PHYS 325 Assignment 8:
Potentials and Fields

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PHYS 325 Computational Physics

1 Problem 5.1: Potential of the Prism Geometry

For this problem, the relaxation method was used to approximate a solution to the field equation for a hollow metallic prism with a solid, metallic inner conductor. The relaxation method was run 200 times, resulting in a precise solution. The potential of the inner conductor was held at 1V, while the outer boundaries of the metallic prism were held at 0V. The relaxation method was run to approximate a solution each time to these initial conditions, with each run lessening the error in the approximation. The results for this approximation are shown in **??**

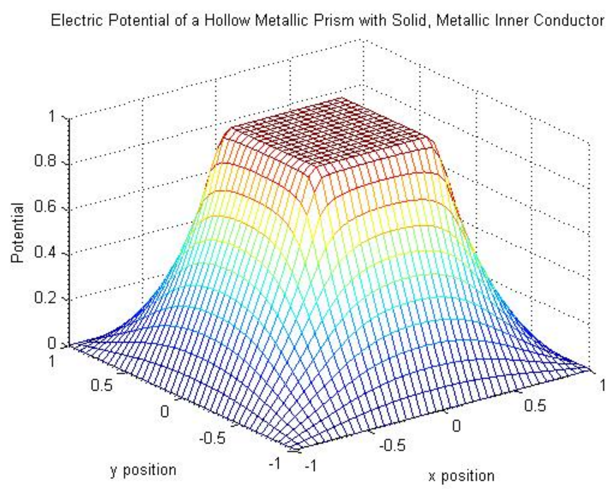

Figure 1: The potential field for a hollow metallic prism with a solid, metallic inner conductor. The potential at the outermost part of the figure was held at 0V, while the prism had a potential of 1V. The potential field was solved using the relaxation method.

2 Problem 5.4: Fringing Field of a Parallel Plate Capacitor

First, the potential field will be shown with varying distances between the capacitor plates. The greatest fringing effects are not seen when the plates are closest, nor when the plates are maximally apart. The fringing effects appear to actually be greatest when the plates are at a distance of $d = 0.5$. Any lesser or further distance results in either the fringing effects being contained too far within the plates themselves, or that the distance between them is sufficient to lessen the E-Field dramatically. Units for the distance between the plates are not given, but neither is the length of the capacitor plates themselves, so the system is relative to the units imposed on the $1x1$ box of the figure.

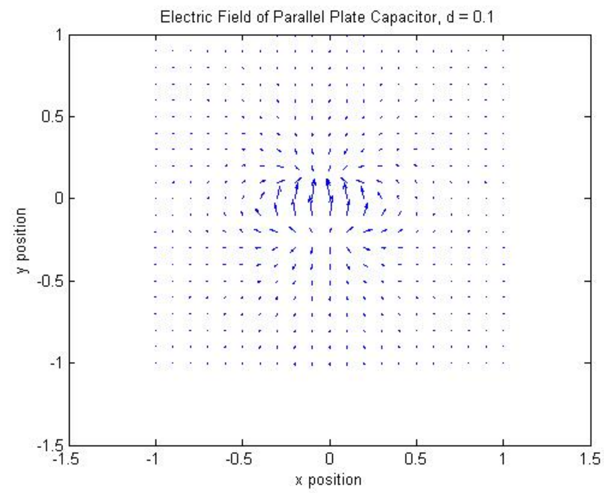

Figure 2: The electric field about two capacitor plates with spacing $d = 0.1$.

