Problem Sheet 1

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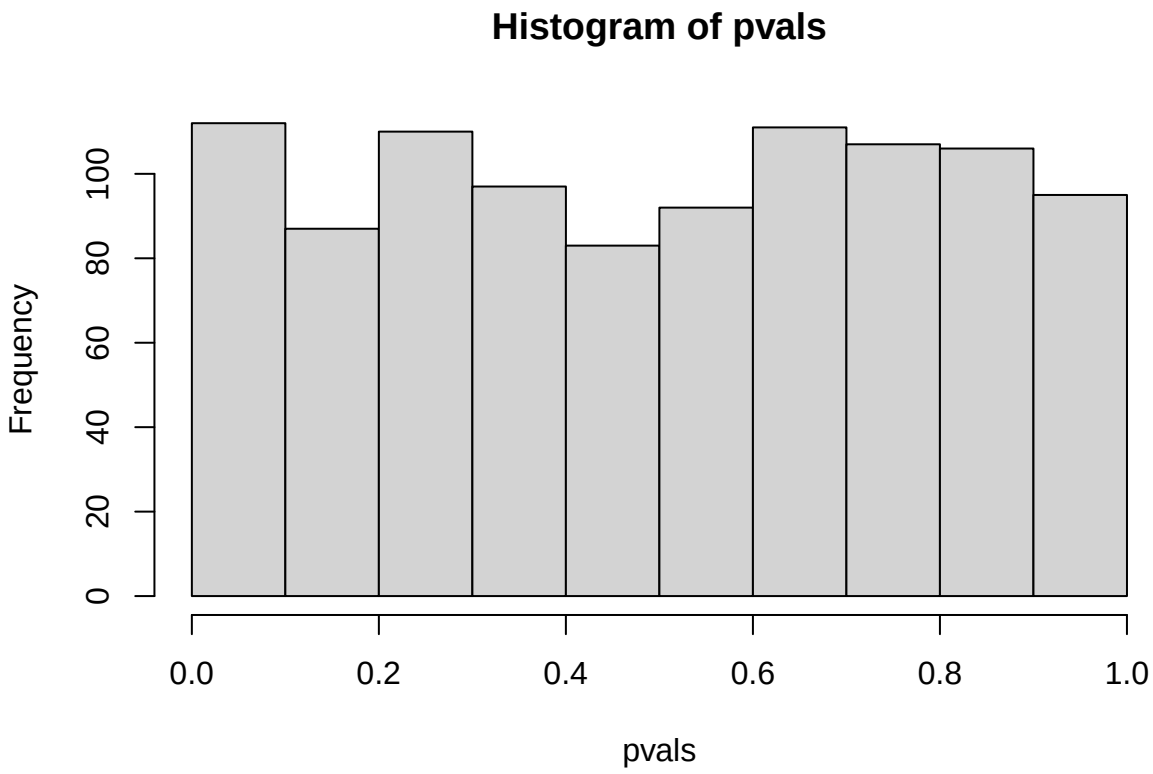
Problem 1

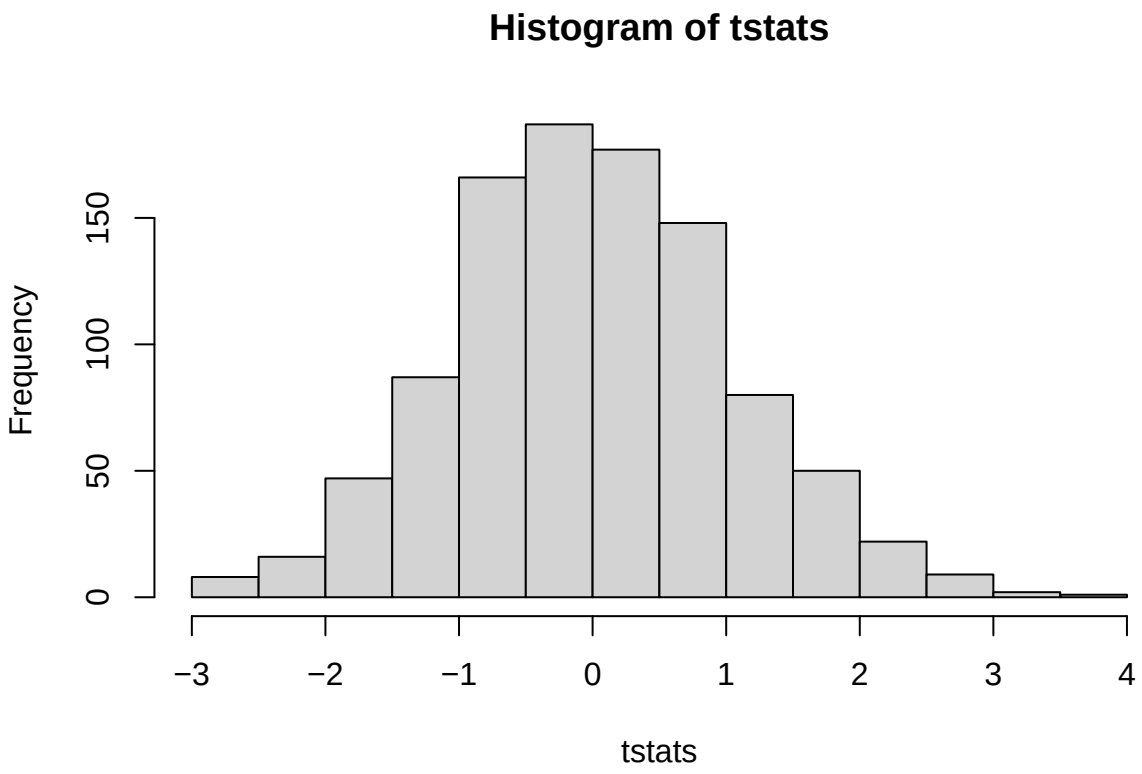
Note throughout I construct p-values, etc., as if the alternative is one-sided, since presumably we don't care if the alpha is negative. ### (a)

62 of the 1000 funds have significantly positive alphas. We'd expect about 5% of the statistics to be significant, which is what we see.

