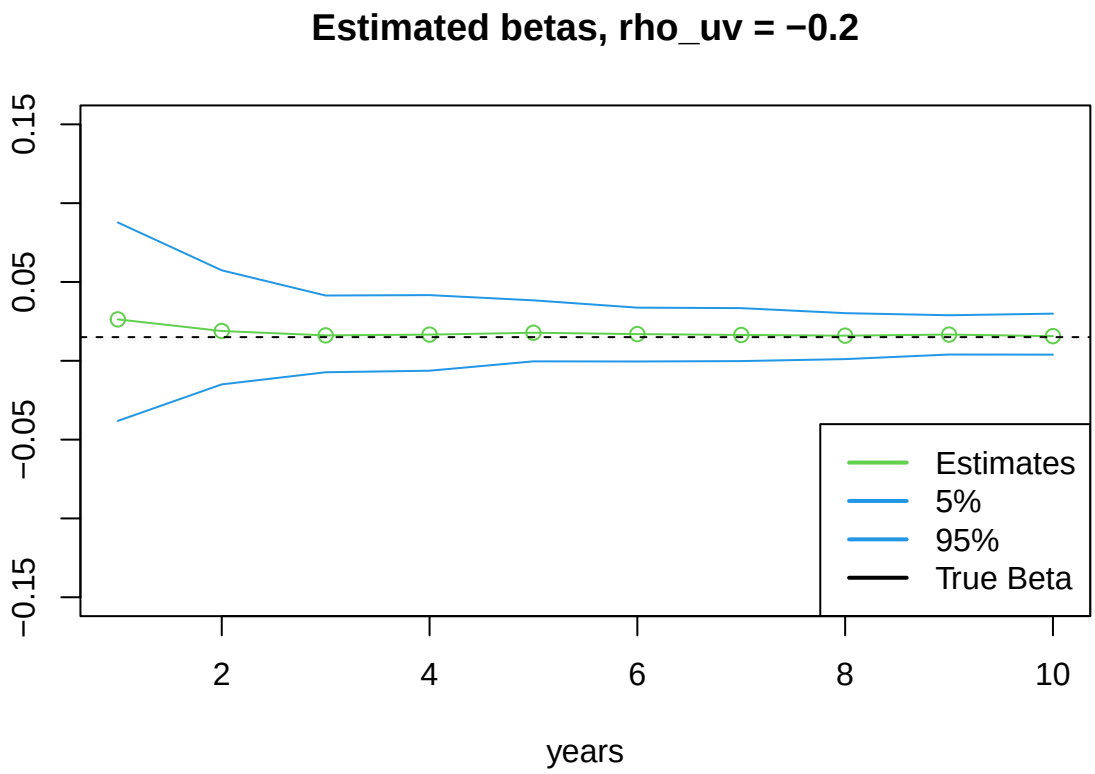


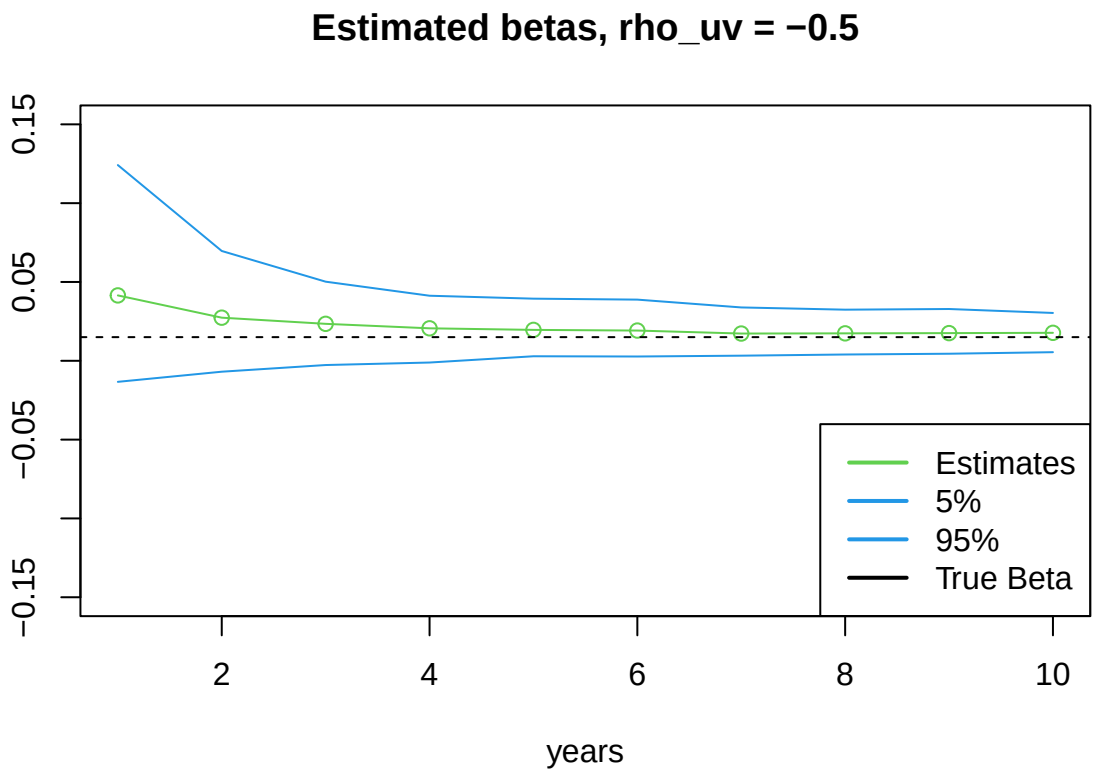
Part 2

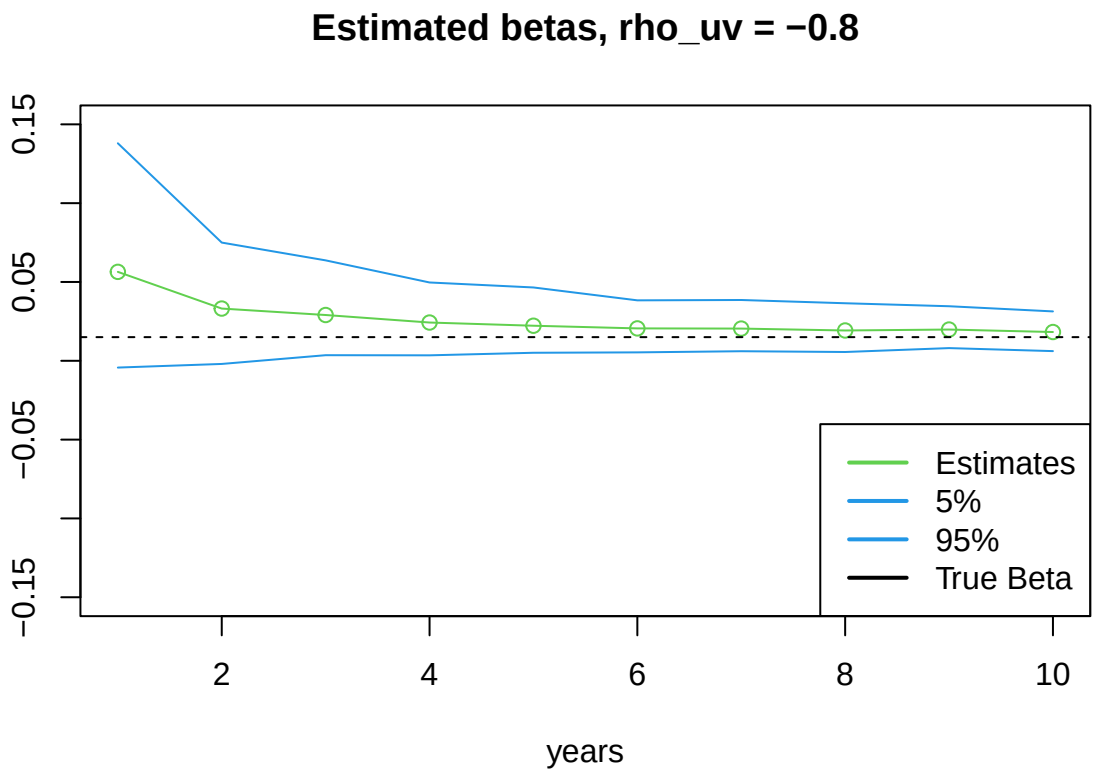
The bias is smaller for smaller magnitudes of the correlation. This is something Stambaugh points out.

```
## same code as above, with an outer loop to loop over values of rho_uv
for (rho_uv in c(-0.2, -0.5, -0.8)){
  sigma_uv = rho_uv*sigma_u*sigma_v
  Sigma <- matrix(c(sigma_u^2, sigma_uv, sigma_uv, sigma_v^2),2,2)
```

```
  for (t in 120*1:10){
    beta_hat = rep(0, B)
    for (b in 1:B){
      rx = matrix(rep(0, (t+1)*2), nrow = t+1)
      uv = mvrnorm(n = t, rep(0, 2), Sigma)
      for (i in 1:t){
        rx[i+1, ] = c(beta, rho)*rx[i, 2] + uv[i,]
      }
      mod <- lm(rx[(2:(t+1)), 1]~rx[(1:t), 2])
      summ <- summary(mod)
      beta_hat[b] <- summ$coefficients[2, 1]
      summ$coefficients[2, 1]
    }
    beta_hat = sort(beta_hat)
