Digital Signal Processing

Module 3.3
Frequency Analysis – Part 3

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Module 3.3 - Frequency Analysis Part 3

Module Objectives:

1. **Review the decibel.**
2. **Discuss time-domain windowing and the window shape and length effects on FFT resolution.**

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Reading for this Module

1. DSP4 chapters 5.3 – 5.4, 5.6

2. Please read handout DSP_Handout_Digital_Sinc.pdf
   There is also posted code with this handout: DSP_Digital_Sinc_Example.m

3. Please review DSP_Handout_Common_Filters_Summary.pdf

4. Orfanidis handouts – these go over the details behind the frequency resolutions
   DSP_Handout_Orfanidis_Reading_Material_Part_I.pdf
   DSP_Handout_Orfanidis_Reading_Material_Part_II.pdf

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In-class computation:

Matlab and Excel.

Canvas:
DSP_FFT_Resolution.m
DSP_FFT_Resolution.fig

DSP_FFT_Resolution_Group_X.xlsx (Edit the one for your group).

You can run code ahead of time but let's not populate the XLSX sheet until we do exercises in class please.

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Announcements

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The Decibel (dB)

The ***Decibel*** (***dB***) is used extensively in many signal processing applications due to the large dynamic ranges of signals involved. The dB is a non-linear, logarithmic scaling of power ratios:

$$X_{dB} = 10 \times log_{10}(x)$$

When dealing with ***voltages or plotting filter magnitudes***, we need power, so the dB is calculated as

$$X_{dB} = 20 \times log_{10}(x)$$

The term x is called a ***natural unit***.

dB	Power Ratio	dB	Power Ratio
100	10,000,000,000 : 1	– 1	0.794 : 1
90	1,000,000,000 : 1	– 3	0.501 (~1/2) : 1
80	100,000,000 : 1	– 6	0.251 : 1
70	10,000,000 : 1	–10	0.1 : 1
60	1,000,000 : 1	–20	0.01 : 1
50	100,000 : 1	–30	0.001 : 1
40	10,000 : 1	– 40	0.000 1 : 1
30	1,000 : 1	–50	0.000 01 : 1
20	100 : 1	–60	0.000 001 : 1
10	10 : 1	–70	0.000 000 1 : 1
6	3.981 : 1	– 80	0.000 000 01 : 1
3	1.995 (~2) : 1	–90	0.000 000 001 : 1
1	1.259 : 1	– 100	0.000 000 000 1 : 1
0	1 : 1		

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The Decibel (dB)

When working with decibels: multiplications in natural units add in dB
divisions in natural units subtract in dB.

Example: Convert 400W to dB (without using a calculator).

$$
\begin{aligned}
&10 \times log_{10}(400) \\
&= 10 \times log_{10}(2 \times 2 \times 10 \times 10) \\
&= 10 \times log_{10}(2) + 10 \times log_{10}(2) + 10 \times log_{10}(10) + 10 \times log_{10}(10) \\
&= 3 + 3 + 10 + 10 \\
&= 26 \ dB
\end{aligned}
$$

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The Decibel (dB)

dB's are also easy to work with when the quantity is raised to a power:

Let

$$10 \times log_{10}(x) = X_{dB}$$

$$\boxed{10 \times log_{10}\left(x^m\right) = m \times X_{dB}}$$

Example: 2 is approximately 3 dB

8 = 2 × 2 × 2 is approximately 3 + 3 + 3 = 9 dB

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The Decibel (dB)

To convert dB back to natural units, use this formula (this is for power, divide by 20 for voltage):