(b)

Here are some plots:





(c)

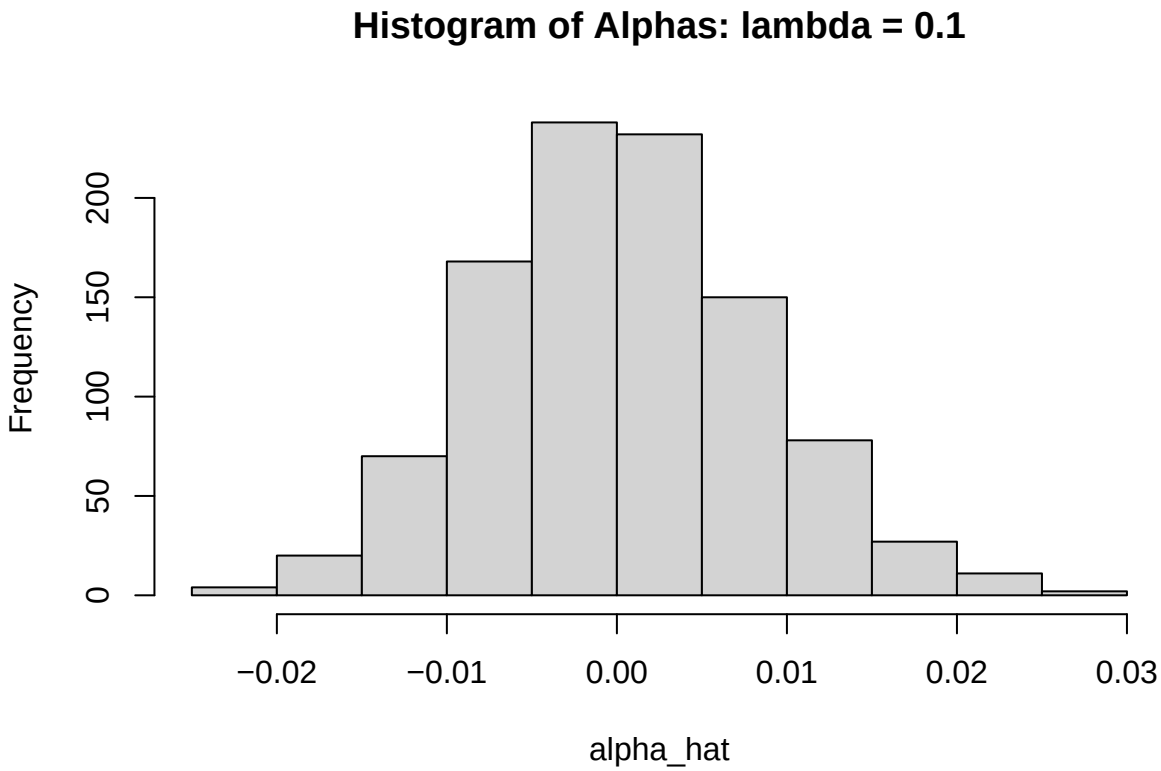
The distribution looks essentially uniform[0, 1]. This is not too surprising, as under the null the coefficients have a t distribution centered at 0, so p values are uniformly distributed. (For any $p \in \mathbb{R}$, if F is the t stat cdf under the null, and the alternative is of the form $tstat > 0$, then by symmetry of the t-distribution cdf,

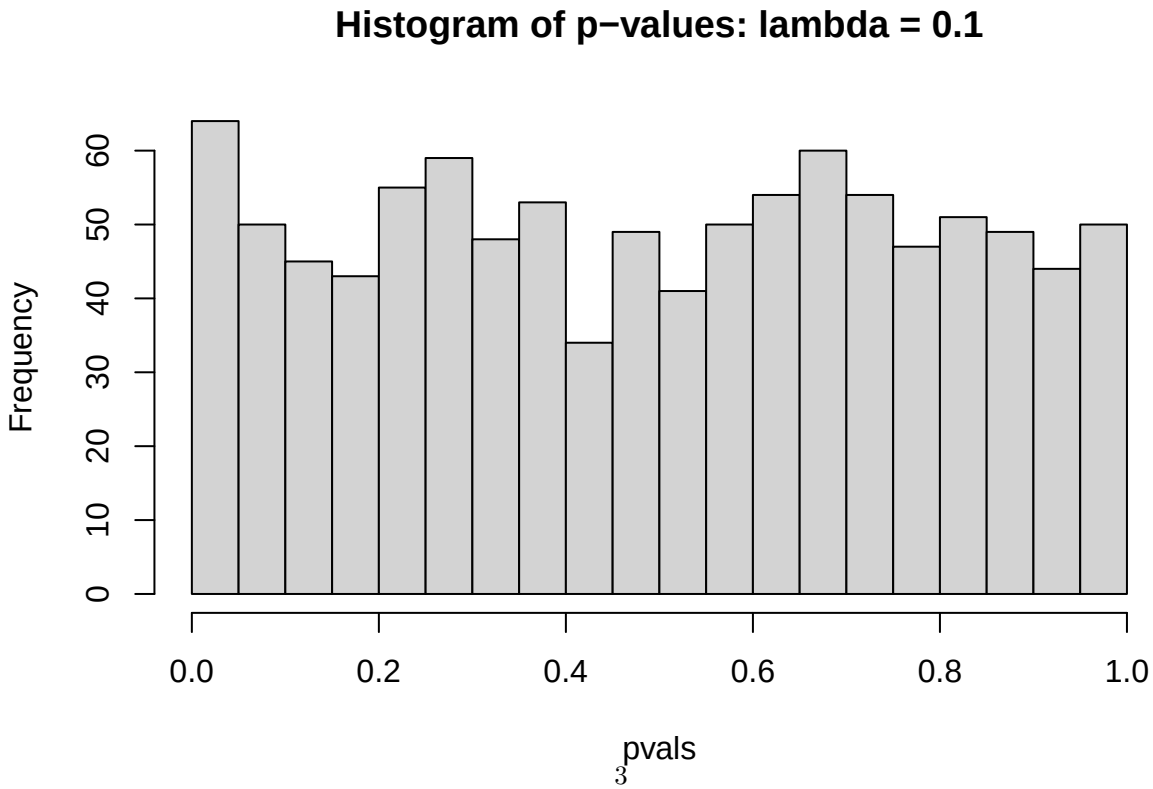
$$Prob(pval < p) = P(F^{-1}(pval) < F^{-1}(p)) = P(1 - tstat < F^{-1}(p)) = 1 - F(1 - F^{-1}(p)) = 1 - (1 - p) = p.$$