    mean_betas[t/120] = mean(beta_hat)
    betas_5[t/120] = beta_hat[round(0.05*B)]
    betas_95[t/120] = beta_hat[round(0.95*B)]
  }
  title = paste0('Estimated betas, rho_uv = ', rho_uv)
  plot(mean_betas, main = title,
       col = 3,
       xlab = 'years', ylab = '',
       ylim = c(-0.15, 0.15))
  lines(x = 1:10, mean_betas, col = 3)
  lines(x = 1:10, y = betas_5, col =20)
  lines(x = 1:10, y = betas_95, col =20)
  abline(h = beta, lty = 'dashed')
  legend(x = 'bottomright', legend = c(NULL,'Estimates', '5%', '95%', 'True Beta'),
         col = c(3, 20, 20, 'black'), lwd = 2)
}
```


Estimated betas, rho_uv = −0.2
0.15
0.05
−0.05
−0.15
2
4
6
8
10
years
Estimates
5%
95%
True Beta


Estimated betas, rho_uv = −0.5
0.15
0.05
−0.05
−0.15
2
4
6
8
10
years
Estimates
5%
95%
True Beta


Estimated betas, rho_uv = −0.8
0.15
0.05
−0.05
−0.15
2
4
6
8
10
years
Estimates
5%
95%
True Beta