$$\boxed{x = 10^{\left(\frac{X_{dB}}{10}\right)}}$$

$$10 \times log_{10}(x) = X_{dB}$$

$$log_{10}(x) = \frac{X_{dB}}{10}$$

$$10^{log_{10}(x)} = x = 10^{\left(\frac{X_{dB}}{10}\right)}$$

Example: 20 dB

$$x = 10^{\left(\frac{20}{10}\right)} = 10^2 = 100$$

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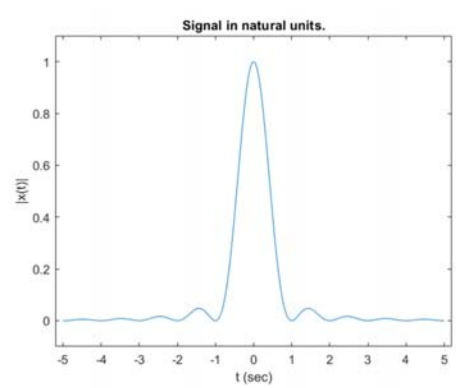
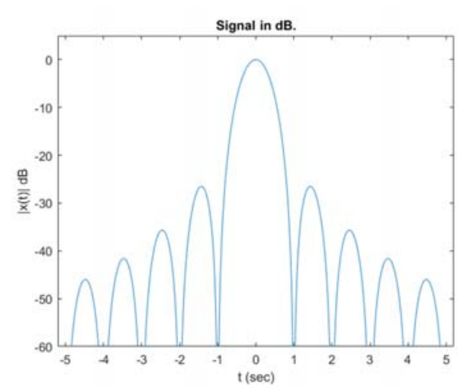
The Decibel (dB)

Plot where details when the signal is small are more evident in dB than natural form.



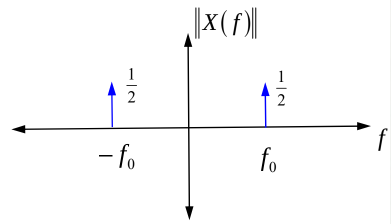


Student Exercise 1

Two minutes.

The Fourier transform of a sinusoid is given below.

$$cos\left(2\pi f_0 t\right) \xleftrightarrow{Fourier} \frac{1}{2}\delta\left(f - f_0\right) + \frac{1}{2}\delta\left(f + f_0\right)$$



What will the DFT of this sinusoid look like?

FFT Resolutions

Multiplication in the time domain is convolution in the frequency domain:

Time domain: element by element multiplication

$$x_w\left[n\right] = x\left[n\right]w\left[n\right]$$

Matlab: `xw = x .* w;`

Frequency domain: convolution

$$X_w\left[k\right] = X\left[k\right] \circledast W\left[k\right]$$

$$X_W\left(e^{j\omega}\right) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X\left(e^{j\omega}\right)W\left(e^{j(\omega-\lambda)}\right)d\lambda$$

$W\left(e^{j\omega}\right)$ **the is the DTFT of** $w\left[n\right]$

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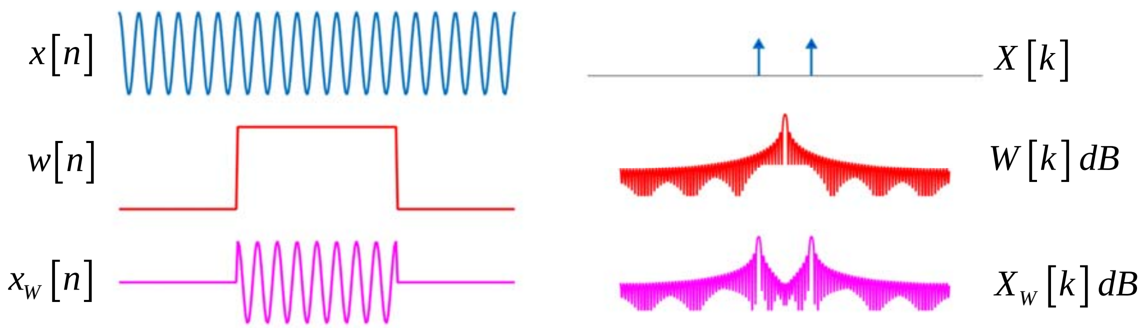
FFT Resolutions

Multiplication in the time domain is convolution in the frequency domain:

$$x_w[n] = x[n]w[n] \qquad X_w[k] = X[k] \circledast W[k]$$



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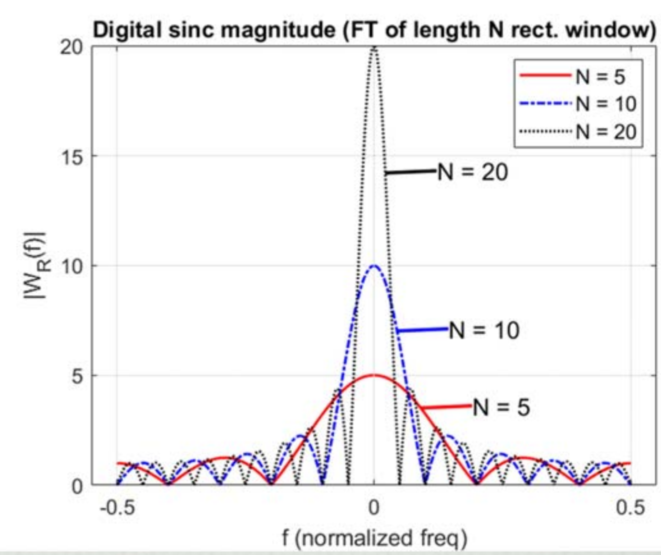
FFT Resolutions