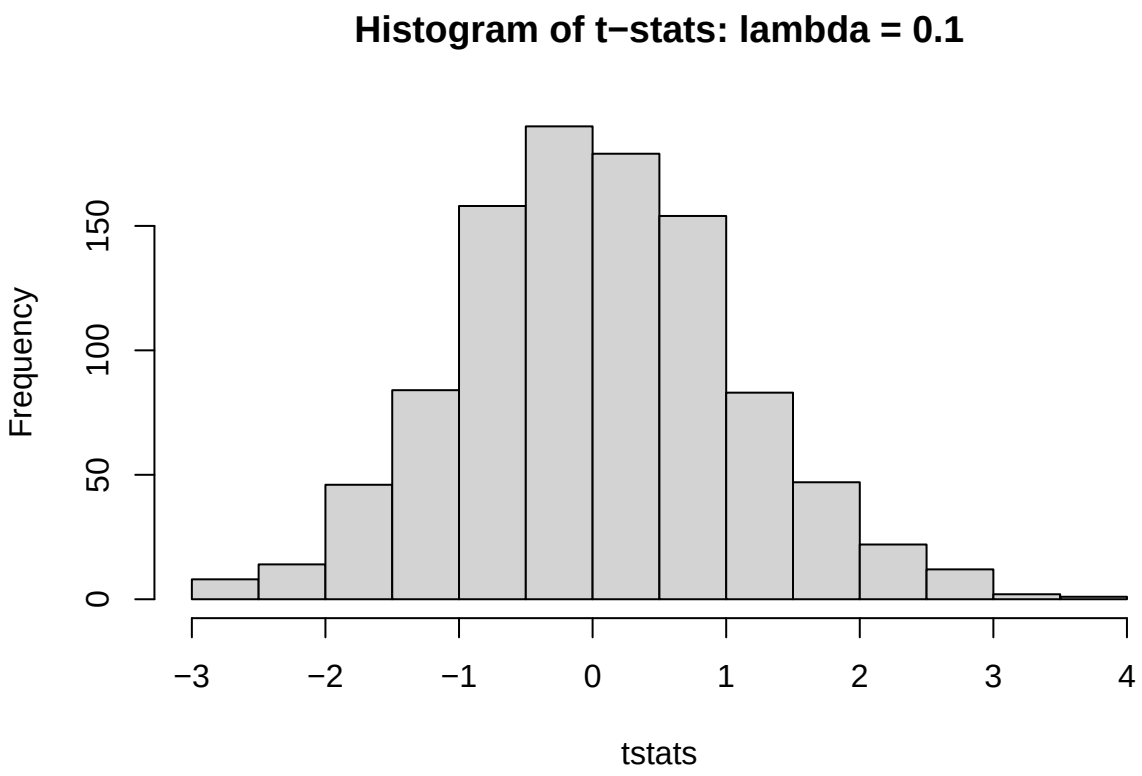
)