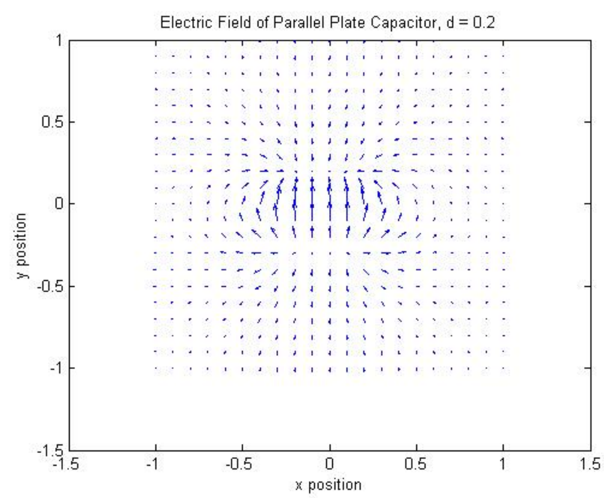

Figure 3: The electric field about two capacitor plates with spacing $d = 0.2$.

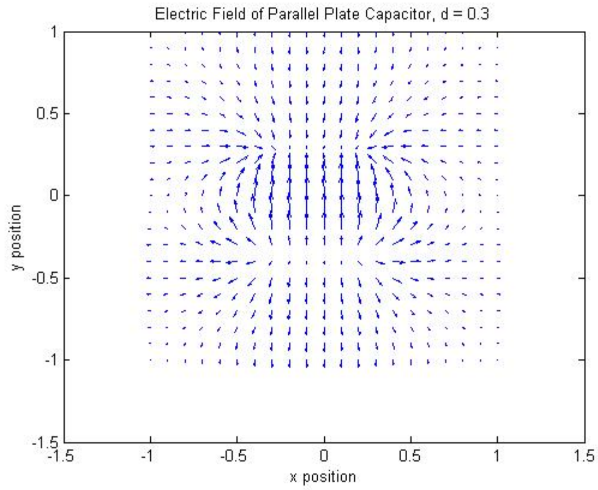

Figure 4: The electric field about two capacitor plates with spacing $d = 0.3$.

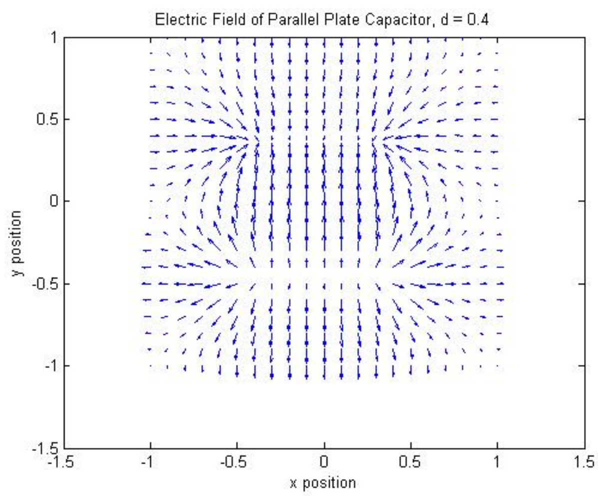

Figure 5: The electric field about two capacitor plates with spacing $d = 0.4$.

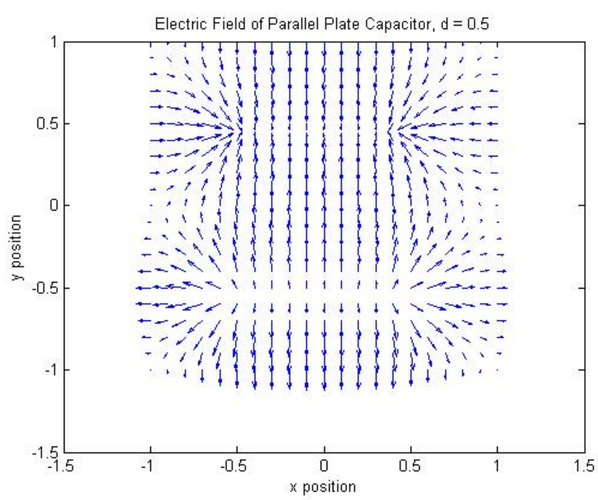

Figure 6: The electric field about two capacitor plates with spacing $d = 0.5$.