The FFT of a rectangular window is called a ***digital sinc***.

Please take time to review the DSP digital sinc handout and code.

DSP_Digital_Sinc_Example.m
DSP_Handout_Digital_Sinc.pdf

The real frequency is the normalized frequency times the sampling rate, f_S.



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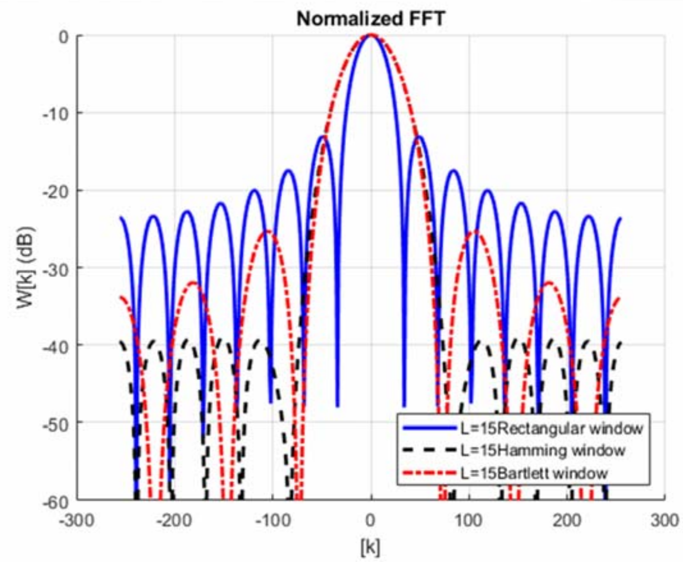
Student Exercise 2

Five minutes. Groups.
Post to discussion board.

Three different L=15 windows FFT responses in dB (normalized to 0 dB peak).

What are differences?



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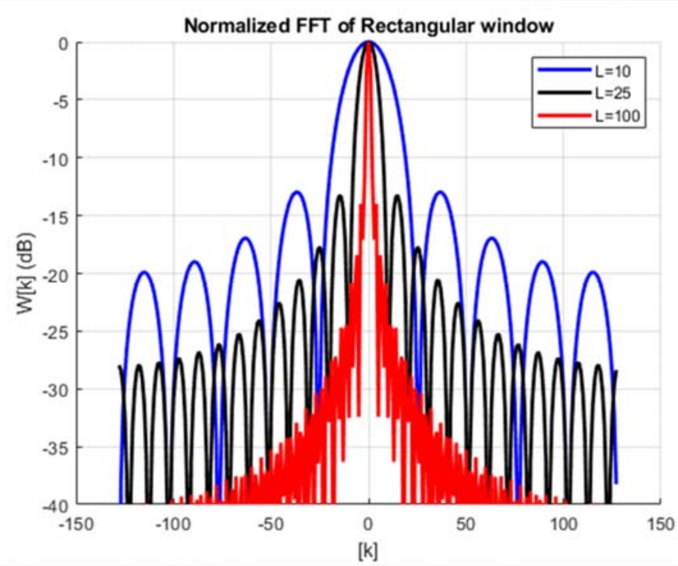
Effect of length.

How about varying L?

These plots show the effects of varying L (10, 25, 100) with N = 128 and a rectangular window.

How does resolution change as L changes?