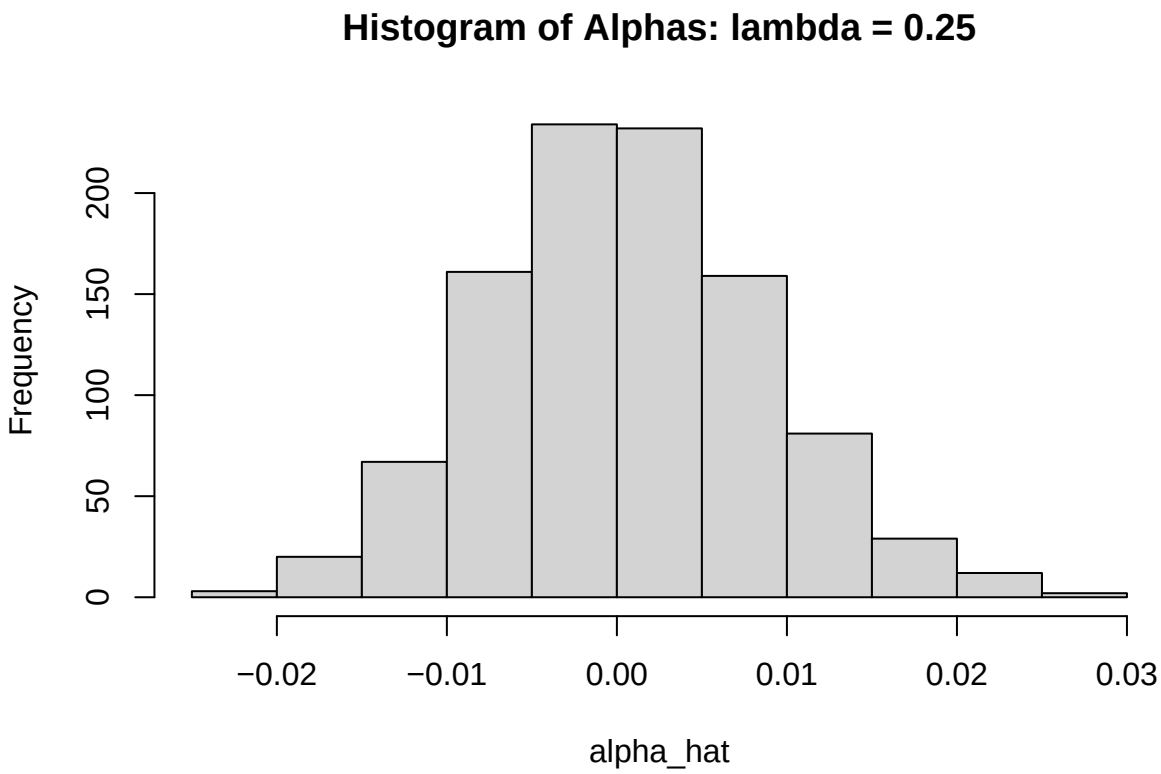
Problem 2

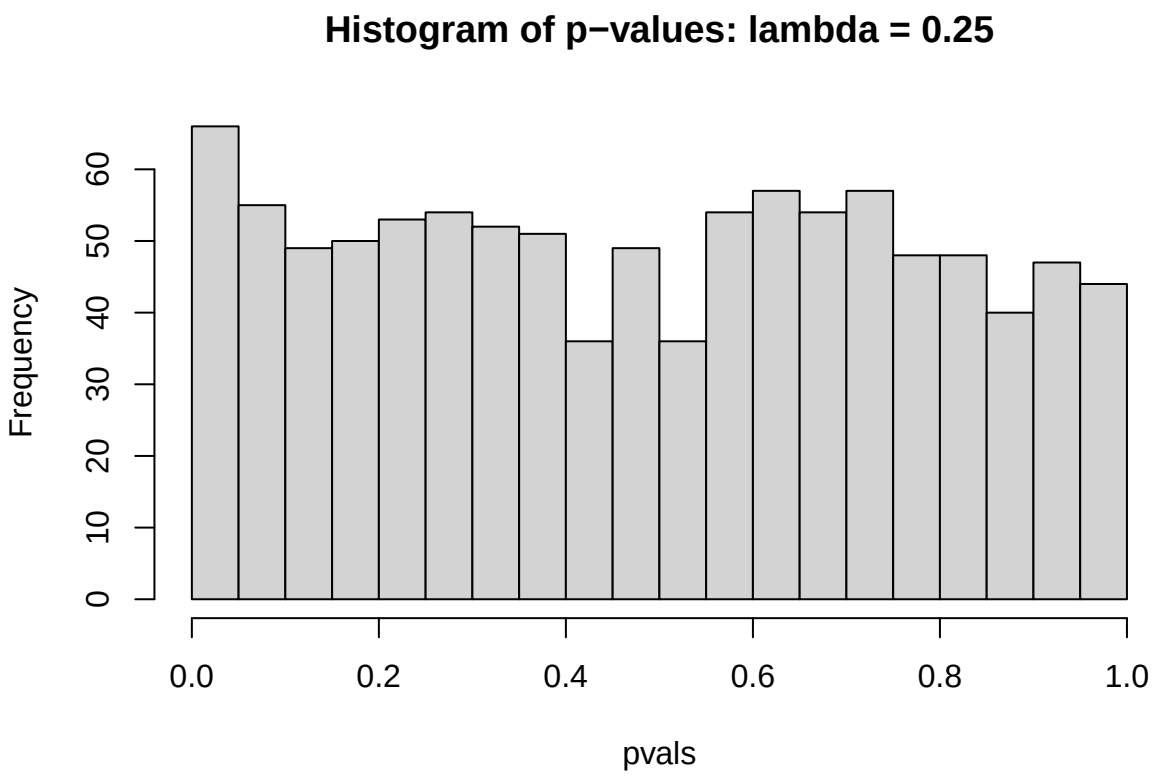
(a)