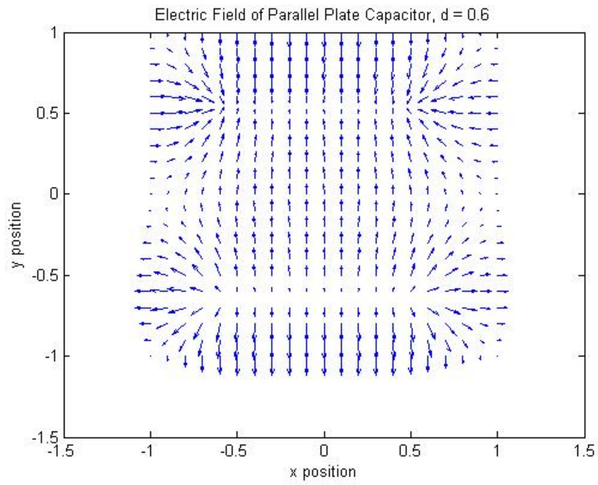

Figure 7: The electric field about two capacitor plates with spacing $d = 0.6$.

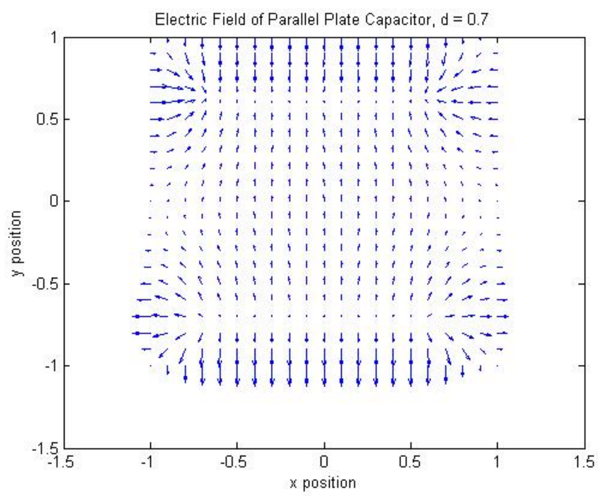

Figure 8: The electric field about two capacitor plates with spacing $d = 0.7$.

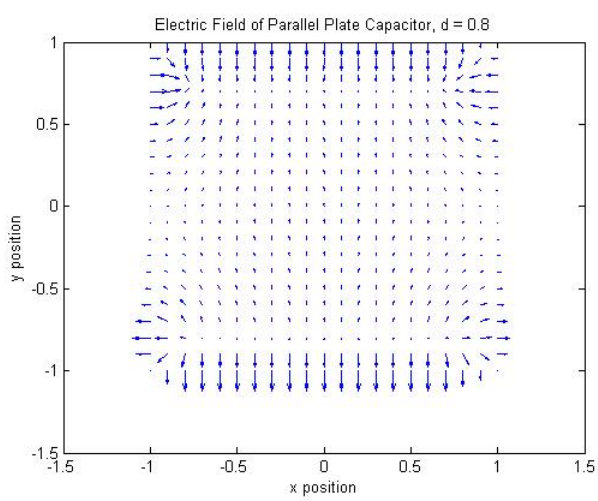

Figure 9: The electric field about two capacitor plates with spacing $d = 0.8$.

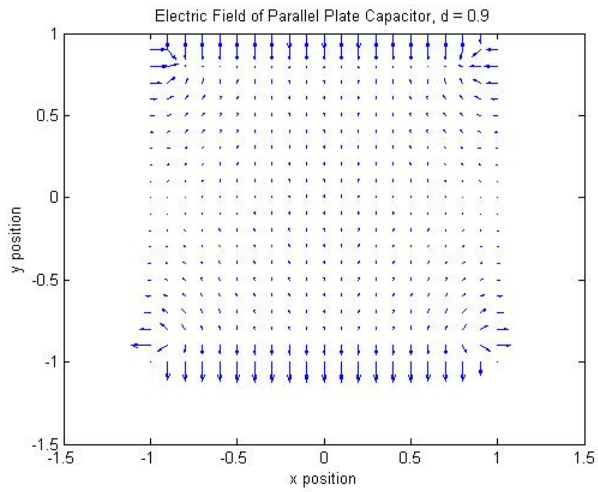

Figure 10: The electric field about two capacitor plates with spacing $d = 0.9$.