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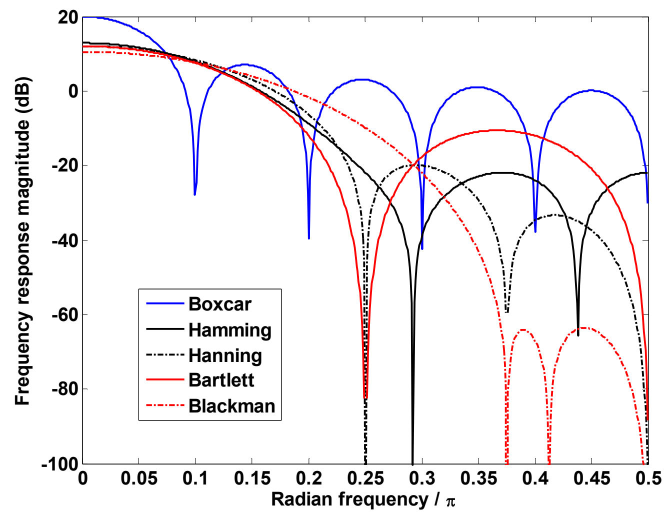
FFT Resolutions

Comparison of freq. responses in dB for several windows.

Please review on your own the common filter handout.



DSP_Handout_Common_Filters_Summary.pdf

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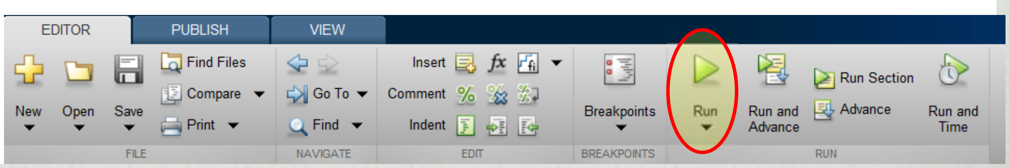
Student Exercise 3

Thirty minutes. Instructor will demo experiment 1 and then you will work together in teams.
We know that the computational FFT resolution is f_S / N. Lets do some experiments to see what the physical frequency resolution of the FFT is. To do this, we will run the following Matlab code and Excel files:

DSP_FFT_Resolution.m
DSP_FFT_Resolution.fig
DSP_FFT_Resolution_Group_X.xlsx – these excel files are on OneDrive. Edit the one for your group.

Open Matlab. Open the file DSP_FFT_Resolution.m in the editor. Click on that file in your menu.
At the top, click the Run icon.



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Student Exercise 3

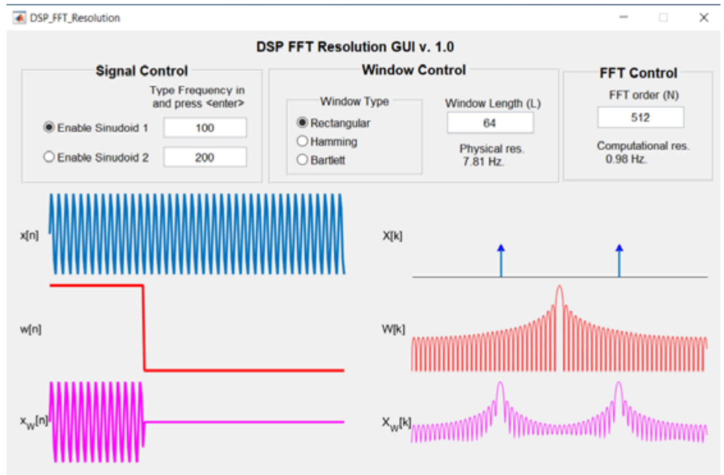
You should see this screen.

The instructor will now demonstrate how to run the GUI for experiment 1 with The whole class.

Once you know, please complete the excel sheet provided for this exercise.

The bottom two frequency plots are in dB to show results more clearly.

The sampling rate is 500 Hz.



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Student Exercise 3

Findings:

FFT Resolution

It can be shown (refer to Orfanidis reading materials) that the ***FFT Physical Frequency Resolution*** is given by

$$\Delta f_{PHYS} = C_{WIN}\frac{f_S}{L} = \frac{C_{WIN}}{LT_S} = \frac{C_{WIN}}{T_L}$$

where L is the window length in samples, f_S is the sampling rate in Hz (samples/second), T_S is the sampling interval (seconds/sample), T_L is the total observation time, that is, $T_L = LT_S$., and C_{WIN} is a constant depending on the window shape. C_{WIN} = 1 for a rectangular and about 2 for a Hamming window.