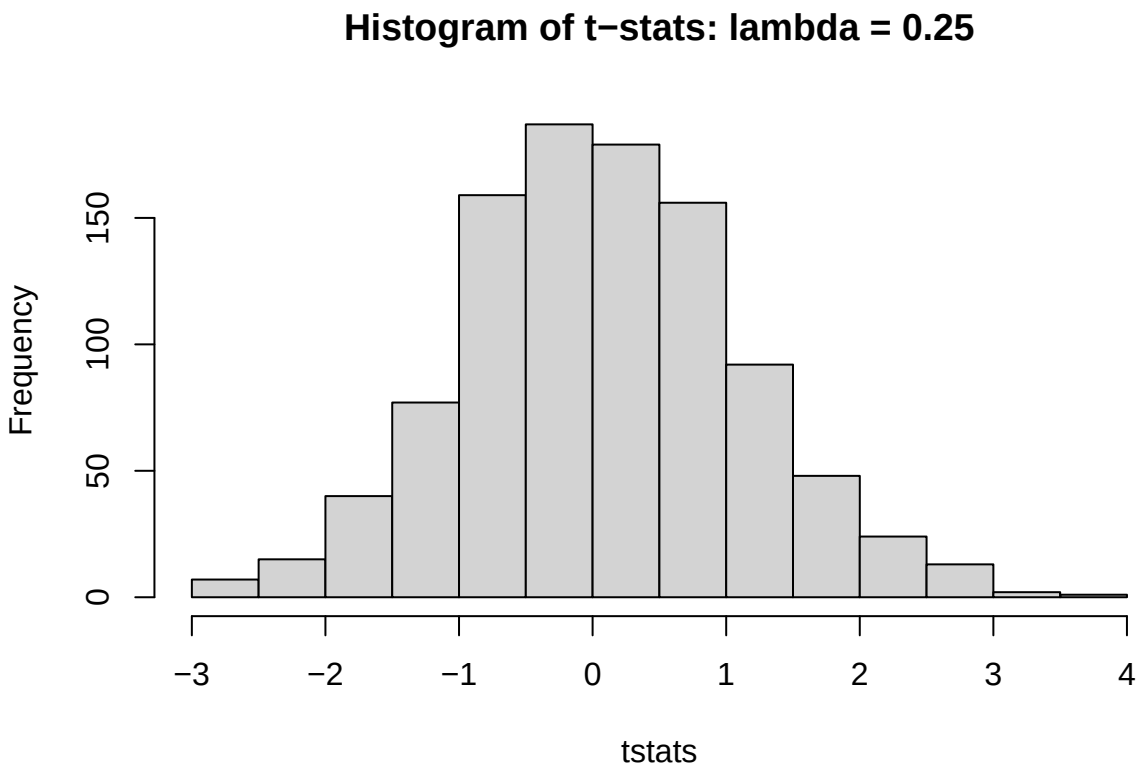

Histogram of Alphas: lambda = 0.1
Frequency
0
50
100
150
200
−0.02
−0.01
0.00
0.01
0.02
0.03
alpha_hat

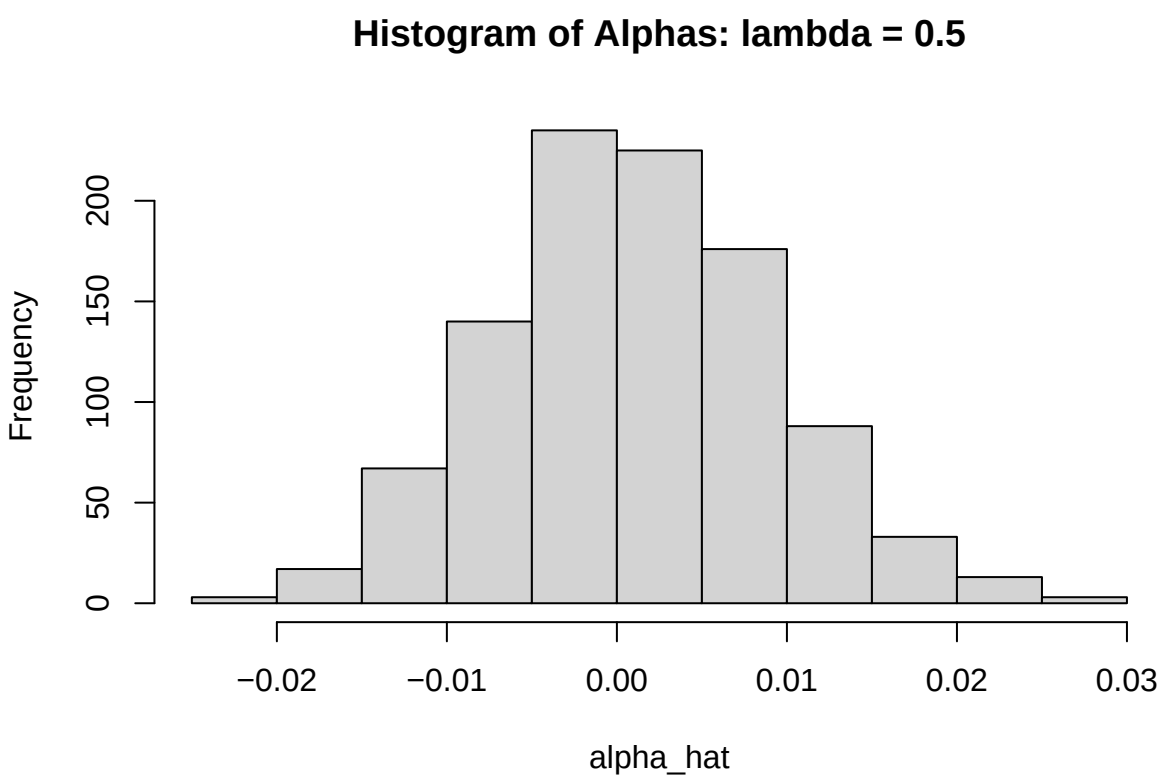

Histogram of p−values: lambda = 0.1
Frequency
0
10
20
30
40
50
60
0.0
0.2
0.4
0.6
0.8
1.0
pvals