The potential can also be viewed similar to the way it was for **??** with the following figures.

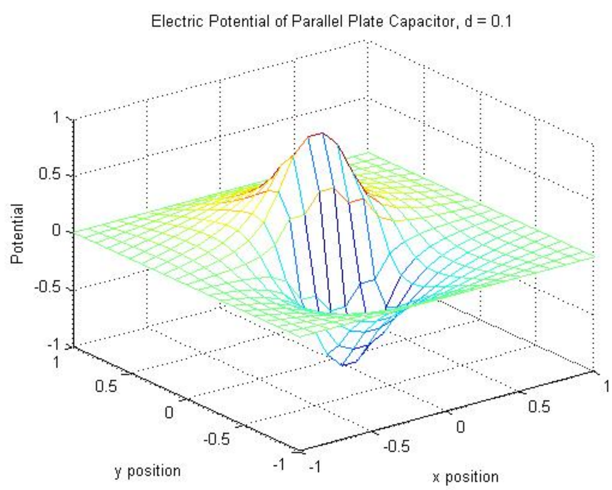

Figure 11: The potential about two capacitor plates with spacing $d = 0.1$.

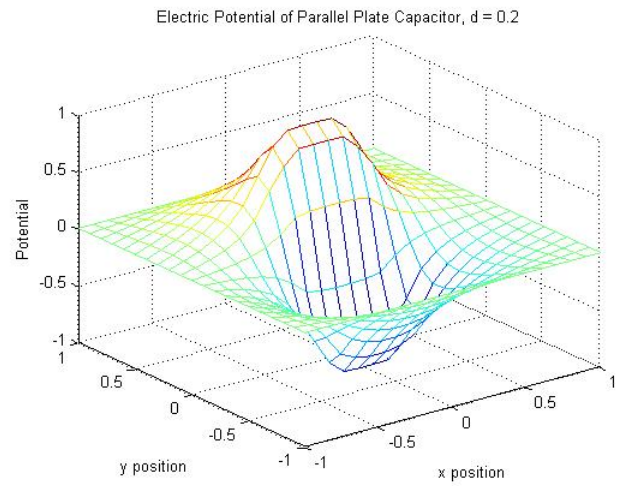

Figure 12: The potential about two capacitor plates with spacing $d = 0.2$.

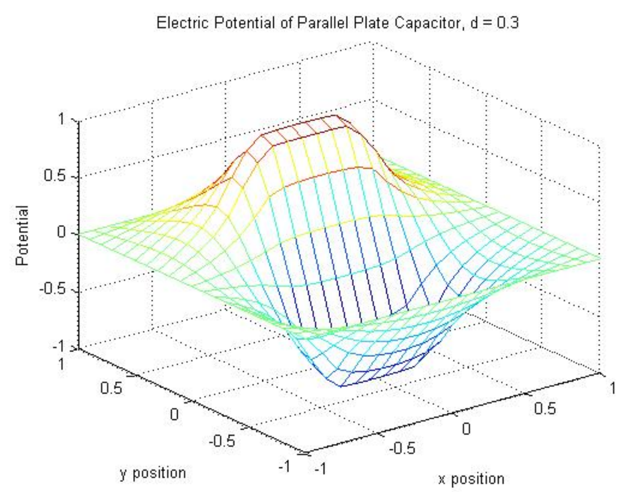

Figure 13: The potential about two capacitor plates with spacing $d = 0.3$.