FFT Resolution

Thus the FFT has two resolutions:

Resolution	Equation	Controlled by
Physical	$\Delta f_{PHYS} = C_{WIN}\frac{f_S}{L}$	1. Sampling rate (f_S) 2. Window type (C_{WIN}) 3. Window length (L)
Computational	$\Delta f_{COMP} = \frac{f_S}{N}$	1. Sampling rate (f_S) 2. FFT Size (N)

Note: A N-point FFT only analyzes N data points in the time domain. This means $0 < L \leq N$.

Minute Paper

Take one minute and fill out anonymous minute paper (on Canvas).

Appendix A

Some Common Window Functions.

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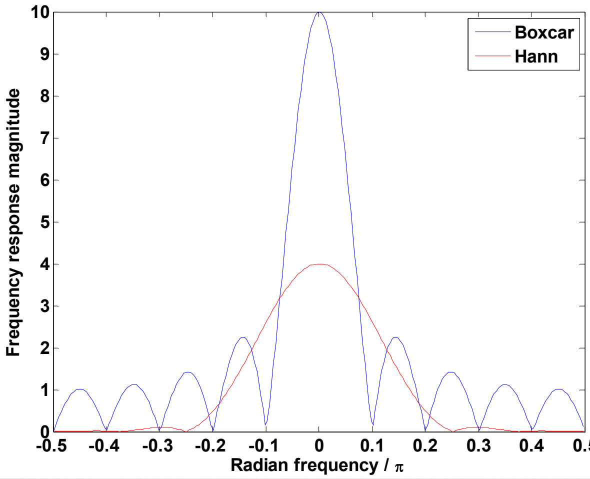
FFT Resolutions

What are some other windows we can use?



Hanning Window

$$w_{HAN}[n] = 0.5\left(1 - cos\left(\frac{2\pi n}{L-1}\right)\right), \quad 0 \le n \le (L-1)$$

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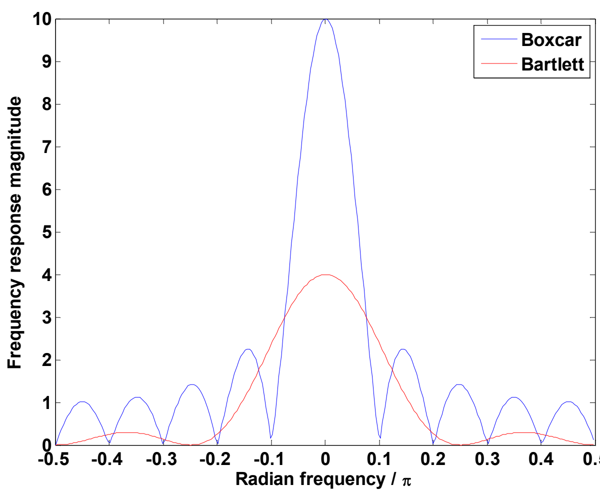
FFT Resolutions



Bartlett Window

$$w_{BARTLETT}[n] = 1 - \left|\frac{n - \frac{L-1}{2}}{\frac{L}{2}}\right|, \quad 0 \le n \le (L-1)$$

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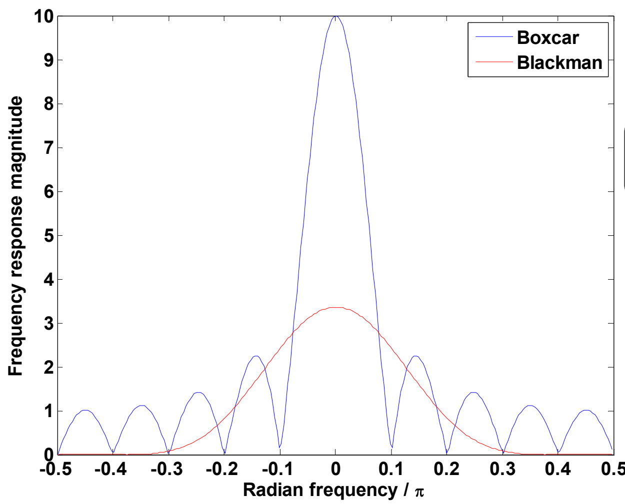
FFT Resolutions



Blackman Window

$$w_{BLACKMAN}[n] = 0.42 - 0.5\,cos\left(\frac{2\pi n}{L-1}\right) + 0.08\,cos\left(\frac{4\pi n}{L-1}\right)$$