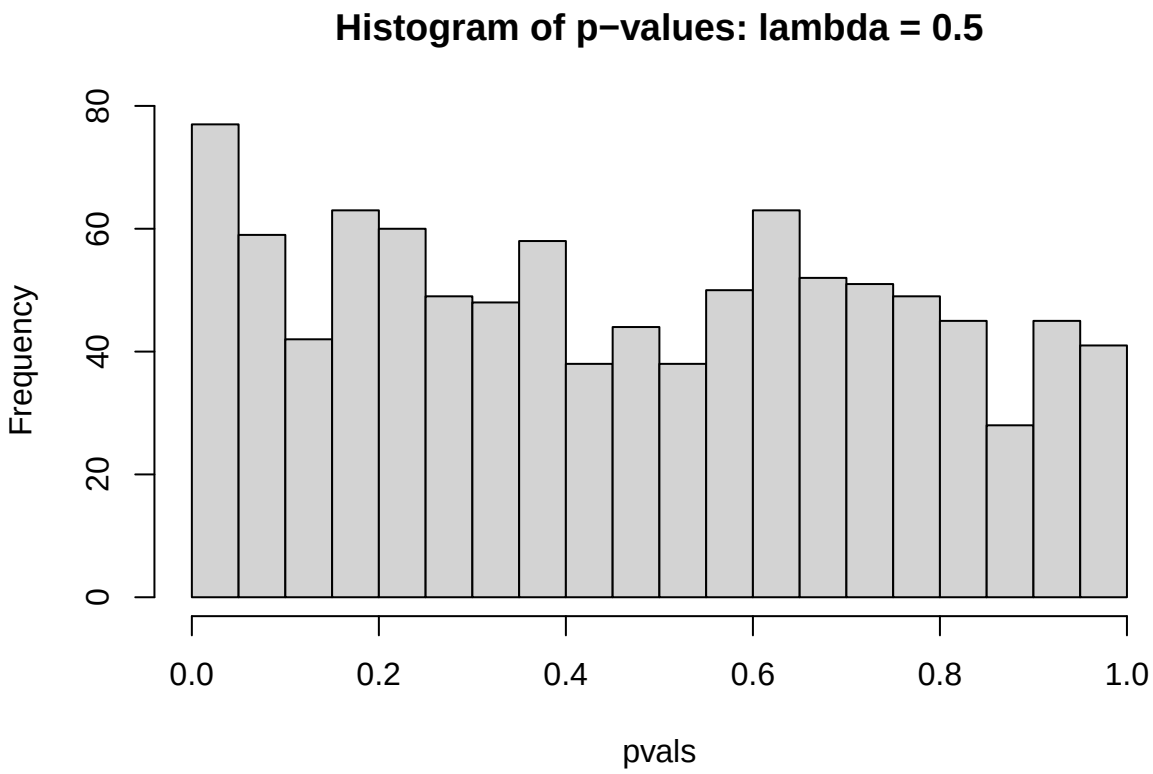

Histogram of t−stats: lambda = 0.1
Frequency
0
50
100
150
−3
−2
−1
0
1
2
3
4
tstats

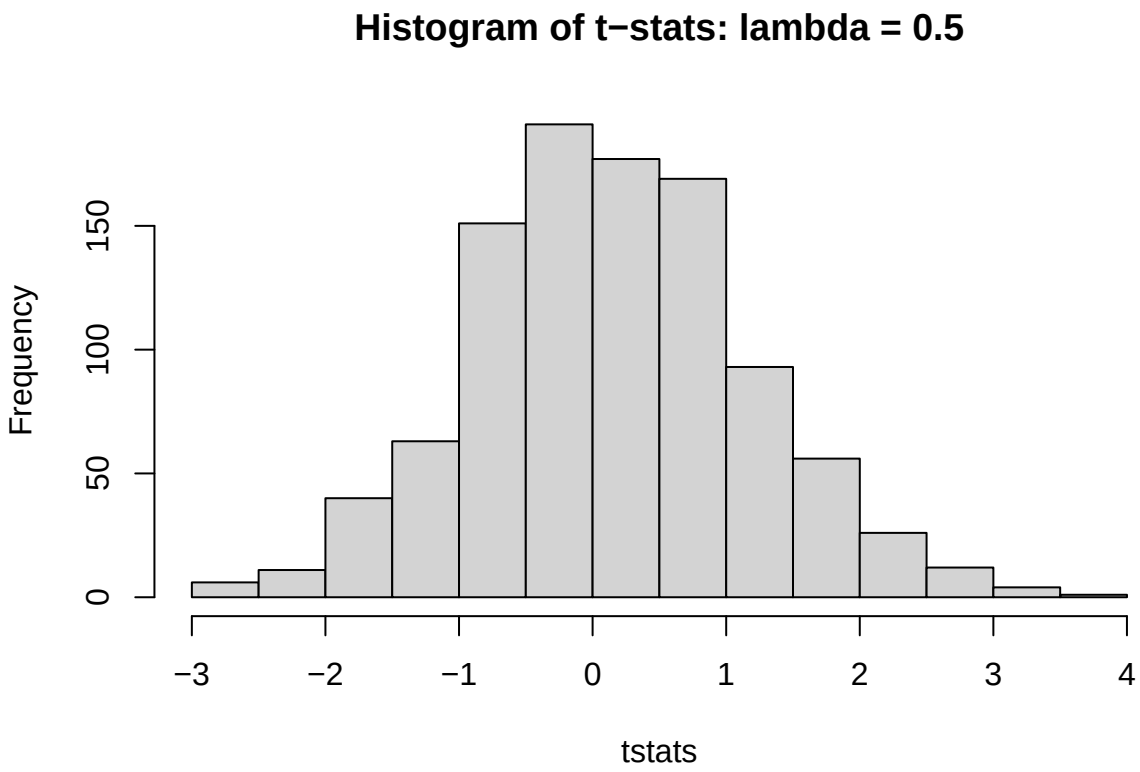

Histogram of Alphas: lambda = 0.25
Frequency
0
50
100
150
200
−0.02
−0.01
0.00
0.01
0.02
0.03
alpha_hat