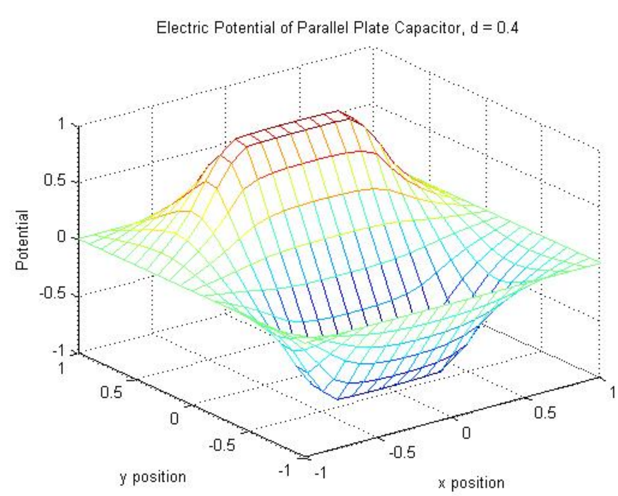

Figure 14: The potential about two capacitor plates with spacing $d = 0.4$.

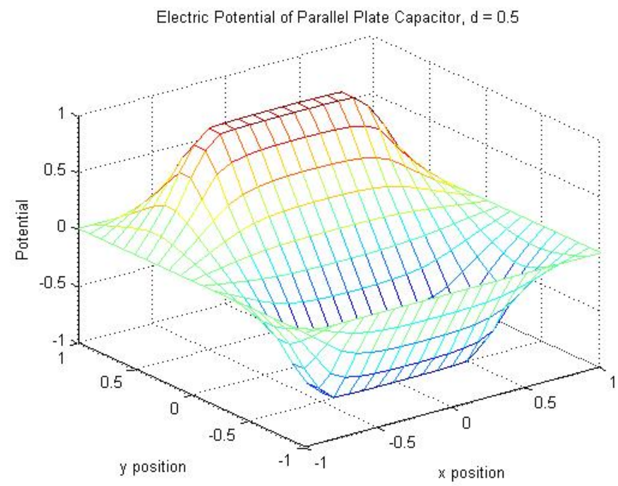

Figure 15: The potential about two capacitor plates with spacing $d = 0.5$.

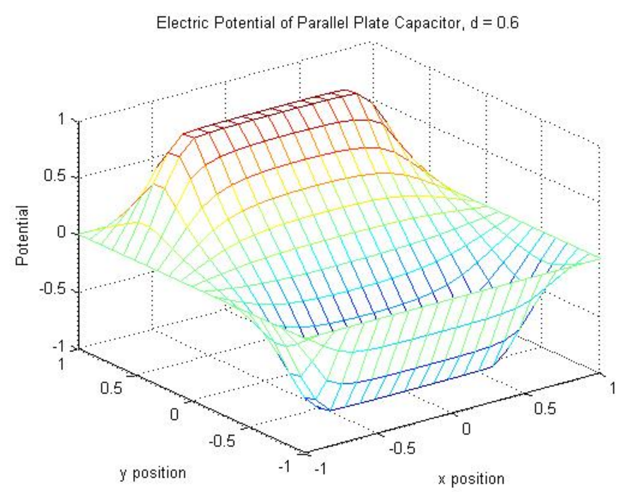

Figure 16: The potential about two capacitor plates with spacing $d = 0.6$.

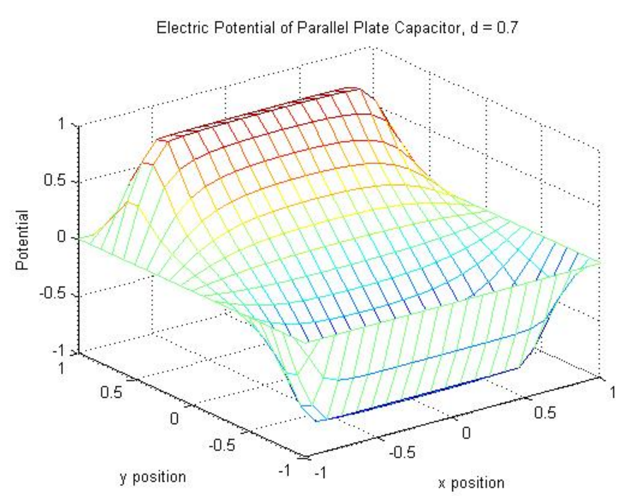

Figure 17: The potential about two capacitor plates with spacing $d = 0.7$.

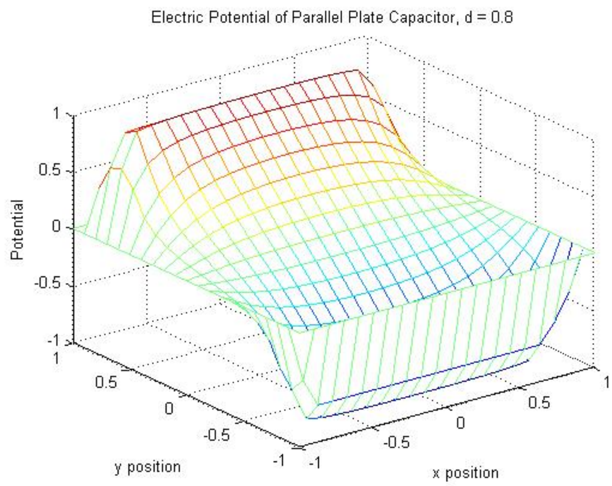

Figure 18: The potential about two capacitor plates with spacing $d = 0.8$.

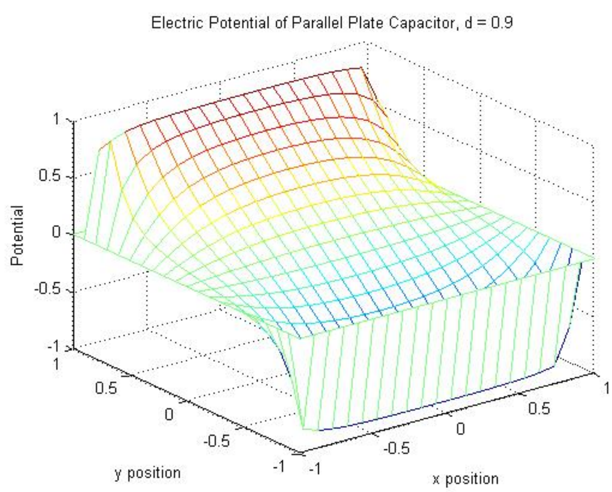

Figure 19: The potential about two capacitor plates with spacing $d = 0.9$.

3 Problem 5.11: Field from a Straight Wire

The most common method for integrating is that of the summation. For a given interval of a function, this function can be split into a series of rectangles (the method of the Riemann Sum) and the areas of these resulting rectangles found to give an approximation of the area under the rectangles. Given an infinite number of rectangles, this approximation gives a definite answer. Computationally, however, this is not a realistic expectation. There are limits to the number of computations that can be computed in a given interval of time. In order to lessen the computation time from the standard summation technique for integrating, other methods exist which use differing geometry to find the areas under the curve of a function. One such method is that of Simpson's Rule, which finds the values of the function for three points, then creates a trapezoid (as opposed to a rectangle) using those three points. The area under the trapezoid is then found, and this is done for points of some interval (δ_x) across the entire span of an interval. This technique can provide an approximation with an error equivalent to that of a fourth order Runge-Kutta method. This magnitude of error was found to be more than sufficient for otherwise pesky computational problems such as simple harmonic motion.

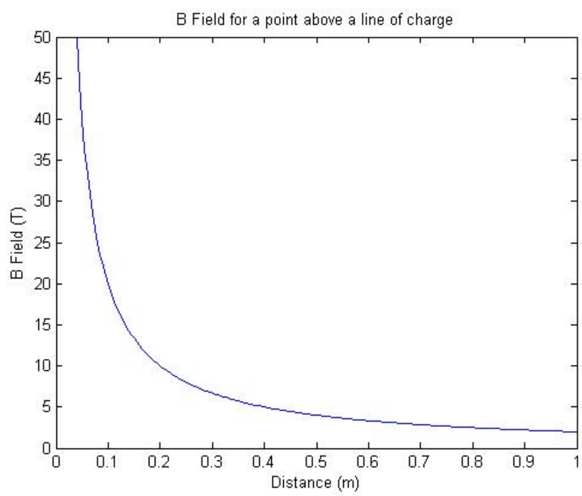

Figure 20: Here, the magnetic field about a straight wire with $\frac{\mu_0 I}{4\pi} = 1$. Here, a $dz = 0.01$ was used, giving a fine approximation for the B-Field about the wire. As expected, the B-field shoots to infinity as the distance approaches 0.