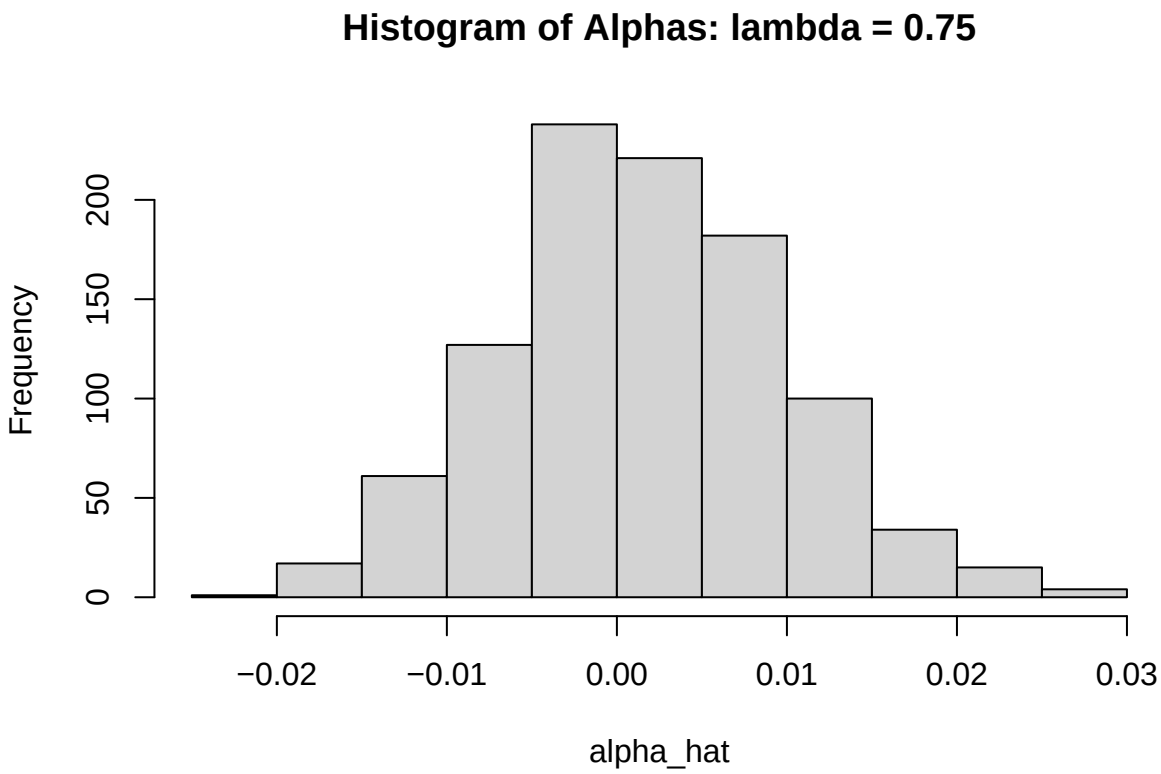

Histogram of p−values: lambda = 0.25
Frequency
0
10
20
30
40
50
60
0.0
0.2
0.4
0.6
0.8
1.0
pvals

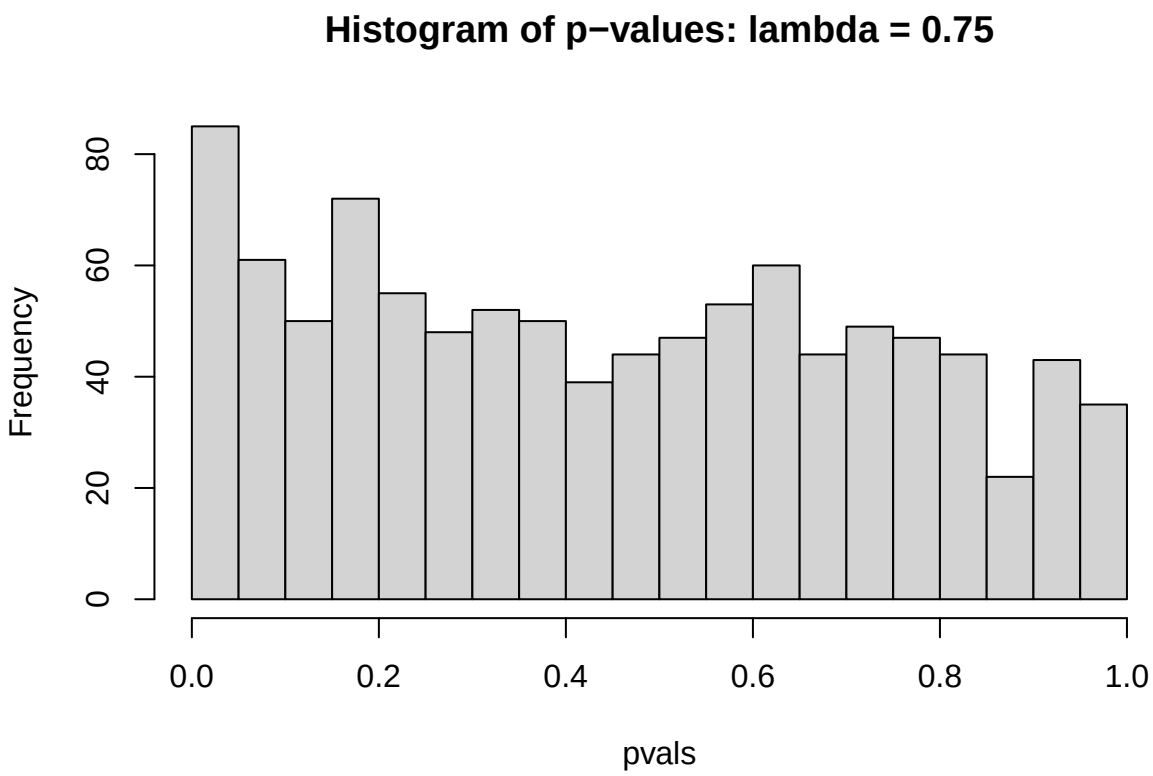

Histogram of t−stats: lambda = 0.25
Frequency
0
50
100
150
−3
−2
−1
0
1
2
3
4
tstats

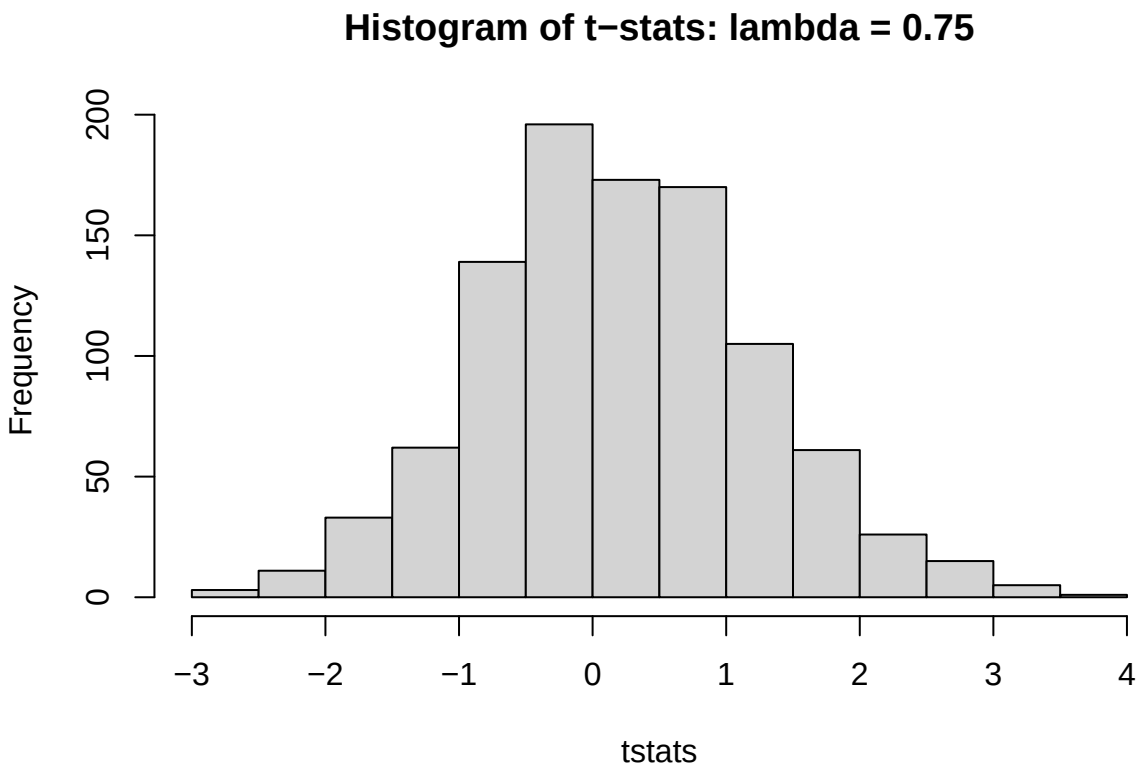

Histogram of Alphas: lambda = 0.5
Frequency
0
50
100
150
200
−0.02
−0.01
0.00
0.01
0.02
0.03
alpha_hat


Histogram of p−values: lambda = 0.5
Frequency
0
20
40
60
80
0.0
0.2
0.4
0.6
0.8
1.0
pvals


Histogram of t−stats: lambda = 0.5
Frequency
0
50
100
150
−3
−2
−1
0
1
2
3
4
tstats


Histogram of Alphas: lambda = 0.75
Frequency
0
50
100
150
200
−0.02
−0.01
0.00
0.01
0.02
0.03
alpha_hat


Histogram of p−values: lambda = 0.75
Frequency
0
20
40
60
80
0.0
0.2
0.4
0.6
0.8
1.0
pvals



The distributions of t-stats and p-values visibly shift to the right as we increase lambda. However, although the p-values do shift a little bit to the left (i.e., towards more slopes being statistically non-zero), the shift isn't massive. This is perhaps not too surprising since there's a lot of noise in the error terms, making skill fairly challenging to detect even when the alpha is half the market risk premium.

(b)

Below I construct four tables. The percentage (multiply by 1000 for the number) of truly skilled funds with insignificant alpha estimates is given in the false negative cell. This number is generally around 90% or maybe a bit more of the total number of skilled funds. The number of truly unskilled funds identified as skilled based on significantly positive alpha estimates is given in the false positive cell, and generally consists of around 5% of the unskilled funds. Lambda = 0.1

Truth\Estimate	Positive	Negative	Total
True	0.013	0.087	0.1
False	0.051	0.849	0.9
Total	0.064	0.936	

Lambda = 0.25

Truth\Estimate	Positive	Negative	Total
True	0.021	0.229	0.25
False	0.045	0.705	0.75
Total	0.066	0.934	

Lambda = 0.5

Truth\Estimate	Positive	Negative	Total
True	0.043	0.457	0.5
False	0.034	0.466	0.5
Total	0.077	0.923	

Lambda = 0.75

Truth\Estimate	Positive	Negative	Total
True	0.067	0.683	0.75
False	0.018	0.232	0.25
Total	0.085	0.915	