4 Conclusions

Overall, the relaxation method is a powerful tool to approximate solutions to field equations. For a small number of relaxations, a good fit solution can be found for simple electric potential problems such as those presented in problems 5.1 and 5.4. Simpson's Rule for integration is a helpful method for more accurately calculating integrals in a computational setting. Similar to the Runge-Kutta method, Simpson's Rule can be applied to lessen the asymmetry present in the approximations of the standard summation integration techniques. While decreasing δ_x can provide a similar solution using standard summation techniques, Simpson's Rule provides a method to reduce computational expense by splitting the intervals into a collection of smaller trapezoids whose areas can then be summed, and a solution with greater accuracy obtained. Using just two extra points in approximating the areas of the trapezoids created in Simpson's Rule, an error similar to that of a fourth order Runge-Kutta method can be achieved.

References

[1] Giordano, Nicholas J., and Hisao Nakanishi. "5. Potentials and Fields Near Electric Charges" Computational Physics. Upper Saddle River, NJ: Pearson/Prentice Hall, 2006. N. pag. Print.

5 MATLAB code

```
%%Problem 5.1
% find the electrostatic potential inside a square box with vertical conducting walls
% Figure 5.2 in Computational Book
% in other words, solve laplace's equation for the boundary conditions
% described above

clear all

%The dimensions of the potential field
%currently, x and y must be equal to eachother.
x_d = 1;
y_d = x_d;
%the prism voltage
prism_v = 1;
%the length of the side of the prism, relative to the dimensions of the
%potential field
scale = 0.4;
prism_size_x = scale*x_d;
prism_size_y = scale*y_d;

dx = 1/20;
x = [-x_d:dx:x_d];
y = [-y_d:dx:y_d];
x_in = [-prism_size_x:dx:prism_size_x];
y_in = [-prism_size_x:dx:prism_size_x];
Vo = zeros(length(x),length(y));
%matrix for inner prism
Vo_in = zeros(length(x_in), length(y_in));
%initial conditions for inner prism
dim_Vo_in = size(Vo_in);
%starting positions to translate this matrix into Vo later
x_b = (length(x) - length(x_in))*0.5;
y_b = (length(y) - length(y_in))*0.5;
x_e = x_b + dim_Vo_in(1);
y_e = y_b + dim_Vo_in(2);
%insert the prism matrix into the Vo matrix
for i = x_b:x_e
   for j = y_b:y_e
       Vo(i,j) = prism_v;
   end
end
%The outer boundary conditions
Vo(1,:) = 0;
Vo(end,:) = 0;
Vo(:,1) = 0;
Vo(:,end) = 0;
```

```
Vn=Vo;

for k=1:200;
    for i=2:length(x)-1;
        for j=2:length(y)-1;
        Vn(i,j)=(1/4)*(Vo(i+1,j)+Vo(i-1,j)+Vo(i,j+1)+Vo(i,j-1));
        end
    end
    Vo=Vn;
    Vo(1,:) = 0;
    Vo(end,:) = 0;
    Vo(:,1) = 0;
    Vo(:,end) = 0;
    for i = x_b:x_e
        for j = y_b:y_e
            Vo(i,j) = prism_v;
        end
    end
end

mesh(x,y,Vn)
title('Electric Potential of a Hollow Metallic Prism with Solid, Metallic Inner Condu
xlabel('x position')
ylabel('y position')
zlabel('Potential')

%%Problem 5.4
% find the electrostatic potential inside a square box with vertical conducting walls
% Figure 5.2 in Computational Book
% in other words, solve laplace's equation for the boundary conditions
% described above

clear all

%The dimensions of the potential field
%currently, x and y must be equal to eachother.
x_d = 1;
y_d = x_d;
%the capacitor voltages
prism_v = 1;
cap1_v = -1;
cap2_v = 1;
%the length of the side of the prism, relative to the dimensions of the
%potential field
scale = 0.1;
prism_size_x = scale*x_d;
prism_size_y = scale*y_d;

dx = 1/10;
x = [-x_d:dx:x_d];
```

```
y = [-y_d:dx:y_d];
x_in = [-prism_size_x:dx:prism_size_x];
y_in = [-prism_size_x:dx:prism_size_x];
Vo = zeros(length(x),length(y));
%matrix for inner prism
Vo_in = zeros(length(x_in), length(y_in));
%initial conditions for inner prism
dim_Vo_in = size(Vo_in);
%starting positions to translate this matrix into Vo later
x_b = (length(x) - length(x_in))*0.5;
y_b = (length(y) - length(y_in))*0.5;
x_e = x_b + dim_Vo_in(1);
y_e = y_b + dim_Vo_in(2);
%insert the prism matrix into the Vo matrix
for i = x_b:x_e
   for j = y_b:y_e
       Vo(i,j) = prism_v;
   end
end
%The outer boundary conditions
Vo(1,:) = 0;
Vo(end,:) = 0;
Vo(:,1) = 0;
Vo(:,end) = 0;
Vn=Vo;

for k=1:1000;
    for i=2:length(x)-1;
        for j=2:length(y)-1;
        Vn(i,j)=(1/4)*(Vo(i+1,j)+Vo(i-1,j)+Vo(i,j+1)+Vo(i,j-1));
        end
    end
    Vo=Vn;
    Vo(1,:) = 0;
    Vo(end,:) = 0;
    Vo(:,1) = 0;
    Vo(:,end) = 0;
    %capacitor plate 1
    for i = x_b:(x_b+1)
        for j = y_b:y_e
            Vo(i,j) = cap1_v;
        end
    end
    %capacitor plate 2
     for i = (x_e-1):(x_e)
        for j = y_b:y_e
            Vo(i,j) = cap2_v;
        end
    end
end
```

```
%plot the electric field of the capacitors
dim_Vo = size(Vo);
[X,Y] = meshgrid(-x_d:dx:x_d);
[U,V] = gradient(Vn);
%quiver(X,Y,U,V)

mesh(x,y,Vn)
title('Electric Potential of Parallel Plate Capacitor, d = 0.1')
xlabel('x position')
ylabel('y position')
zlabel('Potential')
spacing = scale*x_d;
disp('The capacitor spacing is ')
disp(spacing)


%%Problem 5.11
clear all;

L = 10;
dz = 0.01;
dx = 0.1;
z = [-L:dz:L];
x_i = 10;
x = [0:0.5*dz:x_i];


%Using Standard Integration
for j=1:length(x)
B(j)=0;
for i=1:length(z)
    dB(i)= (x(j)*dz)/(((x(j)^2)+(z(i)^2))^(3/2));
    B(j) = B(j) + dB(i);
end
end

% A(1) = 0;
% dx1(1) = 0;
% dx2(1)
% dx3(1)
% dx4(1)
%
%
% %Using Simpson's Rule
% for j=1:length(x)+4
% for i=1:length(z)
%     B(j) = (x(j)*dz)/(((x(j)^2)+(z(i)^2))^(3/2));
%     B(j+1) = (x(j+1)*dz)/(((x(j+1)^2)+(z(i)^2))^(3/2));
%     B(j) = (x(j)*dz)/(((x(j)^2)+(z(i)^2))^(3/2));
```

```
%     B(j) = (x(j)*dz)/(((x(j)^2)+(z(i)^2))^(3/2));
%
% end
% end


plot(x, B)
xlabel('Distance (m)');
ylabel('B Field (T)');
axis([0 1 0 50])
title('B Field for a point above a line of charge');
```